Discrete Probability Distributions

Sections 4.3, 4.4, and 4.6

Review

- A discrete probability has random variables that indicate the number of times an event occurs.
- Each probability is between 0 and 1.
- The probabilities sum to 1.
- Each probability function has a mean, variance, and standard deviation.

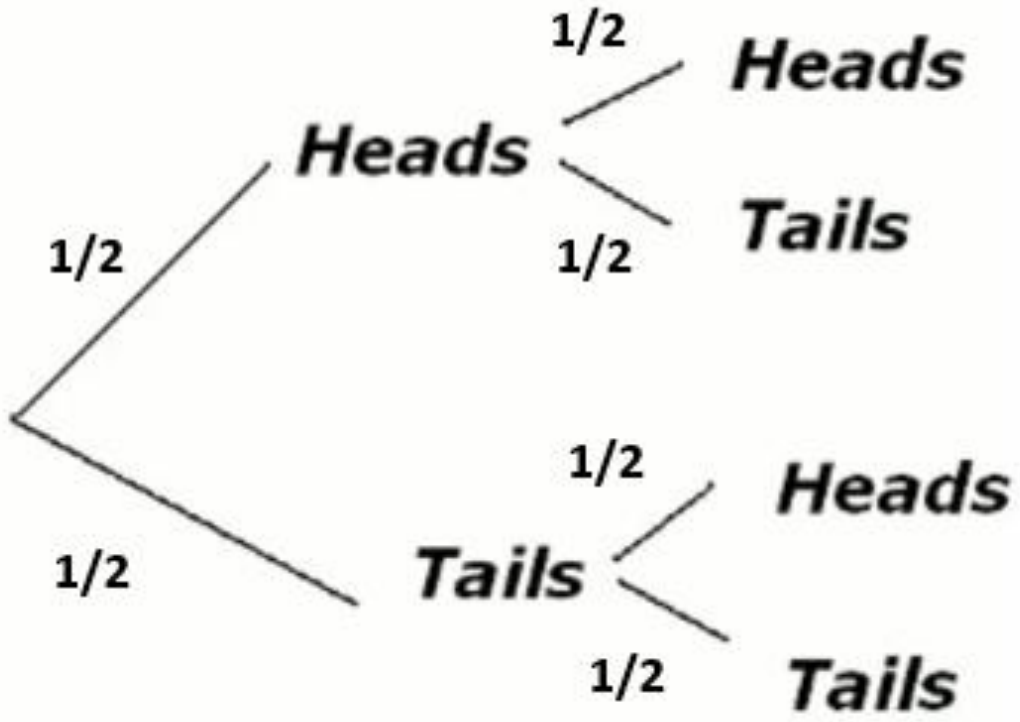
Binomial Probability Distribution

Binomial Probability Distribution

- It occurs a fixed number of times. n
- There are two outcomes “success” or “failure.”
- Each event is an independent event

P(Success) + P(failure) = 1

P(H and H) = (1/2)(1/2) = 1/4



Binomial Distribution

- X = # of success

  n = the total possible successes.

X = 0, …, n

P(Success) = p P(failure) = q where p + q = 1

X ~ B(n, p)

Mean: μ = np Standard deviation: $\sigma = \sqrt{npq}$

Binomial Distribution Formula

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Where

k = # of success

p = probability of success q = 1 - p

n = number of events

In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let *X* = the number of pages that feature signature artists.

a.What values does *x* take on?

X = 0, 1, 2, …, 8

In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let *X* = the number of pages that feature signature artists.

What is the probability distribution?

$X \sim B(100, 8/560)$

$X \sim B(100, 0.0143)$

In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let X = the number of pages that feature signature artists.

What is the probability distribution? $X \sim B(100, 0.0143)$

Find the following probabilities:

a. the probability that two pages feature signature artists

$P(X = 2) = \text{binom.dist}(2, 100, 0.0143, \text{False})$

0.2468

DiscreteProbabilityALL

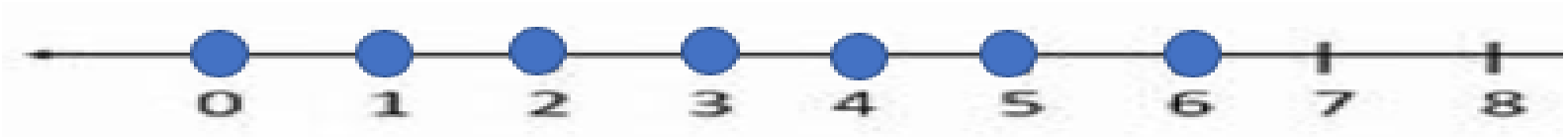
	A	B	C	D	E	F
1	**n**	100			**E(x) = μ**	1.43
2	**p**	0.0143			**Var(x) = σ²**	1.409551
3					**Std dev = σ**	1.19
4	**x**	**P(X = x)**	**P(X <= x)**			
5	0	0.2369	0.2369			
6	1	0.3436	0.5805			
7	2	0.2468	0.8272			
8	3	0.1169	0.9442			
9	4	0.0411	0.9853			
10	5	0.0115	0.9968			
11	6	0.0026	0.9994			
12	7	0.0005	0.9999			
13	8	0.0001	1.0000			

In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let X = the number of pages that feature signature artists.

What is the probability distribution? $X \sim B(100, 0.0143)$

Find the following probabilities:

b. the probability that at most six pages feature signature artists



$P(X \leq 6)$ = binom.dist(6, 100, 0.0143, True)

0.9994

DiscreteProbabilityALL

	A	B	C	D	E	F
1	**n**	100			**E(x) = μ**	1.43
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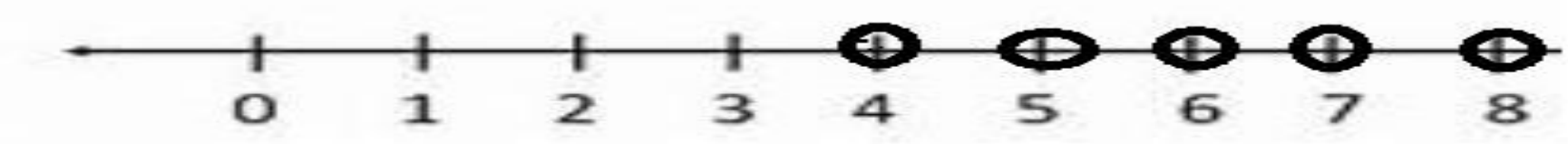
In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let X = the number of pages that feature signature artists.

What is the probability distribution? Find the following probabilities:

a. the probability that more than three pages feature signature artists.



$P(X > 3) = 1 - P(X \leq 3)$

= 1 – binom.dist(3, 100, 0.0143, true)

= 1 - 0.9442

= 0.0558

DiscreteProbabilityALL

	A	B	C	D	E	F
1	**n**	100			**E(x) = μ**	1.43
2	**p**	0.0143			**Var(x) = σ²**	1.409551
3					**Std dev = σ**	1.19
4	**x**	**P(X = x)**	**P(X <= x)**			
5	0	0.2369	0.2369			
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7	2	0.2468	0.8272			
8	3	0.1169	0.9442			
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10	5	0.0115	0.9968			
11	6	0.0026	0.9994			
12	7	0.0005	0.9999			
13	8	0.0001	1.0000			

=1-0.9442 = 0.0558

Discrete Probability General | **Binomial Probability Distrib.** | Poisson Probal ...

In the 2013 *Jerry's Artarama* art supplies catalog, there are 560 pages. Eight of the pages feature signature artists. Suppose we randomly sample 100 pages. Let *X* = the number of pages that feature signature artists.

What is the probability distribution?

Using the formulas, calculate the (i) mean and (ii) standard deviation.

$$\mu = np \quad \sigma = \sqrt{npq}$$

$$\mu = 100(.0143) \quad \sigma = \sqrt{100(0.0143)(1 - .0143)}$$

$$\mu = 1.43 \ \ \sigma = 1.1873$$

#1 If a seed is planted, it has a 85% chance of growing into a healthy plant.

If 8 seeds are planted, what is the probability that exactly 2 doesn't grow? **Round to 4 decimal places.**

P(Growing) = 0.85 P(Not Growing) = 1 – 0.85 = 0.15

X = # that does not grow

P(X = 2) = ?

=binom.dist(2, 8, .15, false)

= 0.2376

How to use the Excel spreadsheet

Binomial Probability Distrib.

	A	B	C	D	E	F
1	n	8			E(x) = μ	1.20
2	p	0.15			Var(x) = σ²	1.02
3					Std dev = σ	1.01
4	x	P(X = x)	P(X <= x)			
5	0	0.2725	0.2725			
6	1	0.3847	0.6572			
7	2	0.2376	0.8948			
8	3	0.0839	0.9786			
9	4	0.0185	0.9971			
10	5	0.0026	0.9998			
11	6	0.0002	1.0000			
12	7	0.0000	1.0000			
13	8	0.0000	1.0000			

#2 Assume that a procedure yields a binomial distribution with a trial repeated n=5 times. Use some form of technology to find the probability distribution given the probability p=0.391 of success on a single trial. $X \sim B(5, 0.391)$

k	P(X=k)
0	0.0838
1	0.2689
2	0.3453
3	0.2217
4	0.0712
5	0.0091

1	n	5	
2	p	0.391	
3			
4	x	P(X = x)	P(X <= x)
5	0	0.0838	0.0838
6	1	0.2689	0.3527
7	2	0.3453	0.6980
8	3	0.2217	0.9197
9	4	0.0712	0.9909
10	5	0.0091	1.0000
11			
12			

... Binomial Probability Distrib. Poisson

#3 The probability that you will win a game is $p=0.24$.

If you play the game 1271 times, what is the most likely number of wins? ***(Round the answer to one decimal place.)***

$$\mu = np$$

$$1271(0.24) = 305.04 \text{ or } 305$$

#3 The probability that you will win a game is p=0.24.

Let X represent the number of games (out of 1271) that you win. Find the standard deviation for the probability distribution of X.
(Round the answer to two decimal places.)

$$\sigma = \sqrt{npq}$$

$$\sigma = \sqrt{1271(0.24)(1 - .24)}$$

$$\sigma = 15.225 \text{ or } 15.23$$

- The *range rule of thumb* specifies that the minimum usual value for a random variable is $\mu-2\sigma$ and the maximum usual value is $\mu+2\sigma$. You already found μ and σ for the random variable X.

Use the range rule of thumb to find the usual range of X values. **Enter the answer as an interval using square-brackets and only whole numbers that contain the two values calculated.**
usual values =

Lower Limit = 305 – 2*15.23 = 279.94

Upper Limit = 305 + 2*15.23 = 330.06

[279, 331]

The lifetime risk of developing pancreatic cancer is about one in 78 (1.28%). Suppose we randomly sample 200 people. Let *X* = the number of people who will develop pancreatic cancer.

a. What is the probability distribution for *X*?

P = .0128 n = 200 X ~B(200, 0.0128)

The lifetime risk of developing pancreatic cancer is about one in 78 (1.28%). Suppose we randomly sample 200 people. Let *X* = the number of people who will develop pancreatic cancer.

a. What is the probability distribution for *X*?

b. Using the formulas, calculate the (i) mean and (ii) standard deviation of *X*.

P = .0128 n = 200 X ~B(200, 0.0128)

$$\mu = 200(0.0128) = 2.56$$

$$\sigma = \sqrt{200(0.0128)(1 - 0.0128)} = 1.5897$$

Excel Spreadsheet

	A	B	C	D	E	F
1	**n**	200			**E(x) = μ**	2.56
2	**p**	0.0128			**Var(x) = σ²**	2.527232
3					**Std dev = σ**	1.59
4	**x**	**P(X = x)**	**P(X <= x)**			
5	0	0.0760	0.0760			
6	1	0.1972	0.2732			
7	2	0.2544	0.5276			
8	3	0.2177	0.7453			
9	4	0.1390	0.8843			
10	5	0.0707	0.9550			
11	6	0.0298	0.9847			
12	7	0.0107	0.9954			
13	8	0.0033	0.9988			

Discrete Probability General | **Binomial Probability Distrib.** | Poisson Probal ...

The lifetime risk of developing pancreatic cancer is about one in 78 (1.28%). Suppose we randomly sample 200 people. Let *X* = the number of people who will develop pancreatic cancer.

Use Excel to find the probability that at most eight people develop pancreatic cancer

$P(X \leq 8)$ = binom.dist(8, 200, .0128, True)

= 0.9988

Excel Spreadsheet

	A	B	C	D	E	F
1	**n**	200			**E(x) = μ**	2.56
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Discrete Probability General | **Binomial Probability Distrib.** | Poisson Probal ...

The lifetime risk of developing pancreatic cancer is about one in 78 (1.28%). Suppose we randomly sample 200 people. Let X = the number of people who will develop pancreatic cancer.

a. What is the probability distribution for X?

Is it more likely that five or six people will develop pancreatic cancer? Justify your answer numerically.

$P(X = 5) = 0.0707$

or

$P(X = 6) = 0.0298$

	A	B	C	D	
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2	**p**	0.0128			**Va**
3					**St**
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Discrete Probability General | Binomial Probability Distrib.