ATCM '13

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GeoGebra

Workshop

by

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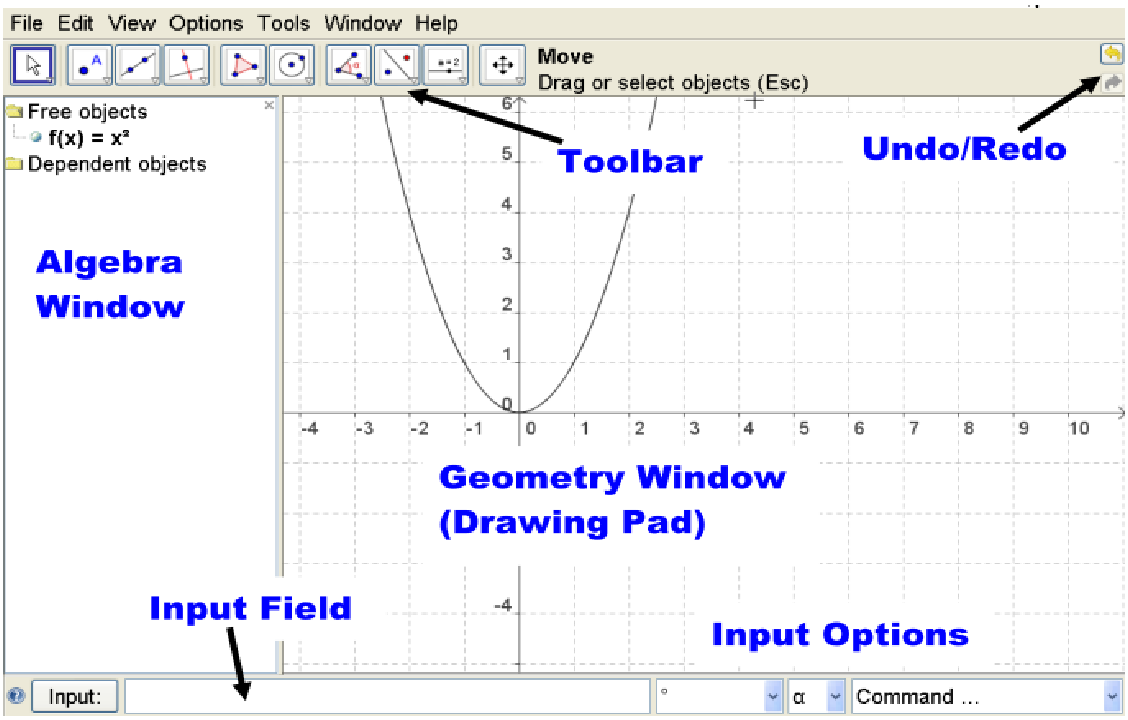
Venue : Computer Lab. Dept. of Mathematics IIT_B

GeoGebra

Introduction

GeoGebra is a dynamic mathematics software that joins geometry , algebra and calculus. GeoGebra is an interactive geometry system. You can do constructions with points , vectors , segments , lines and conic sections as well as functions while changing them dynamically afterwards. At the same time equations and coordinates can be entered directly. Thus Geogebra has the ability to deal with variables for numbers , vectors and points. GeoGebra is also a Computer Algebra System (CAS).

GeoGebra User Interface

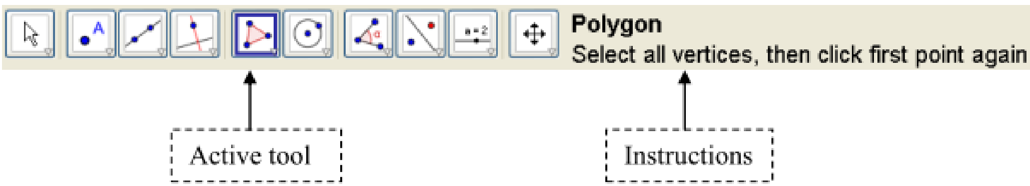


Input Field : Here you can enter the equation for a graph , coordinates of a point or one of Geogebra's commands. A list of commands is viewable by clicking on the ' **Command**' drop down menu near the Input Field.

Geometry Window: This is the window where a graphical representation of your input (graphs, points , lines etc) is displayed.

Algebra Window : This window is on the LHS of the Geometry Window. Every geometrical object will have an algebraic representation in this window. You can open and close the Algebra Window using the View Menu.

Toolbar : This consists of a row of buttons. Which ever button you have selected to use will have a blue border. Each button has a drop down arrow at the bottom right of the button. This drop down arrow reveals more buttons. The active buttons will display some brief instructions on how the button works.

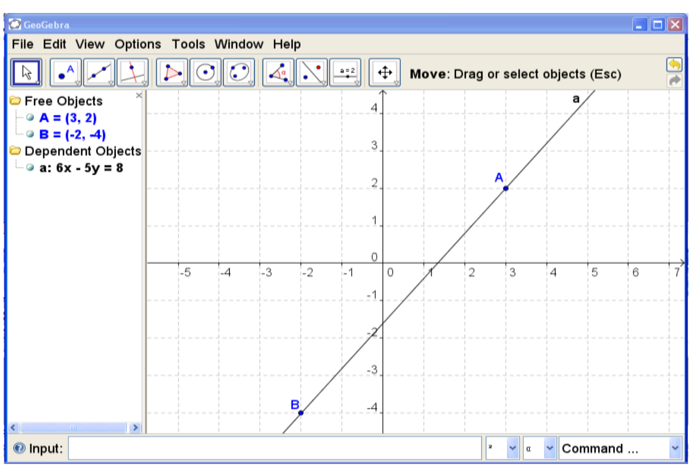


Let us Start : Perform the following actions

1. Go to **Options** , select **Font Size** and choose 18 points
2. Go to **Options** , select **Drawing Pad** , select **Grid** and then select **Grid** check box.
3. Go to **Options** , select **Point Capturing** and then select **On Grid.**
4. Finally to select labeling of points , go to **Options** select **Labelling** then select **New Points Only.**

Activity – 1 : Points and Lines

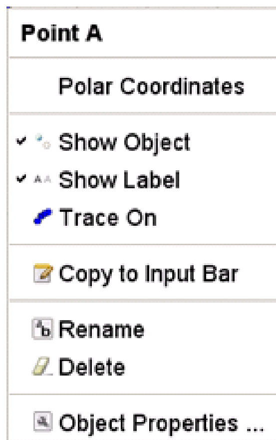
- Click on the **point** button and place a point at (3,2) using the left click on your mouse.
- Click back on the arrow (**Move** Button) and then drag your point around. Notice the coordinates change on the point itself and in the **Algebra** window.
- Click in the **Input Field** and type (-2,-4) and press Enter.
- Click on the **Line through two points** button and then click on the two points on the screen to construct a line through these two points.
- Click on the arrow (**Move** button) and
  - i) click any where on the line and drag the line. You can see the equation of line change in **Algebra** window.
  - ii) Click on any of the two points and drag the line. You will get the same



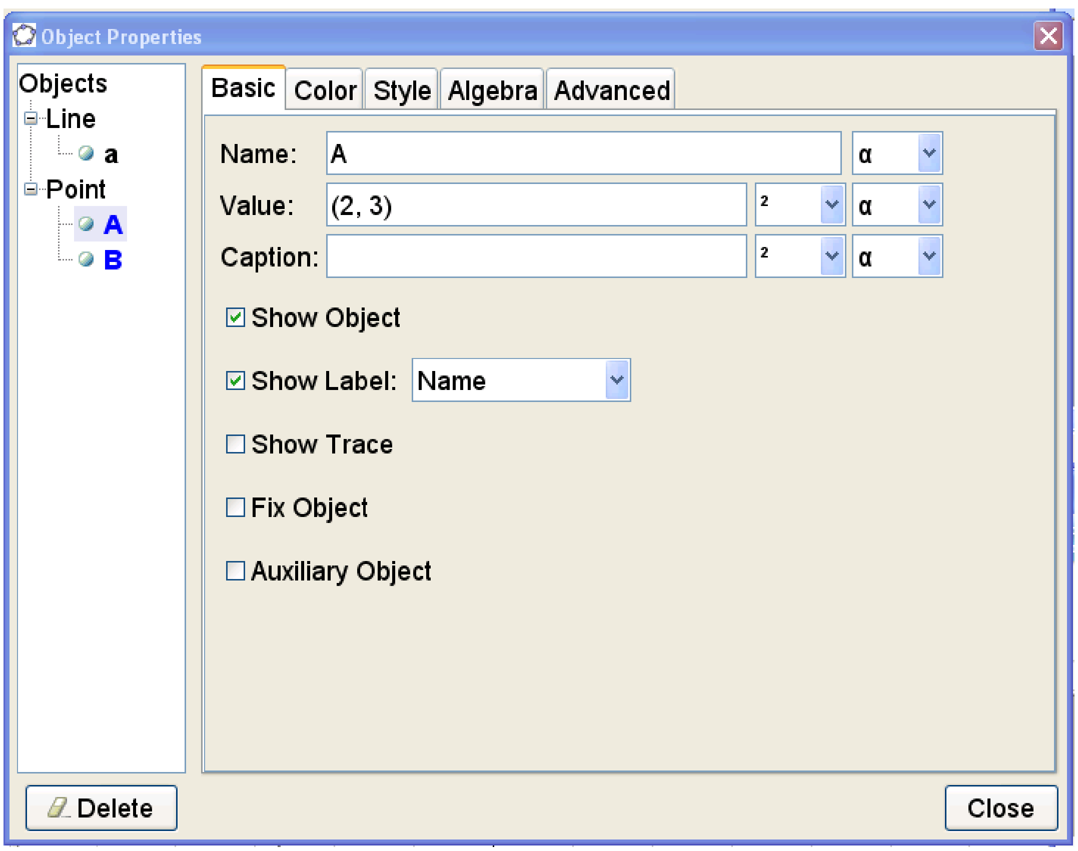
Save your file in a folder. Geogebra by default assigns **.ggb** extension to files.

Activity 2 : Editing objects and their properties

- Right clicking any object in the **Algebra** window or **Geometry** window will show the list of available editing options for that object.
- If you double click on any object in **Algebra** window , Geogebra will allow you to redefine that object. Double click on point A in the **Algebra** window and change coordinates to (2,3).
- Right click on point A in the **Algebra** window to see the available options for that object.



- Right click on any object in the **Algebra** or **Geometry window** and choose **Object Properties**.



With this dialog box you can change color , label , name , caption , thickness etc. for the selected object.

Activity 3 : Moving the screen

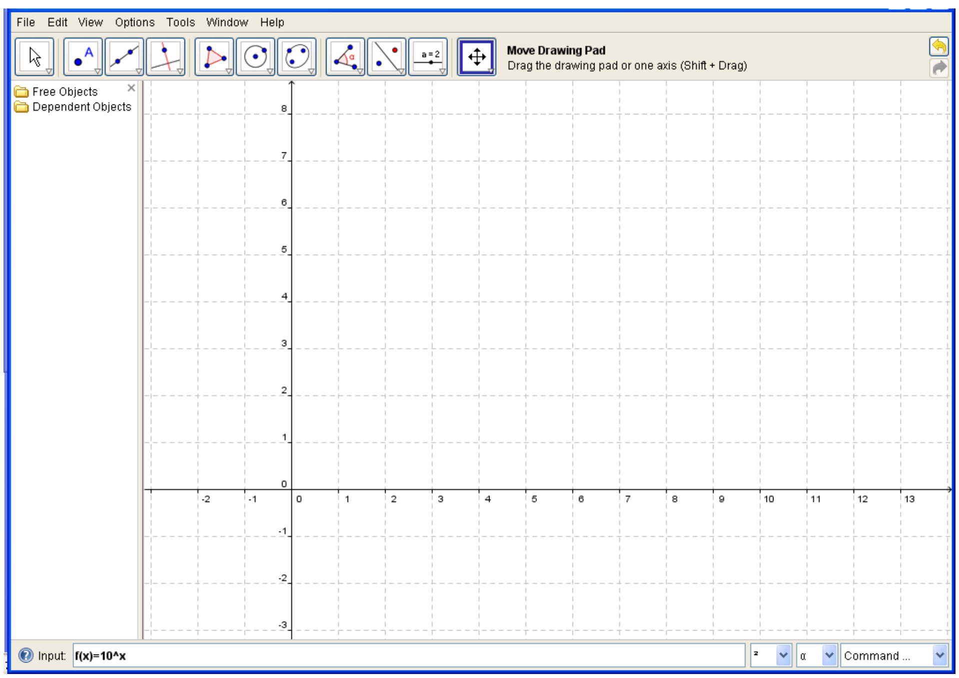
Geogebra allows you to move our construction around into a better position. To do this we use the **Move Drawing Pad** button

- Click on the **Move Drawing Pad** button  and use the left mouse button to drag the screen around.
- Click on the **Move Button**  when moving drawing pad is complete.
- Click on the **Undo** Button (arrow mark on the upper right corner) to move back to the last position. If you keep clicking, it will return to the original position.

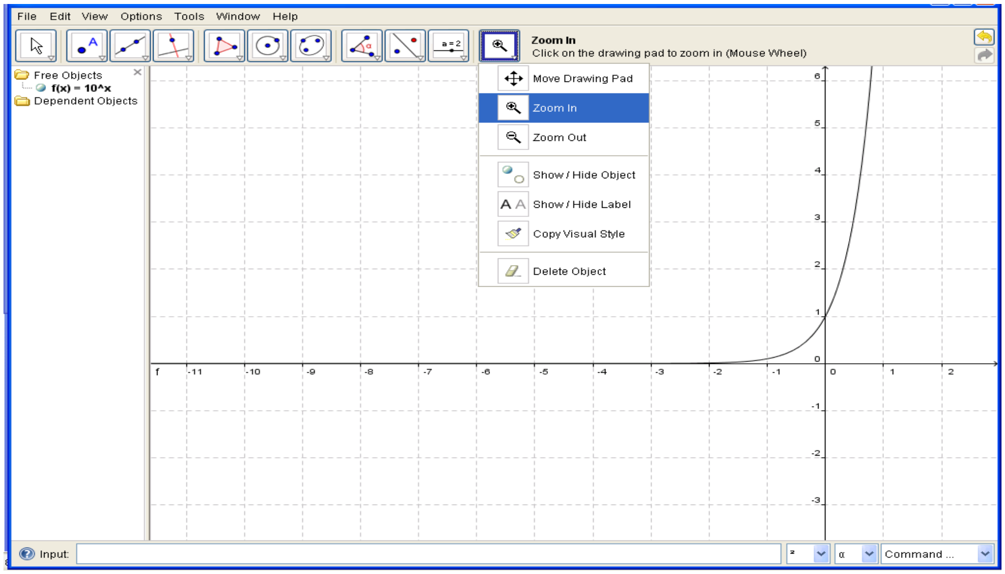
Tip : Hold down the Ctrl key and use the left mouse button to move the screen around. This will save a few clicks.

Activity 4 : Functions and the zoom tool

- Click on **File Menu** and select **New**. (This will open a new Geogebra window)
- Type f(x) = 10^x **(^ symbol = Shift Key + 6)** into the **Input Field.**



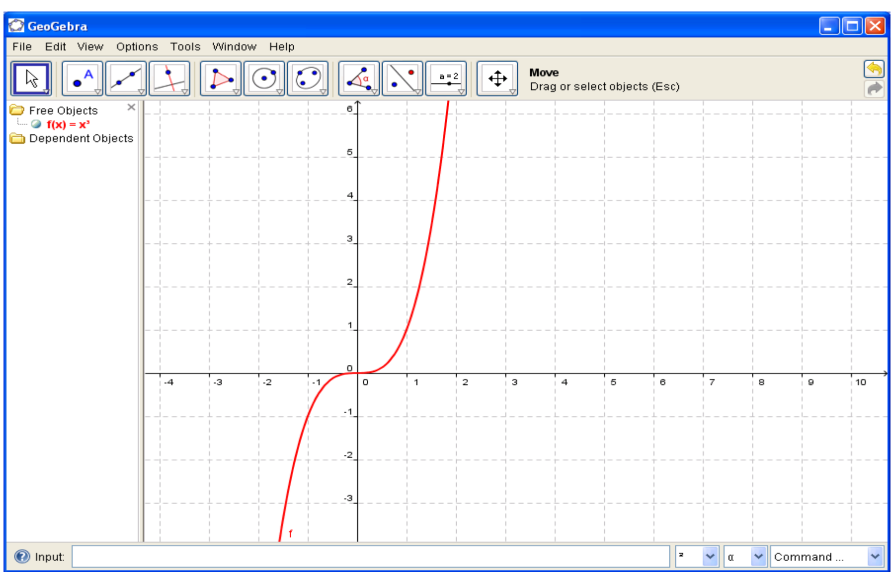
- Click on the **Move Drawing Pad**  button and drag the screen to the right.
- Click on the drop down arrow on the **Move Drawing Pad** button and choose **Zoom In** button 
- Click near the area where function approaches the x-axis to zoom in.



- Another way is to **Right Click and drag a selection box around an area you want to zoom in to. When you release it will zoom in. To return to the original view , Right Click and choose Standard View or use the yellow arrow (undo button at the upper right corner).**

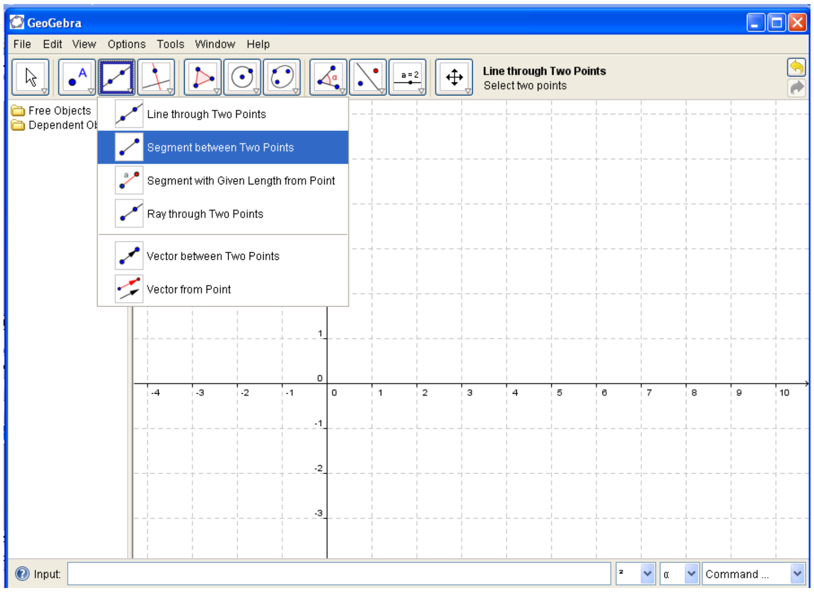
Activity 5 : Translating Functions

- Click on **File Menu** and select **New**. (This will open a new Geogebra window)
- Click in the **Input Field** and type f(x) = x^3
- Drag the curve around to see that its equation changes dynamically.

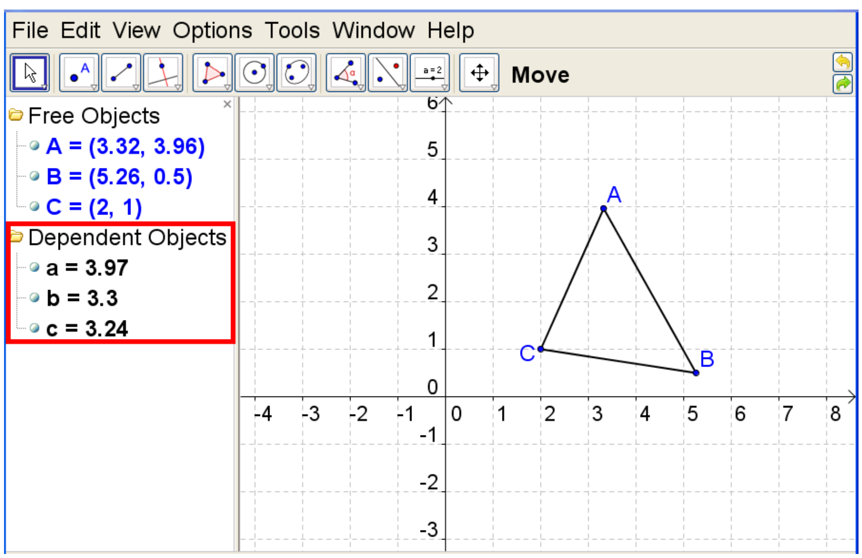


Activity 6 : Angles and Triangles

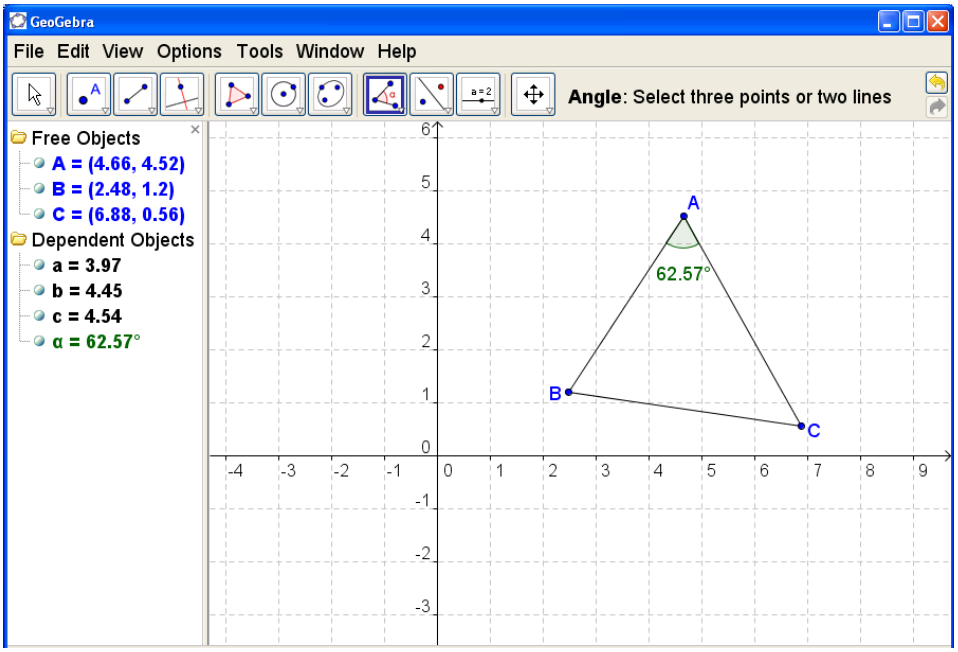
- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Click on the drop down arrow of the **Line through Two Points** and choose the **Segment between Two Points** button



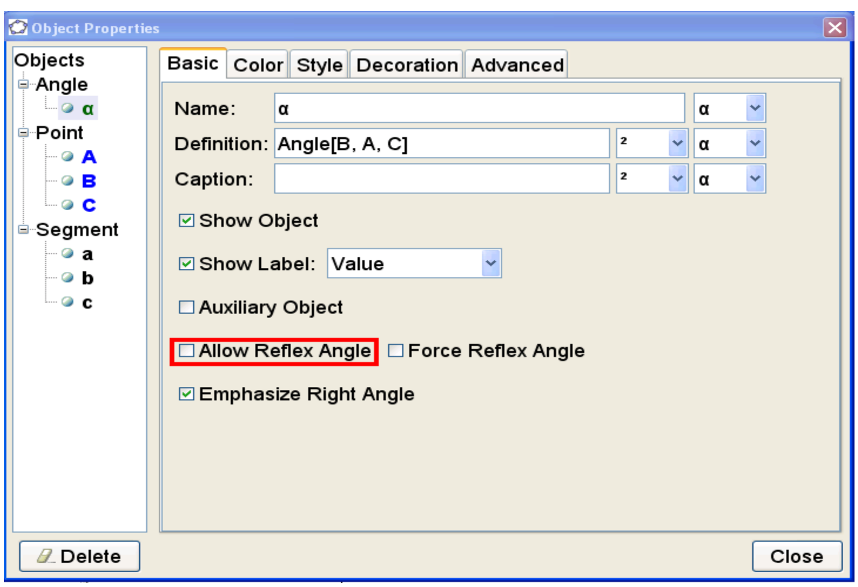
- Construct a triangle using **Segment between Two Points** button. You will see in the **Algebra Window** , the three segments (sides of triangle) are represented by their lengths.



- To see which length corresponds to which side , click on the **Move Tool** (arrow) and hover the mouse over one of the sides to see the corresponding length light up.
- Now to measure internal angles of the triangle , select the **Angle** Button and click once on vertices B , A and C in that order. This will give you measure of $\angle$BAC. (**Make sure that you select the vertices in clockwise direction. Anti-clockwise selection will give you reflex angle.)**

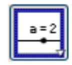


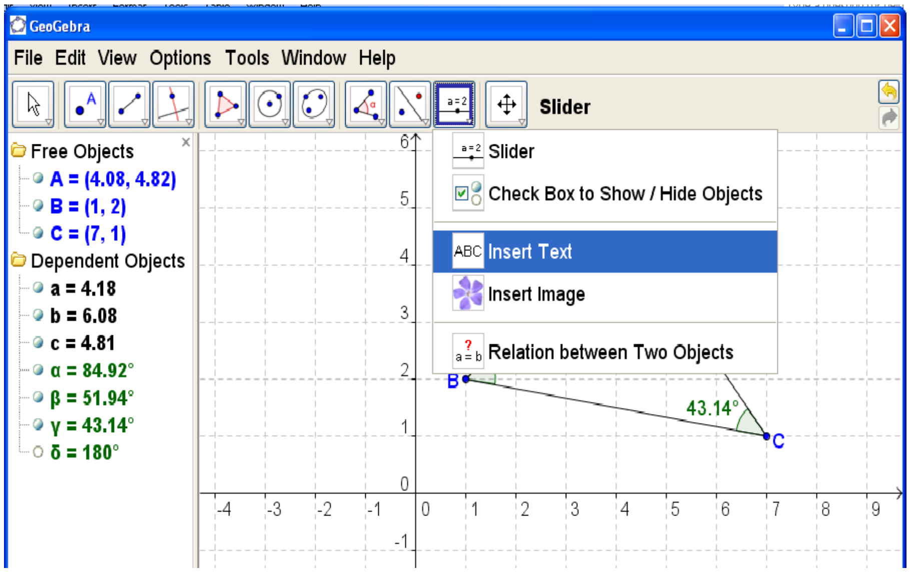
- Similarly, measure ∠CBA and ∠ACB.
- Now, drag the vertices using **Move** Button (Arrow). If you drag enough, you will notice reflex angles appearing. To stop this, right click on an angle in **Algebra or Geometry Window**, and choose **Object Properties**. Under Basic Tab **un-check** the **Allow Reflex Angle** check box and close the **Object Properties box.**



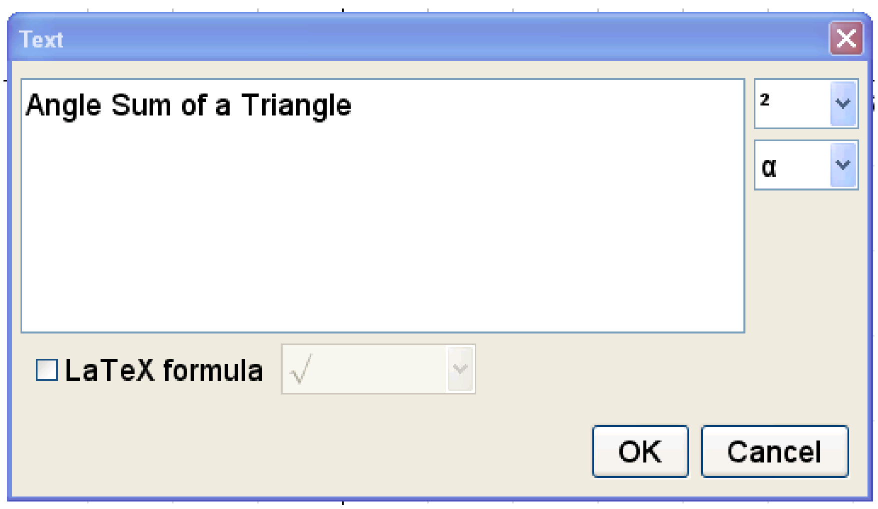
- Check your settings have been applied by dragging the vertices of your triangle.

Activity 7 : Adding Text

- Click on the drop down arrow of the **Slider** Button  and choose the **Insert Text** Button 



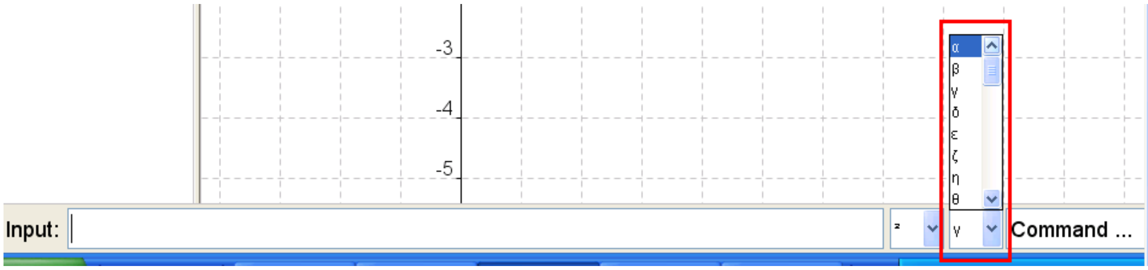
- Click once anywhere on the **Drawing Pad** and type “Angle Sum of a Triangle” in the box that appears. Press **OK** when finished.



- Select **Move** Button (Arrow) and drag the text to a suitable position on the screen.
- Right Click on the text and select **Object Properties**. Using Object Properties dialog box change the color , font size etc of the text. Click **close** when finished.
- To edit text , right click on the text and choose **Edit** , change the text to “ Angle Sum of a Triangle using Geogebra”. Select **OK** when finished.

Activity 8 : Calculation with Input Field

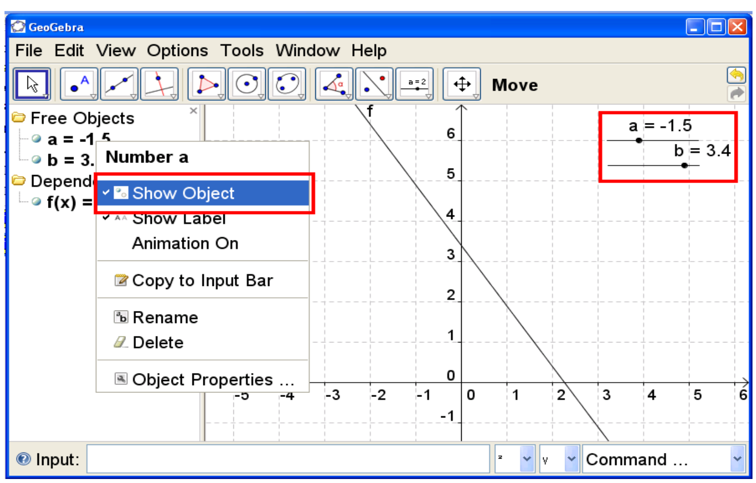
- Click in the **Input Field**
- Click on the **Drop Down Menu** on the right side of **Input Field** which contains Greek Letters.

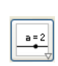


  - Using Greek Letter drop down menu , from the expression α+β+γ and press Enter key from keyboard to calculate the sum.
  - You will notice that δ=180° appears in the **Algebra Window**. Double click on δ=180° to see that this represents α+β+γ

Activity 9 : Sliders

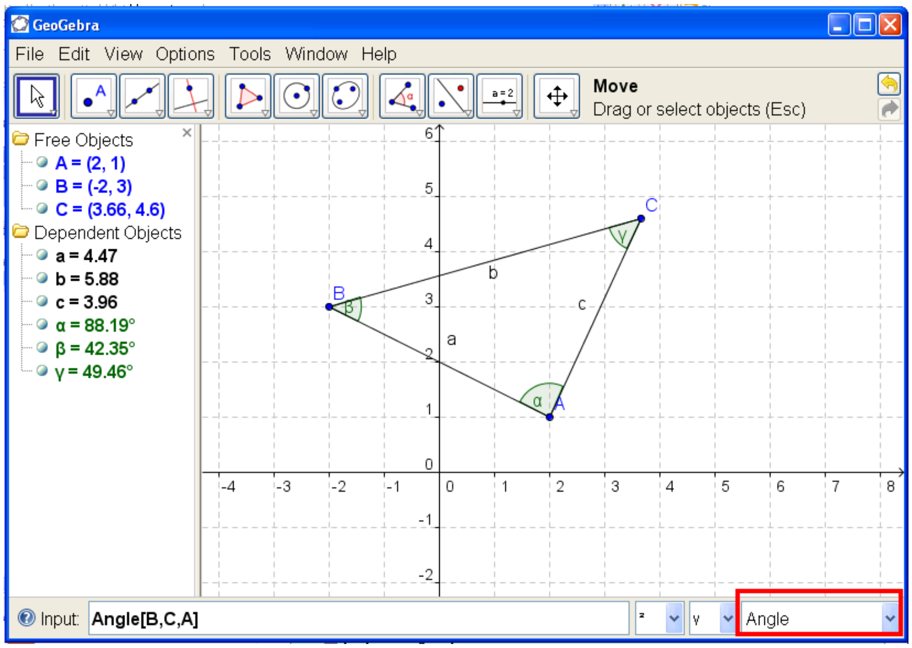
- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Click in the **Input Field**.
- Type a = 1 and press **Enter** key
- Type b = 1 and press **Enter** key
- Type f(x) = a*x + b and press **Enter** key. You can see the equation in the **Algebra Window**.
- In the **Algebra window** , right click on ‘a’ and choose **Show Object** to see slider ‘a’ on the **Drawing Pad.** Repeat the step for ‘b’ also.



- Select **Move** Button (Arrow) and drag the sliders to change the straight line.
- You can change the max , min and increment of the slider by right clicking it and selecting **Object Properties.**
- You can also add sliders by selecting the slider button  and clicking once on the screen.

Activity 10 : Using Commands

- In this activity you will construct a triangle using commands from **Input Field**.
- Click on **File Menu** and select **New.** (This will open a new Geogebra window)
- Click once in the **Input Field**.
- Type **A = (2,2)** and press Enter key. This will create a point A at (2,2)
- Type **B = (-3,2)** and press Enter key. This will create a point B at (-3,2)
- Type **C = (0,5)** and press Enter key. This will create a point C at (0,5)
- Type **Segment[A,B]** to create a segment between A and B
- Type **Segment[B,C]** to create a segment between B and C
- Type **Segment[C,A]** to create a segment between C and A. Your triangle ABC is ready.
- You can drag any of the points A , B or C and see the changes.
- Type **Angle[C,A,B]** to show angle A.
- Repeat the above step to show angles B and C.



- Command can be selected from Command Drop Down List which can be located at the bottom left corner of the Geogebra Window.

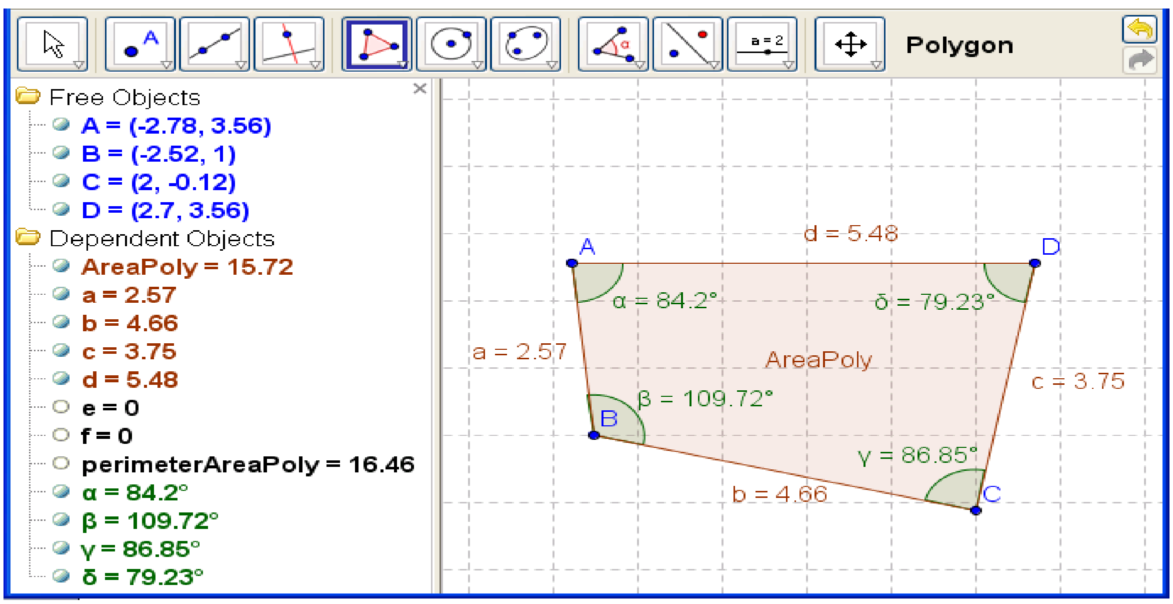
Activity 11 : Polygons

- Click on **File Menu** and select **New**. (This will open a new Geogebra window)
- Select **Polygon** Button  from the menu bar.
- Click on four different points on the drawing pad and then click back on the first point. Make sure that the points are selected in anti-clockwise direction. You will have a 4-sided polygon (quadrilateral)
- Your Algebra Window is now showing
    - The coordinates of four vertices

Lengths of three sides

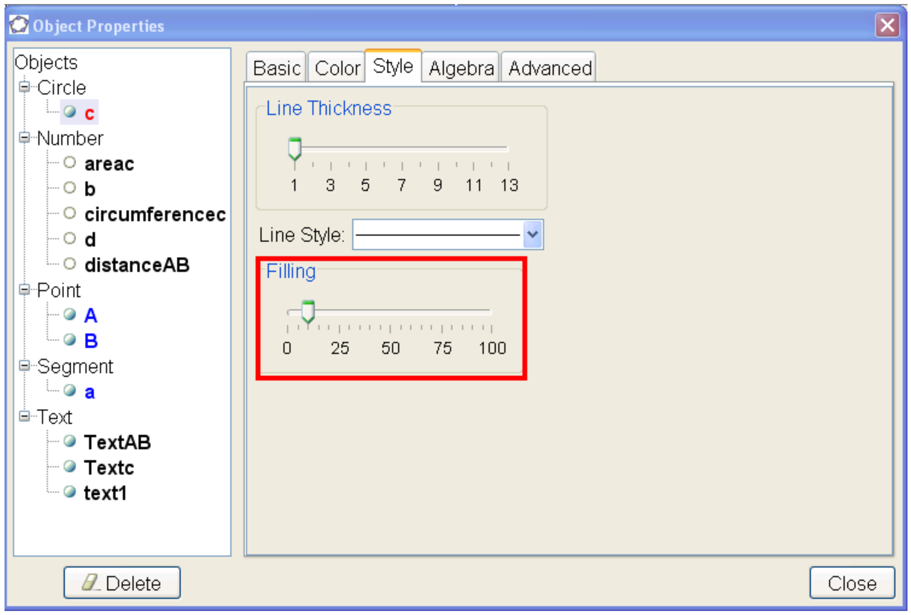
  - The area of the polygon , which is shown as poly1.

- Rename poly1 as AreaPoly1
- Select **Angle** Button and then click once inside the polygon. This will show all the angles of the polygon. Please note , if the vertices are not initially defined anticlockwise , you will see reflex angles.
- From the **Angle** Button drop down menu , select **Distance or Length** Button
- Click on each side of the polygon. This will display the side lengths of polygon.
- Now , with **Distance or Length Button** selected , click inside the polygon to display its perimeter.
- Using **Move** Button (Arrow) you can drag the vertices of polygon around to see the various measurements change.
- You can also move the whole polygon using mouse by dragging out a selection window that contains the polygon.
- Save your work in a **.ggb** file.

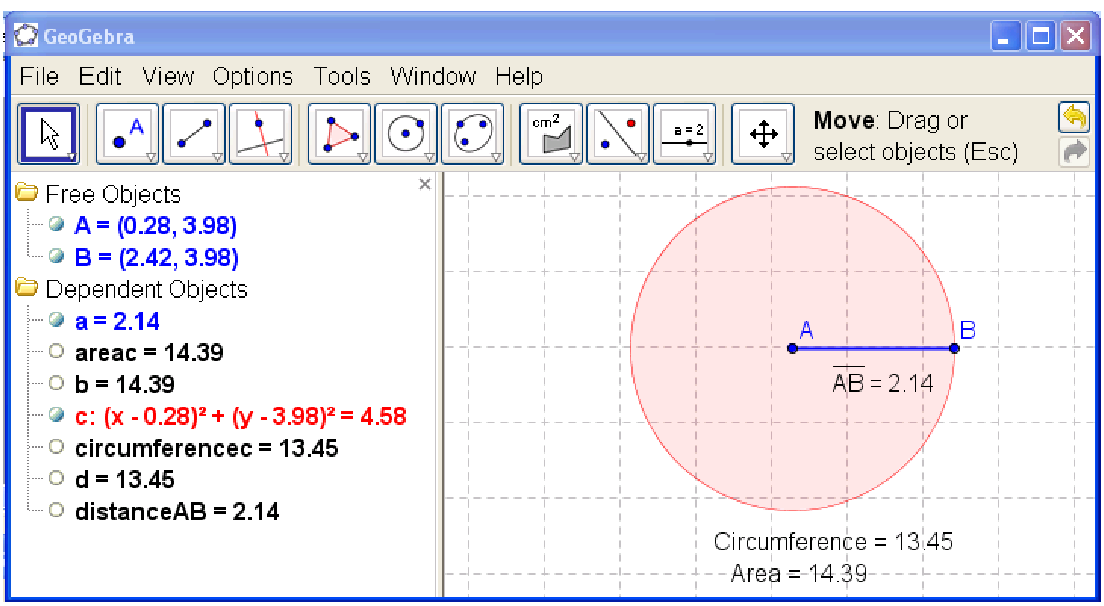


Activity 12 : Circles (Area and Perimeter)

- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Select **Circle** with **Center through Point** Button
- Click once on the **Drawing Pad** to define the center and move the mouse outward and click again to define the second point on the circle. You will have a circle on the drawing pad and corresponding algebraic representation (equation) in the Algebra Window.
- With the help of **Move** Button (Arrow) you can now drag any of the two points or drag circle itself and see how equation changes in the Algebra Window.
- Right click on circumference of the circle and select **Object Properties**. Increase **Filling under Style** tab. This will give you a feeling of circular region.

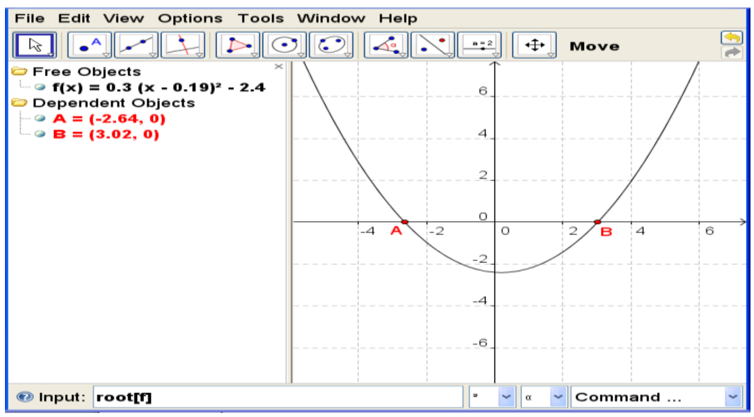


- Select **Distance or Length** Button from the **Angle** drop down button.
- Click once inside the circular region to measure and display Circumference of circle.
- Select **Area** Button from the **Angle** drop down button.
- Click once inside the circular region to measure and display Area of circle
- Again select the **Distance or Length Button** and measure the distance between the center and the point on the circle.
- Using the **Segment between Two Points** Button , construct the radius segment AB.



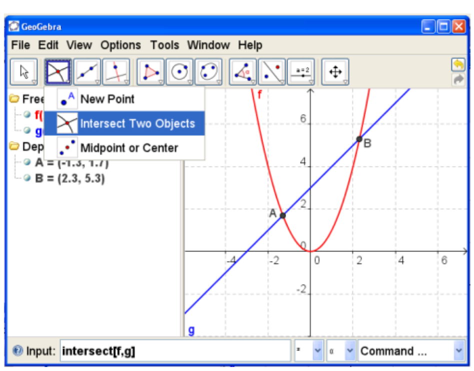
Activity 13 : Graphing

- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Click once in the **Input Field**.
- Enter the y = $0.3x^2$ in the input field. You can enter the expression in following ways
  - y = 0.3 x^2 [Note the space in between 0.3 and x^2]
  - y = 0.3*x^2
  - f(x) = 0.3*x^2
  - 0.3*x^2
- Drag the graph (parabola) to a position where it intersects with x-axis.
- The x-coordinates of points where parabola intersects x-axis are the roots of given polynomial.
- Type **root[f]** in the input box to locate roots of polynomial.
- Drag the parabola to see the roots move with the parabola.



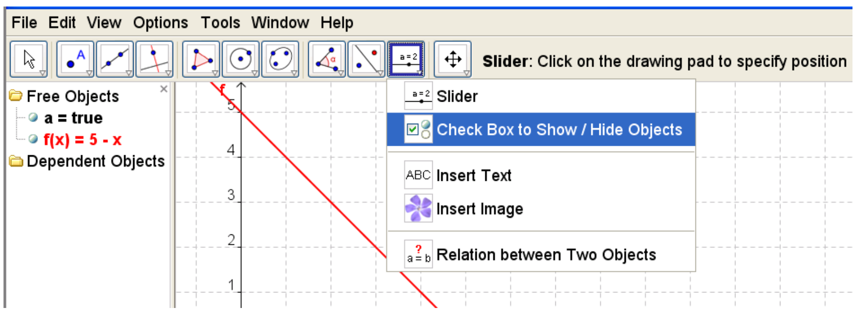
Activity 14 : Intersecting Graphs

- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Click once in the **Input Field.**
- Type f(x) = x^2 and press Enter
- Type g(x) = x + 3 and press Enter
- Click on the **New Point Button** and select the **Intersect Two Objects**  button.
- Click once on each of the two graphs. You will get the points of intersection of two graphs.
- You can also get the points of intersection by typing the command **Intersect[f,g]** in the **Input Field**.

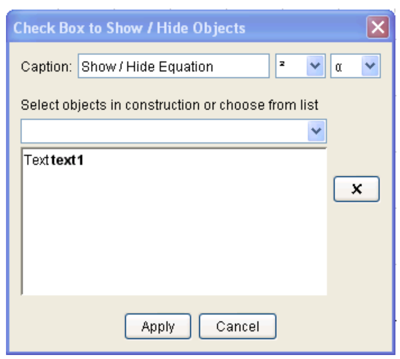


Activity 15 : Using Check Box to Show / Hide Objects

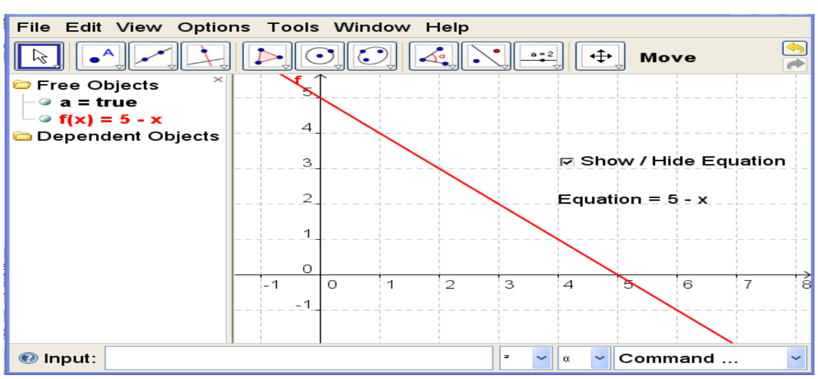
- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Click once in the **Input Field**
- Type 5 - x and press Enter.
- Select **Insert Text** button and click once on the drawing pad.
- Now type "Equation = " in the text box and click any where on the graph in the a **Drawing Pad** and press OK.
- Now select **Check Box** to **Show / Hide Object** Button by clicking on the drop down arrow of **Insert Text** Button



- Click once on the Drawing Pad and
  - Type "Show / Hide Equation " in the Caption section of the box.
  - Now click on "**Select Objects** in construction or choose from list " drop down menu and select Text text1 and press Apply.



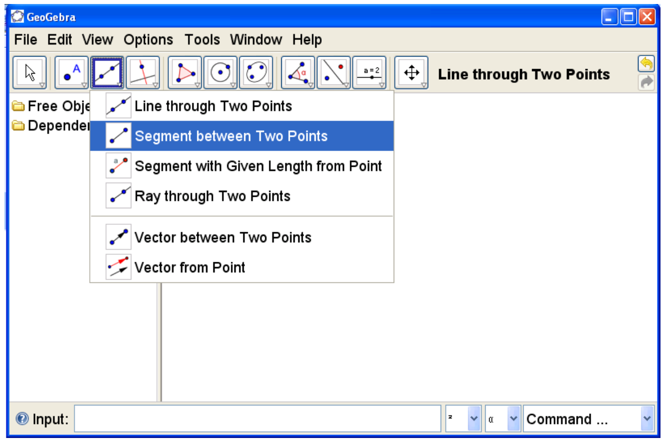
- Now tick **check / uncheck** the check box to show / hide the equation.



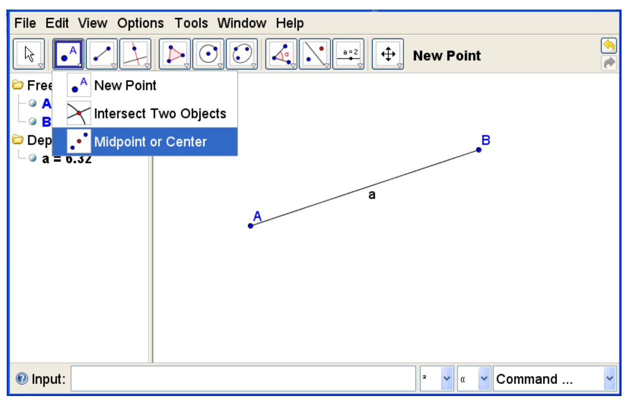
Activity 16 : Parallel and Perpendicular Lines

I . To construct the mid point of a line segment

- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Select **View / Axes** to hide the two axes.
- Follow the command sequence **Options / Labeling / New Points Only**.
- Select **Segment between Two Points Button** .



- Click on any two points on **Drawing Pad** to construct a line segment AB.
- Select **Midpoint or Center** button  under **New Point** tool.  and click any where on the line segment AB to locate its midpoint.

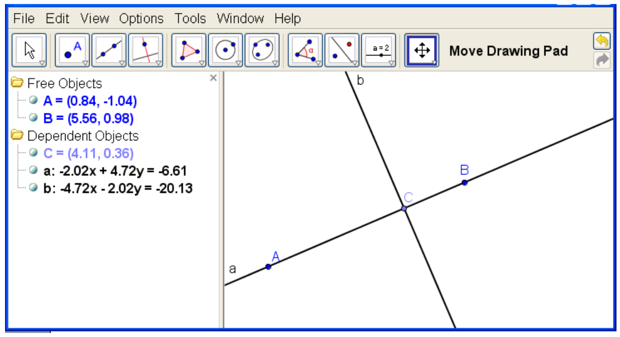


II. To construct a line perpendicular to a given line

A) Through a given point lying on the given line

- Click on **File Menu** and select **New.** (This will open a new GeoGebra window)
- Select **View / Axes** to hide the two axes
- Follow the command sequence **Options / Labeling / New Points Only**.
- Select the **Line Through Two Points** tool  and click anywhere on two points on drawing pad to construct a line.
  - Select **New Point** tool  and click any where on the line constructed in the previous step.

- Select the **Perpendicular Line** tool and click on the point and the line to construct the perpendicular line.

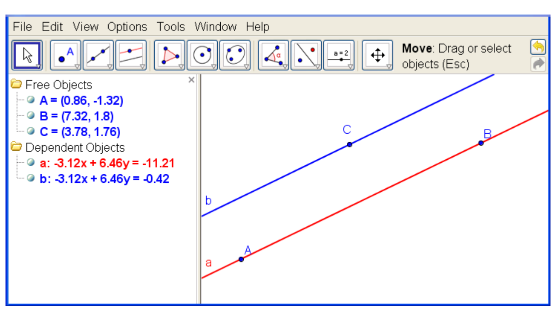


B) Through a given point lying outside the given line

- Click on **File Menu** and select **New.** (This will open a new GeoGebra window)
- Select **View / Axes** to hide the two axes.
- Follow the command sequence **Options / Labeling / New Points Only**.
- Select the **Line Through Two Points** tool and click anywhere on two points on drawing pad to construct a line.
- Select **New Point** tool . and click any where on the drawing pad except the line constructed in the previous step.
- Select the **Perpendicular Line** tool and click on the point and the line to construct the perpendicular line.

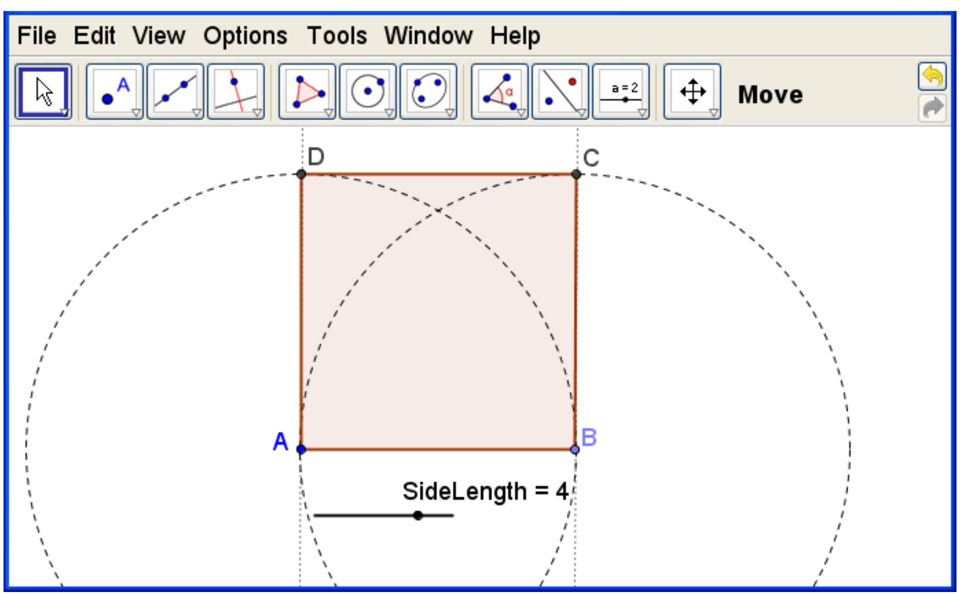
II. To construct a line parallel to a given line

- Click on **File Menu** and select **New**. (This will open a new Geogebra window)
- Select **View / Axes** to hide the two axes.
- Follow the command sequence **Options / Labeling / New Points Only**.
- Select the **Line Through Two Points** tool and click anywhere on two points on drawing pad to construct a line.
- Select **New Point** tool . and click any where on the drawing pad except the line constructed in the previous step.
- Select the **Parallel Line** tool and click on the point and the line to construct the perpendicular line.



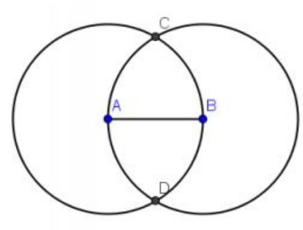
Activity 17 : Construction of a Square

- Click on **File Menu** and select **New.** (This will open a new GeoGebra window)
- Hide algebra window , input field and coordinate axes (View Menu).
- Follow the command sequence **Options / Labeling / New Points Only.**
- Create a slider Side Length with interval 1 to 5 and interval 1.
- Using **Segment with Given Length from Point** tool , construct a segment of length **SideLength.**
- Change value of slider to see how length of segment changes.
- Using **Perpendicular Line** tool , construct two perpendicular lines to constructed line segment at points A and B.
- Using **Circle with Center and Radius** tool construct a circle with center A and B with radius SideLength.
- Using **Intersect Two Objects** tool locate points of intersection of circle and perpendicular lines at points A and B.
- Rename the located points to C and D.
- Hide all construction except points A , B , C and D. (Right click – Show Object)
- Join points A , B , C and D using **Polygon** tool .
- By changing the value of **SideLength ,** we can get squares of different sizes.

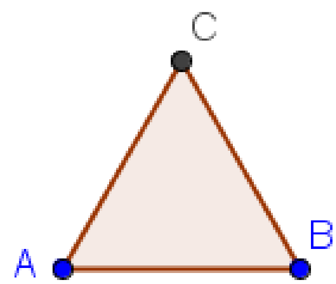


Activity 18: Construction of an Equilateral Triangle

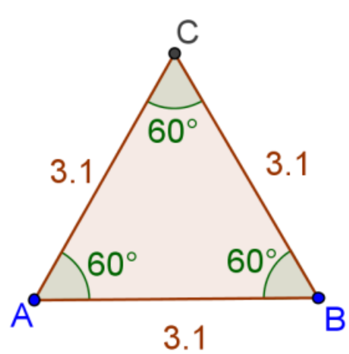
- Click on **File Menu** and select **New**. (This will open a new GeoGebra window)
- Hide algebra window , input field and coordinate axes (View Menu).
- Follow the command sequence **Options / Labeling / New Points Only.**
- Create a slider **SideLength** with interval 1 to 5 and interval 1.
- Using **Segment with Given Length from Point** tool , construct a segment AB of length SideLength.
- Change value of slider to see how length of segment changes.
- Select Circle with **Center through Point** tool , first click on point A and then on point B.
- Next , repeat the previous step , this time first click on point B and then on point A.
- Next select **Intersect Two Objects** tool and click once on two circles constructed in previous two steps. This will locate points of intersection of two circle. Now your construction should look like the following



- We need only three non-collinear points to form a triangle.
- Hide the two circles , point D and segment AB.
- Select **Polygon** tool , and click on points A , B , C and back on point A to make a triangle ABC.

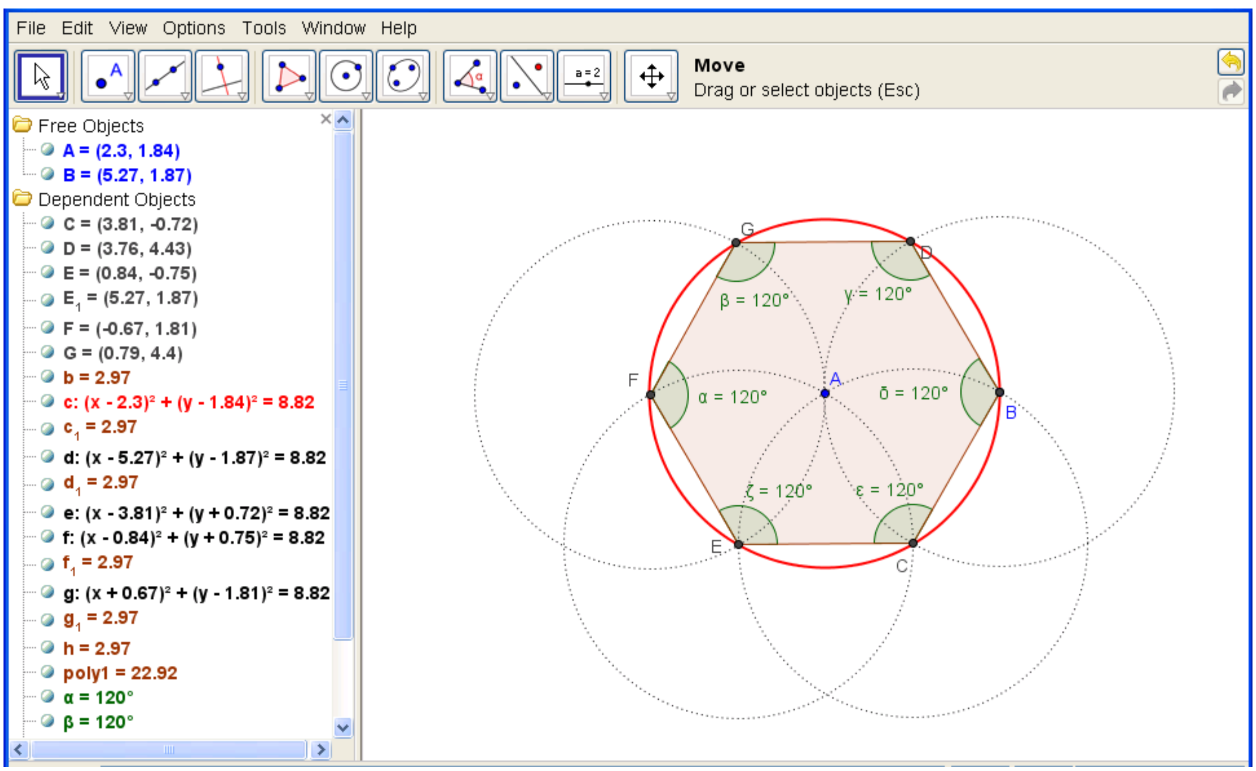


- Select **Angle** tool and click anywhere inside the triangle. This will show measurement of all the interior angles. For an equilateral triangle all the angles should be 60°.
- To verify the lengths of sides , select the **Distance or Length** tool and click on three sides of triangle , this will show length of sides of triangle.



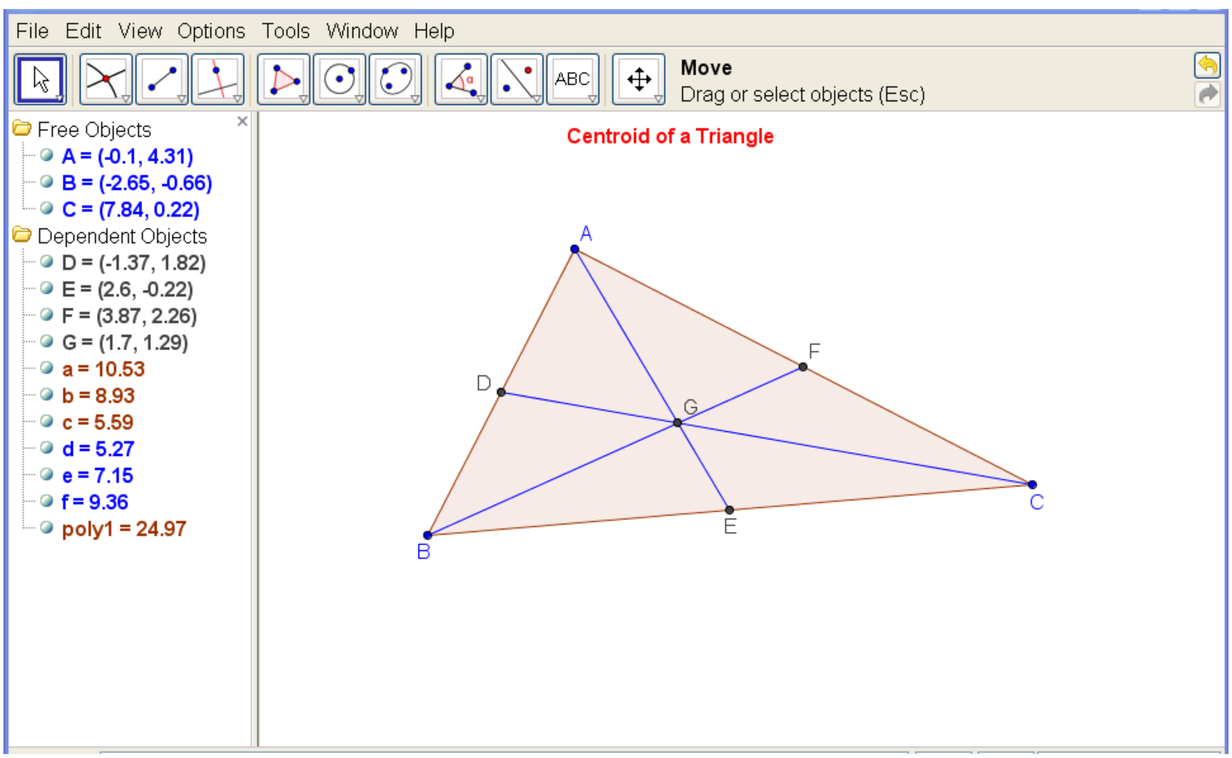
Activity 19 : Construction of a Regular Hexagon

- Click on File Menu and select New. (This will open a new GeoGebra window)
- Hide algebra window , input field and coordinate axes (View Menu).
- Follow the command sequence Options / Labeling / New Points Only.
- Select Circle with Center through Point Tool and construct a circle with center A and passing through point B.
- Construct a circle with Center B and passing through point A.
- Using Intersect Two Objects Tool locate points of intersection C and D of above two circles.
- Now construct a new circle with center C and passing through A.
- Locate point of intersection E of new circle with the first circle.
- Construct circle with center E and passing through A and locate point of intersection F of new circle with first circle.
- Similarly , construct circle with center F and passing through point A and locate point of intersection G of new circle with first circle.
- Using Polygon Tool construct the hexagon BCEFGD. (Make sure to select vertices in anti-clockwise direction).
- Select Angle Tool , and click anywhere inside the hexagon , this will give measurement of all the interior angles.
- Change style , colour , line thickness etc using object properties menu.



Activity 20 : Centroid of a Triangle

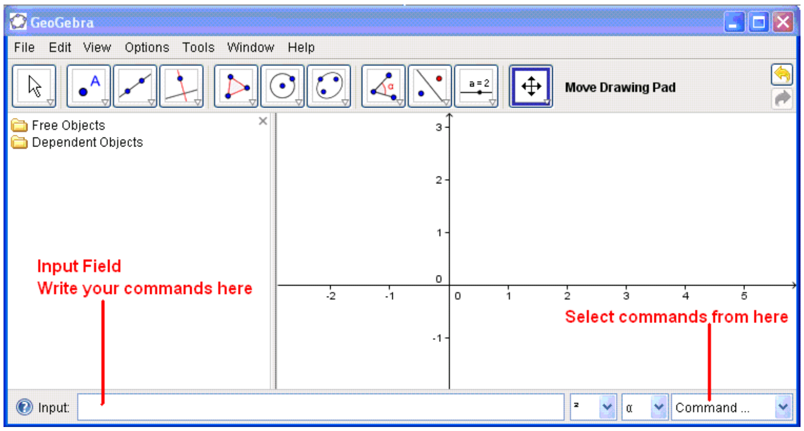
- Click on File Menu and select New. (This will open a new GeoGebra window)
- Hide algebra window, input field and coordinate axes (View Menu).
- Follow the command sequence Options / Labeling / New Points Only.
- Select the Polygon Tool and construct an arbitrary triangle ABC.
- Using Midpoint or Centre Tool , locate mid points D , E and F of three sides AB , BC and CA respectively.
- Using Segment Between Two Points Tool , construct segments AE , BF and CE. These segments are called median of triangle ABC. You can see that the three medians are concurrent i.e. they pass through the same point.
- Locate point of intersection of these medians by using Intersect Two Objects Tool and selecting any of the two medians AE , BF or CE successively.
- The point of intersection G of medians is called the Centroid of the triangle ABC.
- Using Move Tool (Arrow) , drag vertices A , B or C. You will observe that the centroid of a triangle always lies inside the triangle.

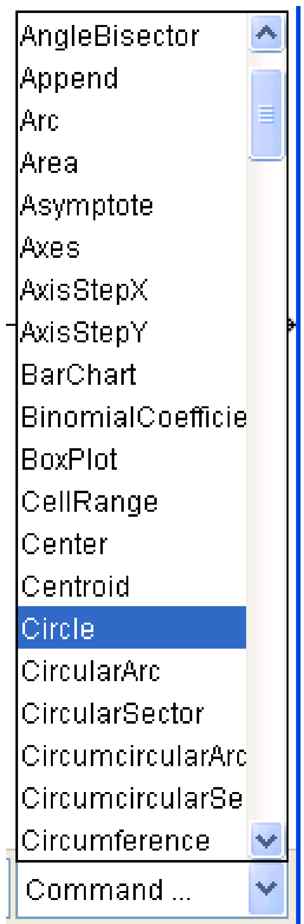


Using Keyboard Commands in GeoGebra

This tutorial explains how to construct Perpendicular Bisector of a Line Segment using keyboard commands of Geogebra entered through Input Field located at the bottom of the GeoGebra window.

List of commands can be found at the lower right corner of the GeoGebra window. Just click on the down arrow of Command list box and you will see the list of commands. See figure below :





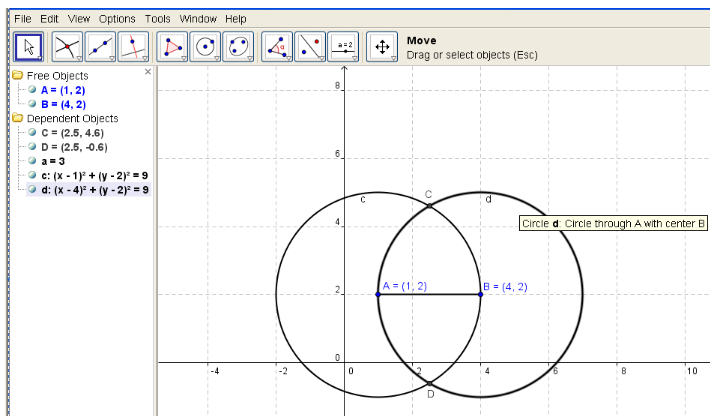
List of Commands

For the purpose of our tutorial, we will write the following commands in order to construct perpendicular bisector of a given line segment. We will also write commands for verification.

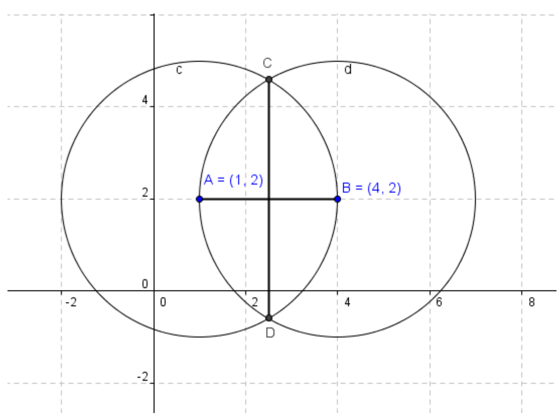
Before starting construction select Options > Labeling > New Points Only to give labels to newly constructed points.

Steps of construction

1. Open GeoGebra
2. To plot first point of our segment AB , write A = (1,2) in Input Field and press enter. Similarly plot point B by writing B = (4,2) in Input Box and pressing enter key.
3. Construct segment AB by typing Segment[A,B]
4. Construct a circle with centre at A and radius AB. For this write Circle[A,B] in input box and press enter.
5. Next construct circle with centre B and radius BA (or AB) by using command Cicle[B,A]
6. Now we will locate the points of intersection of two circles. For this write Intersect[c,d] . c and d are the names given to constructed circles by geogebra. You can see name of any object just by moving mouse over that object as shown below :



7. C and D are the points of intersections of two circles.
8. Now construct segment CD by command Segment[C,D].
9. After this step construction should look like this :



10. CD is the required perpendicular bisector of segment AB

Verification : You can verify the construction yourself with following steps :

1. Locate intersection point E of AB and CD
2. Find lengths of AE and EB (they should be equal)
3. Find the measure of angle AEC or angle BEC (they should be 90°)

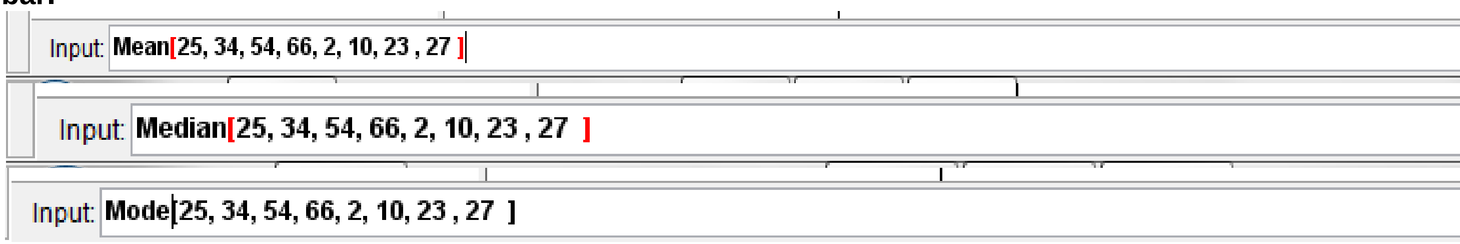
Statistics

Many statistical operations can be done using GeoGebra , with or without using the Spreadsheet View. It has a number of statistical functions and graphing tools.

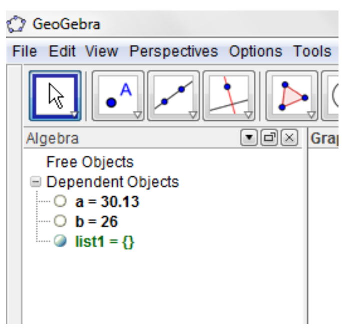
Method 1 :

Using the Input bar only :

To find the Mean , Median and Mode of 25, 34, 54, 66, 2, 10, 23 and 27 **type the following in the Input bar.**



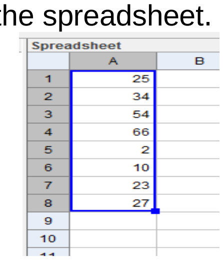
GeoGebra lists the results in the **Algebra** window as Mean = 30.13 , Median = 26 , list1 = {}



Method 2 :

The Input bar and the Spreadsheet view

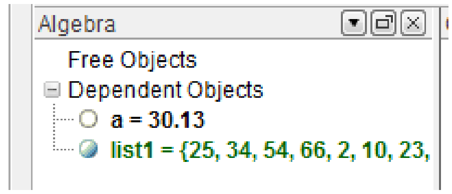
1. Open the **Spreadsheet** view
2. Click on **View menu / Spreadsheet** view
3. Type the data in the first column of the spreadsheet.



Spreadsheet

	A	B
1	25	
2	34	
3	54	
4	66	
5	2	
6	10	
7	23	
8	27	
9		
10		

4. Select the data by dragging the mouse.



5. Right click on the selected block and select **Create list**

   Type the following in Input bar

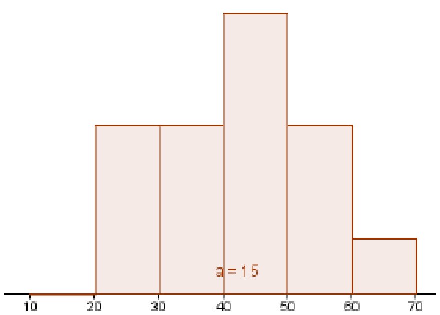
Drawing a histogram

Using the Input Bar

Type the following in the Input bar to creates a histogram using the raw data: Histogram[{List of Class Boundaries}, {List of Raw Data}]

The class boundaries determine the width and position of each bar of the histogram.

GeoGebra will construct the following histogram:



Spreadsheet View

Open the Spreadsheet View:
click the View menu / Spreadsheet View

Spreadsheet View Ctrl+Shift+S

Type the data in the first column of the spreadsheet.

Select the data by dragging the mouse.

Right-click on the selected block and select Create List.

Create List

The following list will appear in the Algebra window:

Free Objects
Dependent Objects
L_1 = {48, 38, 42, 54, 40, 34, 58, 44, 52, 36, 26, 46, 60, 20, 26}

Type the following in the Input bar

Input: Histogram[{10, 20, 30, 40, 50, 60, 70}, L_1]

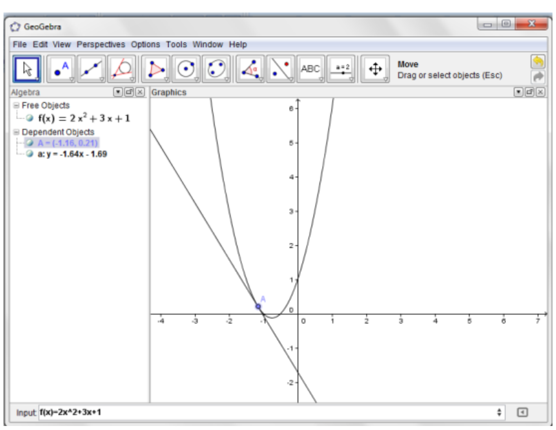
Calculus activities

To construct a tangent to the graph of the function $f(x)= 2x^2+3x+1$

1. Type the equation **$f(x)= 2x^2+3x+1$** in the **Input Bar** and press **Enter.**

Input: f(x)=2x^2+3x+1

2. Select the **New Point** icon from the **ConstructionTools** and click on the graph**.**
3. Select the **Tangents icon** **from the Toolbox ,** click on the **point** and then on the **graph.**
4. **Drag the point** on the graph and observe the tangent.

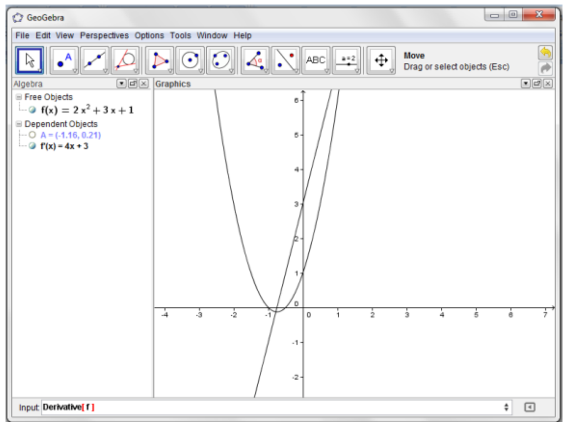


First and Second Derivatives

1. Type the equation **f(x)= 2x^2+3x+1** in the **Input Bar** and press **Enter.**

2. Type the following command in the **Input Bar** and press **Enter.**

GeoGebra will calculate the derivative in the Algebra View and display the function f'(x) in the Graphics View as shown



GeoGebra can also calculate the Second Derivative f''(x)

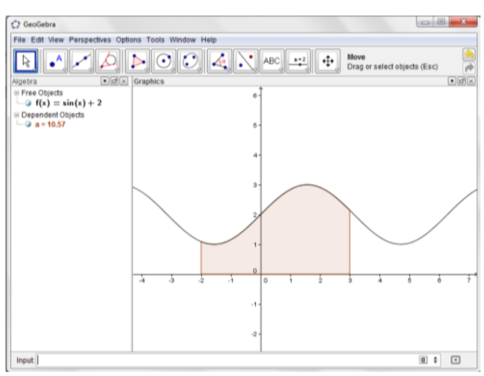
Definite Integrals

To find the area under a curve

1. Type the equation of the **function** in the **Input Bar** and press **Enter**

2. Type the following command.

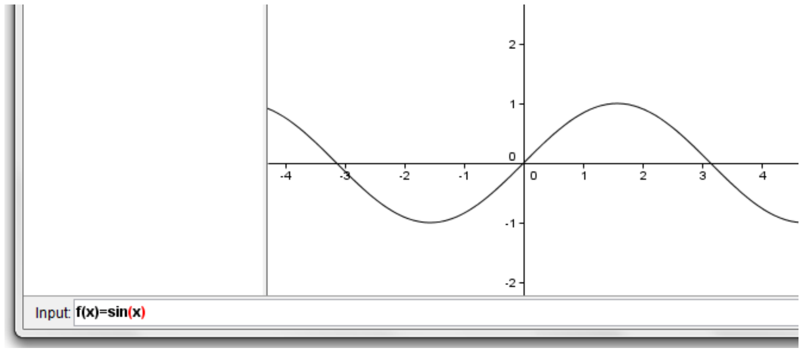
GeoGebra calculates the definite integral of the function in the interval [-2 , 3] and shows it in the Algebra View and displays the image in the Graphics View.

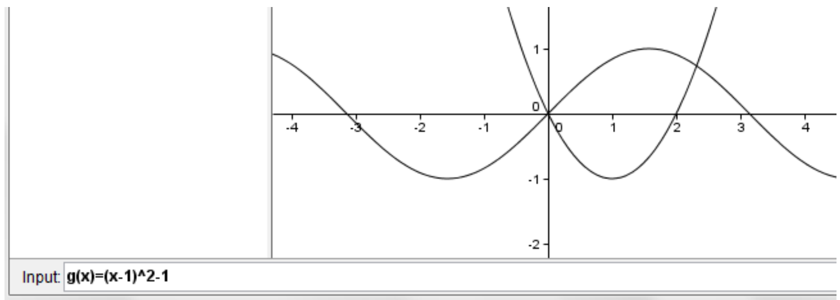


Area between two curves

To find the area between the curves f(x)=sin(x) and g(x)=(x-1)^2-1 between their points of intersection

1. Enter the equations in the **Input Bar** one by one **.**

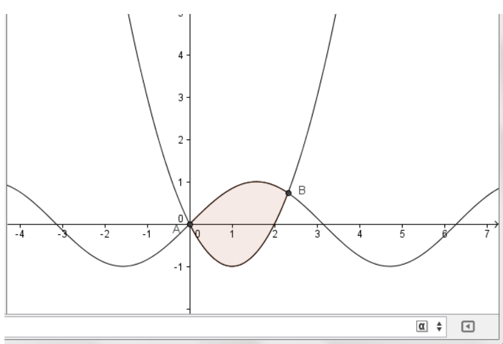




2. From the **Construction Tools** select **Intersect Two Objects** and **click on both functions.** You will find the **points of intersections as A and B.**

3. Input the following command

GeoGebra will construct and calculate the area between the curves within the interval [abscissa of A , abscissa of B] and return the value as **a = 2.9** in the Algebra View.



Activity for self study

Find the area bounded by the functions $f(x) = x^2$ and $g(x) = |x| + 1$

Find the area bounded by the curves $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$