Week 8 Homework

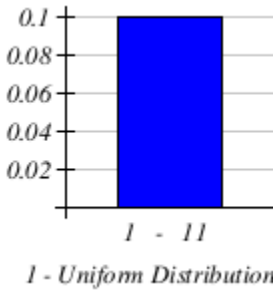
DS 23 Business Statistics

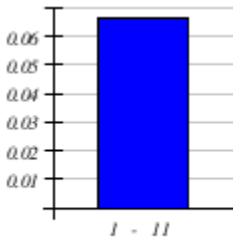
1. (1 pts)

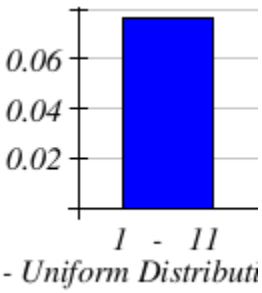
A random number generator picks a number from one to eleven in a uniform manner.

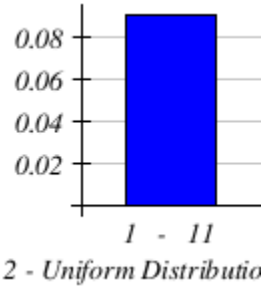
A uniform distribution is the area of a box. Like other distributions, the sum of the probabilities is 1. Therefore, the area of the rectangle is 1. If a random variable is

a) select the correct graph for the probability distribution.

- 

- 

- 

- 

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The height of the rectangle is k where k*(11-1) = 1

k*10 = 1

Divide both sides by 10 to get k = 1/10 = .10

The height must be 0.10.

Graph 1 – has a height of 0.10.

b) What is the probability function? exact answer

f(x) = 1/(11-1) = 1/10, 1 ≤ x ≤ 11

c) Compute the mean μ round to four decimals

E(x) = μ = (1 + 11)/2 = 6.0

d) Compute the standard deviation σ round to four decimals

Std Dev = σ = (11 – 1)/$\sqrt{12}$ = 2.8868

e) What is the probability of P(3.5< x < 9) = round to four decimals

P(3.5< x < 9) = (9 – 3.5)/(11-1) = 5.5/10 = 0.55

f) What is the probability of P(x > 3) round to four decimals

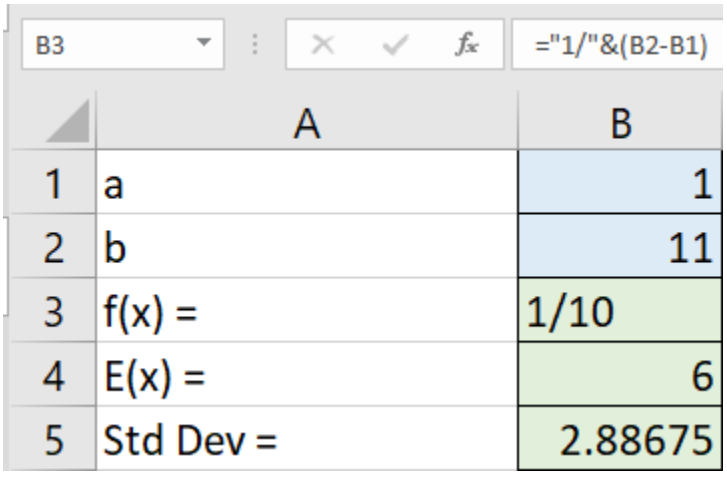
P(X > 3) = (11 – 3)/(11 – 1) = 0.8000

g) What is the probability of P(x > 5.5|x > 1) round to four decimals

P(x > 5.5|x > 1) = (11- 5.5)/ (11 – 1) = 5.5/10 = 0.55

Using the Excel Spreadsheet provided, I would enter 1 in cell B1 and 11 in cell B2.

The mean and standard deviation is computed based on the formulas above.



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For e) enter 3.5 in cell B6 and 9 in cell B7. The probability, P(3.5 < X < 9) is in cell B11, 0.5500.

For f) P(X > 3) enter 3 in cell B6. The probability is in cell B10, 0.80

	A	B	C	D	E
1	a	1		Percentile	0.83
2	b	11		83%	9.3
3	f(x) =	1/10			
4	E(x) =	6			
5	Std Dev =	2.88675			
6	c	3.5			
7	d	9			
8	f				
9	P(X < 3.5)	0.2500			
10	P(X > 3.5)	0.7500			
11	P(3.5 < X < 9)	0.5500			

for g) P(X > 5.5 | X > 3), in this case we change the a value to 3 in cell B1 and enter 5.5 in cell B6. You will find the probability in cell B10, 0.6875.

	A	B	C	D	E
1	a	3		Percentile	0.83
2	b	11		83%	9.64
3	f(x) =	1/8			
4	E(x) =	7			
5	Std Dev =	2.3094			
6	c	5.5			
7	d	9			
8	f				
9	P(X < 5.5)	0.3125			
10	P(X > 5.5)	0.6875			
11	P(5.5 < X < 9)	0.4375			

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2. (1 pts)

According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between **10 and 21** pounds a month until they approach trim body weight. Let's suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month.

a) f(x) = **1/(21-10) = 1/11**

b) μ = **(10 + 21)/2 = 31/2 = 15.5**

c) σ = **(21-10)/$\sqrt{12}$ = 3.1754**

d) Find the probability that the individual lost more than 20 pounds in a month. (Round to 4 decimal places)

P(X > 20) = (21 -20)/(21-10) = 1/11 = 0.0909

For problems a) thru d) above you can use the Excel Spreadsheet provided. Enter 10 in cell B1. Enter 21 in cell B2. Enter 20 in cell B6. The probability function is in cell B3, 1/11. The mean is in cell B4, 15.5. The standard deviation is in cell B5, 3.1754. The P(X>20) is in cell B10, 0.0909.

	A	B
1	a	10
2	b	21
3	f(x) =	1/11
4	E(x) =	15.5
5	Std Dev =	3.17543
6	c	20
7	d	9
8	f	16
9	P(X < 20)	0.9091
10	P(X > 20)	0.0909

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e) Suppose it is known that the individual lost more than 16 pounds in a month. Find the probability that he lost less than 18 pounds in the month. (Round to 4 decimal places)

P(X < 18 | X>16) = (18 – 16)/(21 – 16) = 2/5 = 0.4000

Using the Excel Spreadsheet provided, enter 16 in cell B1. Then enter 18 in cell B6. The probability is in cell B9, 0.4000.

	A	B	C	D	E
1	a	16		Percentile	0.83
2	b	21		83%	20.15
3	f(x) =	1/5			
4	E(x) =	18.5			
5	Std Dev =	1.44338			
6	c	18			
7	d	9			
8	f				
9	P(X < 18)	0.4000			
10	P(X > 18)	0.6000			

f) P(11 < x < 15 | x > 14) = (Round to 4 decimal places)

P(11 < X < 15 | x > 14) = (15-14)/(21 – 14) = 1/7 = 0.1429

To use the Excel spreadsheet, enter 14 in cell B1, 14 in cell B6 (X > 14 therefore we start measuring at x = 14), and 15 in cell B7, 0.1429. The probability that X is between 11 and 15 is actually P(14 < X< 15|x > 14).

	A	B	C	D	E
1	a	14		Percentile	0.83
2	b	21		83%	19.81
3	f(x) =	1/7			
4	E(x) =	17.5			
5	Std Dev =	2.02073			
6	c	14			
7	d	15			
8	f				
9	P(X < 14)	0.0000			
10	P(X > 14)	1.0000			
11	P(14 < X < 15)	0.1429			

3. (1 pts)

A distribution is given as X ~ Exp(0.64). Round to 4 decimal places.

a) What is the mean?

m = 0.64

E(x) = μ = 1/m = 1/0.64 = 1.5625

b) What is the standard deviation?

Std Dev = σ = 1/0.64 = 1.5625

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Using the Excel Spreadsheet provided(Exponential Distribution tab at the bottom of the spreadsheet), enter 0.64 in cell B1. The mean is in cell B2. The standard deviation is in cell B3.

	A	B
1	**m**	0.64
2	**E(x)**	1.5625
3	**Std Dev**	1.5625

c) $P(19 < X < 20)$ = **$P(X < 20) - P(X < 19) = (1 - e^{-.64*20}) - (1-e^{-.64*19})$**
$= e^{-.64*19} - e^{-.64*20} = 2.47498E\text{-}06 = .00000247498 = 0.0000$

Using the Excel Spreadsheet provided, find the probability for x = 20, in cell B26. Also, find the probability for x = 19, in cell B25. Subtract the two values, 1.0000 – 1.0000 = 0.

d) $P(X > 6) = e^{-.64*6}$ **= 0.0215**

Using technology enter the following formula in the Excel Spreadsheet

P(X > 6) = 1 – P(X < 6)
= 1 – expon.dist(6, 0.64, True)
= 0.0215

See Excel Spreadsheet provided, Look up x = 6 in cell A12, and the corresponding probability P(X < 6) in cell B12.

	A	B
1	**m**	0.64
2	**E(x)**	1.5625
3	**Std Dev**	1.5625
4		
5	**x**	**P(X<x)**
6	0	0.0000
7	1	0.4727
8	2	0.7220
9	3	0.8534
10	4	0.9227
11	5	0.9592
12	6	0.9785

e) Find the 30th percentile.

$0.3 = P(X < k) = 1 - e^{-.64*k}$

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$$.3 = 1 - e^{-.64*k}$$
$$e^{-.64*k} = .7$$
$$\ln e^{-.64*k} = \ln 0.7$$
$$-.64k = \ln 0.7$$
$$k = \ln 0.7/(-.64) = .5573$$

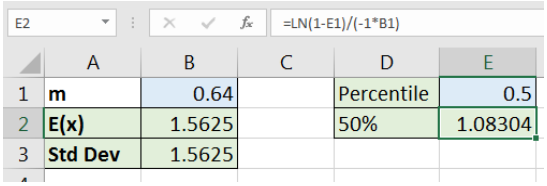
Using the Excel Spreadsheet provided, enter 0.30 in cell E1 to get the percentile in cell E2.

	A	B	C	D	E
1	**m**	0.64		Percentile	0.3
2	**E(x)**	1.5625		30%	0.5573046
3	**Std Dev**	1.5625			

f) Find the median.
The median is the 50th percentile.

$$0.5 = P(X < k) = 1 - e^{-.64*k}$$
$$.5 = 1 - e^{-.64*k}$$
$$e^{-.64*k} = .5$$
$$\ln e^{-.64*k} = \ln 0.5$$
$$-.64k = \ln 0.5$$
$$k = \ln 0.5/(-.64) = 1.08304247$$

Using the Excel Spreadsheet provided, enter 0.5 in cell E1. The 50th percentile is in cell E2, 1.0830.



Using the Excel Spreadsheet, enter 0.50 in cell E1. The fiftieth percentile is 1.0830.

4. (1 pts)

The time (in years) after reaching age 52 that it takes an individual to retire is approximately exponentially distributed with a **mean of about 1 years**. Suppose we randomly pick one retired individual. We are interested in the time after age 52 to

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retirement.
The mean is 1 year.
a) μ = **1**
b) σ = **1**
m = 1/1 = 1
X ~ Exp(1)

c) Find the probability that the person retired after age 62. Rounded to 4 decimal places

P(X > t+r| X>t) = P(X > r)

P(X > 62|X>52) = P(X > 10 + 52| X> 52) = P(X > 10)
= e^{-1*10} =4.53999E-05 = .0000453999 = 0

Using technology
P(X > 10) = 1 – P(X < 10)
= 1- Expon.dist(10, 1, True) = 4.53999E-05 = .0000453999 = 0

Using the Excel Spreadsheet that I provided look up x =10 in column A (A16) and find the probability in column B (B16).
P(X>10) = 1- P(X<10)
= 1 – 1 = 0

	A	B
1	**m**	1
2	**E(x)**	1
3	**Std Dev**	1
4		
5	**x**	**P(X<x)**
6	0	0.0000
7	1	0.6321
8	2	0.8647
9	3	0.9502
10	4	0.9817
11	5	0.9933
12	6	0.9975
13	7	0.9991
14	8	0.9997
15	9	0.9999
16	10	1.0000

d) Find the probability that the person retired before age 53. Rounded to 4 decimal places

P(X < 53 | X > 52) = P(X < 52 + 1 | X > 52) = P(X < 1)
= (1 – $e^{-1(1)}$)

Using Technology
= Expon.dist(1, 1, True) = 0.6321

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Using the Excel Spreadsheet provided, I look up x = 1 in column A(A7) and find the probability in column B(B7).

	A	B
1	**m**	1
2	**E(x)**	1
3	**Std Dev**	1
4		
5	**x**	**P(X<x)**
6	0	0.0000
7	1	0.6321

0.6321

e) Find the probability that the person retired after age 53. Rounded to 4 decimal places

$P(X > 53 \mid X > 52) = P(X > 1) = 1 - P(X < 1) = e^{-1(1)} = 0.3679$

Using Technology

=1 – Expon.dist(1, 1, True) = 0.3679

Using the Excel Spreadsheet provided

Look up X = 1 in column A (A7) and find the probability in column B(B7).

1 - 0.6321 = .3679

f) In a room of 5000 people over age 73, how many do you expect will NOT have retired yet? Rounded to 2 decimal places

$P(X > 73 \mid X > 52) = P(X > 52 + 21 \mid X > 52) = P(X > 21)$

$= e^{-1(21)} = 7.58256043E\text{-}10 = 0.000000000758256043$

5000(7.58256043E-10) = 0

Using technology

$P(X > 21) = 1 - P(X < 21)$

= 1 – Expon.dist(21, 1, True) = 7.58256E-10 = .000000000758256 = 0

5000 * 0 = 0

5. (1 pts)

If X is distributed normally with mean 58 and standard deviation 6. Round your answers to 4 decimal places.

Use technology to find the probability.

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$X \sim N(58, 6)$

a) What is the P(X > 61)?
P(X > 61) = 1 – P(X < 61)
= 1-norm.dist(61, 58, 6, True) = 0.3085

Use the Excel spreadsheet provided
Enter mean, 58, in cell B1.
Enter standard deviation, 6, in cell B2.
Enter 61 in cell B5.
Find the probability in cell, B7. P(X > 61)

b) What is the P(X < 63)?
= norm.dist(63, 58, 6, True) = 0.7977

Use the Excel spreadsheet provided
Enter 63 in cell B5.
Find the probability in cell, B8. P(X< 63) = 0.7977

c) What is the P(44 < X < 59)?
= norm.dist(59, 58, 6, True) – norm.dist(44, 58, 6, True)
= 0.5564

Use the Excel spreadsheet provided.
In cell B5, enter 44.
In cell B6, enter 59.
In cell B9 is the probability P(44< X< 59) = 0.5564

d) 68% of the population is between and .
Mean = 58
Standard Deviation = 6
58 – 6 = 52
58 + 6 = 64

Using the Excel Spreadsheet provided
In Excel click into cell C11, and enter the following formula
= B1 – B2

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The upper limit, click in cell D11 and enter the following formula
= B1 + B2

e) 95% of the population is between and .
Mean = 58
Standard deviation = 6
Lower Limit = 58 – 2* 6 = 46
Upper Limit = 58 + 2* 6 = 70

Using the Excel spreadsheet provided, click in cell C12 and enter the formula.
=B1 – 2*B2 Lower Limit
In cell D12
=B1 + 2*B2

f) 99.7% of the population is between and .
Mean = 58
Standard deviation = 6
Lower Limit = 58 – 3* 6 = 40
Upper Limit = 58 + 3* 6 = 76

Using the Excel spreadsheet provided, click in cell C13 and enter the formula.
=B1 – 3*B2 Lower Limit
In cell D13
=B1 + 3*B2

6. (1 pts)

The distribution of cell phones calls is normally distributed with mean 74 and a standard deviation of 6.

a) 70 is standard deviations below the mean.

Calculate the Z score
Z = (70 – 74)/6 = -4/6 = -0.67

Using the Excel Spreadsheet provided enter mean in cell B1. The standard deviation in B2. Enter 70 in cell B3. The Z-score is in cell B4, -0.67.

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b) 78 is standard deviations above the mean.
Calculate the Z Score
Z = (78-74)/ 6 = 4/6 = 0.67

Using the Excel Spreadsheet provided enter mean in cell B1. The standard deviation in B2. Enter 70 in cell B3. The Z-score is in cell B4, 0.67.

c) -1.22 standard deviations below the mean is .

-1.22 = (X – 74)/6
Multiply both sides by 6
(6)(-1.22) = X – 74
Add 74 to both sides
74 + (6) (-1.22) = X = 66.68

Using Excel
= 74+(-1.22)*6

X = Mean + ZScore*Standard Deviation
74 + (-1.22)*6 = 66.68
d) 0.71 standard deviations above the mean is .

X = Mean + Z score* Standard Deviation
X = 74 + 0.71*6 = 74.426

7. (1 pts)

In the country of United States of Heightlandia, the height measurements of ten-year-old children are approximately normally distributed with a mean of 54.5 inches, and standard deviation of 4.6 inches.

A) What is the probability that a randomly chosen child has a height of **less** than 65.8 inches?

Answer= (Round your answer to 3 decimal places.)
P(X < 65.8)
Mean = 54.5
Standard deviation = 4.6
= norm.dist(65.8, 54.5, 4.6, True)

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= 0.9930

Using the Excel Spreadsheet provided, enter mean in cell B1, the standard deviation in cell B2, enter 65.8 in cell b5. The probability is in cell B8, 0.9930

B) What is the probability that a randomly chosen child has a height of **more** than 52.8 inches?

Answer= (Round your answer to 3 decimal places.)
P(X > 52.8)
= 1 – norm.dist(52.8, 54.5, 4.6, True)
=0.6441

Using the Excel Spreadsheet provided, enter mean in cell B1, the standard deviation in cell B2, enter 52.8 in cell b5. The probability is in cell B7, 0.6441.

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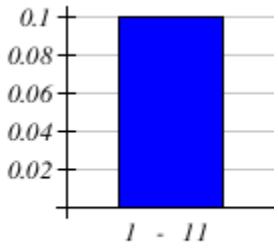
Key - Form 1



1. ~ 1/10 ~ 6 ~ 2.8868 ~ 0.55 ~ 0.8 ~ 0.55
2. 1/11 ~ 15.5 ~ 3.1754 ~ 0.0909 ~ 0.4 ~ 0.1429
3. 1.5625 ~ 1.5625 ~ 0 ~ 0.0215 ~ 0.5573 ~ 1.083
4. 1 ~ 1 ~ 0 ~ 0.6321 ~ 0.3679 ~ 0
5. 0.3085 ~ 0.7977 ~ 0.5564 ~ 52 ~ 64 ~ 46 ~ 70 ~ 40 ~ 76
6. -0.67 ~ 0.67 ~ 66.68 ~ 78.26
7. 0.993 ~ 0.644