

Solution of Mini-Assignment

Question 1

```
for(int i = 1;i<=n;i++)
{
    for(int j = n;j>=1;j--)
    {
        System.out.println("CSIT");
    }
}
```

Here inner loop complexity is **n** because the inner loop runs n times for any value of n.
Here the outer loop also runs n times.

So, at iteration i = 1, j loop runs n times

So, at iteration i = 2, j loop runs n times

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So, at iteration i = n, j loop runs n times, so total time is

$n+n+\dots n$ times

$n \cdot n$

n^2

So maximum power of n is 2

$$T(n) = O(n^2)$$

Question 2

```
for(int i = 1;i<=n;i++)
{
    System.out.println("CSIT");
}
for(int j = n;j>=1;j--)
{
    System.out.println("CSIT");
}
```

Here the first loop complexity is **n** because the loop runs n times for any value of n. Similarly, the complexity of the second loop is **n** because the loop runs n times for any value of n.

So total time is

n+n

2n

So maximum power of n is 1

$$T(n) = O(n)$$

Question 3

```
for(int i = 1; i <= n; i++)
{
    System.out.println("CSIT");
}
for(int i = 1; i <= n; i++)
{
    for(int j = 1; j <= m; j = j*2)
    {
        System.out.println("CSIT");
    }
}
```

Here the first loop complexity is **n** because the loop runs n times for any value of n. Similarly, the complexity of the second loop is **n** because the loop runs n times for any value of n. But the inner loop runs 2^{i-1} time

At iteration 1, j loop runs 1 times = 2^{1-1}

At iteration 2, j loop runs 2 times = 2^{2-1}

At iteration 3, j loop runs 3 times = 2^{3-1}

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At iteration k, j loop runs 2^{k-1}

As per the statement

$$2^{k-1} = n$$

$$(k-1)\log_2 2 = \log_2 n$$

$$K = \log_2 n + 1$$

$$\text{Total complexity} = n + n(\log_2 n + 1) = 2n + n\log_2 n$$

So maximum power of n is $\log_2 n$

$$T(n) = O(n\log_2 n)$$

Question 4

```
for (i = 0; i < N; i++)  
{  
    a = a + rand();  
}  
for (j = 0; j < M; j++)  
{  
    b = b + rand();  
}
```

Here the first loop complexity is **N** because the loop runs N times for any value of N .

Similarly, the complexity of the second loop is **M** because the loop runs M times for any value of M .

So total time is

$$N+M$$

Here the value of N and M is user input. So

$$T(n) = O(N+M)$$

Question 5

```
for (i = 0; i < N; i++)  
{  
    for (j = N; j > i; j--)  
    {  
        a = a + i + j;  
    }  
}
```

Here the outer loop complexity is **N** because the loop runs N (0 to $N-1$) times for any value of N .

But inner loop runs based on the value of i in outer loop

when $i = 0$, j runs N times
when $i = 1$, j runs $N - 1$ times

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When $i = n - 1$, j runs 1 times

So total time is

$N + (N - 1) + (N - 2) + \dots + 2 + 1$

$(N(N + 1))/2$

$N^2/2 + N/2$ maximum power on N is 2

$$T(n) = O(N^2)$$

Question 6

```
for(int i = 1; i <= n; i *= 5)
{
    System.out.println("CSIT");
}
```

Here, the inner loop runs 5^{i-1} time

At iteration 1, j loop runs 1 times = 5^{1-1}

At iteration 2, j loop runs 5 times = 5^{2-1}

At iteration 3, j loop runs 25 times = 5^{3-1}

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At iteration k , j loop runs 5^{k-1}

As per the statement

$5^{k-1} = n$

$(k - 1)\log_5 5 = \log_5 n$

$K = \log_5 n + 1$

So maximum power of n is $\log_5 n$

$$T(n) = O(\log_5 n)$$

Question 7

```
for(int i = 1; i <= n; i++)
{
    for(int j = 1; j <= m; j = j*2)
    {
        for(k=j; k >= 1; k--)
        {
            System.out.println("CSIT");
        }
    }
}
```

Here the outer loop runs n time.
First inner loop runs $\log_2 m$ times.
Inner loop also runs $\log_2 m$ times.
Total complexity $n * \log_2 m * \log_2 m$

$$T(n) = O(n * \log_2 m * \log_2 m) \text{ or } O(n * \log_2 m^2)$$

Question 8

```
for(int j = 1; j < i; j *= 2)
{
    for(int k = j; k >= 1; k /= 2)
    {
        System.out.println("DSA");
    }
}
```

Outer loop runs 2^{i-1} time

At iteration 1, j loop runs 1 times = 2^{1-1}

At iteration 2, j loop runs 2 times = 2^{2-1}

At iteration 3, j loop runs 4 times = 2^{3-1}

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At iteration p , j loop runs 2^{p-1}

As per the statement

$$2^{p-1} = i$$

$$(p-1)\log_2 2 = \log_2 i$$

$$p = \log_2 i + 1$$

Inner loop depends on the value of j variable

At iteration 1, j loop runs 2

Outer loop runs 2^{i-1} time

At iteration 1, j loop runs 1 times = 2^{1-1}

At iteration 2, j loop runs 2 times = 2^{2-1}

At iteration 3, j loop runs 4 times = 2^{3-1}

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At iteration k, j loop runs 2^{k-1}

As per the statement

$$2^{k-1} = n$$

$$(k-1)\log_2 2 = \log_2 n$$

$$k = \log_2 n + 1$$

Total complexity

$$p * k$$

$$(\log_2 i + 1) * (\log_2 n + 1)$$

$$= \log_2 n * \log_2 i + \log_2 n + \log_2 i + 1$$

$$T(n) = O(\log_2 i * \log_2 n)$$