IIT2019173 IVP Tutorial 12/09/2021

1 - Let $y(t) = x(t) * h(t)$

* Deriving Fourier transform of $y(t)$:

$$Y(j\omega) = \mathcal{F}\{y(t)\} = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} (x * h)(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} x(\tau) h(t-\tau) e^{-j\omega t}\, dt\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau}\, d\tau \int h(t-\tau) e^{-j\omega(t-\tau)}\, dt\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau} H(j\omega)\, d\tau$$

$$= \left(\int_{-\infty}^{\infty} x(\tau) e^{-j\omega\tau}\, d\tau\right) H(j\omega)$$

$$= X(j\omega)\, H(j\omega)$$

Proved

2 - Properties of linear function: $f(x+y) = f(x) + f(y)$

$f(cx) = c f(x)$

To Prove: $FT[a\, f_1(t) + b\, f_2(t)] = a F_1(j\omega) + b F_2(j\omega)$

$$\Rightarrow \int_{-\infty}^{\infty} (a f_1(t) + b f_2(t)) e^{-j\omega t}\, dt$$

$$= a\int_{-\infty}^{\infty} f_1(t) e^{-j\omega t}\, dt + b\int_{-\infty}^{\infty} f_2(t) e^{-j\omega t}\, dt \quad \text{(Linearity of Integrals)}$$

$$= a F_1(j\omega) + b F_2(j\omega)$$

Proved

3- Given $H(u,v)$ is real and symmetric

$$H(u,v) = H^*(u,v) = H^*(-u,-v) = H(-u,-v)$$

The filter in spatial domain is

$$h(x,y) = \mathcal{J}^{-1}[H(u,v)] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} H(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Then

$$h^*(x,y) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} H^*(u,v)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} H^*(-u,-v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} H(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$= h(x,y)$ Similarly for $h(-x,-y)$

4- $$g(x,y) = \frac{1}{4}\left[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)\right]$$

$$G(u,v) = \frac{1}{4}\left[e^{j2\pi v/N} + e^{j2\pi u/M} + e^{-j2\pi u/M} + e^{-j2\pi v/N}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{2\pi u}{M}\right) + \cos\left(\frac{2\pi v}{N}\right)\right] F(u,v)$$

Filter transfer function in freq. domain

5 - $X(k) = \sum_{n=0}^{n=3} x(n) e^{-j\frac{2\pi nk}{4}}$

$$= x(0) + x(1) e^{-\frac{j\pi k}{2}} + x(2) e^{-j\pi k} + x(3) e^{-j\frac{\pi 3k}{2}}$$

$$X(k) = e^{-\frac{j\pi k}{2}} + 2e^{-j\pi k} + 3e^{-j\frac{3\pi k}{2}}$$

Now, $X(0) = 1+2+3 = 6 \angle 0$

$$X(1) = e^{-j\pi/2} + 2(e^{-j\pi}) + 3(e^{-j\frac{3\pi}{2}})$$

$$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} + 2(\cos\pi - j\sin\pi) + 3(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2})$$

$$= 0 - j + 2(-1-0) + 3(0+1) = -2+2j$$

$\angle = \tan^{-1}\frac{2}{-2} \Rightarrow \angle = 135^\circ$

Mag $= 2\sqrt{2}$

$$X(2) = e^{-j\pi} + 2(e^{-j2\pi}) + 3e^{-j3\pi}$$

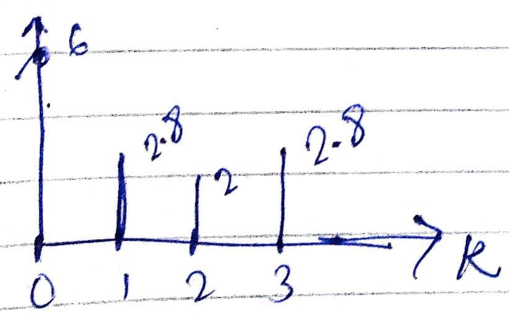
$$= \cos\pi - j\sin\pi + 2(\cos 2\pi - j\sin 2\pi) + 3(\cos 3\pi - j\sin 3\pi)$$

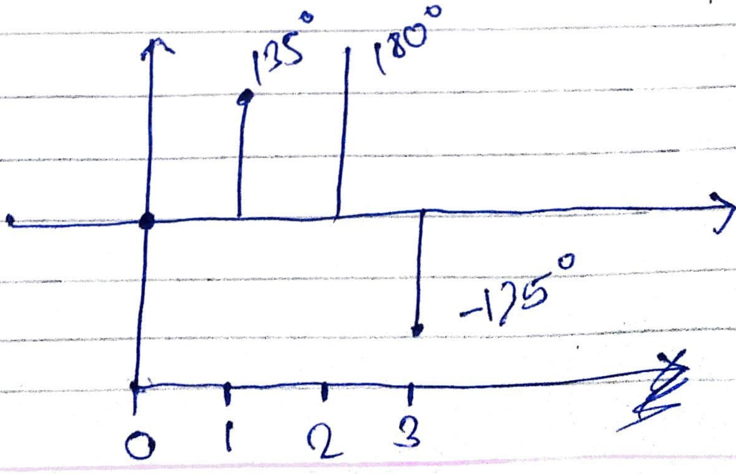
$= -2$ ~~Mg~~ Mag $= 2$ $\angle = 180^\circ$

$$X(3) = e^{-j\frac{3\pi}{2}} + 2(e^{-j3\pi}) + 3e^{-j\frac{3\pi 3}{2}} = -2-2j$$

~~Mag~~ Mag $= 2\sqrt{2}$ $\angle = -135^\circ$

$$X(k) = \{6, -2+2j, -2, -2-2j\}$$





G Let image $= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ Kernel $= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

2D DFT = Kernel × image × Kernel^T

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$