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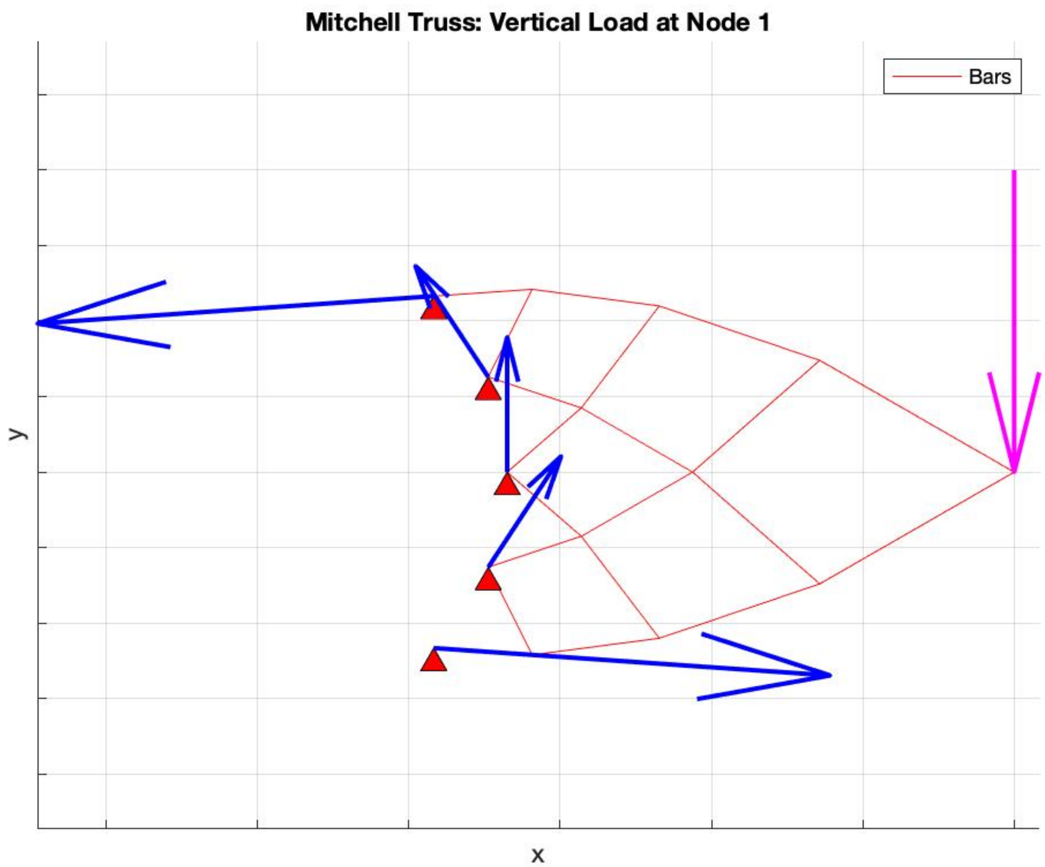
MAE 290A Homework 1: Tensegrity Structure Statics

Case 1) Mitchel Truss

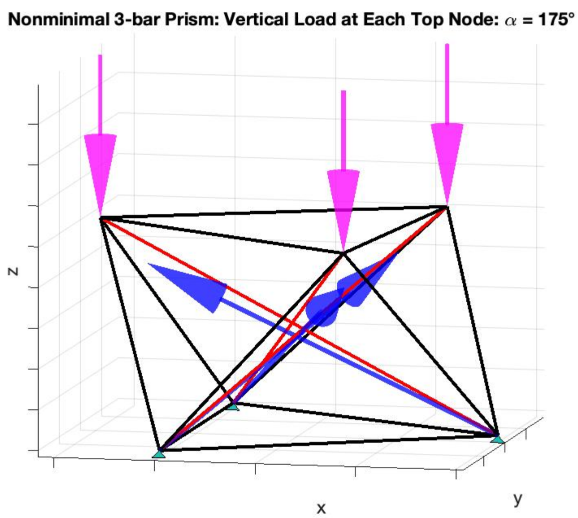
The structure is not potentially inconsistent, since m^, n^, and r are equal to 20. Furthermore, the system is not underdetermined, suggesting that the structure is not tensionable. There are as many equations as unknowns, and a single solution exists for static equilibrium. The reaction loads corresponding to that solution are illustrated in the figure below. The structure is well defined without any soft modes, leading to the conclusion that under finite disturbances to the applied load, the structure will neither collapse nor wobble between different deformed configurations.

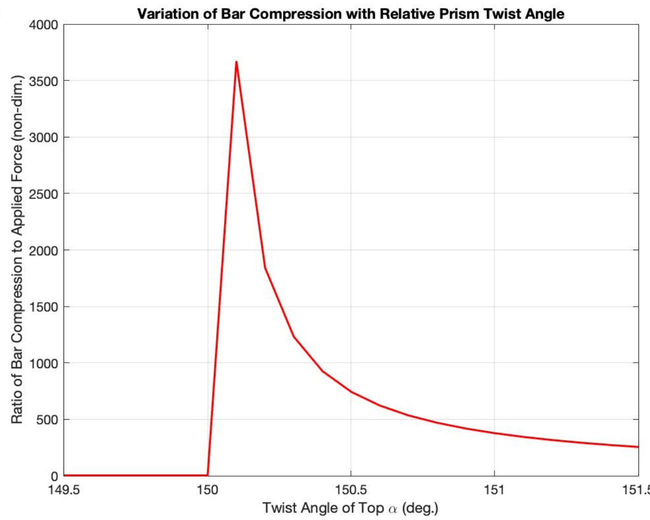


However, some bars are not under compression and could be replaced with strings in order to sustain tension properly. With the absence of strings, it is not possible to pretension the structure to correct the issue of bars under tension. Possible solutions to this would be to alter the direction of the force and apply it along the x-direction, or to replace the bars under tensions with tensionable strings.

Case 2) Non-minimal 3-bar Prism

To build up to the case of the 4-bar non-minimal tensegrity prism, the 3-bar case is considered first. The structure is not potentially inconsistent since r is equal to m-hat, and the matrix A_{se} has orthogonal rows. Although all bars are in compression, the system is underdetermined since m^ exceeds r and n^ (15 > 9 = 9). This can be expected since the structure is non-minimal. All six of the extra cross-strings lack tension under load. However, by enforcing a minimum tension of 0.1 in all strings with no load, the structure is seen to be pretensionable! Mathematically, this constitutes determining a unique solution that enforces positive tension greater than a prescribed minimum in each string, even in an non-loaded environment. For the pretensioned structure, the applied load now results in a proper system with all strings in tension and all bars in compression. Each string supports tension greater than or equal to 0.1.

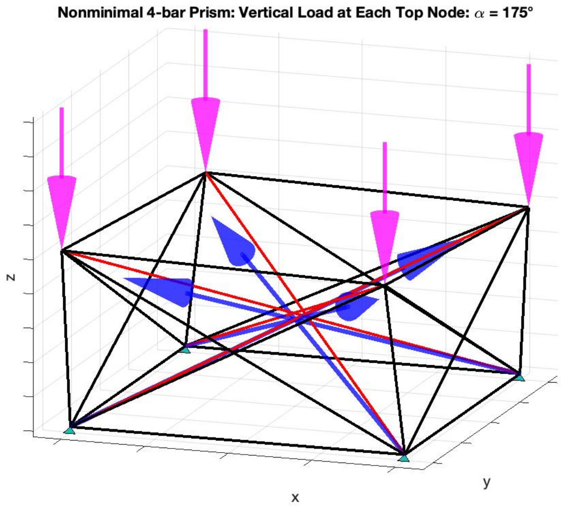


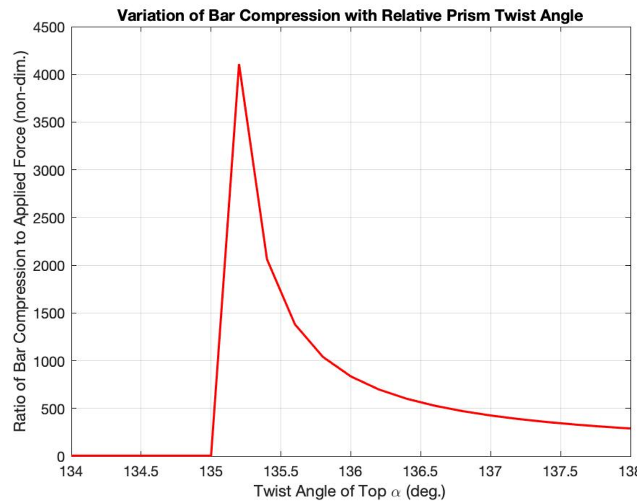


An interesting phenomenon is the relationship between the compressive load in each bar (equal due to the apparent 3-way symmetry), and the twist angle of the top triangle with respect to the base triangle. As the twist angle varies from 0 radians, to π/2 and eventually π, the compressive load experiences a singularity at a specific angle, that is found by inspection to be π/2 + π/3 radians (*see figure below*). The load in the bars remain consistent with the order of magnitude of the applied force below this critical angle α_c, but spikes to infinity at α_c and then decays toward still-relatively-large values past α_c. It is thus advised to fix the tensegrity structure at angles below α_c = 5π/6 to avoid catastrophic buckling failure in the bars.

Case 3) Non-minimal 4-bar Prism

Similar to the 3-bar prism, the 4-bar non-minimal prism also is not potentially inconsistent: *in this case, r = m^ = 12, n^ = 24*. No additional strings are necessary. Once more, the bars all sustain equal compressive load, but only four of the 20 strings are in tension (*the ones that belong to the minimal case*). With more unknowns than available equations, the system is underdetermined and lends itself to the possibility of pretensioning under no load. Enforcing a minimum tension of 0.1 leads to the conclusion that the structure is in fact pretensionable (*no strings that are slack*).





Two results are of interest in this configuration. As previously explored, a singular value of the twist angle appears, but this at $\alpha_c = \pi/2 + \pi/4$. It is observable from the two cases explored that the critical angle of singularity corresponds to the number of bars in the fully non-minimal (*maximum number of cross-strings*) tensegrity prism through the relationship below. Again, it is advised to use operation angles less than α_c to avoid buckling failure in the bars.

For a *p-bar* non-minimal prism: $$\alpha_c = \frac{\pi}{2} + \frac{\pi}{p} = \pi\left(\frac{p+2}{2p}\right)$$

Another interesting note is the distribution of tensile stresses among the cross-strings spanning the top and bottom squares. For both the 3-bar and 4-bar prisms (*denoted as a general p-bar prim henceforth*), the tensile loads in only the p minimal cross-strings are observed to be non-zero without pretensioning. Once the structure is pretensioned, p more cross-strings (B) accept significant load, while the remaining $(p)(p+1) - 2p = p^2 - p$ strings only sustain the minimum enforced load of 0.1. When the load is then added, the minimal strings accept the greatest load, commensurate to the order of magnitude of the applied load. The aforementioned p non-minimal strings accept a little more load but of significantly lesser magnitude. The remaining $p^2 - p$ strings still *only sustain the minimum tensile load of 0.1*! Due to these properties, the non-minimal prism is seen to be a complex tensegrity structure that can be applied with care in several applications to leverage the presence of uniaxial tensile members, in place of rigid-body structures.