

Data Mining:

Concepts and Techniques

(3rd ed.)

— Chapter 3 —

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Data Reduction Strategies

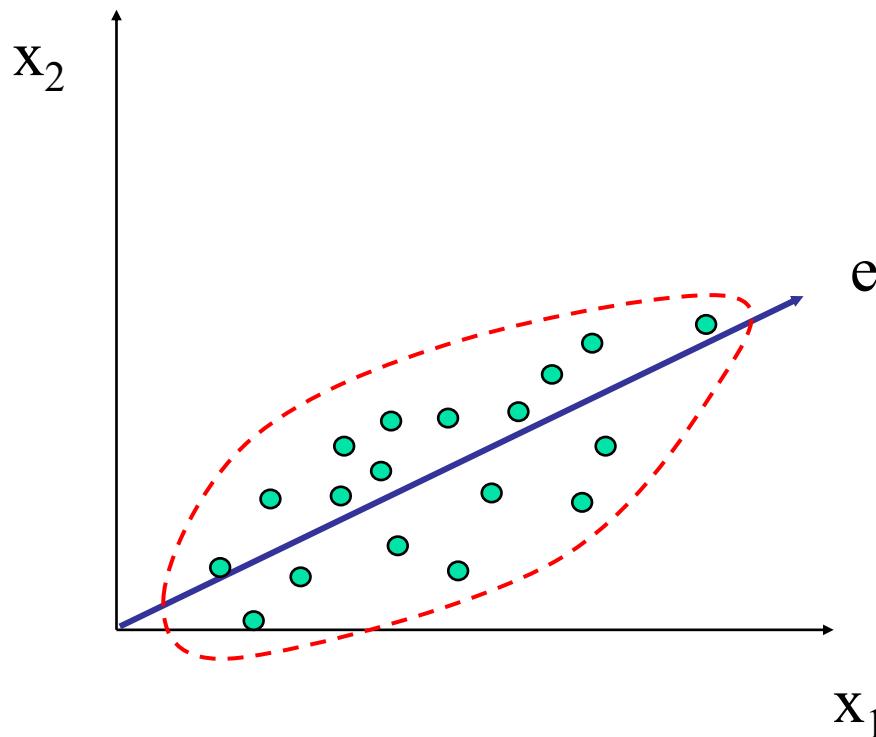
- **Data reduction:** Obtain a reduced representation of the data set that is much smaller in volume but yet produces the same (or almost the same) analytical results
- Why data reduction? — A database/data warehouse may store terabytes of data. Complex data analysis may take a very long time to run on the complete data set.
- Data reduction strategies
 - Dimensionality reduction, e.g., remove unimportant attributes
 - Wavelet transforms
 - Principal Components Analysis (PCA)
 - Feature subset selection, feature creation
 - Numerosity reduction (some simply call it: Data Reduction)
 - Regression and Log-Linear Models
 - Histograms, clustering, sampling
 - Data cube aggregation
 - Data compression

Data Reduction 1: Dimensionality Reduction

- **Curse of dimensionality**
 - When dimensionality increases, data becomes increasingly sparse
 - Density and distance between points, which is critical to clustering, outlier analysis, becomes less meaningful
 - The possible combinations of subspaces will grow exponentially
- **Dimensionality reduction**
 - Avoid the curse of dimensionality
 - Help eliminate irrelevant features and reduce noise
 - Reduce time and space required in data mining
 - Allow easier visualization
- **Dimensionality reduction techniques**
 - Wavelet transforms
 - Principal Component Analysis
 - Supervised and nonlinear techniques (e.g., feature selection)

Principal Component Analysis (PCA)

- Find a projection that captures the largest amount of variation in data
- The original data are projected onto a much smaller space, resulting in dimensionality reduction. We find the eigenvectors of the covariance matrix, and these eigenvectors define the new space



Principal Component Analysis (Steps)

- Given N data vectors from n -dimensions, find $k \leq n$ orthogonal vectors (*principal components*) that can be best used to represent data
 - Normalize input data: Each attribute falls within the same range
 - Compute k orthonormal (unit) vectors, i.e., *principal components*
 - Each input data (vector) is a linear combination of the k principal component vectors
 - The principal components are sorted in order of decreasing “significance” or strength
 - Since the components are sorted, the size of the data can be reduced by eliminating the *weak components*, i.e., those with low variance (i.e., using the strongest principal components, it is possible to reconstruct a good approximation of the original data)
- Works for numeric data only

Attribute Subset Selection

- Another way to reduce dimensionality of data
- Redundant attributes
 - Duplicate much or all of the information contained in one or more other attributes
 - E.g., purchase price of a product and the amount of sales tax paid
- Irrelevant attributes
 - Contain no information that is useful for the data mining task at hand
 - E.g., students' ID is often irrelevant to the task of predicting students' GPA

Heuristic Search in Attribute Selection

- There are 2^d possible attribute combinations of d attributes
- Typical heuristic attribute selection methods:
 - Best single attribute under the attribute independence assumption: choose by significance tests
 - Best step-wise feature selection:
 - The best single-attribute is picked first
 - Then next best attribute condition to the first, ...
 - Step-wise attribute elimination:
 - Repeatedly eliminate the worst attribute
 - Best combined attribute selection and elimination
 - Optimal branch and bound:
 - Use attribute elimination and backtracking

Attribute Creation (Feature Generation)

- Create new attributes (features) that can capture the important information in a data set more effectively than the original ones
- Three general methodologies
 - Attribute extraction
 - Domain-specific
 - Mapping data to new space (see: data reduction)
 - E.g., Fourier transformation, wavelet transformation, manifold approaches (not covered)
 - Attribute construction
 - Combining features (see: discriminative frequent patterns in Chapter 7)
 - Data discretization

Data Reduction 2: Numerosity Reduction

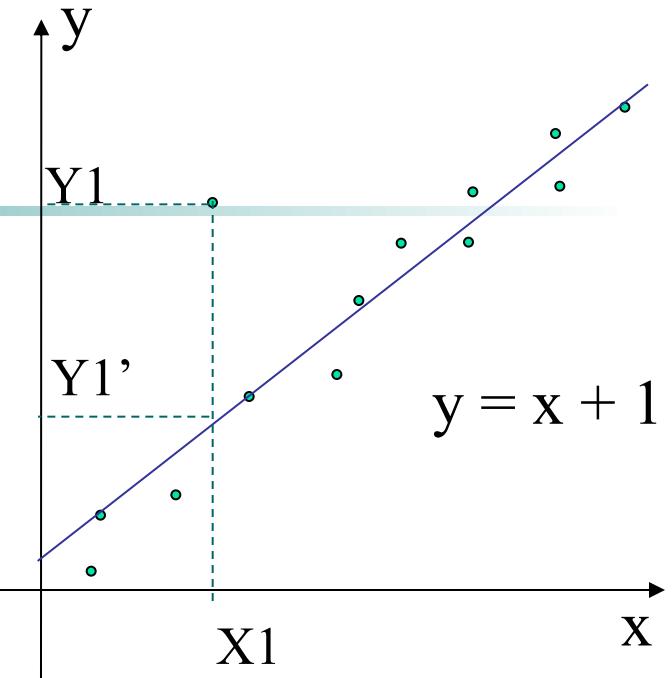
- Reduce data volume by choosing alternative, *smaller forms* of data representation
- **Parametric methods** (e.g., regression)
 - Assume the data fits some model, estimate model parameters, store only the parameters, and discard the data (except possible outliers)
 - Ex.: Log-linear models—obtain value at a point in m -D space as the product on appropriate marginal subspaces
- **Non-parametric** methods
 - Do not assume models
 - Major families: histograms, clustering, sampling, ...

Parametric Data Reduction: Regression and Log-Linear Models

- **Linear regression**
 - Data modeled to fit a straight line
 - Often uses the least-square method to fit the line
- **Multiple regression**
 - Allows a response variable Y to be modeled as a linear function of multidimensional feature vector
- **Log-linear model**
 - Approximates discrete multidimensional probability distributions

Regression Analysis

- Regression analysis: A collective name for techniques for the modeling and analysis of numerical data consisting of values of a **dependent variable** (also called **response variable** or *measurement*) and of one or more *independent variables* (aka. **explanatory variables** or **predictors**)



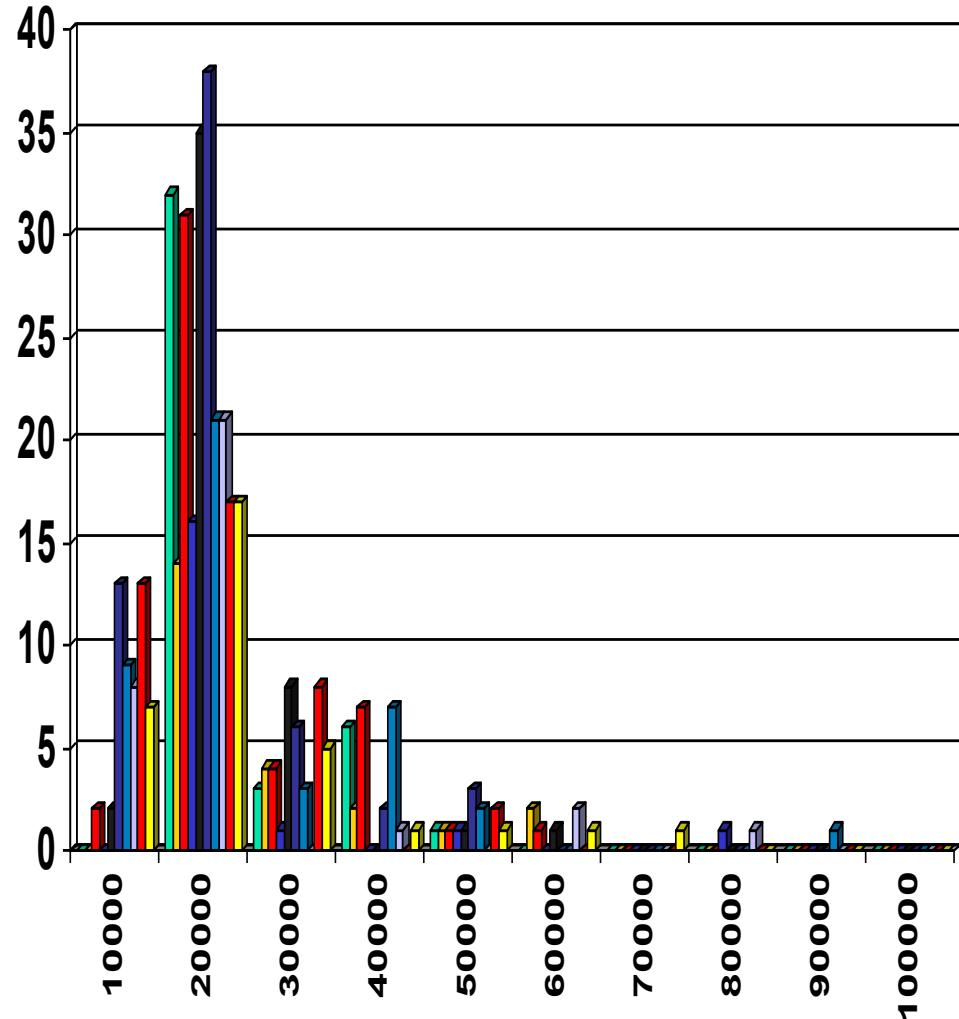
- The parameters are estimated so as to give a "**best fit**" of the data
- Most commonly the best fit is evaluated by using the **least squares method**, but other criteria have also been used
- Used for prediction (including forecasting of time-series data), inference, hypothesis testing, and modeling of causal relationships

Régress Analysis and Log-Linear Models

- Linear regression: $Y = wX + b$
 - Two regression coefficients, w and b , specify the line and are to be estimated by using the data at hand
 - Using the least squares criterion to the known values of $Y_1, Y_2, \dots, X_1, X_2, \dots$
- Multiple regression: $Y = b_0 + b_1 X_1 + b_2 X_2$
 - Many nonlinear functions can be transformed into the above
- Log-linear models:
 - Approximate discrete multidimensional probability distributions
 - Estimate the probability of each point (tuple) in a multi-dimensional space for a set of discretized attributes, based on a smaller subset of dimensional combinations
 - Useful for dimensionality reduction and data smoothing

Histogram Analysis

- Divide data into buckets and store average (sum) for each bucket
- Partitioning rules:
 - Equal-width: equal bucket range
 - Equal-frequency (or equal-depth)



Clustering

- Partition data set into clusters based on similarity, and store cluster representation (e.g., centroid and diameter) only
- Can be very effective if data is clustered but not if data is “smeared”
- Can have hierarchical clustering and be stored in multi-dimensional index tree structures
- There are many choices of clustering definitions and clustering algorithms
- Cluster analysis will be studied in depth in Chapter 10

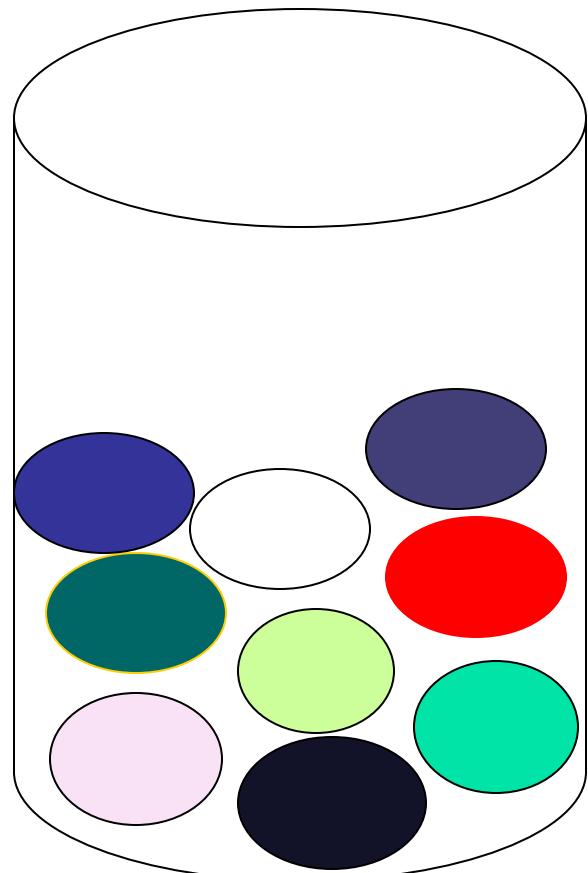
Sampling

- Sampling: obtaining a small sample s to represent the whole data set N
- Allow a mining algorithm to run in complexity that is potentially sub-linear to the size of the data
- Key principle: Choose a **representative** subset of the data
 - Simple random sampling may have very poor performance in the presence of skew
 - Develop adaptive sampling methods, e.g., stratified sampling:
- Note: Sampling may not reduce database I/Os (page at a time)

Types of Sampling

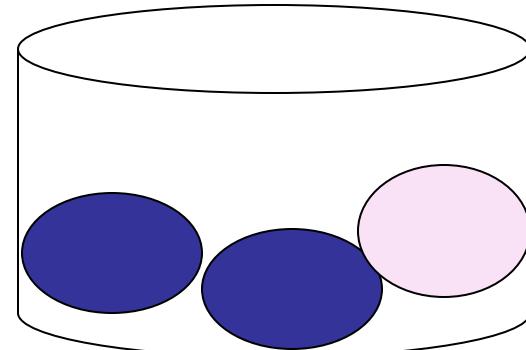
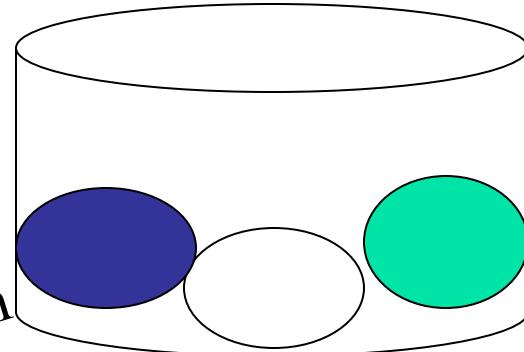
- **Simple random sampling**
 - There is an equal probability of selecting any particular item
- **Sampling without replacement**
 - Once an object is selected, it is removed from the population
- **Sampling with replacement**
 - A selected object is not removed from the population
- **Stratified sampling:**
 - Partition the data set, and draw samples from each partition (proportionally, i.e., approximately the same percentage of the data)
 - Used in conjunction with skewed data

Sampling: With or without Replacement



SRSWOR
(simple random
sample without
replacement)

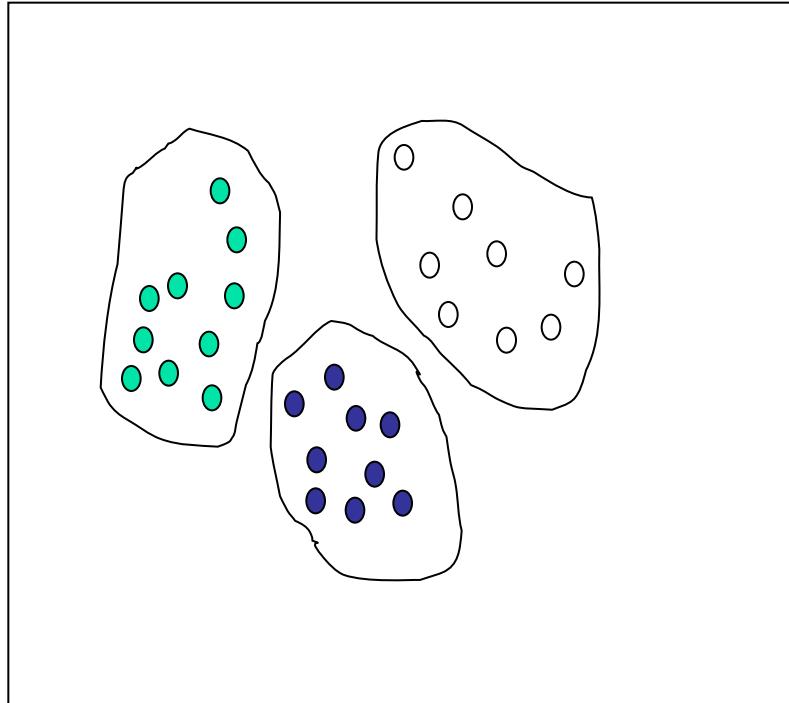
SRSWR



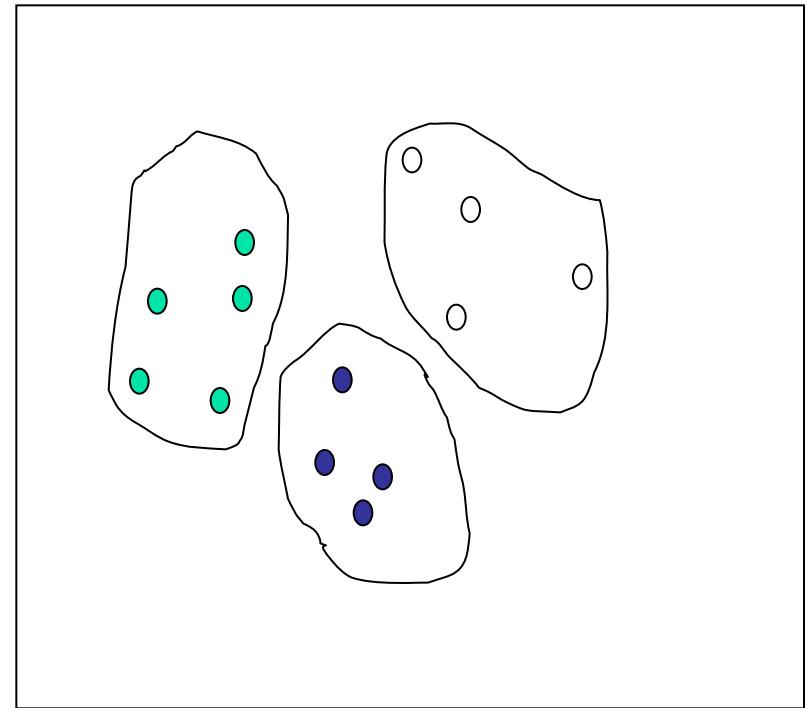
Raw Data

Sampling: Cluster or Stratified Sampling

Raw Data



Cluster/Stratified Sample



Data Cube Aggregation

- The lowest level of a data cube (base cuboid)
 - The aggregated data for an **individual entity of interest**
 - E.g., a customer in a phone calling data warehouse
- Multiple levels of aggregation in data cubes
 - Further reduce the size of data to deal with
- Reference appropriate levels
 - Use the smallest representation which is enough to solve the task
- Queries regarding aggregated information should be answered using data cube, when possible

Data Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values s.t. each old value can be identified with one of the new values
- Methods
 - Smoothing: Remove noise from data
 - Attribute/feature construction
 - New attributes constructed from the given ones
 - Aggregation: Summarization, data cube construction
 - Normalization: Scaled to fall within a smaller, specified range
 - min-max normalization
 - z-score normalization
 - normalization by decimal scaling
 - Discretization: Concept hierarchy climbing

Normalization

- **Min-max normalization:** to $[new_min_A, new_max_A]$

$$v' = \frac{v - min_A}{max_A - min_A} (new_max_A - new_min_A) + new_min_A$$

- Ex. Let income range \$12,000 to \$98,000 normalized to [0.0, 1.0]. Then \$73,000 is mapped to $\frac{73,600 - 12,000}{98,000 - 12,000} (1.0 - 0) + 0 = 0.716$

- **Z-score normalization** (μ : mean, σ : standard deviation):

$$v' = \frac{v - \mu_A}{\sigma_A}$$

- Ex. Let $\mu = 54,000$, $\sigma = 16,000$. Then $\frac{73,600 - 54,000}{16,000} = 1.225$

- **Normalization by decimal scaling**

$$v' = \frac{v}{10^j} \quad \text{Where } j \text{ is the smallest integer such that } \text{Max}(|v'|) < 1$$

Examples

Example: Student Test Scores

Normalizing test scores from different subjects with varying scales to a [0,1] range:

Subject	Original Score	Min	Max	Normalized Score
Math	85	45	100	0.73
Physics	42	20	50	0.73
History	78	30	90	0.80
English	65	40	80	0.63

Notice how the normalized scores allow for fair comparison across subjects with different scales.

Examples

Example: Student Heights (cm)

Normalizing student heights using z-score normalization:

Given: Mean (μ) = 170 cm, Standard Deviation (σ) = 10 cm

Student	Height (cm)	Z-Score	Interpretation
A	155	-1.5	1.5 SD below mean
B	168	-0.2	0.2 SD below mean
C	172	0.2	0.2 SD above mean
D	190	2.0	2.0 SD above mean

Note: In a normal distribution, approximately 68% of values fall within ± 1 SD, 95% within ± 2 SD, and 99.7% within ± 3 SD from the mean.

Decimal Scaling Normalization

Example: Revenue Data (in millions)

Given data: -10, 201, 301, -401, 501, 601, 701

Step 1: Maximum absolute value = 701

Step 2: Smallest j where $\max(|v'|) < 1$ is $j = 3$ ($10^3 = 1000$)

Step 3: Divide all values by $10^3 = 1000$

Original Value	Normalized Value ($v/10^3$)
-10	-0.01
201	0.201
301	0.301
-401	-0.401
501	0.501
601	0.601
701	0.701

Discretization

- Three types of attributes
 - Nominal—values from an unordered set, e.g., color, profession
 - Ordinal—values from an ordered set, e.g., military or academic rank
 - Numeric—real numbers, e.g., integer or real numbers
- Discretization: Divide the range of a continuous attribute into intervals
 - Interval labels can then be used to replace actual data values
 - Reduce data size by discretization
 - Supervised vs. unsupervised
 - Split (top-down) vs. merge (bottom-up)
 - Discretization can be performed recursively on an attribute
 - Prepare for further analysis, e.g., classification

Data Discretization Methods

- Typical methods: All the methods can be applied recursively
 - Binning
 - Top-down split, unsupervised
 - Histogram analysis
 - Top-down split, unsupervised
 - Clustering analysis (unsupervised, top-down split or bottom-up merge)
 - Decision-tree analysis (supervised, top-down split)
 - Correlation (e.g., χ^2) analysis (unsupervised, bottom-up merge)

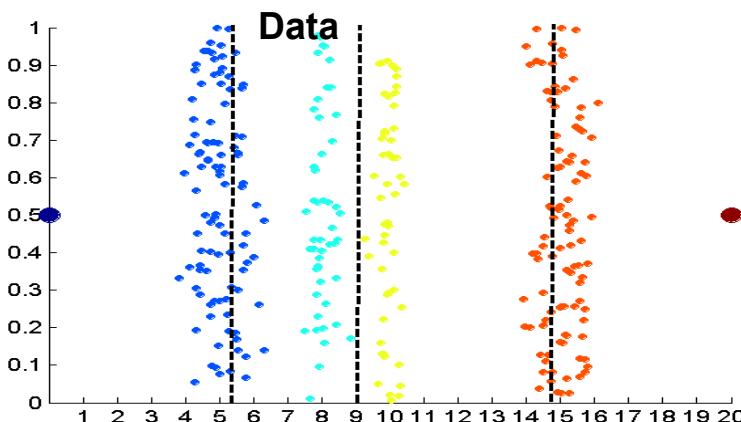
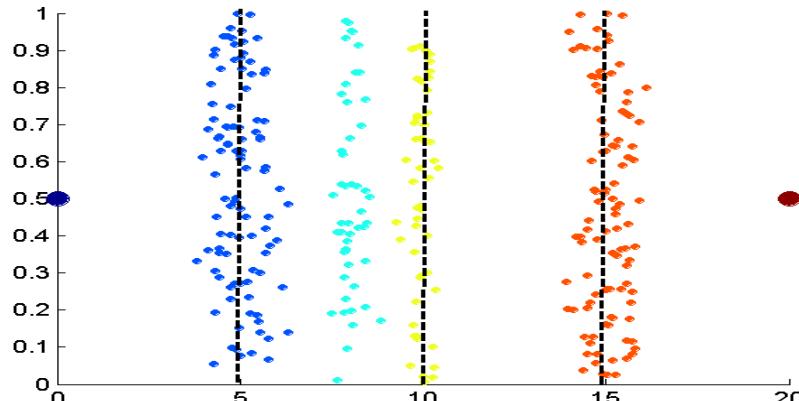
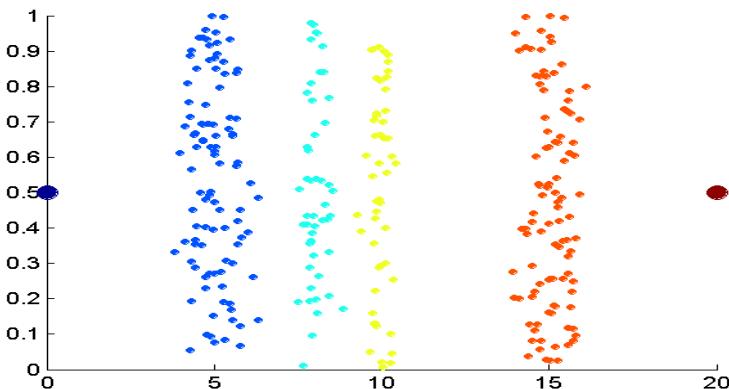
Simple Discretization: Binning

- Equal-width (distance) partitioning
 - Divides the range into N intervals of equal size: uniform grid
 - if A and B are the lowest and highest values of the attribute, the width of intervals will be: $W = (B - A)/N$.
 - The most straightforward, but outliers may dominate presentation
 - Skewed data is not handled well
- Equal-depth (frequency) partitioning
 - Divides the range into N intervals, each containing approximately same number of samples
 - Good data scaling
 - Managing categorical attributes can be tricky

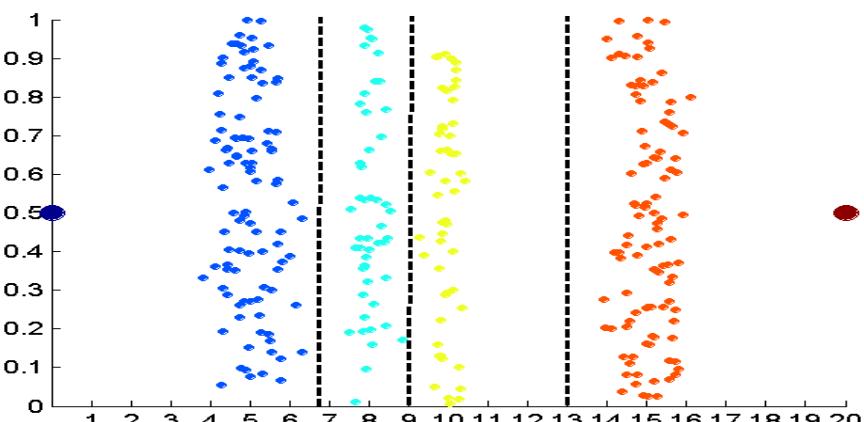
Binning Methods for Data Smoothing

- Sorted data for price (in dollars): 4, 8, 9, 15, 21, 21, 24, 25, 26, 28, 29, 34
- * Partition into equal-frequency (**equi-depth**) bins:
 - Bin 1: 4, 8, 9, 15
 - Bin 2: 21, 21, 24, 25
 - Bin 3: 26, 28, 29, 34
- * Smoothing by **bin means**:
 - Bin 1: 9, 9, 9, 9
 - Bin 2: 23, 23, 23, 23
 - Bin 3: 29, 29, 29, 29
- * Smoothing by **bin boundaries**:
 - Bin 1: 4, 4, 4, 15
 - Bin 2: 21, 21, 25, 25
 - Bin 3: 26, 26, 26, 34

Discretization Without Using Class Labels (Binning vs. Clustering)



Equal frequency (binning)



K-means clustering leads to better results

Discretization by Classification & Correlation Analysis

- Classification (e.g., decision tree analysis)
 - Supervised: Given class labels, e.g., cancerous vs. benign
 - Using *entropy* to determine split point (discretization point)
 - Top-down, recursive split
- Correlation analysis (e.g., Chi-merge: χ^2 -based discretization)
 - Supervised: use class information
 - Bottom-up merge: find the best neighboring intervals (those having similar distributions of classes, i.e., low χ^2 values) to merge
 - Merge performed recursively, until a predefined stopping condition

Concept Hierarchy Generation

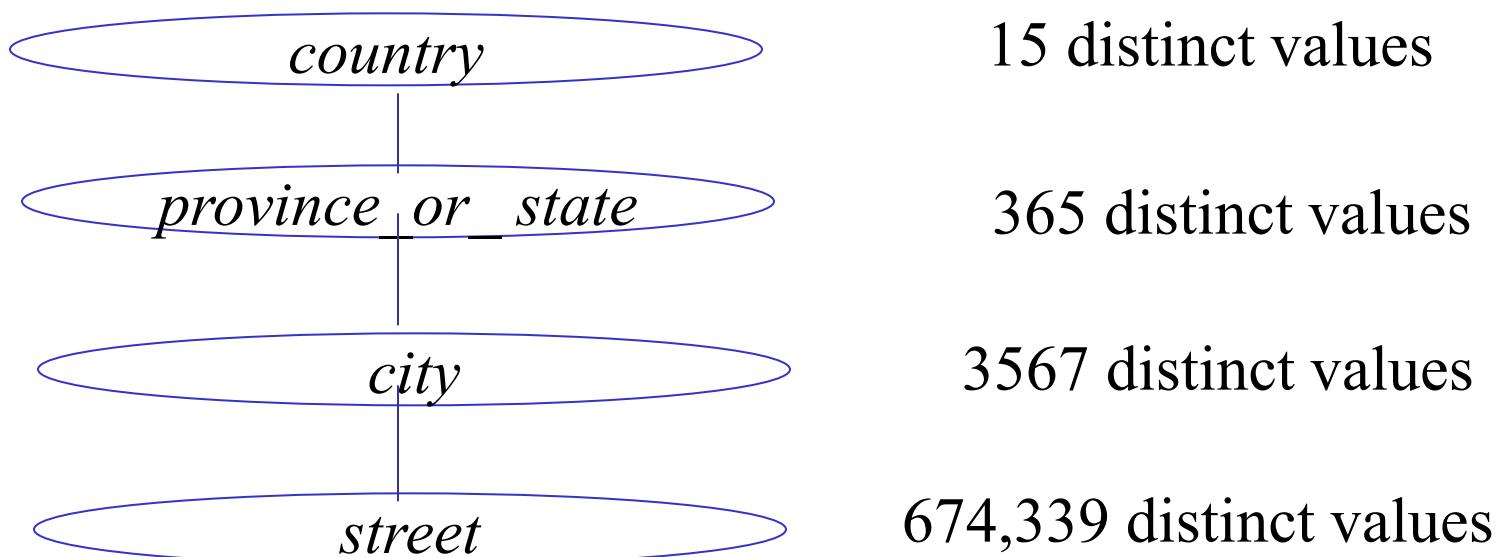
- **Concept hierarchy** organizes concepts (i.e., attribute values) hierarchically and is usually associated with each dimension in a data warehouse
- Concept hierarchies facilitate drilling and rolling in data warehouses to view data in multiple granularity
- Concept hierarchy formation: Recursively reduce the data by collecting and replacing low level concepts (such as numeric values for *age*) by higher level concepts (such as *youth*, *adult*, or *senior*)
- Concept hierarchies can be explicitly specified by domain experts and/or data warehouse designers
- Concept hierarchy can be automatically formed for both numeric and nominal data. For numeric data, use discretization methods shown.

Concept Hierarchy Generation for Nominal Data

- Specification of a partial/total ordering of attributes explicitly at the schema level by users or experts
 - $\textit{street} < \textit{city} < \textit{state} < \textit{country}$
- Specification of a hierarchy for a set of values by explicit data grouping
 - $\{\text{Urbana, Champaign, Chicago}\} < \text{Illinois}$
- Specification of only a partial set of attributes
 - E.g., only $\textit{street} < \textit{city}$, not others
- Automatic generation of hierarchies (or attribute levels) by the analysis of the number of distinct values
 - E.g., for a set of attributes: $\{\textit{street}, \textit{city}, \textit{state}, \textit{country}\}$

Automatic Concept Hierarchy Generation

- Some hierarchies can be automatically generated based on the analysis of the number of distinct values per attribute in the data set
 - The attribute with the most distinct values is placed at the lowest level of the hierarchy
 - Exceptions, e.g., weekday, month, quarter, year



Summary

- **Data quality:** accuracy, completeness, consistency, timeliness, believability, interpretability
- **Data cleaning:** e.g. missing/noisy values, outliers
- **Data integration** from multiple sources:
 - Entity identification problem
 - Remove redundancies
 - Detect inconsistencies
- **Data reduction**
 - Dimensionality reduction
 - Numerosity reduction
 - Data compression
- **Data transformation and data discretization**
 - Normalization
 - Concept hierarchy generation