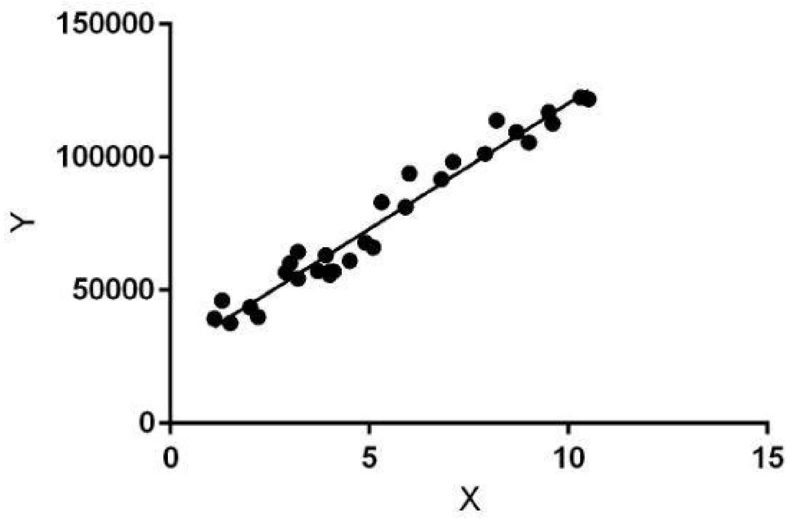
Experiment No. 1
Analyze the Boston Housing dataset and apply appropriate Regression Technique
Date of Performance: 27-07-2023
Date of Submission: 17-08-2023

Aim: Analyze the Boston Housing dataset and apply appropriate Regression Technique.

Objective: Ability to perform various feature engineering tasks, apply linear regression on the given dataset and minimize the error.

Theory:

Linear Regression is a machine learning algorithm based on supervised learning. It performs a regression task. Regression models a target prediction value based on independent variables. It is mostly used for finding out the relationship between variables and forecasting. Different regression models differ based on – the kind of relationship between dependent and independent variables they are considering, and the number of independent variables getting used.



Linear regression performs the task to predict a dependent variable value (y) based on a given independent variable (x). So, this regression technique finds out a linear relationship between x (input) and y(output). Hence, the name is Linear Regression.

In the figure above, X (input) is the work experience and Y (output) is the salary of a person. The regression line is the best fit line for our model.

Dataset:

The Boston Housing Dataset

The Boston Housing Dataset is derived from information collected by the U.S. Census Service concerning housing in the area of Boston MA. The following describes the dataset columns:

CRIM - per capita crime rate by town
ZN - proportion of residential land zoned for lots over 25,000 sq.ft.
INDUS - proportion of non-retail business acres per town.
CHAS - Charles River dummy variable (1 if tract bounds river; 0 otherwise)
NOX - nitric oxides concentration (parts per 10 million)
RM - average number of rooms per dwelling
AGE - proportion of owner-occupied units built prior to 1940
DIS - weighted distances to five Boston employment centers
RAD - index of accessibility to radial highways
TAX - full-value property-tax rate per $10,000
PTRATIO - pupil-teacher ratio by town
B - 1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
LSTAT - % lower status of the population
MEDV - Median value of owner-occupied homes in $1000's

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.graphics.gofplots import ProbPlot
import sklearn.datasets
from sklearn.model_selection import train_test_split
from statsmodels.formula.api import ols
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import MinMaxScaler
```

```
df = pd.read_csv('housing.csv')
```

```
print(df)
```

```
        CRIM    ZN  INDUS  CHAS    NOX     RM   AGE     DIS  RAD  TAX  \
0    0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296
1    0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242
2    0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242
3    0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222
4    0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222
..       ...   ...    ...   ...    ...    ...   ...     ...  ...  ...
501  0.06263   0.0  11.93     0  0.573  6.593  69.1  2.4786    1  273
502  0.04527   0.0  11.93     0  0.573  6.120  76.7  2.2875    1  273
503  0.06076   0.0  11.93     0  0.573  6.976  91.0  2.1675    1  273
504  0.10959   0.0  11.93     0  0.573  6.794  89.3  2.3889    1  273
505  0.04741   0.0  11.93     0  0.573  6.030  80.8  2.5050    1  273

     PTRATIO       B  LSTAT  MEDV
0       15.3  396.90   4.98  24.0
1       17.8  396.90   9.14  21.6
2       17.8  392.83   4.03  34.7
3       18.7  394.63   2.94  33.4
4       18.7  396.90   5.33  36.2
..       ...     ...    ...   ...
501     21.0  391.99   9.67  22.4
502     21.0  396.90   9.08  20.6
503     21.0  396.90   5.64  23.9
504     21.0  393.45   6.48  22.0
505     21.0  396.90   7.88  11.9

[506 rows x 14 columns]
```

```
df.head()
```

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.90	5.33	36.2

```
#The price of the house indicated by the variable MEDV is the target variable and the rest are the independent variables based on which w
```

```
# Info of dataframe
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   CRIM     506 non-null    float64
 1   ZN       506 non-null    float64
 2   INDUS    506 non-null    float64
 3   CHAS     506 non-null    int64
 4   NOX      506 non-null    float64
 5   RM       506 non-null    float64
 6   AGE      506 non-null    float64
 7   DIS      506 non-null    float64
 8   RAD      506 non-null    int64
 9   TAX      506 non-null    int64
 10  PTRATIO  506 non-null    float64
 11  B        506 non-null    float64
 12  LSTAT    506 non-null    float64
 13  MEDV     506 non-null    float64
```

```
dtypes: float64(11), int64(3)
memory usage: 55.5 KB
```

```
# checking the number of rows and Columns in the data frame
df.shape
```

```
(506, 14)
```

```
# check for missing values
df.isnull().sum()
```

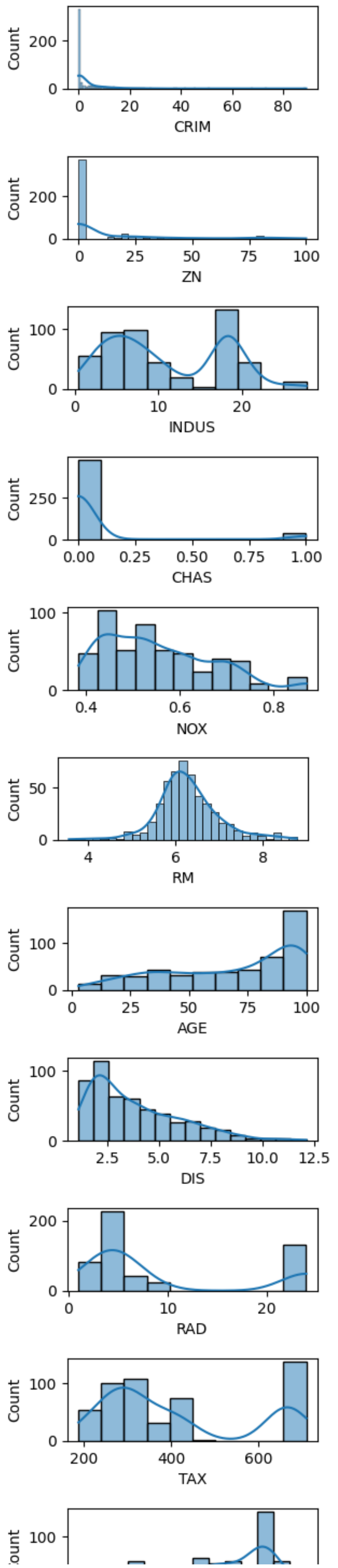
```
CRIM       0
ZN         0
INDUS      0
CHAS       0
NOX        0
RM         0
AGE        0
DIS        0
RAD        0
TAX        0
PTRATIO    0
B          0
LSTAT      0
MEDV       0
dtype: int64
```

```
# statistical measures of the dataset
df.describe()
```

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.
75%	3.677083	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.

```
correlation = df.corr()
```

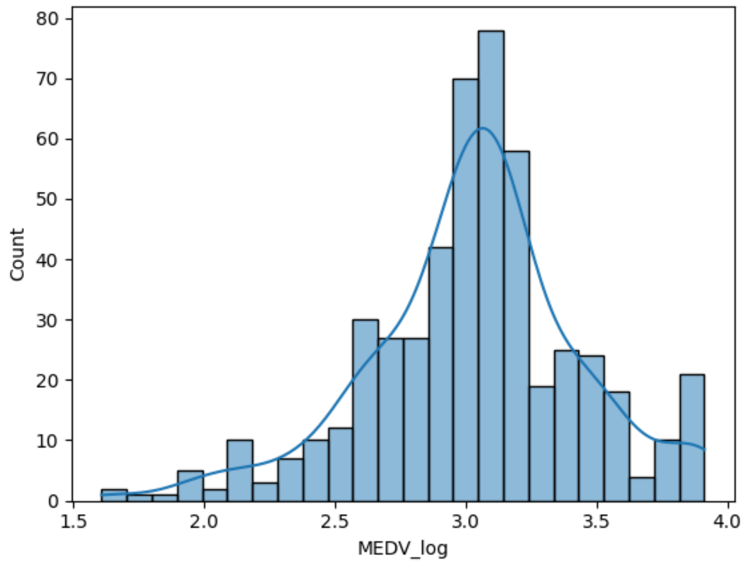
```
#plot all the columns to look at their distributions
for i in df.columns:
    plt.figure(figsize=(3, 1))
    sns.histplot(data=df, x=i, kde = True)
    plt.show()
```




```
#The dependent variable MEDV seems to be slightly right skewed, apply a log transformation on the 'MEDV' column and check the distributic
df['MEDV_log'] = np.log(df['MEDV'])
```

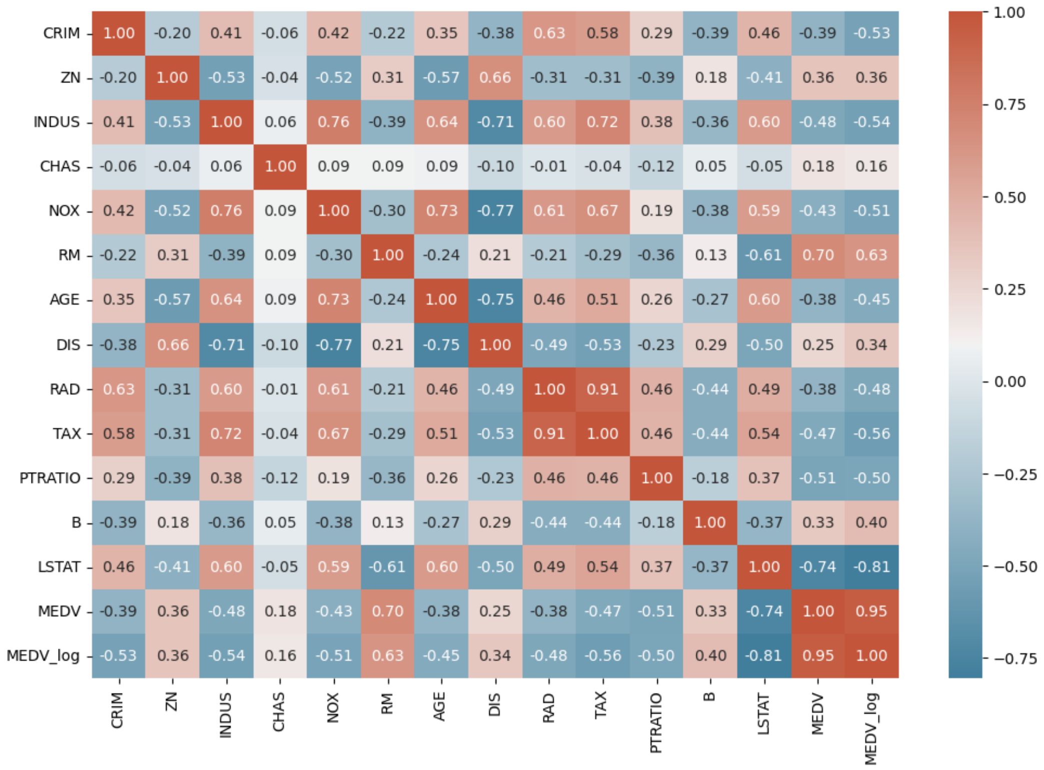
```
sns.histplot(data=df, x='MEDV_log', kde = True)
```

```
<Axes: xlabel='MEDV_log', ylabel='Count'>
```



The log-transformed variable (MEDV_log) appears to have a nearly normal distribution without skew, and hence we can proceeed.

```
plt.figure(figsize=(12,8))
cmap = sns.diverging_palette(230, 20, as_cmap=True)
sns.heatmap(df.corr(),annot=True,fmt='.2f',cmap=cmap )
plt.show()
```




```
# separate the dependent and indepedent variable
Y = df['MEDV_log']
X = df.drop(columns = {'MEDV', 'MEDV_log'})

# add the intercept term
X = sm.add_constant(X)
```

```
# splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.30 , random_state=1)


#Check for Multicollinearity
#use the Variance Inflation Factor (VIF), to check if there is multicollinearity in the data.
#Features having a VIF score > 5 will be dropped/treated till all the features have a VIF score < 5

from statsmodels.stats.outliers_influence import variance_inflation_factor

# function to check VIF
def checking_vif(train):
    vif = pd.DataFrame()
    vif["feature"] = train.columns

    # calculating VIF for each feature
    vif["VIF"] = [
        variance_inflation_factor(train.values, i) for i in range(len(train.columns))
    ]
    return vif


print(checking_vif(X_train))
```

```
    feature         VIF
0     const  585.099960
1      CRIM    1.993439
2        ZN    2.743911
3     INDUS    4.004462
4      CHAS    1.078490
5       NOX    4.430555
6        RM    1.879494
7       AGE    3.155351
8       DIS    4.361514
9       RAD    8.369185
10      TAX   10.194047
11  PTRATIO    1.948555
12        B    1.385213
13    LSTAT    2.926462
```

There are two variables with a high VIF - RAD and TAX. Remove TAX as it has the highest VIF values and check the multicollinearity again.

```
# create the model after dropping TAX
X_train = X_train.drop(['TAX'],1)

# check for VIF
print(checking_vif(X_train))
```

```
    feature         VIF
0     const  581.372515
1      CRIM    1.992236
2        ZN    2.483521
3     INDUS    3.277778
4      CHAS    1.052841
5       NOX    4.397232
6        RM    1.876243
7       AGE    3.154114
8       DIS    4.339453
9       RAD    2.978247
10  PTRATIO    1.914523
11        B    1.384927
12    LSTAT    2.924524
<ipython-input-17-31a12e8753ff>:2: FutureWarning: In a future version of pandas all arguments of DataFrame.drop except for the argu
  X_train = X_train.drop(['TAX'],1)
```

```
#create the linear regression model using statsmodels OLS and print the model summary.
model1 = sm.OLS(y_train, X_train).fit()

# get the model summary
model1.summary()
```

OLS Regression Results

Dep. Variable:	MEDV_log	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.763
Method:	Least Squares	**F-statistic:**	95.56
Date:	Tue, 01 Aug 2023	**Prob (F-statistic):**	2.97e-101
Time:	01:45:20	**Log-Likelihood:**	78.262
No. Observations:	354	**AIC:**	-130.5
Df Residuals:	341	**BIC:**	-80.22
Df Model:	12		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	4.4999	0.253	17.767	0.000	4.002	4.998
CRIM	-0.0122	0.002	-7.005	0.000	-0.016	-0.009
ZN	0.0010	0.001	1.417	0.157	-0.000	0.002
INDUS	-0.0002	0.003	-0.066	0.947	-0.006	0.005
CHAS	0.1164	0.039	3.008	0.003	0.040	0.193
NOX	-1.0297	0.187	-5.509	0.000	-1.397	-0.662
RM	0.0569	0.021	2.734	0.007	0.016	0.098
AGE	0.0003	0.001	0.390	0.697	-0.001	0.002
DIS	-0.0496	0.010	-4.841	0.000	-0.070	-0.029
RAD	0.0080	0.002	3.885	0.000	0.004	0.012
PTRATIO	-0.0458	0.007	-6.762	0.000	-0.059	-0.033
B	0.0002	0.000	1.796	0.073	-2.35e-05	0.001

Independent variables (ZN, AGE, and INDUS) have a high p-value and low t, which implies a minimum significance. Drop insignificant variables from the above model and create the regression model again

Skew:	0.387	**Prob(JB):**	1.34e-22

```
# create the model after dropping TAX
Y = df['MEDV_log']

X = df.drop(columns = {'MEDV', 'MEDV_log', 'ZN', 'AGE', 'INDUS', 'TAX'})
X = sm.add_constant(X)

#splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.30 , random_state=1)

# create the model
model2 = sm.OLS(y_train, X_train).fit()

# get the model summary
model2.summary()
```

OLS Regression Results

Dep. Variable:	MEDV_log	**R-squared:**	0.769
Model:	OLS	**Adj. R-squared:**	0.763
Method:	Least Squares	**F-statistic:**	127.5
Date:	Tue, 01 Aug 2023	**Prob (F-statistic):**	6.21e-104
Time:	01:45:24	**Log-Likelihood:**	77.190
No. Observations:	354	**AIC:**	-134.4
Df Residuals:	344	**BIC:**	-95.69
Df Model:	9		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	4.5147	0.252	17.925	0.000	4.019	5.010
CRIM	-0.0119	0.002	-6.909	0.000	-0.015	-0.009
CHAS	0.1165	0.039	3.016	0.003	0.041	0.192
NOX	-1.0234	0.168	-6.086	0.000	-1.354	-0.693
RM	0.0622	0.020	3.089	0.002	0.023	0.102
DIS	-0.0434	0.008	-5.488	0.000	-0.059	-0.028
RAD	0.0083	0.002	4.092	0.000	0.004	0.012
PTRATIO	-0.0490	0.006	-7.936	0.000	-0.061	-0.037
B	0.0002	0.000	1.824	0.069	-1.95e-05	0.001
LSTAT	-0.0287	0.002	-12.577	0.000	-0.033	-0.024

Omnibus:	35.608	**Durbin-Watson:**	1.927
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	104.246
Skew:	0.425	**Prob(JB):**	2.31e-23
Kurtosis:	5.519	**Cond. No.**	9.76e+03

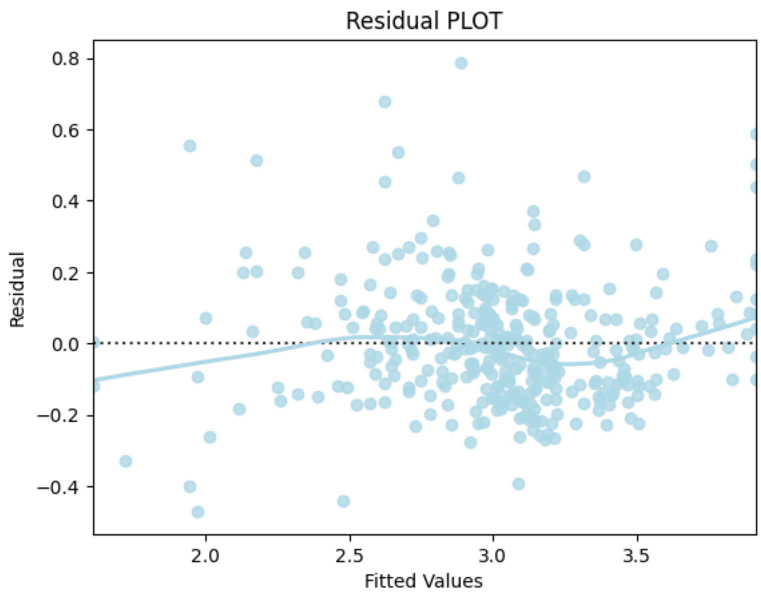
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 9.76e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

```
residuals = model2.resid
residuals.mean()
```

    8.154180406851432e-17

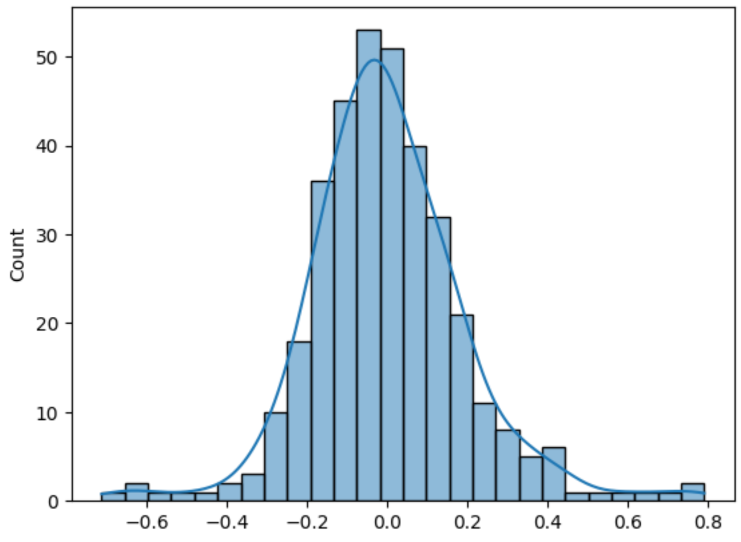
```
# predicted values
fitted = model2.fittedvalues

#sns.set_style("whitegrid")
sns.residplot(x = y_train, y = residuals , color="lightblue", lowess=True)
plt.xlabel("Fitted Values")
plt.ylabel("Residual")
plt.title("Residual PLOT")
plt.show()
```



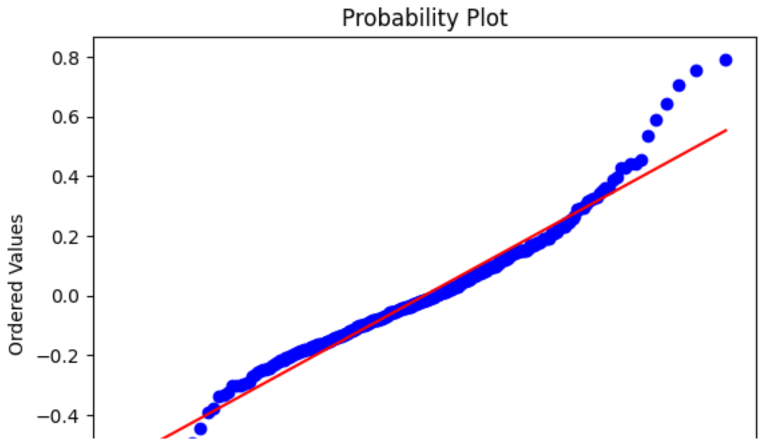

```
# Plot histogram of residuals
sns.histplot(residuals, kde=True)
```

    <Axes: ylabel='Count'>




```
# Plot q-q plot of residuals
import pylab
import scipy.stats as stats

stats.probplot(residuals, dist="norm", plot=pylab)
plt.show()
```




```
# RMSE
def rmse(predictions, targets):
    return np.sqrt(((targets - predictions) ** 2).mean())


# MAPE
def mape(predictions, targets):
    return np.mean(np.abs((targets - predictions)) / targets) * 100


# MAE
def mae(predictions, targets):
    return np.mean(np.abs((targets - predictions)))


# Model Performance on test and train data
def model_pref(olsmodel, x_train, x_test):

    # Insample Prediction
    y_pred_train = olsmodel.predict(x_train)
    y_observed_train = y_train

    # Prediction on test data
    y_pred_test = olsmodel.predict(x_test)
    y_observed_test = y_test

    print(
        pd.DataFrame(
            {
                "Data": ["Train", "Test"],
                "RMSE": [
                    rmse(y_pred_train, y_observed_train),
                    rmse(y_pred_test, y_observed_test),
                ],
                "MAE": [
                    mae(y_pred_train, y_observed_train),
                    mae(y_pred_test, y_observed_test),
                ],
                "MAPE": [
                    mape(y_pred_train, y_observed_train),
                    mape(y_pred_test, y_observed_test),
                ],
            }
        )
    )

# Checking model performance
model_pref(model2, X_train, X_test)
```

```
    Data      RMSE       MAE      MAPE
0  Train  0.194565  0.141729  4.919107
1   Test  0.191732  0.146199  5.069304
```

The errors have increased slightly on the test data. This suggested further investigation to improve the performance on general data.

```
# import the required function

from sklearn.model_selection import cross_val_score

# build the regression model and
linearregression = LinearRegression()
```

```
cv_Score11 = cross_val_score(linearregression, X_train, y_train, cv = 10)
cv_Score12 = cross_val_score(linearregression, X_train, y_train, cv = 10, scoring = 'neg_mean_squared_error')


print("RSquared: %0.3f (+/- %0.3f)" % (cv_Score11.mean(), cv_Score11.std() * 2))
print("Mean Squared Error: %0.3f (+/- %0.3f)" % (-1*cv_Score12.mean(), cv_Score12.std() * 2))
```

```
RSquared: 0.726 (+/- 0.251)
Mean Squared Error: 0.041 (+/- 0.024)
```

Get model Coefficients in a pandas dataframe with column 'Feature' having all the features and column 'Coefs' with all the corresponding Coefs. Write the regression equation.

```
coef = pd.Series(index = X_train.columns, data = model2.params.values)

coef_df = pd.DataFrame(data = {'Coefs': model2.params.values }, index =  X_train.columns)
coef_df
```

	Coefs
const	4.514720
CRIM	-0.011919
CHAS	0.116497
NOX	-1.023431
RM	0.062203
DIS	-0.043391
RAD	0.008288
PTRATIO	-0.049038
B	0.000249
LSTAT	-0.028659

```
#Write the equation of the fit
Equation = "log (Price) ="
print(Equation, end='\t')
for i in range(len(coef)):
    print('(', coef[i], ') * ', coef.index[i], '+', end = ' ')
```

```
log (Price) =   ( 4.514720483568433 ) *  const + ( -0.011918775173037938 ) *  CRIM + ( 0.11649715902151694 ) *  CHAS + ( -1.0234312
```

✓ 0s completed at 7:15 AM

Conclusion:

Selection of some important features:

1. CRIM: The crime rate can affect housing prices; areas with higher crime rates may have lower property values.
2. CHAS: Proximity to the Charles River can be an attractive feature, potentially increasing the house price.
3. NOX: Air pollution (nitric oxides concentration) can influence property value; areas with higher pollution levels may have lower prices.
4. RM: The number of rooms in a dwelling is positively related to the house price; larger homes tend to have higher values.
5. DIS: Shorter distances to employment centers are often preferred, leading to higher demand and potentially higher prices.
6. RAD: Better accessibility to radial highways can be desirable, affecting the housing demand and, consequently, prices.
7. PTRATIO: A lower pupil-teacher ratio is often considered desirable, indicating better educational resources in the area.
8. B: The proportion of Black residents can influence housing prices in certain locations due to historical segregation patterns.
9. LSTAT: The percentage of lower-status population may be indicative of the overall economic condition of the area, affecting property values.

The chosen features appear to be relevant and meaningful in estimating the price of a house.

- Mean Squared Error (MSE): 0.041 (+/- 0.024)
- The MSE value of 0.041 indicates that, on average, the squared difference between the predicted house prices and the actual house prices is 0.041. This value is relatively low, which suggests that the model is performing well and producing accurate predictions for house prices.
- The standard deviation of the MSE is given as +/- 0.024. A low standard deviation indicates that the model's performance is consistent across the folds, which is a positive sign.
- Mean Squared Error of 0.041 is indicative of a well-performing linear regression model for the given data.