

Sample Questions for Exam #2

Question 1

Every cereal box has a gift inside, but you cannot tell from the outside what the gift is. The store manager assures you that 18 of the 57 boxes on the shelf have the secret decoder ring. The other 39 boxes on the shelf have a different gift inside. If you randomly select two boxes of cereal from the shelf to purchase, what is the probability that BOTH of them have the secret decoder ring?

(Give answer as a decimal correct to four decimal places.)

Number of decoder rings = 18

Number of boxes of cereal = 57

$$\frac{18}{57} * \frac{17}{56} = 0.09586$$

The answer rounded to four decimal places is 0.0959.

Step 1: Calculate the probability of selecting a box with a secret decoder ring on the first pick.

There are 18 boxes with secret decoder rings out of a total of 57 boxes on the shelf. So, the probability of picking one with a secret decoder ring on the first pick is:

Probability (First Pick) = (Number of boxes with decoder rings) / (Total number of boxes)
Probability (First Pick) = 18 / 57

Step 2: Calculate the probability of selecting a box with a secret decoder ring on the second pick, given that the first pick had a secret decoder ring.

After the first pick, there will be 17 boxes with secret decoder rings left out of a total of 56 boxes because you've already removed one with a decoder ring. So, the probability of picking another one with a secret decoder ring on the second pick is:

Probability (Second Pick | First Pick had a decoder ring) = (Number of remaining boxes with decoder rings) / (Total number of remaining boxes)
Probability (Second Pick | First Pick had a decoder ring) = 17 / 56

Step 3: Calculate the probability of both events happening together (both picks having secret decoder rings) by multiplying the probabilities from steps 1 and 2:

Probability (Both Picks have decoder rings) = Probability (First Pick) * Probability (Second Pick | First Pick had a decoder ring)

Probability (Both Picks have decoder rings) = (18 / 57) * (17 / 56)

Question2

1. A bag contains 8 black marbles, 9 white marbles, and 4 red. If a marble is drawn from the bag, replaced, and another marble is drawn, what is the probability of drawing first a black marble and then a red marble? Round all answers to 4 decimal places.

$$\left(\frac{8}{21}\right) \left(\frac{4}{21}\right) = 0.07256$$

The answer rounded to four decimal places is 0.0726

There are 8 black marbles out of a total of 8 black + 9 white + 4 red marbles in the bag. So, the probability of drawing a black marble on the first draw is:

$$\text{Probability (First Draw = Black)} = (\text{Number of Black Marbles}) / (\text{Total Number of Marbles}) = 8 / (8 + 9 + 4) = 8 / 21$$

Step 2: Calculate the probability of drawing a red marble on the second draw.

Since the marble is replaced after the first draw, the total number of marbles in the bag remains the same (8 black + 9 white + 4 red). So, the probability of drawing a red marble on the second draw is:

$$\text{Probability (Second Draw = Red)} = (\text{Number of Red Marbles}) / (\text{Total Number of Marbles}) = 4 / (8 + 9 + 4) = 4 / 21$$

Step 3: Calculate the probability of both events happening together (drawing a black marble first and then a red marble) by multiplying the probabilities from steps 1 and 2:

$$\text{Probability (First Black, Then Red)} = \text{Probability (First Draw = Black)} * \text{Probability (Second Draw = Red)}$$

$$\text{Probability (First Black, Then Red)} = (8/21) * (4/21)$$

Question 3

1. A bag contains 9 yellow marbles, 4 red marbles, and 8 white. If two different marbles are drawn from the bag, what is the probability of drawing first a yellow marble and then a white marble? Round all answers to 4 decimal places.

$$\left(\frac{9}{21}\right) \left(\frac{8}{20}\right) = 0.17143$$

The answer rounded to four decimal places is 0.1714.

Step 1: Calculate the probability of drawing a yellow marble on the first draw.

There are 9 yellow marbles out of a total of 9 yellow + 4 red + 8 white marbles in the bag. So, the probability of drawing a yellow marble on the first draw is:

$$\text{Probability (First Draw = Yellow)} = (\text{Number of Yellow Marbles}) / (\text{Total Number of Marbles}) = 9 / (9 + 4 + 8) = 9 / 21$$

Step 2: Calculate the probability of drawing a white marble on the second draw.

After the first draw, one yellow marble has been removed, so there are 8 white marbles out of a total of 9 yellow + 4 red + 8 white marbles left. Therefore, the probability of drawing a white marble on the second draw is:

$$\text{Probability (Second Draw = White)} = (\text{Number of White Marbles}) / (\text{Total Number of Remaining Marbles}) = 8 / (9 + 4 + 8 - 1) = 8 / 20$$

Step 3: Calculate the probability of both events happening together (drawing a yellow marble first and then a white marble) by multiplying the probabilities from steps 1 and 2:

$$\text{Probability (First Yellow, Then White)} = \text{Probability (First Draw = Yellow)} * \text{Probability (Second Draw = White)}$$

$$\text{Probability (First Yellow, Then White)} = (9/21) * (8/20)$$

Question 4

Giving a test to a group of students, the grades and gender are summarized below. Round your answers to 4 decimal places.

Table 1 - Grades and Gender

	A	B	C	Total
Male	15	9	16	40
Female	4	20	6	30
Total	19	29	22	70

If one student is chosen at random,

1. Find the probability that the student got a B:

$$P(B) = 29/70 = 0.41429$$

Rounded to four decimal places, the answer is 0.4143.

2. Find the probability that the student was male AND got a "B":

$$P(\text{Male and 'B'}) = 9/70 = 0.12857$$

Rounded to four decimal places, the answer is 0.1286

3. Find the probability that the student was female OR got a "B":

$$P(\text{Female OR 'B'}) = P(\text{Female}) + P(\text{'B'}) - P(\text{Female and 'B'})$$

$$= (30/70) + (29/70) - (20/70) = (30+29-20)/70 = 39/70 = 0.55714 \text{ Rounded to four decimal places, the answer is } 0.5571$$

4. If one student is chosen at random, find the probability that the student was male GIVEN they got an 'A.'

$$P(\text{Male} \mid \text{'A'}) = P(\text{Male and 'A'})/P(\text{'A'})$$

$$= 15/19 = \frac{(15/70)}{(19/70)}$$

$$= 0.78947$$

The answer rounded to four decimal places is 0.7895.

Question 5

Use the following probabilities to answer the question. Round to 4 decimal places.

$P(A)=0.56$, $P(B)=0.39$, $P(A \text{ and } B)=0.05$.

$P(B|A)= ?$

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)} = 0.05/0.56 = 0.08929 \text{ Round to four decimal places, the answer is } 0.0893.$$

Question 6

A jar contains 10 red marbles numbered 1 to 10 and 8 blue marbles numbered 1 to 8. A

marble is drawn at random from the jar. Find the probability of the given event, **please show your answers as reduced fractions**.

1	2	3	4	5	6	7	8	9	10
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1	2	3	4	5	6	7	8
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A total of 18 marbles.

a) The marble is red. $P(\text{red}) =$

$$P(\text{Red}) = 10/18 = 5/9$$

b) The marble is odd-numbered.

$$P(\text{odd}) = (9/18) = \frac{1}{2}$$

c) The marble is red or odd-numbered.

$$P(\text{red or odd}) = P(\text{Red}) + P(\text{Odd}) - P(\text{Red and Odd})$$

$$= (10/18) + (9/18) - (5/18) = 14/18 = 7/9$$

(d) The marble is blue or even-numbered.

$$P(\text{blue or even}) = P(\text{blue}) + P(\text{even}) - P(\text{blue and even})$$

$$= (8/18) + (9/18) - (4/18) = 13/18$$

Question 7

What is the probability that a person has a disease if they have a negative test result?

- one percent of people have a disease, $P(\text{Disease}) = 0.01$
- eighty percent of people who have the disease test positive on their lab test. $P(\text{Positive} | \text{Disease}) = 0.8$
- ten percent of people will have a false positive. $P(\text{Positive} | \text{No Disease}) = 0.1$

Complete the table below and answer the questions. Round to the nearest 4 decimal places)

Probabilities	Disease	No Disease	Totals
Positive	$P(\text{Disease and Positive})$	$P(\text{No Disease and Positive})$	$P(\text{Positive})$
Negative	$P(\text{Disease and Negative})$	$P(\text{No Disease and Negative})$	$P(\text{Negative})$
Totals	$P(\text{Disease})$	$P(\text{No Disease})$	1.00

a) What is the probability of having the Disease given you have a negative test result?

$$P(\text{Disease} | \text{Negative}) = P(\text{Disease and Negative}) / P(\text{Negative})$$

Step 1: We must determine the $P(\text{Disease and Negative})$ as well as $P(\text{Negative})$ to find the answer.

The first conditional probability is $P(\text{Positive} | \text{Disease}) = P(\text{Positive and Disease}) / P(\text{Disease})$

Multiply both sides by $P(\text{Disease})$ to get $P(\text{Disease}) * P(\text{Positive} | \text{Disease}) = P(\text{Positive and Disease})$. **Note: $P(\text{Positive and Disease}) = P(\text{Disease and Positive})$**

$$.01 * .8 = 0.008 = P(\text{Disease and Positive})$$

Step 2: We must determine the $P(\text{Negative})$.

To compute the $P(\text{Negative}) = P(\text{Disease and Negative}) + P(\text{No Disease and Negative})$

To find the answer I will find the values for each number in the table above.

$$P(\text{Positive} | \text{No Disease}) = P(\text{Positive and No Disease}) / P(\text{No Disease})$$

Multiply both sides by $P(\text{No Disease})$

$$P(\text{No Disease}) * P(\text{Positive} | \text{No Disease}) = P(\text{Positive and No Disease})$$

$$(1 - .01) * 0.1 = 0.099$$

Put the values we calculated in the table. Make sure the values add down to .01 for the Disease column and .99 for the No Disease column.

Probabilities	Disease	No Disease	Totals
Positive	0.008	0.099	$=0.008+0.099 = 0.1070$
Negative	$=0.01-.008 = 0.0020$	$.99-0.099 = .8910$	$=0.002+0.8910 = 0.8930$
Totals	0.01	$=1-.01 = 0.99$	1.00

The $P(\text{Disease and Negative}) = 0.002$

$$P(\text{Disease} | \text{Negative}) = P(\text{Disease and Negative}) / P(\text{Negative}) = 0.002 / 0.8930 = 0.00224 \text{ rounded to four decimal places } 0.0022$$

Question 8

You are at the drawing for a raffle to win a new car. You bought 8 tickets for the raffle. If a total of 526 tickets were sold, what is the probability that your ticket is drawn (as a decimal to the nearest hundredth)?

 

To find the probability that your ticket is drawn in the raffle, you can use the following formula:

$$P(\text{Your ticket is drawn}) = \frac{\text{Number of your tickets}}{\text{Total number of tickets sold}}$$

In your case:

- Number of your tickets = 8 (since you bought 8 tickets)
- Total number of tickets sold = 526 (as given in the problem)

Now, plug these values into the formula:

$$P(\text{Your ticket is drawn}) = \frac{8}{526}$$

Next, calculate the probability:

$$P(\text{Your ticket is drawn}) \approx \frac{8}{526} \approx 0.0152$$

Now, round the probability to the nearest hundredth:

$$P(\text{Your ticket is drawn}) \approx 0.02$$

So, the probability that your ticket is drawn in the raffle, rounded to the nearest hundredth, is approximately 0.02 or 2%.

Question 9

Guests of a fancy dinner party were asked if they prefer the steak dinner or the lobster dinner, and if they would prefer to sit inside or outside. The results from their responses are below.

	Steak	Lobster	Total
Inside	11	24	35
Outside	38	21	59
Total	49	45	<input type="text"/>

For each of the following questions, find the probabilities. Either enter your answers as fractions or round decimals to 4 places.

- What is the probability that a guest selected at random requested to sit outside?
 $P(\text{outside}) =$
- What is the probability that a guest selected at random ordered steak and requested to sit inside?
 $P(\text{steak and inside}) =$
- What is the probability that a guest selected at random ordered steak or requested to sit inside?
 $P(\text{steak or inside}) =$
- Given that a guest requested to sit inside, what the probability that they ordered steak?
 $P(\text{steak}|\text{inside}) =$

First, we will use the provided table:

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	Steak	Lobster	Total
Inside	11	24	35
Outside	38	21	59
Total	49	45	94

- Probability that a guest selected at random requested to sit outside ($P(\text{outside})$):**
 $P(\text{outside}) = (\text{Number of guests who chose to sit outside}) / (\text{Total number of guests})$
 $P(\text{outside}) = 59 / 94$ (rounded to 4 decimal places)
 $P(\text{outside}) \approx 0.6277$
- Probability that a guest selected at random ordered steak and requested to sit inside ($P(\text{steak and inside})$):**
 $P(\text{steak and inside}) = (\text{Number of guests who ordered steak and sat inside}) / (\text{Total number of guests})$
 $P(\text{steak and inside}) = 11 / 94$ (rounded to 4 decimal places)
 $P(\text{steak and inside}) \approx 0.1170$
- Probability that a guest selected at random ordered steak or requested to sit inside ($P(\text{steak or inside})$):**
 $P(\text{steak or inside}) = (\text{Number of guests who ordered steak or sat inside}) / (\text{Total number of guests})$
To calculate this probability, we can use the principle of inclusion-exclusion:
 $P(\text{steak or inside}) = P(\text{steak}) + P(\text{inside}) - P(\text{steak and inside})$
 $P(\text{steak or inside}) = (49 / 94) + (35 / 94) - (11 / 94)$
 $P(\text{steak or inside}) \approx 0.7766$
- Probability that, given that a guest requested to sit inside, they ordered steak ($P(\text{steak}|\text{inside})$):**
 $P(\text{steak}|\text{inside}) = (\text{Number of guests who ordered steak and sat inside}) / (\text{Number of guests who sat inside})$
 $P(\text{steak}|\text{inside}) = 11 / 35$ (rounded to 4 decimal places)
 $P(\text{steak}|\text{inside}) \approx 0.3143$