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Excellence in
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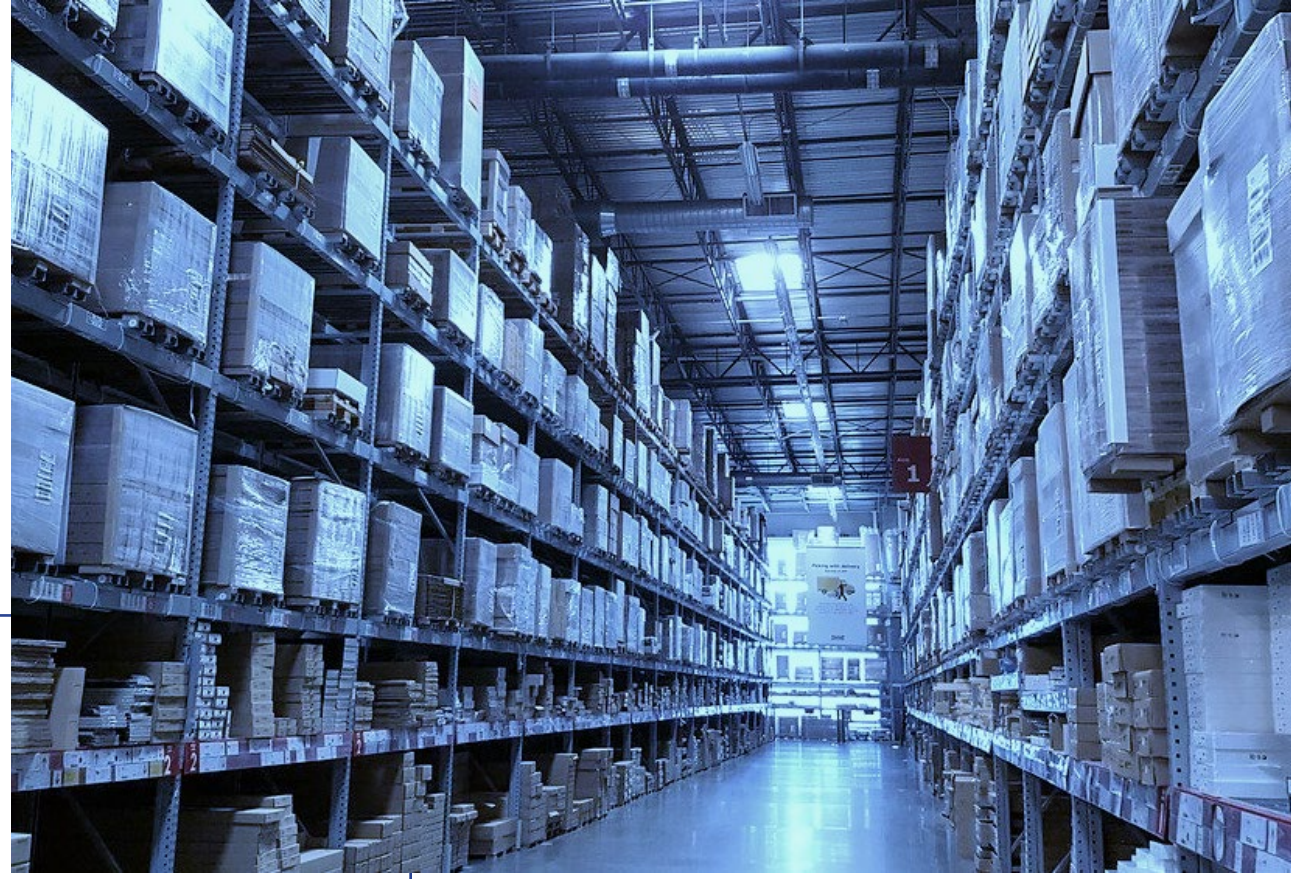
Education



Prof. Dr. Carl Marcus Wallenburg
Chair of Logistics and Services Management

LOGISTICS and SUPPLY CHAIN MANAGEMENT

Module 3 – Warehousing and Inventory Management

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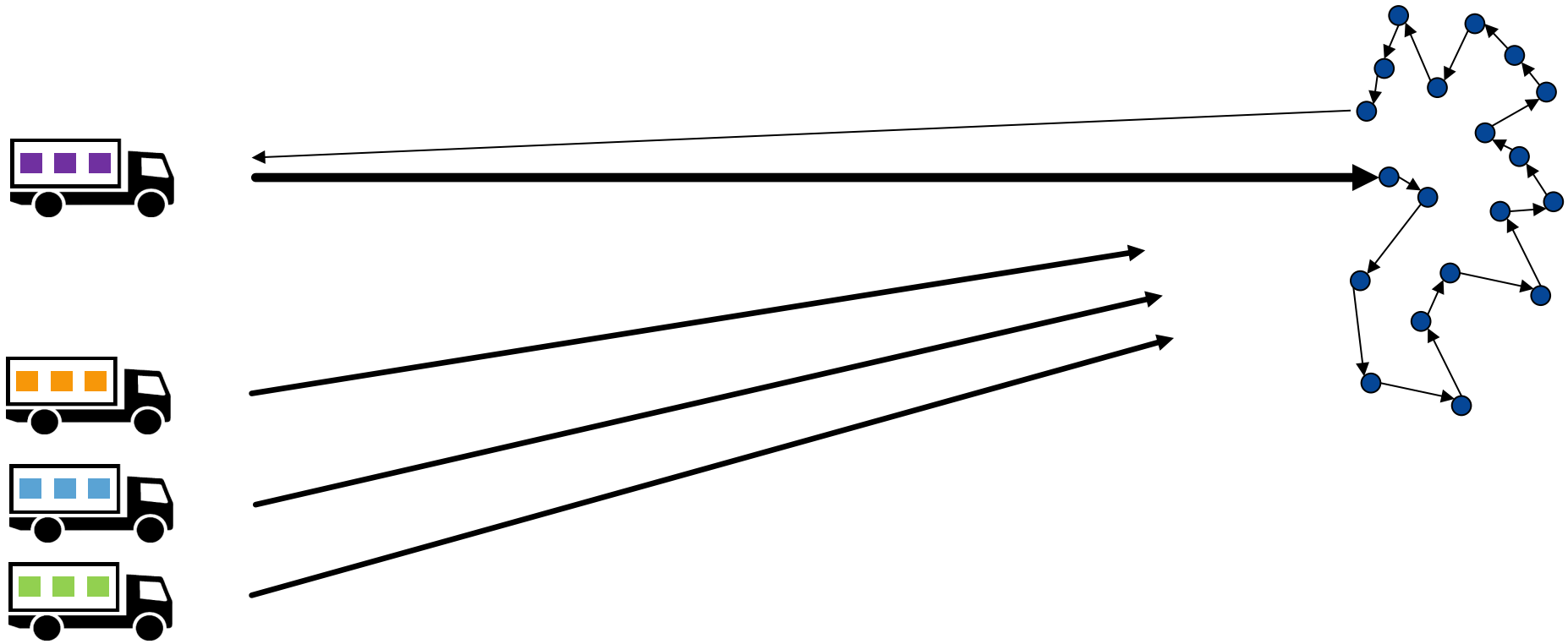
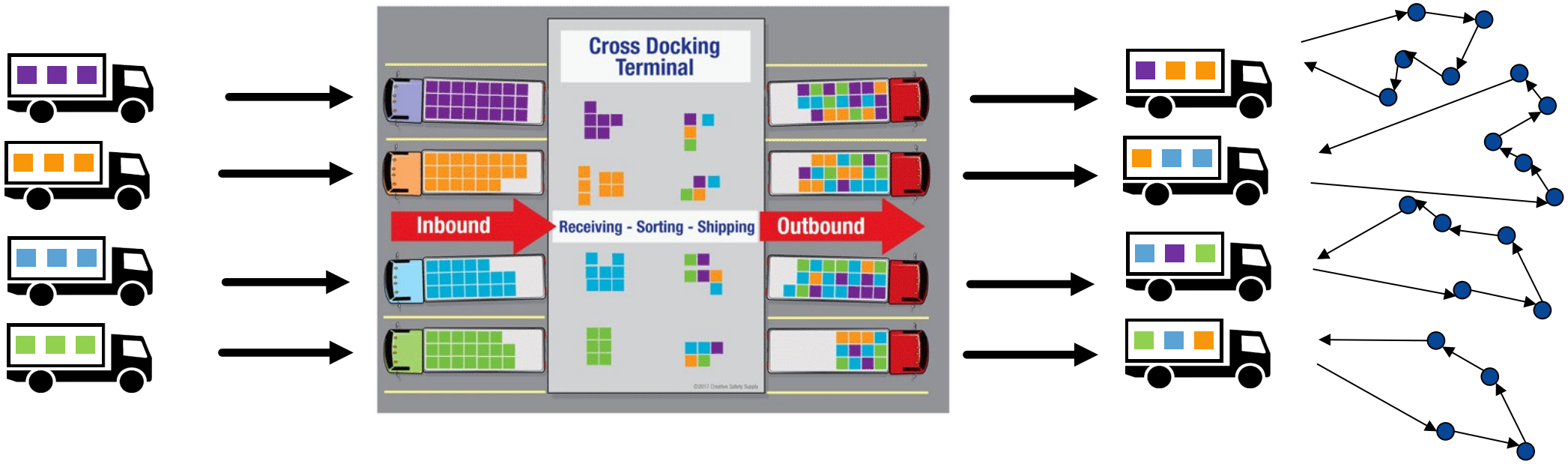
1 **Warehousing and Transshipment**
- 1.1 **Basic forms of Transshipment**
- 1.2 Order Picking

2 Inventory Management
- 2.1 Cycle Inventory
- 2.2 Safety Inventory
- 2.3 Newsboy model
- 2.4 Impact of coordination on inventory
- 2.5 The Bullwhip Effect

Connecting to the customer: The milk run



Crossdocking – Transshipment with unchanged loading device



Crossdocking (cont.)

Merge-in-transit

Merge-in-transit

Merge-in-transit is defined as the process of consolidating different suppliers‘ component shipments at one customer to fulfill his order.

Core of merge-in-transit

The identification of all components is necessary to fulfill one customers‘ order at a merging terminal and securing their delivery at the same time.

Crossdocking is related to process efficiency

To ensure that incoming shipments of one customer are shipped as soon as capacities are free.

Operative efficiency is better compared to merge-in-transit.

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Purpose of Warehouses

1. Holding

- Use of warehouses for holding of inventory
- Modes: Long-term storage (e.g., aging liquors), general-purpose storage, temporary storage (e.g., trucking terminal)

2. Consolidation

- Transportation rates influence the use of storage facilities

→ Reduction of transportation costs via consolidation

3. Break-Bulk

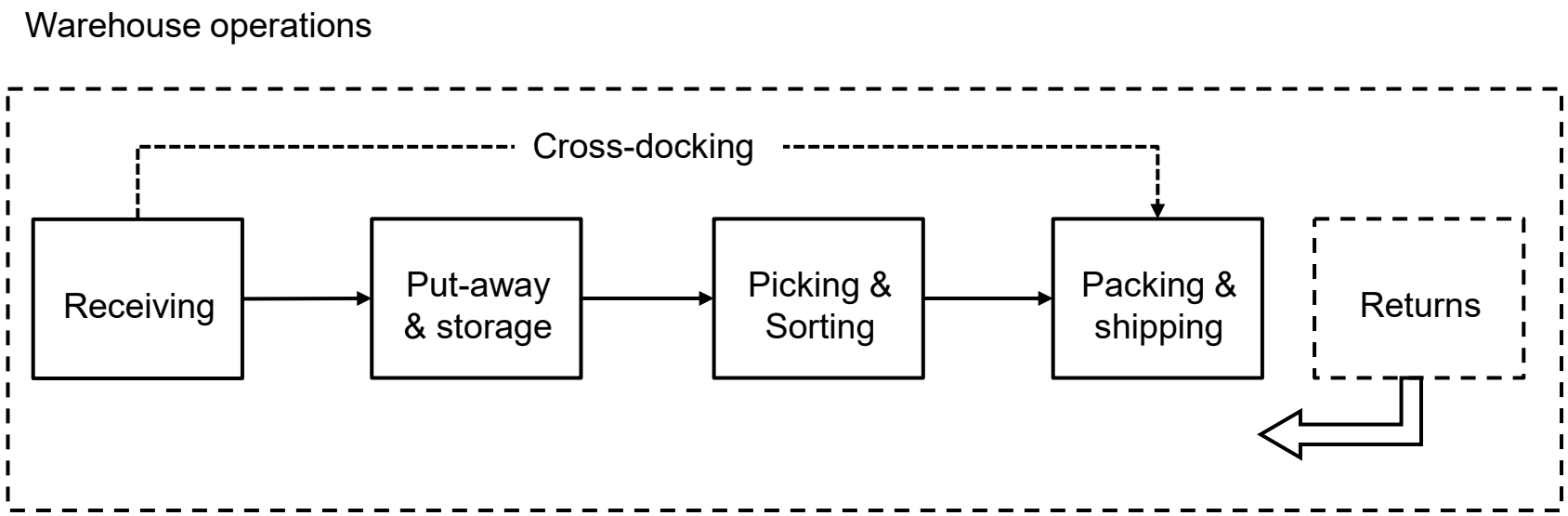
- High volume shipments with low transport rates are broken down and reshipped in smaller quantities

4. Mixing

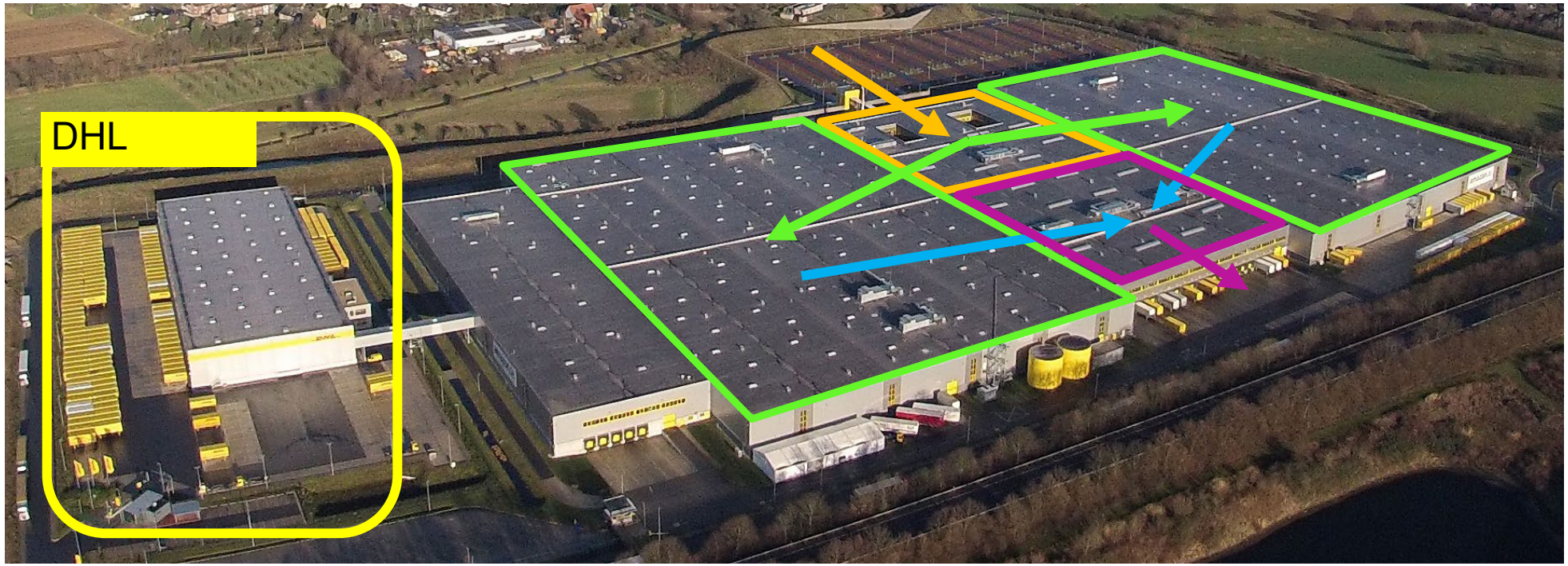
- High transportation rates on small volumes occurring when customers order directly from manufacturers

→ High volume shipments of single goods to a mixing point where different products are assembled to orders and re-shipped to customers

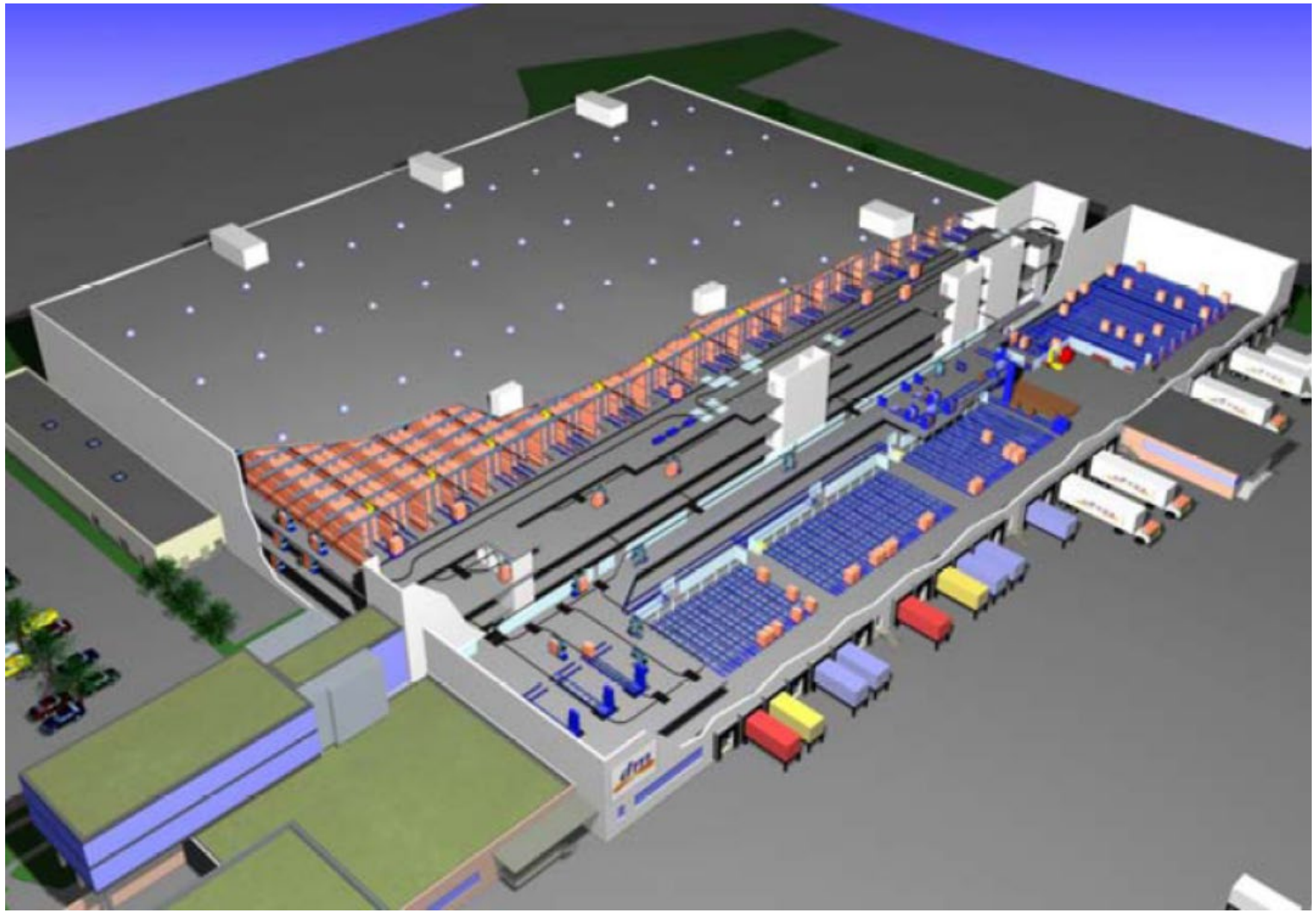
Framework for warehousing



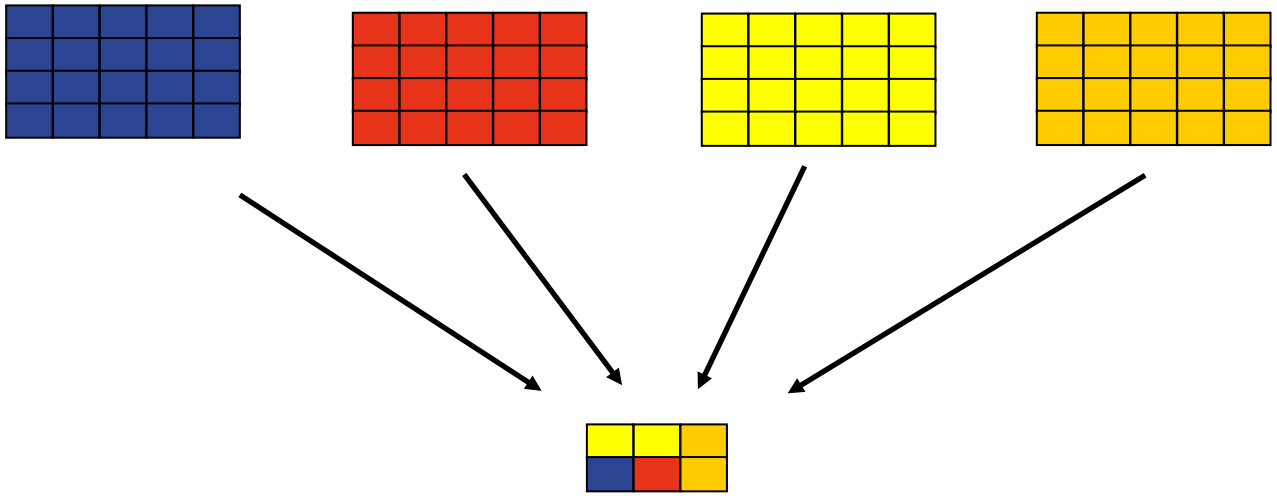
Amazon fulfillment center in Rheinberg



Layout of a DM Drogeriemarkt warehouse



Order picking bundles products according to specific orders



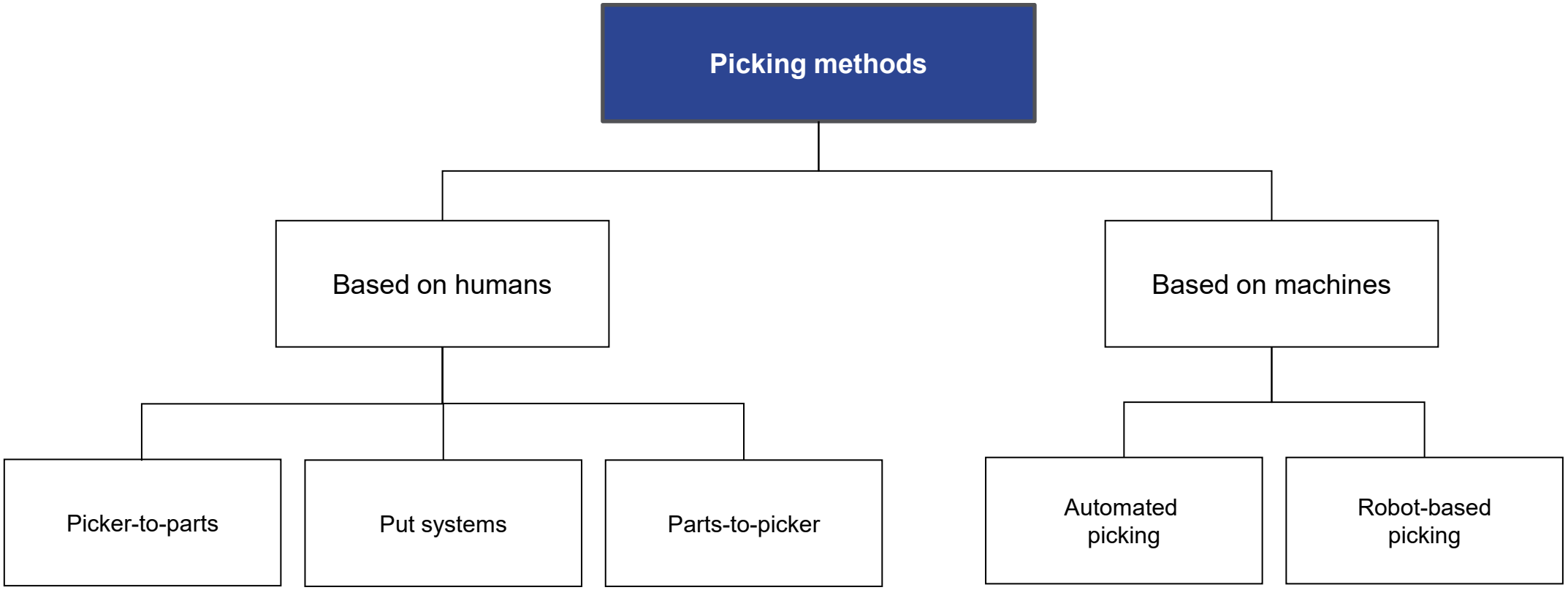
Warehousin

Inside an Amazon FC

Order-picking at Amazon

Order-picking at Amazon

Classifications for order picking

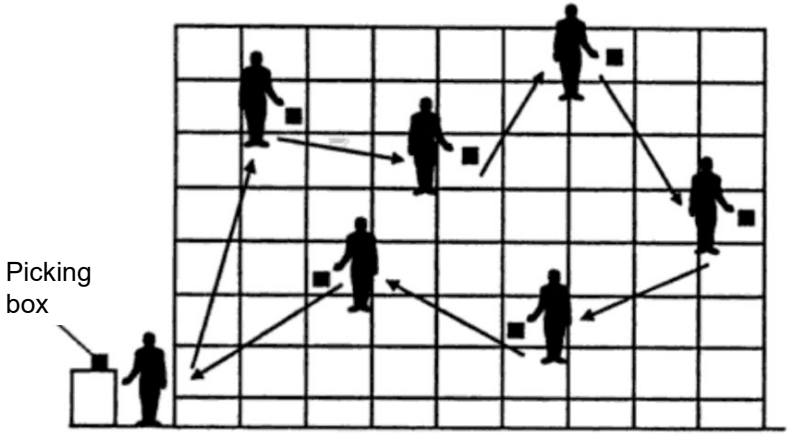


Principles of order picking

Picker-to-parts (static provision of goods)

→ The picker moves to the storage place and manually releases the respective good.

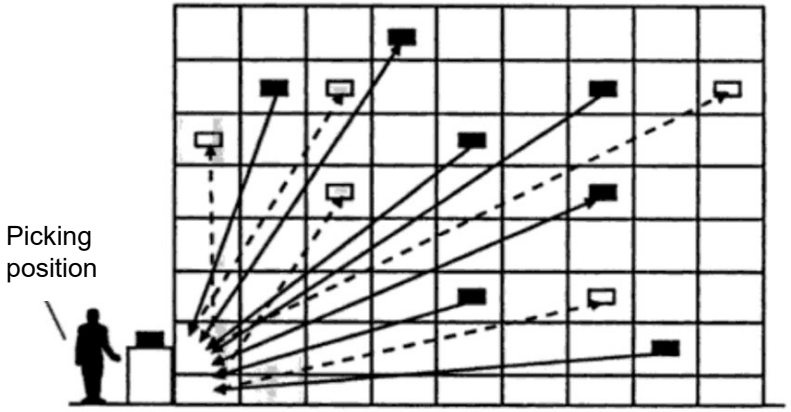
Goods can be shaped very differently.



Parts-to-picker (dynamic provision of goods)

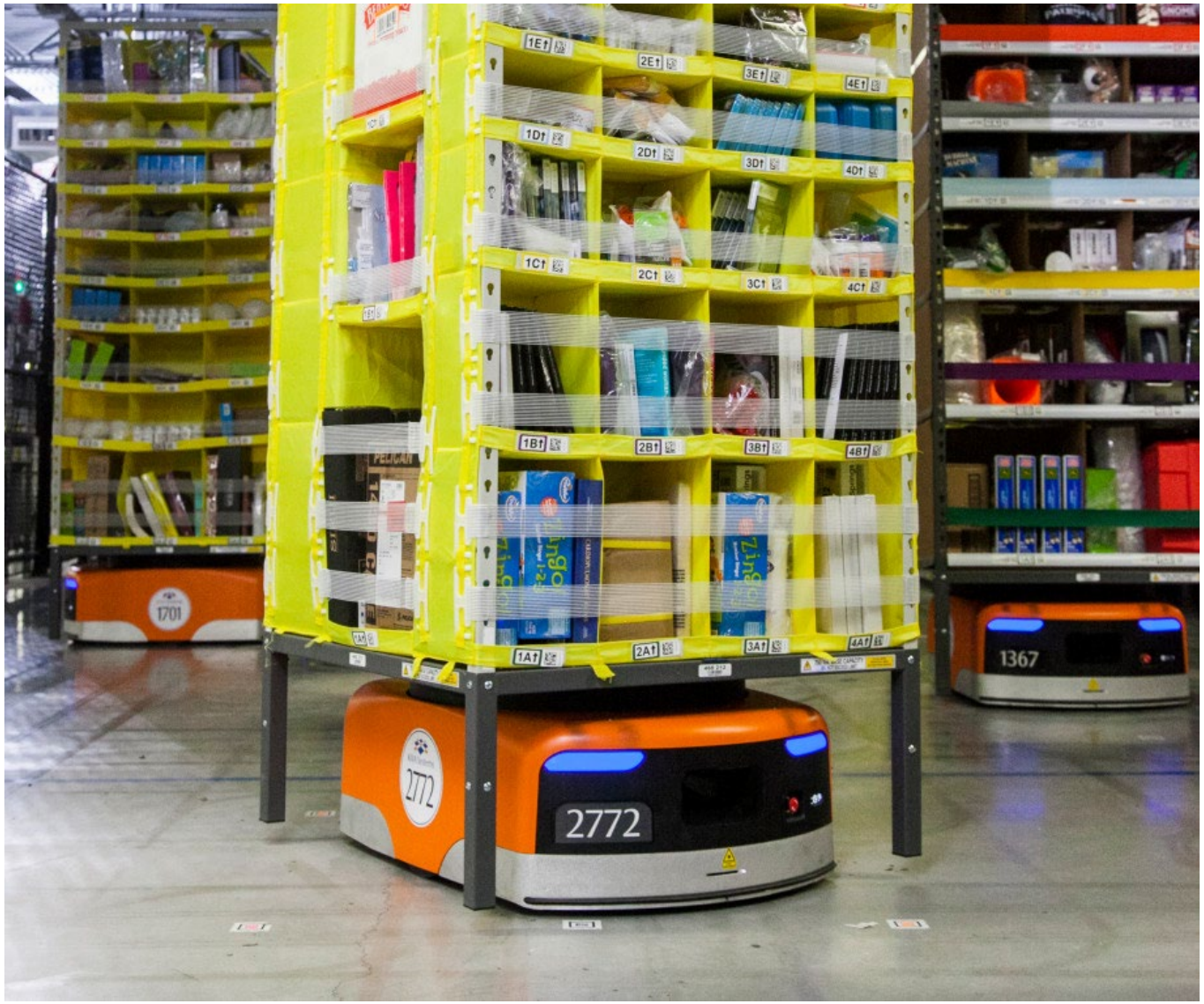
→ Automatic release of whole unit of goods from storage.
After manual picking, units are moved back to storage.

Goods need to be rather standardized and uniform



Kiva robots at Amazon

Order Picking



Order Picking

Guiding the picker to the right items

Pick-by-voice	The information provided via headset with the possibility to report missing parts
Pick-by-light	Signal lights and displays are attached to the storage shelf. The releasing of goods is countersigned electronically.
Pick-by-vision	The information is displayed via AR on data glasses with the possibility to report problems.
Pick-by-lists	electronic: find and release goods, scanning of barcode paper-based: providing type of good, number, and quantity

Pick-by-voice

Pick-by-light

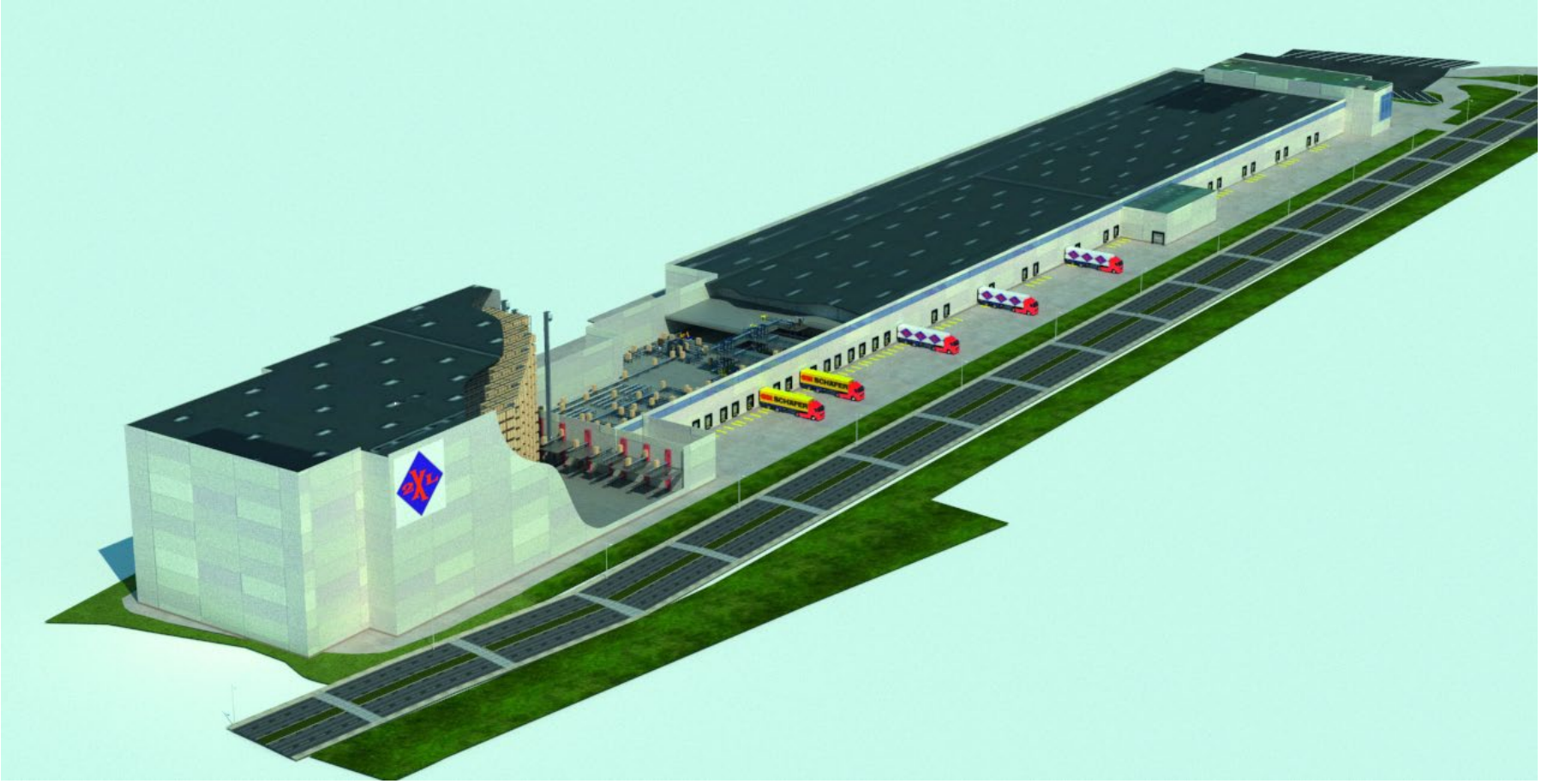
Pick-by-vision

Using Walmart-Superstores in the USA for Online Retailing

Internal transportation

Forklifts carry the bulk of movements

Automated warehouse at 2XL



Video: Automated warehouse at 2XL

Why is the 2XL solution highly automized, despite the industry standard being warehouses with a low degree of automatization?

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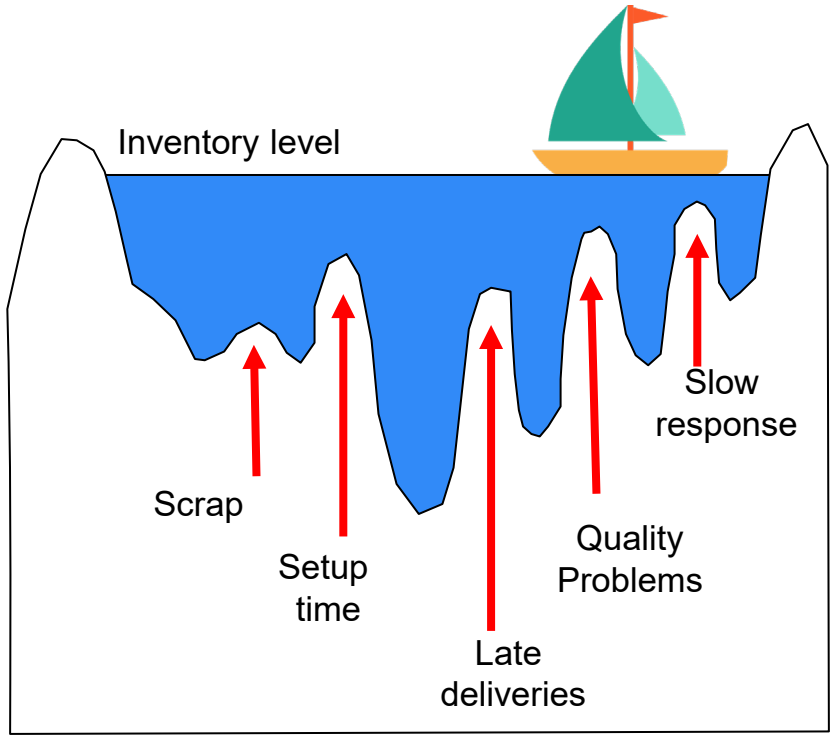
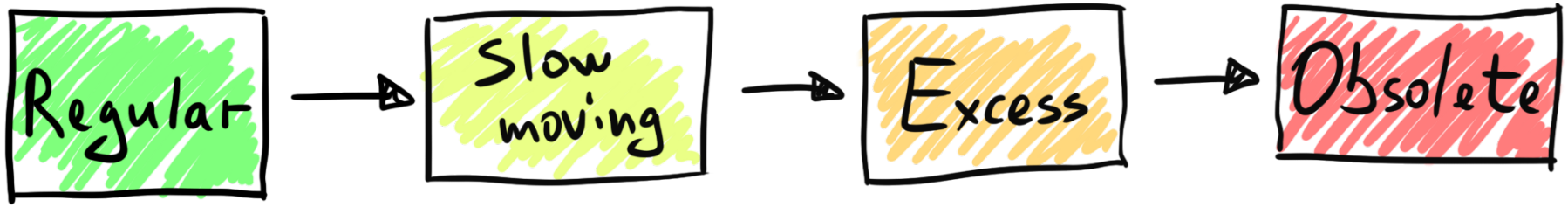
Inventory is good in masking operational problems

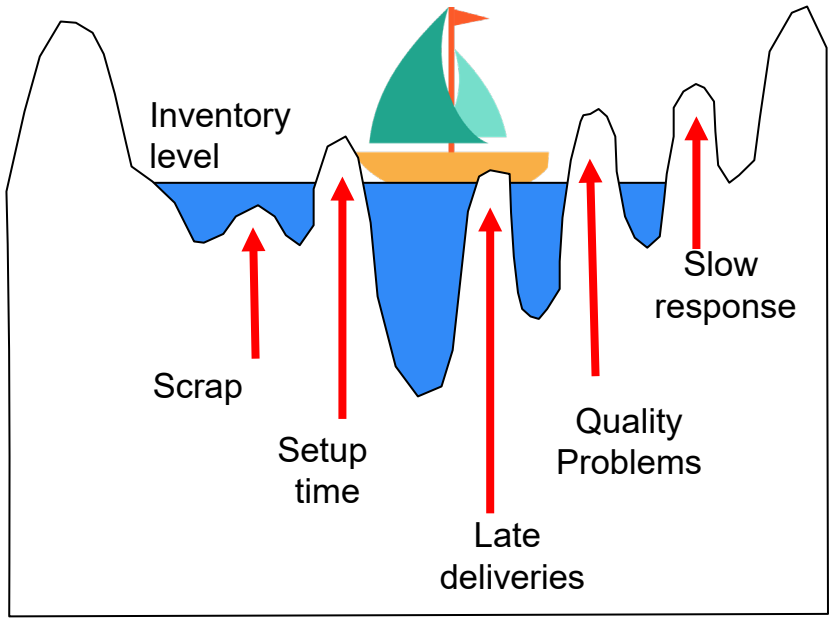


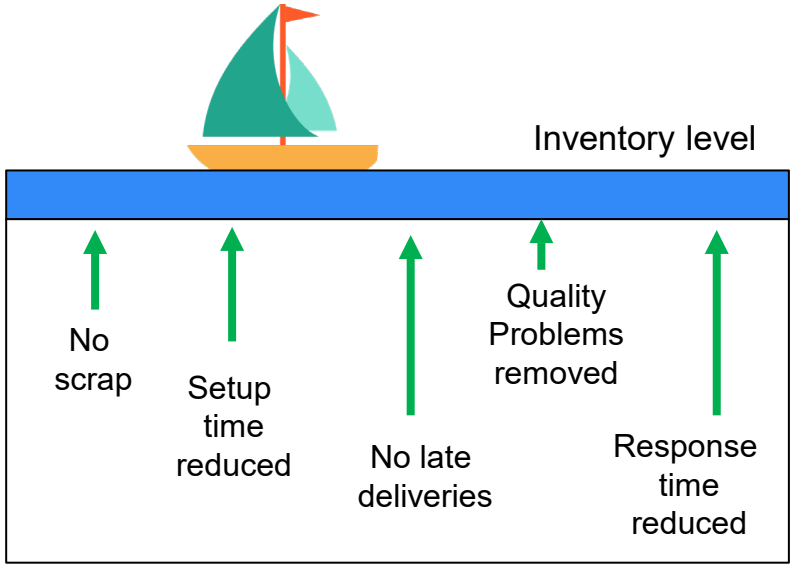
Inventory creates problems of its own



$$Weeks\ on\ hand\ (WOH) = \frac{Avg.Inventory\ Level}{Weekly\ Usage\ of\ Item}$$

Lowering inventory allows detecting and removing operational problems





Why do we have inventory in a supply chain?

Different types of inventory

Cycle inventory

- Inventory due to economies of scale
- Lot sizes that are larger than customer demand
- Assumes full predictability of supply and demand

Answers the question **How much to order?**

Safety inventory

- Inventory due to uncertainty of supply and demand
- Carried to prevent stock-out situations
- Level depends on uncertainty and desired product availability

Answers the question **When to order?**

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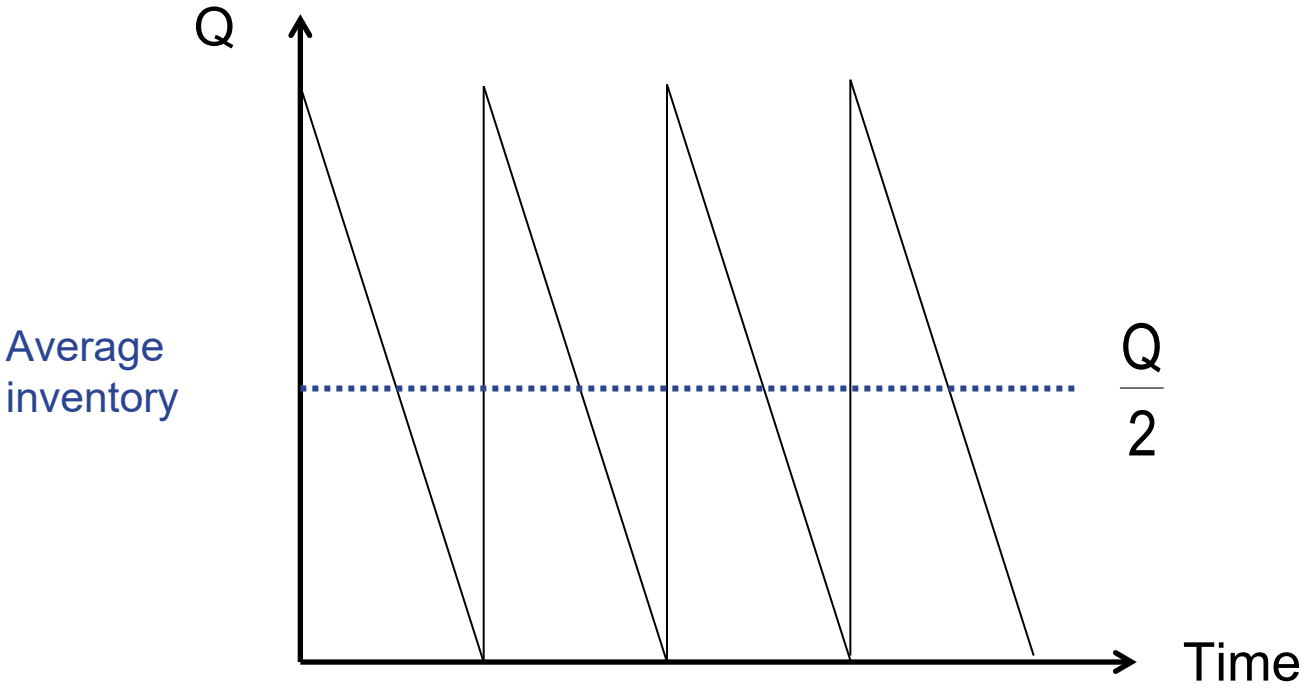
Cycle inventory



Relevant costs in determining cycle inventory

Fixed ordering cost = S

The Fixed cost is associated with product`s acquisition for inventory`s replenishment and include costs for processing (incl. buyer's time), transmitting, & handling order (incl. updating inventory records), etc. The fixed ordering cost refers to all costs that don`t vary with size of order but are incurred each time an order is placed.

Holding cost = H = h C

The Holding cost is the cost of carrying one unit in inventory for a specified period of time. It includes costs for storage and handling, costs of capital & obsolescence and usually assumed to be proportional to average qty/value of available products.

Material cost = C

The Material cost is the average price paid per unit purchased and a key factor in lot-sizing decisions.

Economic order quantity (EOQ) – the basic cost components

Total annual cost = Annual Material Costs (AMC)
+ Annual Ordering Costs (AOC)
+ Annual Holding Costs (AHC)

$$\text{AMC} = \text{Material Cost} * \text{Annual Demand} = C * D \qquad [\text{AMC}] = \frac{\$}{\text{year}}$$

$$\text{AOC} = \text{Number of Orders per Year} * \text{Fixed Ordering Cost} = \frac{D}{Q} * S \qquad [\text{AOC}] = \frac{\$}{\text{year}}$$

$$\text{AHC} = \text{Average inventory} * \text{Inventory holding cost} * \text{Material Cost} = \frac{Q}{2} * h * C \qquad [\text{AHC}] = \frac{\$}{\text{year}}$$

$$\boxed{\text{TC} = \frac{D}{Q} * S + \frac{Q}{2} * h * C} \qquad [\text{TC}] = \frac{\$}{\text{year}}$$

Economic order quantity (EOQ) – finding the optimum lot size

The optimal lot size is one that minimizes the total cost TC

$$TC = C * D + \frac{D}{Q} * S + \frac{Q}{2} * h * C \qquad \rightarrow \text{Min!}$$

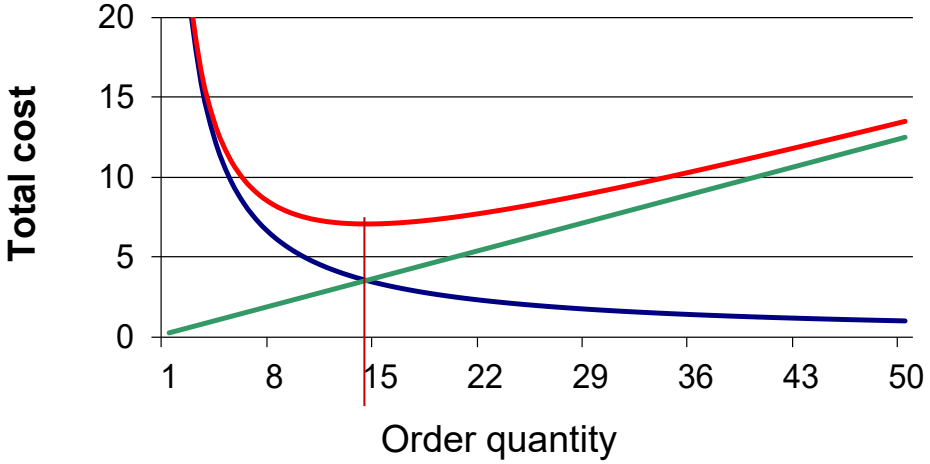
Take the first derivative with respect to the lot size & set it to zero!

$$\frac{d(TC)}{dQ} = 0 - D * S * Q^{-2} + \frac{h * C}{2} \overset{!}{=} 0$$

$$\Rightarrow \quad D * S * Q^{-2} = \frac{h * C}{2} \quad \Rightarrow \quad Q^2 = \frac{2 * D * S}{h * C} \quad \Rightarrow \quad \boxed{Q* = \sqrt{\frac{2 * D * S}{h * C}}}$$

At optimal quantity Q*, annual ordering costs and holding costs are equal

Optimized order quantity depends on fixed and variable costs



$$\left(\frac{D}{Q} * S\right) = \frac{Q}{2} * h * C$$

Exercise: Inventory Planning

What`s the optimal ordering quantity for B&M?

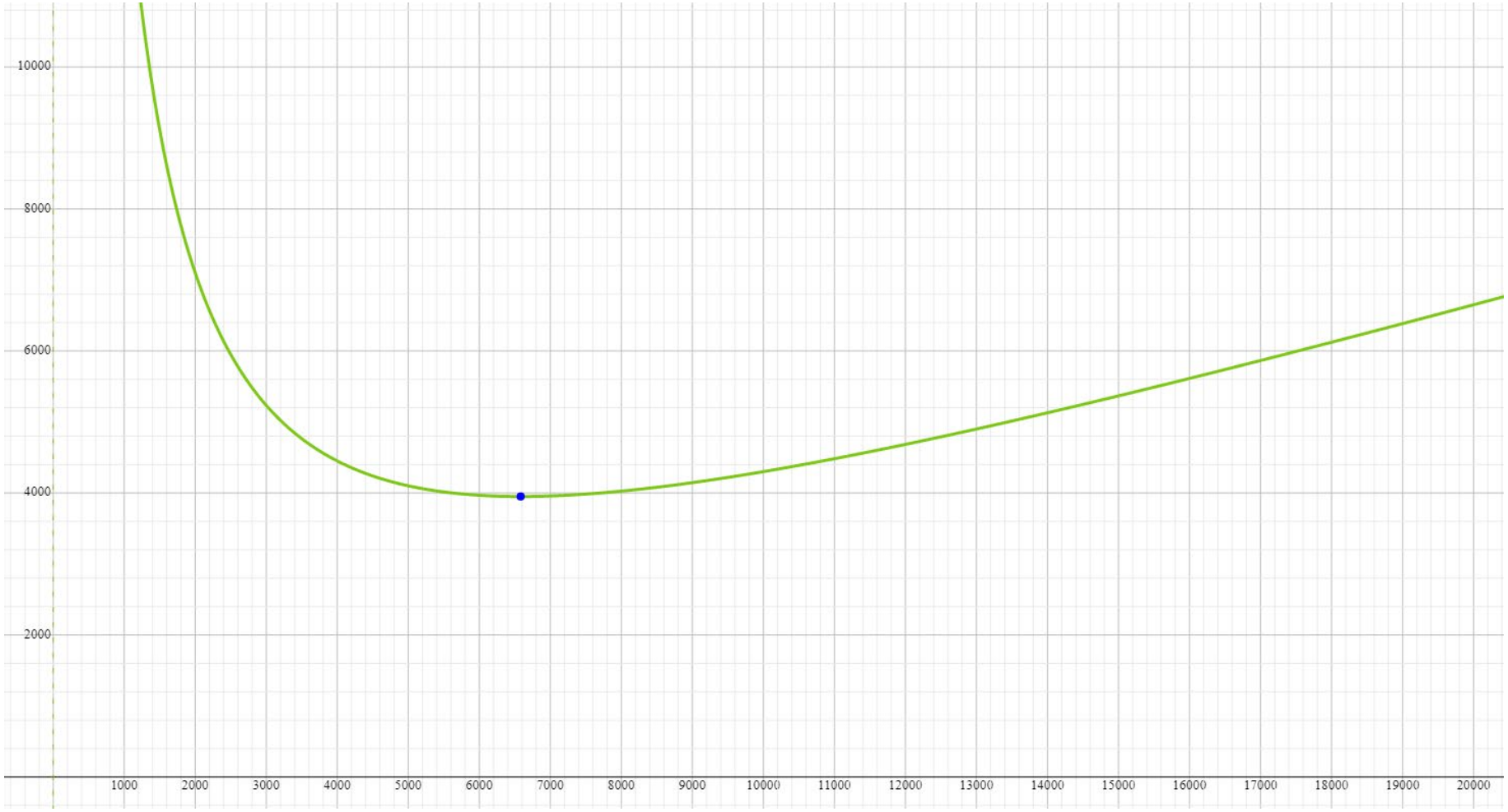
Demand for flip-chart blocks is 2,500 per week at B&M Office Supplies.

B&M incurs a fixed order placement, transportation, and receiving cost of $100 for each order of flip-charts it places at the manufacturer.

B&M incurs inventory holding costs of 20 percent per year. The buying price is $3 for each purchased flip-chart block. B&M's operating margin lies at 7%.

EOQ-analysis of B&Ms optimal lot size

EOQ-analysis of B&Ms optimal lot size



Exercise: Inventory Planning

What`s the optimal ordering quantity for B&M?

Assume B&M would get a 0.10$ discount per flip chart if each order would exceed 15,000 items.

EOQ-analysis with quantity discounts

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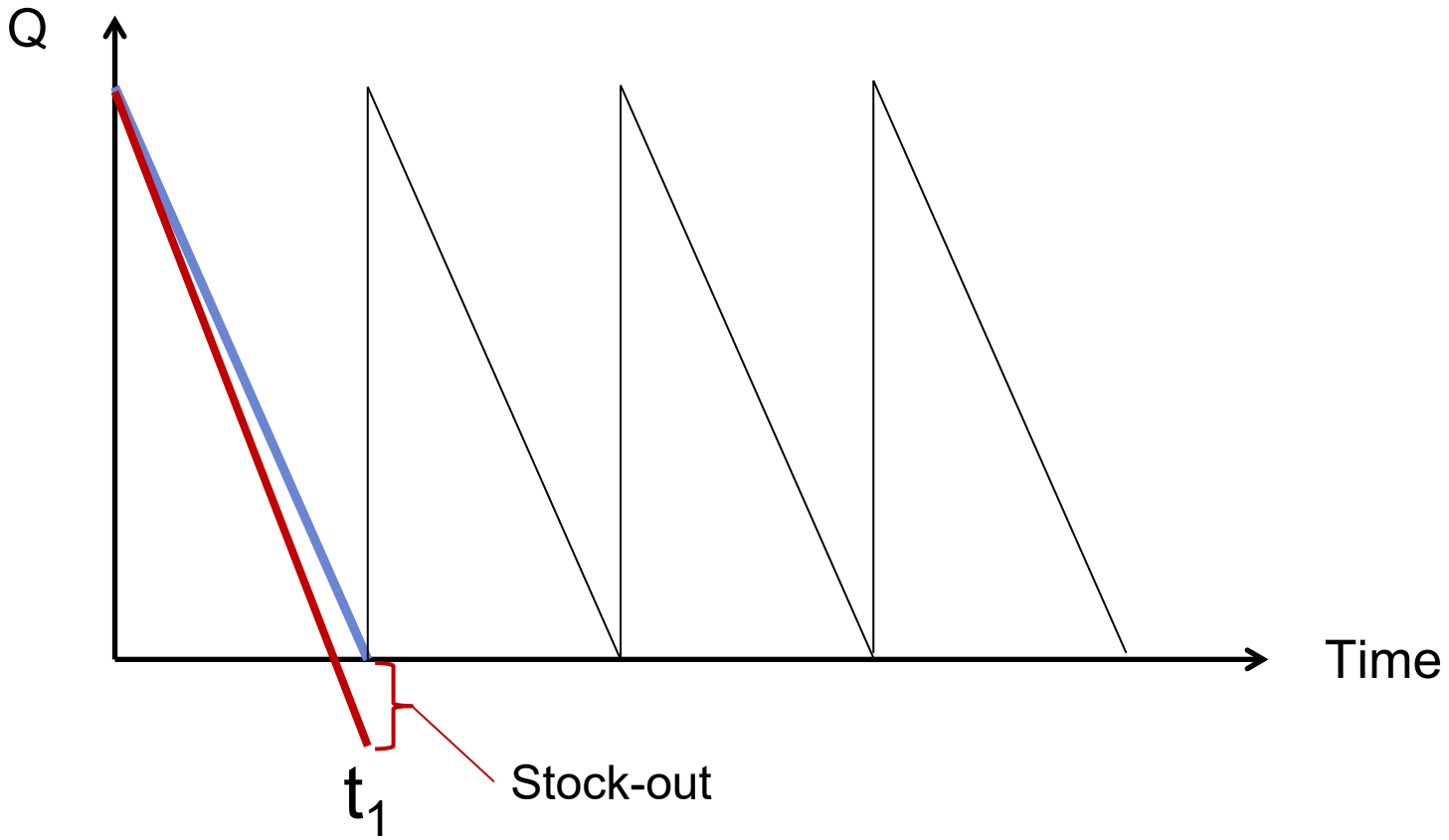
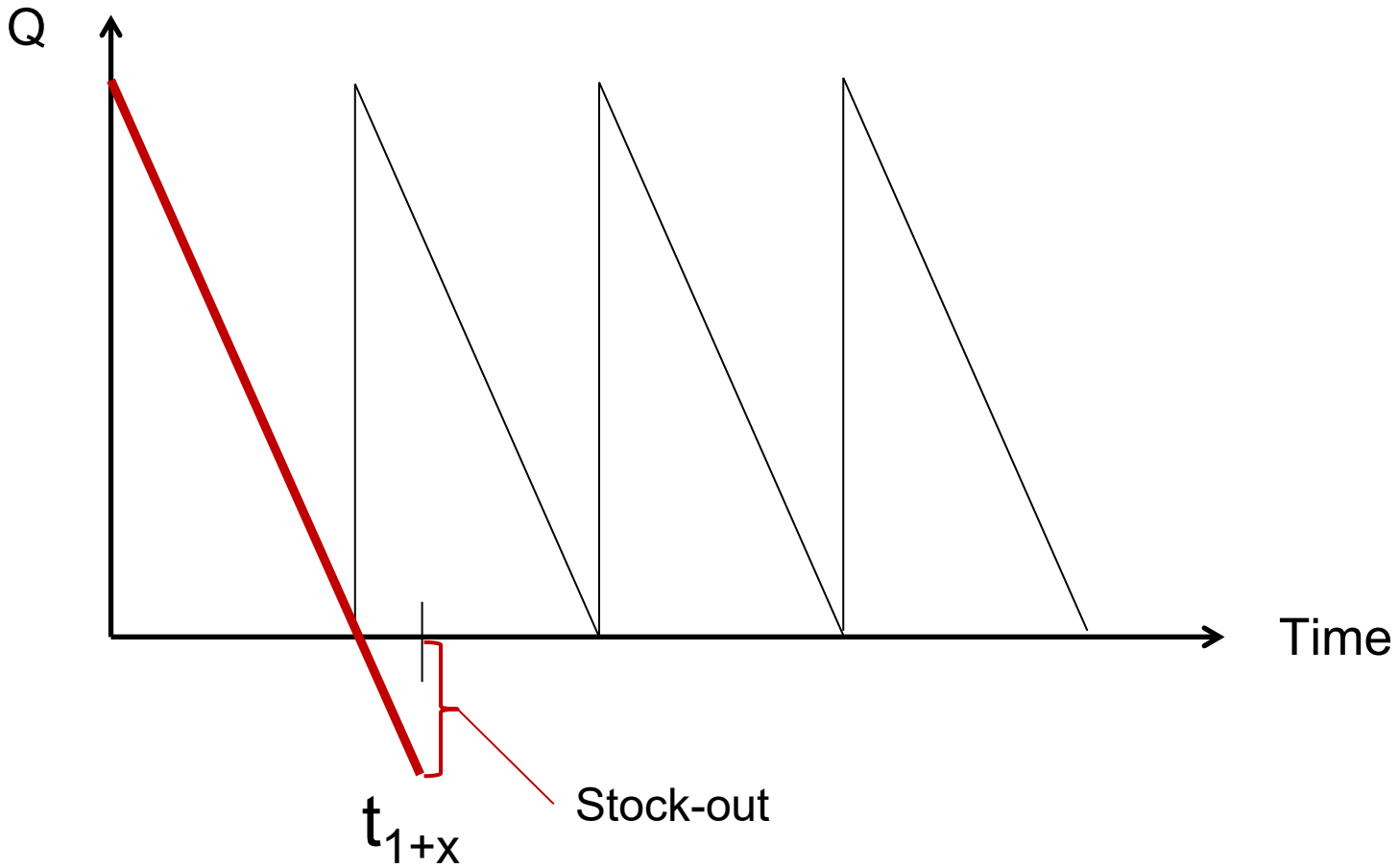
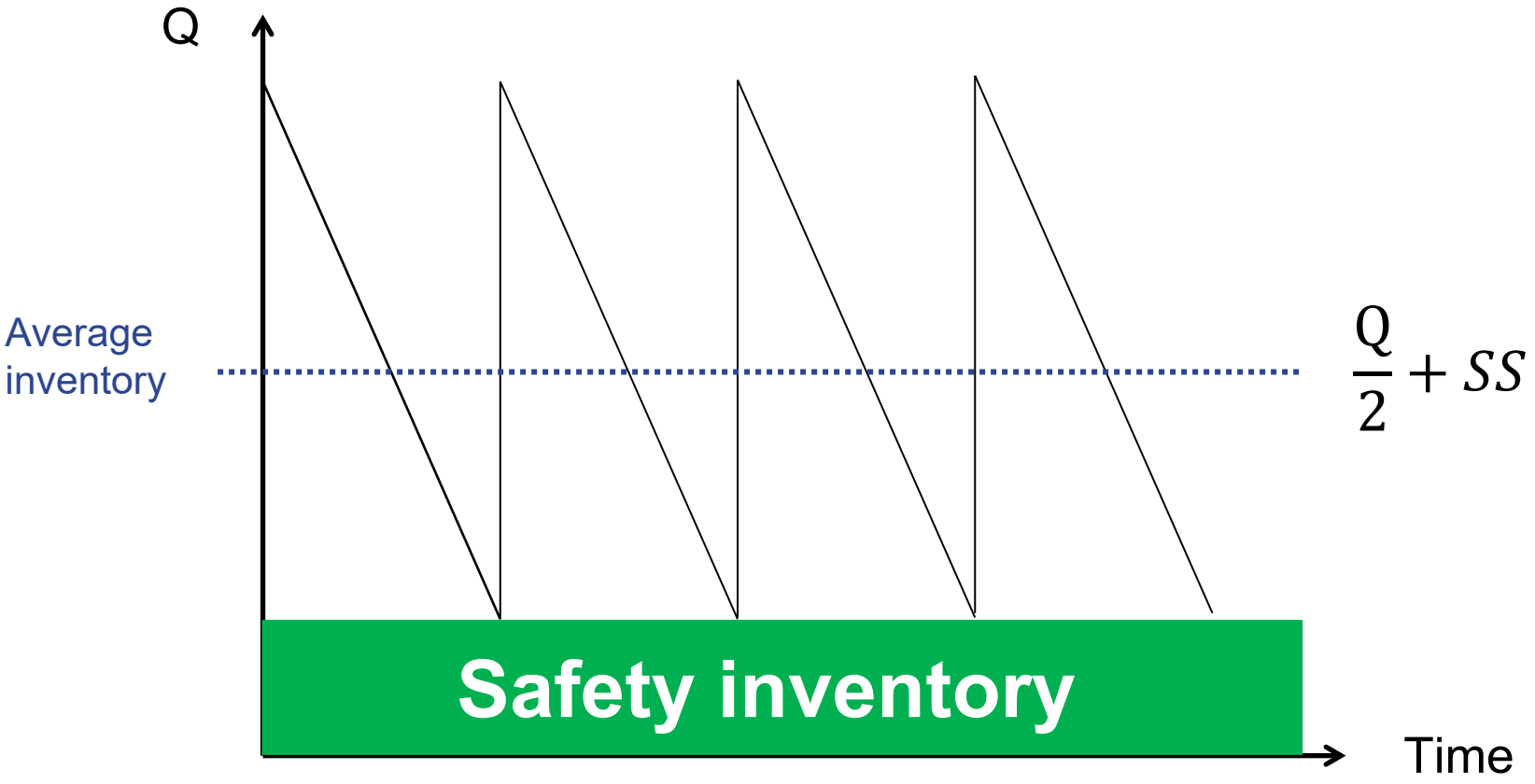
Cycle inventory with demand uncertainty



Cycle inventory with supply uncertainty



Safety inventory to reduce stockouts



Demand uncertainty & possible stock-outs

1. **What is the safety inventory and the average inventory carried by B&M?**

2. **What is value for Weeks On Hand?**

Assume the weekly demand for flip-chart blocks at B&M to be normally distributed, with a mean of 2,500 units and a standard deviation of 500 units.

The manufacturer takes two weeks to fill an order placed by the B&M manager. The store manager currently orders 10,000 flip-chart blocks when the inventory on hand drops to 6,000 units.

Analysis

Analysis

Analysis

- Average demand per week, D = 2,500 unit/week
- Standard deviation of weekly demand, σ_D= 500 unit
- Average lead time for replenishment, L = 2 week
- Reorder point, ROP = 6,000 unit
- Average lot size Q = 10,000 unit

Safety inventory = ROP – D * L
= 6,000 unit – 2 ~~week~~ * 2,500 unit/~~week~~
= 1,000 unit

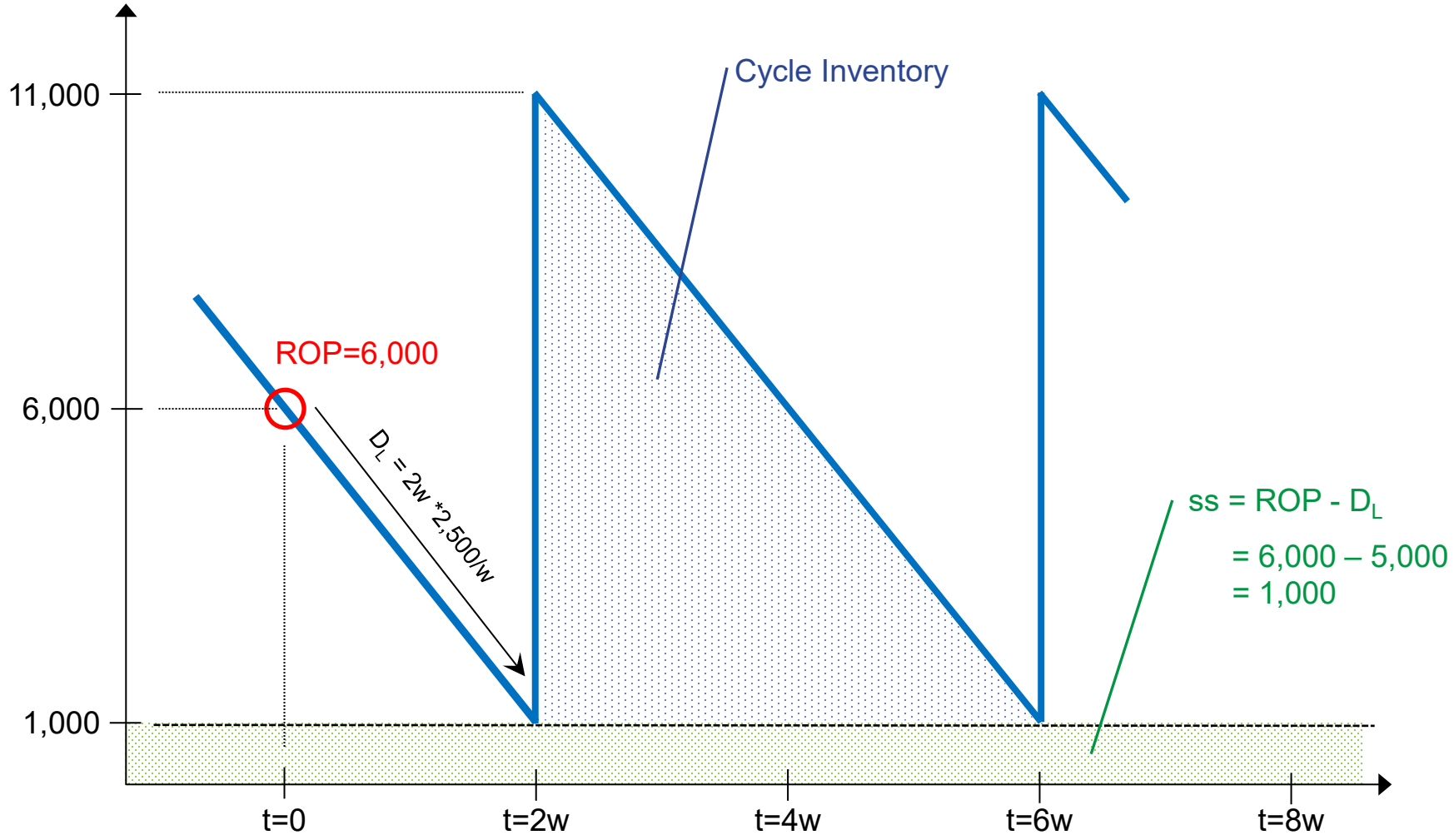
Cycle inventory $= \frac{Q}{2} = \frac{10{,}000 \text{ unit}}{2} = 5{,}000 \text{ unit}$

Average inventory = cycle inv. + safety inv. = 5,000 unit + 1,000 unit = 6,000 unit

$$\text{Weeks on hand} = \frac{\text{average inventory}}{\text{average demand per week}} = \frac{6{,}000 \text{ unit week}}{2{,}500 \text{ unit}} = 2.4 \text{ weeks}$$

On average, the inventory lasts for 2.4 weeks

Analysis



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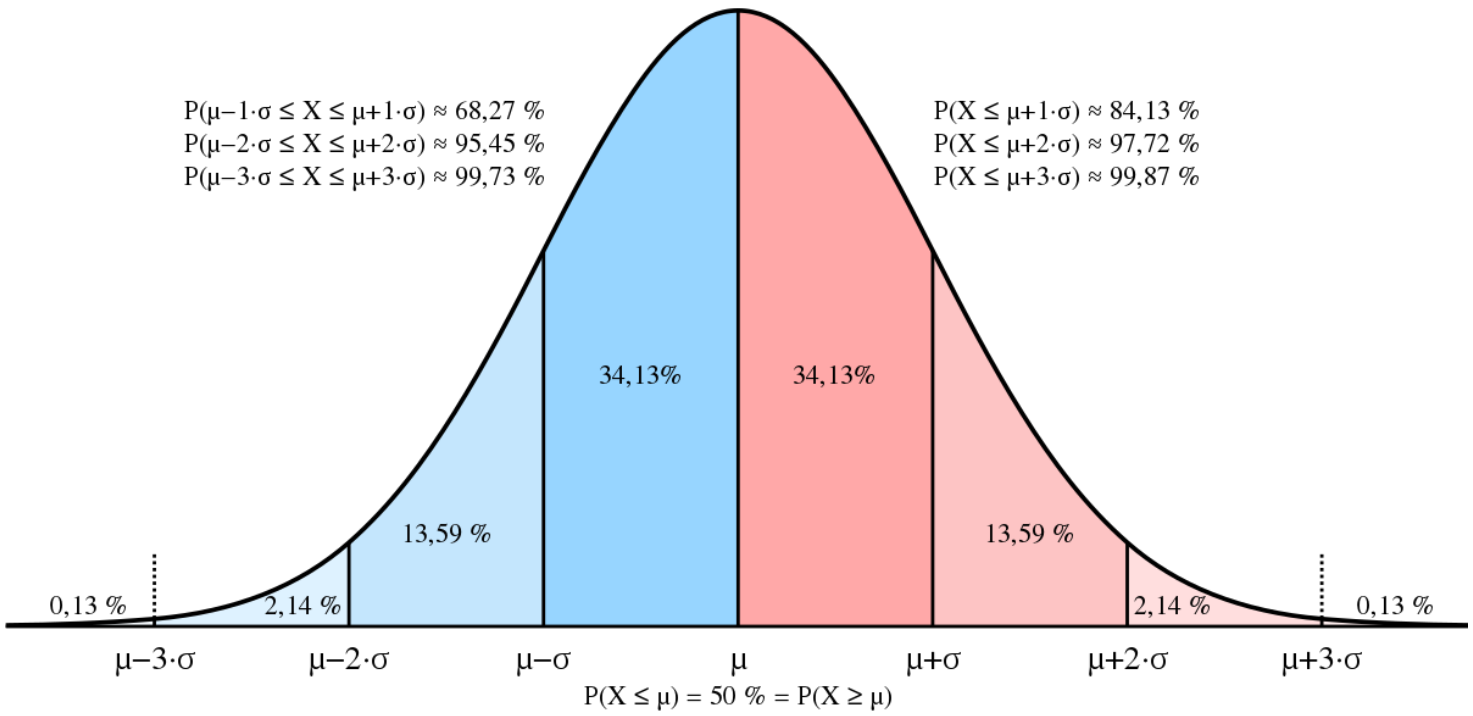
Assume that demand is independent form one week to the next.

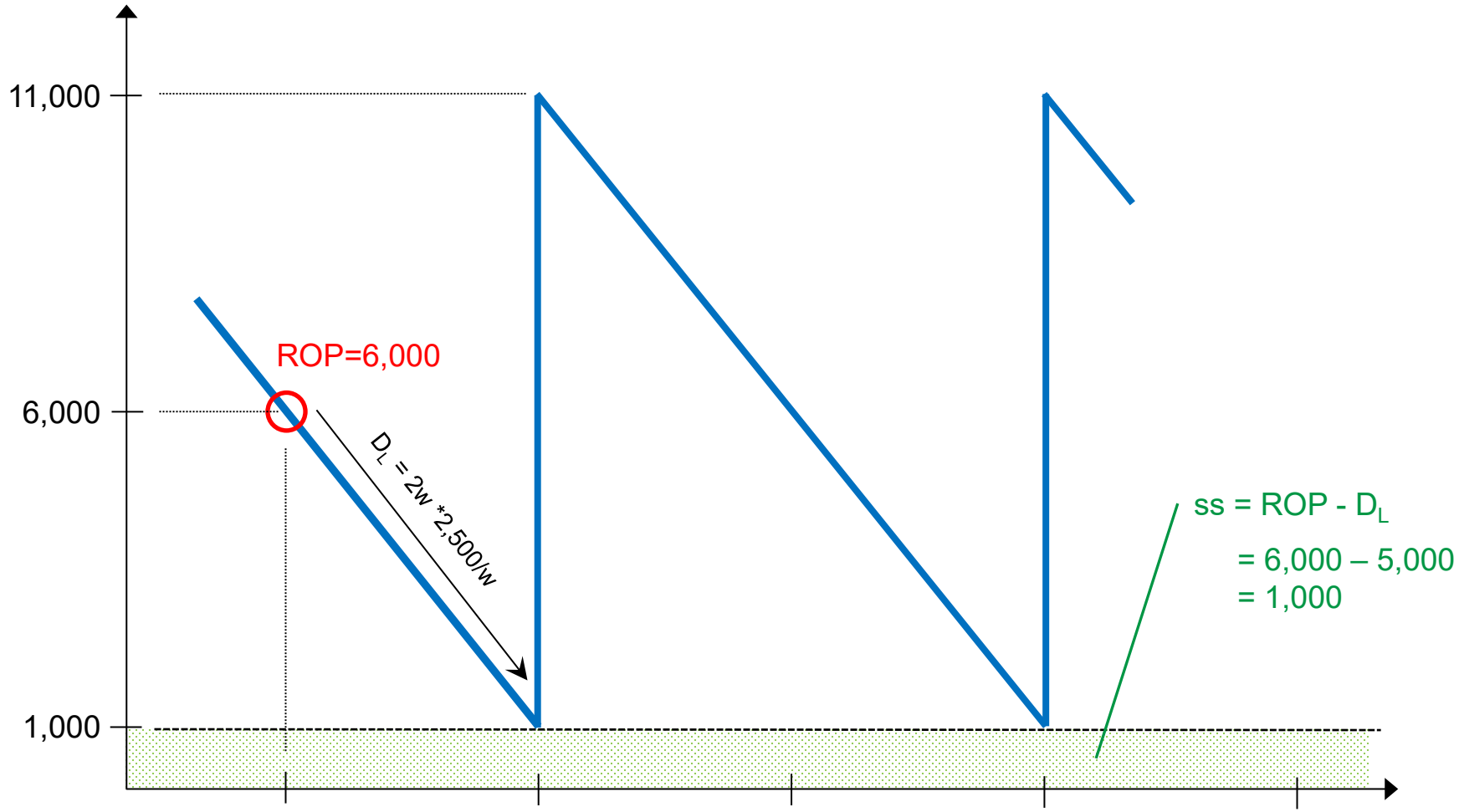
How likely will a stock-out situation occur during the reorder period?

Analysis

Analysis


P(μ−1·σ ≤ X ≤ μ+1·σ) ≈ 68,27 %
P(μ−2·σ ≤ X ≤ μ+2·σ) ≈ 95,45 %
P(μ−3·σ ≤ X ≤ μ+3·σ) ≈ 99,73 %
P(X ≤ μ+1·σ) ≈ 84,13 %
P(X ≤ μ+2·σ) ≈ 97,72 %
P(X ≤ μ+3·σ) ≈ 99,87 %
34,13%
34,13%
13,59 %
13,59 %
0,13 %
2,14 %
2,14 %
0,13 %
μ−3·σ
μ−2·σ
μ−σ
μ
μ+σ
μ+2·σ
μ+3·σ
P(X ≤ μ) = 50 % = P(X ≥ μ)

Analysis



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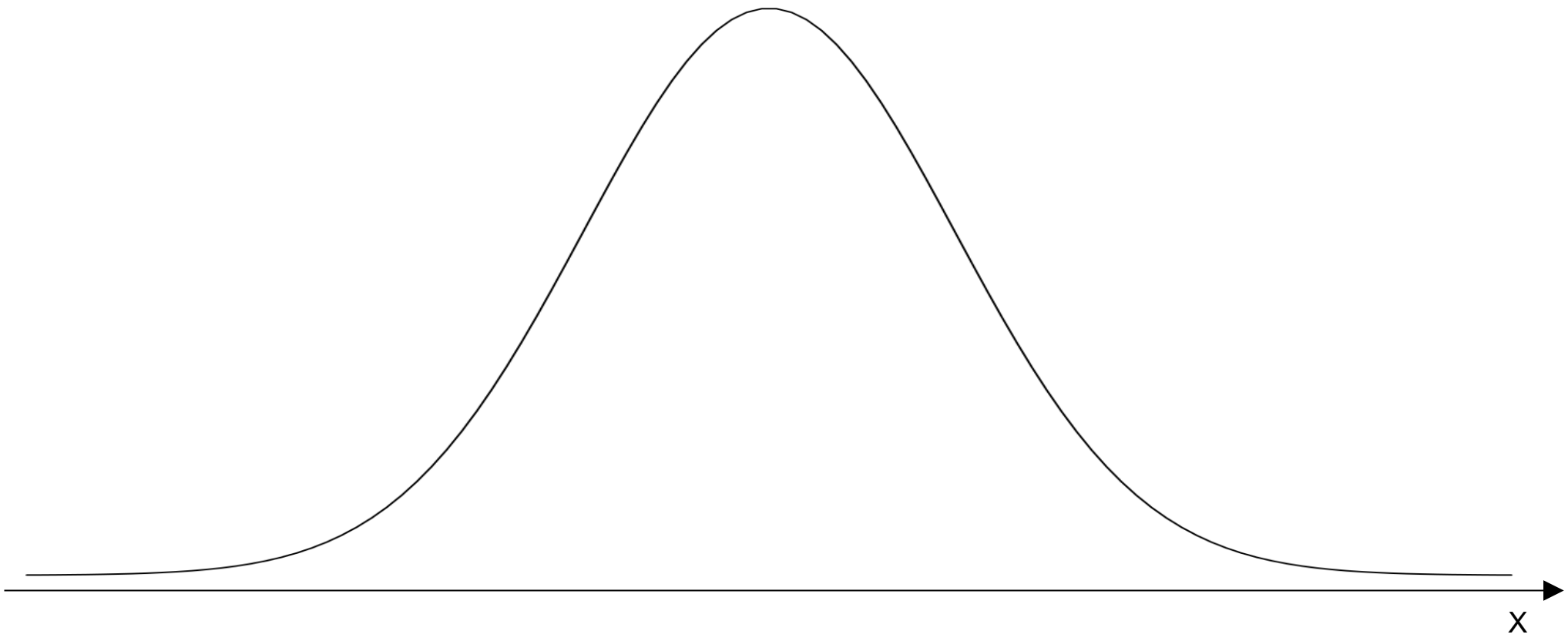
Analysis



Analysis

Analysis

z_c	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

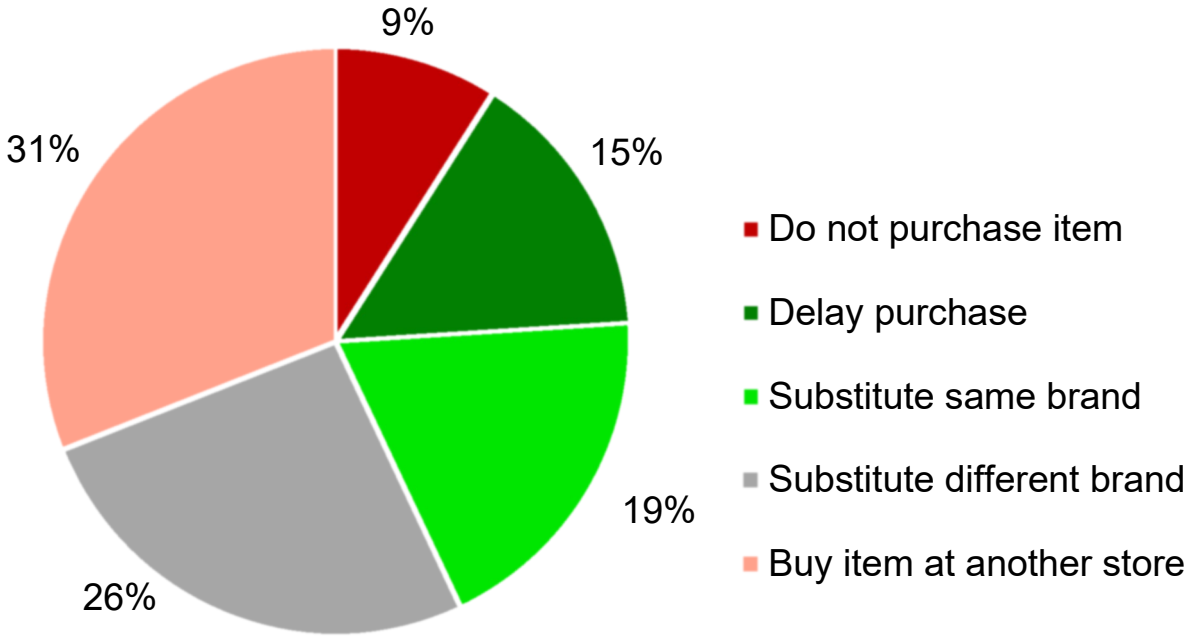
Should B&M Office Supplies be satisfied with their out-of-stock figure%?

Safety Inventory

What happens when a stock-out occurs?

Results of stock-outs

Shopper behavior when faced with a stock out



Source: Christopher (2005:51)

40% risk to the retailer

35% risk to the brand owner

Italy: 2014-supermarket study

35% risk to the retailer

73% risk to the brand owner

Safety Inventory

Safety Inventory

Expected shortage per replenishment cycle

ESC is calculated as follows:

$$ESC = -ss * \left(1 - F_S\left(\frac{ss}{\sigma_L}\right)\right) + \sigma_L * \left(f_S\left(\frac{ss}{\sigma_L}\right)\right)$$

$\Rightarrow$

F_S: Standard cumulative distribution
f_S: Standard normal density function

$$ESC = -1{,}000\text{ unit} * \left(1 - F_S\left(\frac{1{,}000\text{ unit}}{707\text{ unit}}\right)\right) + 707\text{unit} * f_S\left(\frac{1{,}000\text{ unit}}{707\text{ unit}}\right)$$

$\Rightarrow$

$$ESC = -1{,}000\text{ unit} * (1 - 0.9207) + 707\text{ unit} * (0{,}1476) \approx \underline{25\text{ unit}}$$

<u>In Excel this is calculated as follows:</u>

ESC = -1000 * [1 – NORMDIST(1000/707;0;1;1)] + 707 * NORMDIST(1000/707;0;1;0)]

(in the German version of Excel use NORM.VERT instead of NORMDIST)

Standard Normal Density Function

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3980	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3712	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.3467	0.3448	0.3429	0.3410	0.3391	0.3372	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2920
0.8	0.2897	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.2203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1456	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.1040	0.1023	0.1006	0.0989	0.0973	0.0957
1.7	0.0940	0.0925	0.0909	0.0893	0.0878	0.0863	0.0848	0.0833	0.0818	0.0804
1.8	0.0790	0.0775	0.0761	0.0748	0.0734	0.0721	0.0707	0.0694	0.0681	0.0669
1.9	0.0656	0.0644	0.0632	0.0620	0.0608	0.0596	0.0584	0.0573	0.0562	0.0551
2.0	0.0540	0.0529	0.0519	0.0508	0.0498	0.0488	0.0478	0.0468	0.0459	0.0449
2.1	0.0440	0.0431	0.0422	0.0413	0.0404	0.0396	0.0387	0.0379	0.0371	0.0363
2.2	0.0355	0.0347	0.0339	0.0332	0.0325	0.0317	0.0310	0.0303	0.0297	0.0290
2.3	0.0283	0.0277	0.0270	0.0264	0.0258	0.0252	0.0246	0.0241	0.0235	0.0229
2.4	0.0224	0.0219	0.0213	0.0208	0.0203	0.0198	0.0194	0.0189	0.0184	0.0180
2.5	0.0175	0.0171	0.0167	0.0163	0.0158	0.0154	0.0151	0.0147	0.0143	0.0139

The fill rate

$$\text{ESC} = -1{,}000 \text{ unit} * (1 - 0.9207) + 707 \text{ unit} * (0{,}1476) \approx \underline{25 \text{ unit}}$$

Accordingly, the **fill rate (f_r)** is:

$$f_r = 1 - \frac{\text{ESC}}{Q} \quad \Rightarrow \quad f_r = 1 - \frac{25 \text{ unit}}{10{,}000 \text{ unit}} = \underline{0.9975}$$

Different measures of product availability

Product Fill Rate
β service level

Refers to the portion of total demand that can be met from the existing inventory.

Characteristics:

- Volume driven measure
- Often used in practice

Cycle Service Level
α service level

Refers to the probability that demand for a product during one replenishment cycle can fully be met from the existing inventory.

Characteristics:

- Event driven measure

Looking at it from a sales/order fulfillment perspective

Fill rate β - service level	**Cycle Service Level** α - service level	**OTIF service level** On time in full
?	?	?

Safety Inventory

Calculating safety stock with variable demand & variable lead time

Assume the replenishment lead time for B&M to be normally distributed with a mean of 2 weeks and a standard dev. of half a week.

1. What is the impact of variable lead time on the EOQ?
2. The manager at B&M is still satisfied with a cycle service level of 92,07%. Calculate the safety stock and corresponding reorder point.

Analysis with variable demand & variable lead time (1)

To account for variable lead time, the formula is slightly adapted:

$$ROP = \bar{d}\,\bar{L} + safety\ stock$$

$$Safety\ stock = z\,\sigma_{dLT}$$

$$\sigma_{dLT} = \sqrt{\bar{L}\,\sigma_d^2 + \bar{d}^2 {\sigma_{LT}}^2}$$

where

$\bar{d}$	= Average demand
$\bar{L}$	= Average lead time
σ_d	= Standard deviation of demand
σ_{LT}	= Standard deviation of lead time
σ_{dLT}	= Standard deviation of demand during lead time

Analysis with variable demand & variable lead time (1)

$\bar{d}$ = 2,500 units per week

$\bar{L}$ = 2 weeks

σ_d = 500 units per week

σ_{LT} = 0.5 weeks

z = 1.41

Safety stock

$$= z\sqrt{\bar{L}\,\sigma_d^2 + \bar{d}^2\sigma_{LT}{}^2}$$

$$= 1.41\sqrt{2 * 500^2 + 2{,}500^2 * 0.5^2}$$

$$= 1.41 * 1{,}436 = \mathbf{2{,}025}$$

ROP

$$= \bar{d}\,\bar{L} + SS$$

$$= 2{,}500 * 2 + 2{,}025$$

$$= \mathbf{7{,}025}$$

Table of contents.

1 Warehousing and Transshipment
- 1.1 Order Picking
- 1.2 Basic forms of Transshipment

2 Inventory Management
- 2.1 Cycle Inventory
- 2.2 Safety Inventory
- 2.3 **Newsboy model**
- 2.4 Impact of coordination on inventory
- 2.5 The Bullwhip Effect

The Newsboy Model

How many newspapers should Jack order for his daily sales?

- Jack buys each newspaper for 30 cents
- He sells the newspaper for 90 cents
- Unsold copies can be returned to the publisher for 15 cents each (salvage value)

Deciding on the Optimal Quantity

Cost of Excess Inventory
(Overstocking → Overage cost)

Cost of Lost Sales
(Understocking → Underage cost)

Objective: Minimize the expected costs
Decision variable: Optimal Purchasing Quantity
(or starting inventory level)

How many newspapers should the newsboy order?

If the newsboy orders more than actual demand (unsold units) he will have cost of

$$C_0 * (Q - D)$$

C_0: overage cost (i.e. loss for each unsold unit)

If the newsboy order less than actual demand (stock-out) he will have opportunity costs of

$$C_u * (D - Q)$$

C_u: underage cost (i.e. forgone profit for each unit out of stock)

The key to solving this dilemma is the critical ratio

$$F(Q*) = \frac{C_u}{C_u + C_0}$$

- The quantity should be set so that the cycle service level is equal to the critical ratio.

$$F(Q*) = \frac{C_u}{C_u + C_0} = \frac{60cent}{60cent + 15cent} = 0{,}8 \Rightarrow 80\%$$

Week 1	15	19	20	9	12	7	15
Week 2	1	15	17	6	21	5	6
Week 3	7	8	9	9	12	17	15
Week 4	12	11	19	17	13	20	21
Week 5	18	17	15	11	13	12	14

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$$F(Q*) = \frac{C_u}{C_u + C_0} = \frac{60cent}{60cent + 15cent} = 0{,}8 \Rightarrow 80\%$$

Week 1	15	19	20	9	12	7	15
Week 2	1	15	17	6	21	5	6
Week 3	7	8	9	9	12	17	15
Week 4	12	11	19	17	13	20	21
Week 5	18	17	15	11	13	12	14

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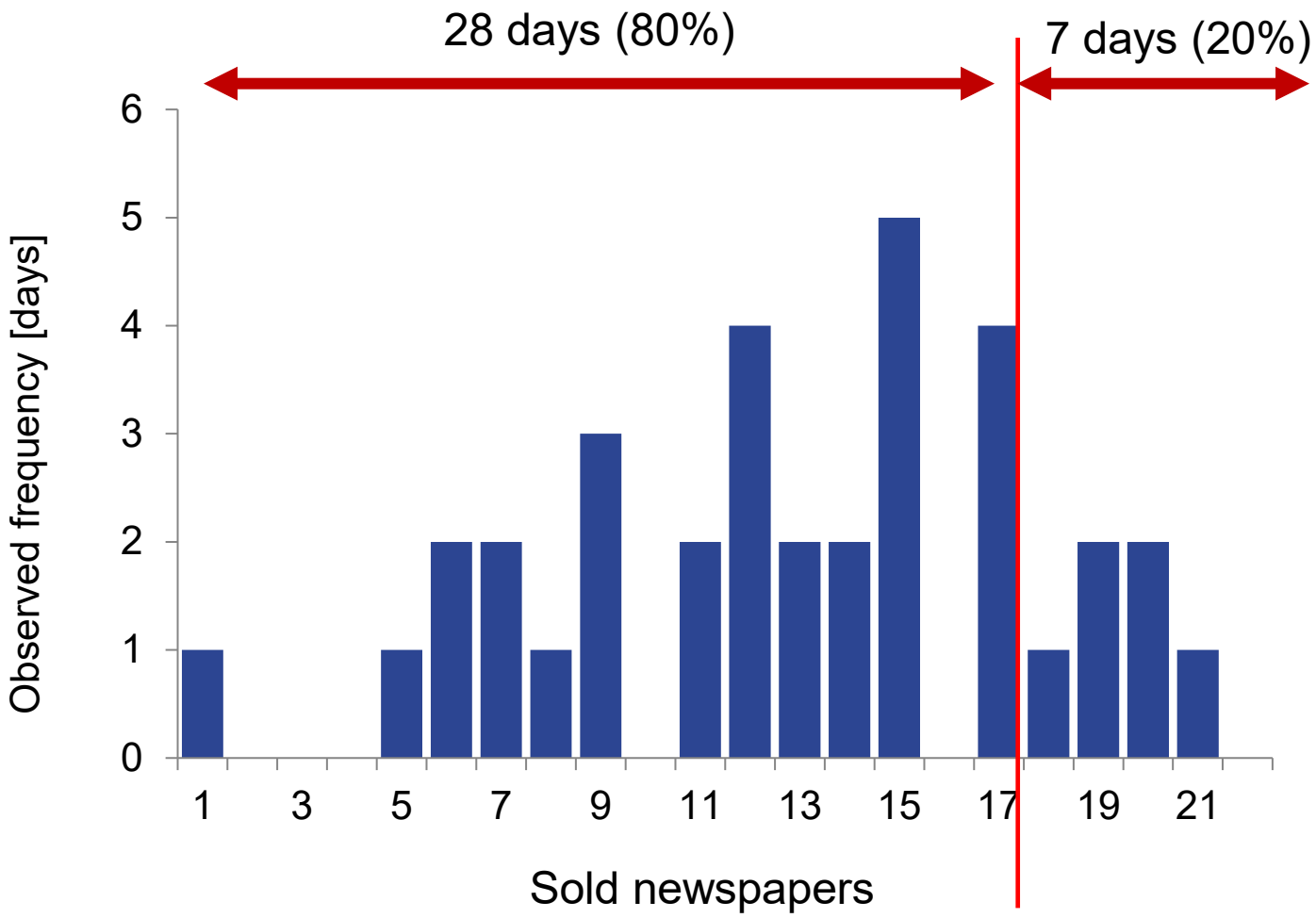

28 days (80%)
7 days (20%)
Observed frequency [days]
0
1
2
3
4
5
6
1
3
5
7
9
11
13
15
17
19
21
Sold newspapers

Q* assuming demand to be normally distributed (1)

Determine mean μ & standard deviation σ from sample

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\mu = \frac{1}{35} * 458 = 13.09$$

15	19	20	9	12	7	15
1	15	17	6	21	5	6
7	8	9	9	12	17	15
12	11	19	17	13	20	21
18	17	15	11	13	12	14

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2}$$

$$\sigma = \sqrt{\frac{1}{34}\sum_{i=1}^{n}(x_i - 13.09)^2} = 4.98$$

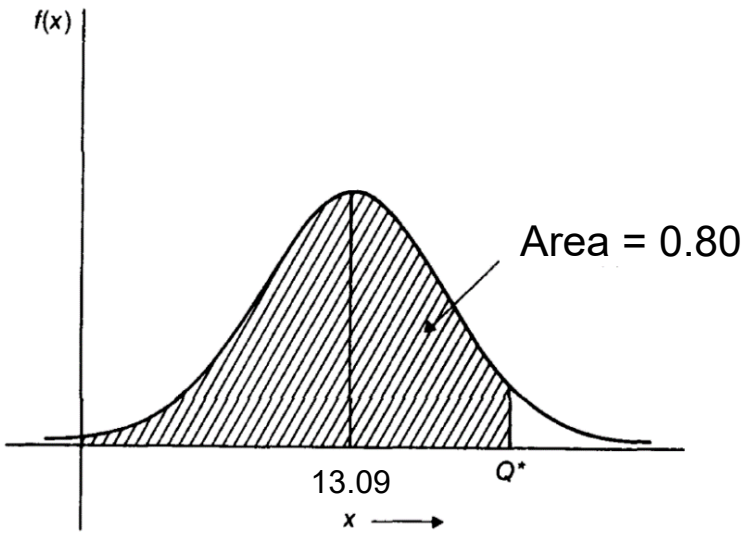
<u>In the following:</u>
assume normally distributed demand with mean μ and variance σ^2.
Notation: $D \sim N(\mu, \sigma^2)$

Jack‘s Q* when demand is normally distributed (2)

Again, determine Optimal Service Level

$$C_0 = C - S \Rightarrow 30cent - 15cent = 15cent$$
$$C_u = P - C \Rightarrow 90cent - 30cent = 60cent$$

$$F(Q*) = \frac{C_u}{C_u + C_0} \Rightarrow \frac{60cent}{60cent + 15cent} = 0{,}8 \Rightarrow 80\%$$



Use standard normal transformation to determine Q*

To determine optimal quantity you need to find the corresponding value that fulfill given share of demand (cumulative distribution function)

$$Q * = \mu + \sigma * z$$

Values for z are readily available in standard normal tables

$$Q * = \mu + \sigma * z = 13.09 + 4.98 * 0.85 = 17.32$$

z-table on next slide
(the z-value for 80% is 0.85)

Z-Table

$z=0.85$

z_c	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

In Class Exercise

You are helping a Christmas tree retailer determine how many trees to order for the upcoming season.

You can assume from past experience that the average demand for Christmas trees in this area is 500 trees for the season, normally distributed with a standard deviation of the demand is 100 trees.

If this person was able to buy each tree at a cost of €5 and sell it at €50, then how many trees do you recommend that he orders?

How does the Fisher-model suggest that we act when being in Jack's situation?

In Class Exercise

What are the fundamental differences in determining the optimal inventory level for mineral water compared to newspapers?