

Unit - 4 Probabilistic Graphical Models

1) NAIVE BAYES ALGORITHM:

The Naive Bayes algorithm is a classification technique based on Bayes' Theorem. It assumes that all features (attributes) are conditionally independent given the class label.

$$P(c|x) = \frac{P(x|c) P(c)}{P(x)}$$

where,

$C \rightarrow$ class

$x \rightarrow$ feature vector

Since $P(x)$ is constant the decision is based on maximizing $P(x|c) P(c)$.

Applications:

- * Spam Detection
- * Medical ^aDiagnosis
- * Sentiment analysis

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2) BAYESIAN BELIEF NETWORK : (BBN)

A Bayesian Belief Network (BBN) also known as a Bayesian Network, is a graphical model that represents probabilistic relationships among a set of random variables.

It is represented as a directed Acyclic Graph (DAG).

- * Nodes represent random variables.
- * Edges represents conditional dependencies.
- * Each node has a conditional probability Table (CPT) that quantifies the effect of parent nodes.

Example:

Consider medical diagnosis network:

Node 1 : Flu

Node 2 : Fever

Node 3 : Cough

Here Fever and Cough depends on Flu.

The network compactly encodes these dependencies.

Advantages:

- * Captures casual relationships.
- * Handles incomplete data.
- * Supports both inference (predict unknowns) and learning (update probabilities)

Applications:

- * Medical diagnosis (diseases & symptoms)
- * Fault detection in engineering systems.
- * Decision support system.

3) Hidden Markov Model (HMM)

A Hidden Markov Model (HMM) is a statistical model for systems that evolve over time but where underlying states are hidden. Instead, we observe outcomes generated probabilistic from the hidden states.

Components:

- a) Hidden states (S)
- b) Observations (O)
- c) Transition Probabilities (A)
- d) Emission probabilities (B)
- e) Initial state distribution (π)

Key problems solved by HMM:

- * Evaluation : Compute probability of observation sequence given model.
- * Decoding : Determine most likely sequence of hidden states (Viterbi algorithm)
- * Learning : Estimate parameters.

Application :

- * speech recognition
- * Natural language processing
- * Bio Informatics.

4. BAYESIAN INFERENCE:

Bayesian Inference is a method of statistical inference where probability is used to represent uncertainty about parameters. Unlike frequentist methods, Bayesian methods update beliefs based on new evidence.

$$P(H|D) = \frac{P(D|H) \cdot P(H)}{P(D)}$$

where :

$H \rightarrow$ hypothesis

$D \rightarrow$ observed data

Advantages:

- * Incorporates prior knowledge
- * Naturally handles uncertainty
- * Produces full probability distributions.

PROBLEM BASED ON NAIVE BAYES:

Problem:

A spam filter uses the Naive Bayes algorithm.
Consider the word "Offer" appearing in emails.

$$P(\text{spam}) = 0.4$$

$$P(\text{Not spam}) = 0.6$$

$$P(\text{Offer} | \text{spam}) = 0.8$$

$$P(\text{Offer} | \text{not spam}) = 0.2$$

If an email ~~class~~ contains the word "Offer",
classify it as Spam or Not Spam.

Solution:

$$P(\text{Spam} | \text{Offer}) = \frac{P(\text{Offer} | \text{Spam}) P(\text{Spam})}{P(\text{Offer})}$$

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$$\begin{aligned} P(\text{Offer}) &= P(\text{Offer} | \text{Spam}) + P(\text{Offer} | \text{NotSpam}) P(\text{NotSpam}) \\ &= (0.8)(0.4) + (0.2)(0.6) \\ &= 0.32 + 0.12 \\ &= 0.44 \end{aligned}$$

Now,

$$P(\text{Spam} | \text{Offer}) = \frac{0.8 \times 0.4}{0.44} = 0.727$$

$$P(\text{NotSpam} | \text{Offer}) = \frac{0.2 \times 0.6}{0.44} = 0.273$$

Hence the email is classified as "Spam".