Phys 442

ELECTRICITY & MAGNETISM 3

University of Waterloo

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Disclaimer

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Latest versions of all my course notes are available at **www.tcfraser.com/coursenotes**.

1 Coordinates and Symmetry

A clever choice of coordinates systems typically makes solving a problem considerably easier. Mathematically, this is due to *Noether's Theorem*. A typical three dimensional Lagrangian will have three dependent generalized coordinates $L = L(x, y, z) = L(s, \theta, \zeta) = \cdots$. However, if one can identify generalized coordinates q that make the Lagrangian invariant $\frac{\partial L}{\partial q} = 0$, then the *Euler-Lagrange* equations are considerably similar,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \implies \frac{\partial L}{\partial \dot{q}} = \text{const.} \implies L \propto \dot{q}$$

As such, the number of equations that remain to solved has been reduced.

2 First Assignment?

A1.1: Use cylindrical coordinates with ζ along the axis of the cable,

$$V(\zeta) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{C}} \frac{\mathrm{d}\rho}{\mathscr{r}}$$

Where $\vec{\mathscr{r}} = \vec{r} - \vec{r}'$, $\vec{r}'$ is the source point and $\vec{r}$ is the field point. The entire cylinder is the set of all source points $\vec{r}'$ that are contained inside $|\vec{r}'| \leq R$.

$$\vec{r} = \zeta\hat{\zeta}$$

$$\vec{r}'r = s'\hat{s}' + \zeta'\hat{\zeta}$$

$$V(\zeta) = \frac{\rho}{4\pi\epsilon_0}\int_{\mathcal{C}} \frac{\mathrm{d}V}{|\vec{r} - \vec{r}'|}$$

Where $\mathrm{d}V = s\mathrm{d}s\mathrm{d}\theta\mathrm{d}\zeta$. One can then find the electric *field* by doing $\vec{E} = -\vec{\nabla}V = E_\zeta\hat{\zeta} = -\frac{\partial V}{\partial \zeta}\hat{\zeta}$

A1.2:

Between the two conductors, there will be a radial electric field $\vec{E} = E(s)\hat{s}$ and parallel magnetism field $\vec{B} = B(s)\hat{\zeta}$. Outside the two conductors, there will be no electric or magnetic field.

$$E^{\parallel}_{\text{vac}} = 0$$

$$E^{\perp}_{\text{vac}} = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Todo (TC Fraser): Figure out O'Donovan