UNIT – II
OPTICS FOR ENGINEERS

Syllabus

- **Prerequisite:**

  Wave front and Huygens's principle, reflection and refraction, interference, Constructive and destructive interference, Young's double slit experiment, diffraction, comparison of Fresnel diffraction and Fraunhofer diffraction

- **Interference:**

  Thin Film Interference: Introduction (division of amplitude), Stoke's relation, Interference in thin film of constant thickness in reflected light, Formation of colors in thin film (point source and extended source); Interference in Wedge shaped film; Formation of Newton's rings; Applications (Antireflecting & High reflecting films).

- **Diffraction:**

  Introduction (distinguish between interference and diffraction), Fresnel and Fraunhofer diffraction, Fraunhofer diffraction at single slit & double slit (qualitative), Diffraction Grating, Absent spectra, Resolving power of a grating & Dispersive power of a grating (qualitative), Applications.

Interference

Definition:

The modification of intensity due to superposition of two or more waves of light is called **interference.**

Superposition:

When two or more waves arrive at a point in a space simultaneously, the resultant intensity at that point is vector sum of their individual intensity.

Condition for interference:

- At least two waves are required.
- Both waves are identical & coherent.

But coherent waves are not exists in nature.

There are following methods to produce coherent waves.

(a) Division of amplitude:

The incident beam is divided into two or more beams due to partial reflection at the surface of thin film.

The two or more beams generated are identical and coherent.

The amplitude and therefore the intensity of the original wavefront get divided therefore it called **division of amplitude**.

Stokes Relation:

The phase change of π or path difference $\lambda/2$ occurs when light waves are reflected from the surface of denser medium and no change in phase occurs when light waves are reflected at the surface of rarer medium.

Type of Interference:

Constructive Interference:

When two or more waves arrive at a point are in phase with each other then resultant intensity is increases after mixing is called constructive interference.

It gives bright band or maximum.

Destructive Interference:

When two or more waves arrive at a point are out of phase with each other then resultant intensity is decreases after mixing is called destructive interference.

It gives dark band or minimum.

Interference in Thin Film:

Que: Derive the condition for a thin transparent film of constant thickness to appear bright and dark when viewed in reflected light.

Consider monochromatic light 'AB' of wavelength 'λ' from an extended source is incident at B, on the upper surface of thin film of thickness 't' and refractive index 'μ' as shown in fig.

Let 'i' be the angle of incidence. Incident ray is multiply reflected and refracted at point B, C, D, E & gives out BR_1, DR_2 reflected & CT_1, ET_2 as transmitted rays.

Thus we get a set of parallel reflected rays BR_1, DR_2 & a set of parallels transmitted rays CT_1, ET_2,

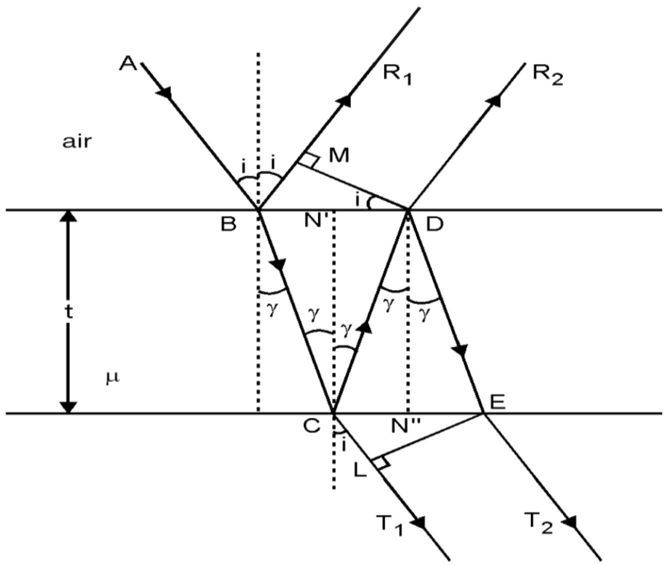


Fig. Interference in thin films

For Reflected system:

To find the path difference between BR_1, and DR_2, draw DM perpendicular to BR_1. The paths travelled by the beams beyond DM are equal.

Hence the optical path difference between them is

$$\Delta = \text{Path BCD in film} - \text{Path BM in air}$$

$$\Delta = \mu\,(BC + CD) - BM \text{ ------- (1)}$$

From Fig. in ∠BCD,

$$\cos r = \frac{CN'}{BC}$$

But, CN' = t

$$\therefore BC = CD = \frac{t}{\cos r}$$

$$\therefore \quad \mu\,(BC + CD) = \frac{2\,\mu\, t}{\cos r} \text{ ------- (2)}$$

From Fig. in ∠BDM,

$$\sin i = \frac{BM}{BD} = \frac{BM}{2BN'}$$

$$BM = 2\,BN'\sin i$$

Again in ∠BCD,

$$\tan r = \frac{BN'}{CN'} = \frac{BN'}{t}$$

$$\therefore BN' = CN' \tan r = t \cdot \tan r$$

$$BM = 2t \tan r \sin i$$

$$BM = 2t\frac{\sin r}{\cos r}\sin i$$

Multiply & divide above equation by sin r we get,

$$BM = 2t\frac{\sin^2 r}{\cos r}\frac{\sin i}{\sin r} \qquad (\because \frac{\sin i}{\sin r} = \mu)$$

$$BM = \frac{2\,\mu\, t}{\cos r}\sin^2 r \;\text{......... (3)}$$

∴ Put equation (2) & (3) in equation (1) the optical path difference between the rays is,

$$\Delta = \frac{2\,\mu\, t}{\cos r} - \frac{2\,\mu\, t}{\cos r}\sin^2 r$$

$$\Delta = \frac{2\,\mu\, t}{\cos r}\,(1 - \sin^2 r)$$

$$\therefore \boldsymbol{\Delta = 2\,\mu\, t \cos r}$$

The film is optically denser than the surrounding medium air. Hence the ray BR_1 originating by reflection at the denser medium suffers a phase change of π or a path change of $\frac{\lambda}{2}$ due to reflection at B. (No such change of phase occurs for ray DR_2)

Hence effective path difference between BR_1 & DR_2 is

$$\boldsymbol{\Delta = 2\,\mu\, t \cos r + \frac{\lambda}{2}} \text{ ------- } \mathbf{(4)}$$

Condition for maxima or constructive interference in Reflected Light:

The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

$$\Delta = n\lambda$$

$$2\,\mu\, t \cos r + \frac{\lambda}{2} = n\,\lambda$$

$$\mathbf{2\,\mu\, t \cos r = (2\,n - 1)\,\frac{\lambda}{2}} \text{ ------- (5)}$$

Where, n = 1, 2, 3, 4,

or $\mathbf{2\,\mu\, t \cos r = (2\,n + 1)\,\frac{\lambda}{2}}$

where, n = 0, 1, 2, 3,….

Condition for Minima or destructive interference in Reflected Light:

The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ i.e.

$$\Delta = (2n + 1)\,\lambda/2$$

$$2\,\mu\, t \cos r + \frac{\lambda}{2} = (2\,n + 1)\,\frac{\lambda}{2}$$

$$\mathbf{2\,\mu\, t \cos r = n\,\lambda} \text{ ------- (6)}$$

where n = 0, 1, 2, 3, ...

Interference in Wedge Shaped Film:

Consider a film of non-uniform thickness as shown in Fig.

Let OX and OX′ are two surfaces inclined by an angle θ. The angle θ between the surfaces OX and OX′ is known as the **angle of wedge**.

The thickness of the film gradually increases from O to X. Such a film of non-uniform thickness is known as wedge shaped film. The point O at which the thickness is zero is known as the edge of the wedge.

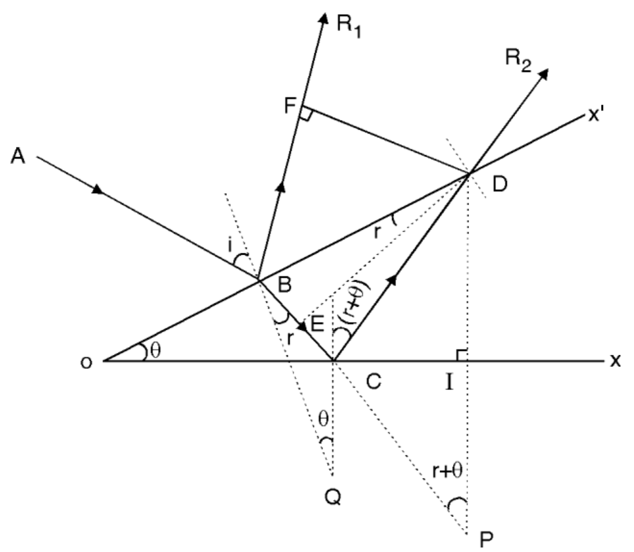


Fig. Interference in wedge shaped film

Consider a beam of monochromatic (AB) light of wavelength ‘λ’ is incident at an angle ‘i’ on the upper surface of the film. It is reflected, refracted & gives out BR1 and DR_2 as a two coherent beam. When BR1 and DR_2 mixed with each other Interference is occur.

To find the path difference between these two rays, draw DF perpendicular to BR_1.

The optical path difference between the rays BR_1 and DR_2 is

$$\Delta = \mu (BC + CD) - BF \text{ ------- (1)}$$

Draw a perpendicular to surface OX at point C.

We have the perpendicular to OX′ at point B. Both these perpendiculars will meet at point Q as shown in diagram.

$$\therefore \angle BQC = \angle XOX' = \theta$$

Draw a perpendicular DE on BC from D.

As θ is small enough, BE = EC

Also from diagram ∠ QBE = r = ∠ BDE

Draw a perpendicular from D on OX such that intersects OX at I and BC produced at P. Also we get CP = CD.

Equation (1) can be written as

$$\Delta = \mu (BC + CD) - BF$$

$$\Delta = \mu (BE + EC + CP) - BF$$

From diagram

$$\mu = \frac{\sin i}{\sin r} = \frac{BF}{BE}$$

or $BF = \mu BE$

$$\therefore \Delta = \mu (BE + EC + CP) - \mu BE$$

$$\Delta = \mu EP \quad (\text{as } E - C - P)$$

Now consider ΔDPC

as CP = CD, ∠ CPD = r + θ

and Δ DPE is a right angle triangle

$$\therefore \cos (r + \theta) = \frac{EP}{DP}$$

$$\therefore EP = DP \cos (r + \theta) = 2t \cos (r + \theta)$$

where DP = 2 DI = 2t , t = thickness of film at point D

$$\therefore \Delta = \mu EP$$

$$\mathbf{\Delta = 2 \mu t \cos (r + \theta)}$$

Due to reflection at B, an additional path change of $\frac{\lambda}{2}$ occurs for the ray BR_1. Hence the total path difference between the interfering rays is,

$$\mathbf{\Delta = 2 \mu t \cos (r + \theta) + \frac{\lambda}{2}} \text{ ------- (2)}$$

Condition for maxima or constructive interference:

$$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = n\lambda$$

$$\mathbf{2 \mu t \cos (r + \theta) = (2n - 1) \frac{\lambda}{2}} \text{ ------- (3)}$$

n = 1, 2, 3, 4 .....

Condition for minima or destructive interference:

$$2 \mu t \cos (r + \theta) + \frac{\lambda}{2} = (2n - 1) \frac{\lambda}{2}$$

$$\mathbf{2 \mu t \cos (r + \theta) = n\lambda} \text{ ------- (4)}$$

n = 0, 1, 2, 3,.....

Case-1

For normal incidence and air film, r = 0 and μ = 1

Total path difference $\mathbf{\Delta} = 2t \cdot \cos \theta + \frac{\lambda}{2}$

$2t \cos \theta = (2n - 1) \frac{\lambda}{2}$, For maxima

and $2t \cos \theta = n\lambda$, For minima

Case-2 For very small angle of the wedge,

As $\theta \to 0, \cos \theta \to 1$

$2t = (2n - 1) \frac{\lambda}{2}$ For maxima

$2t = n\lambda$ For minima

Spacing between Two Consecutive Bright Bands in wedge shape film:

Que: Derive the expressions for fringe width in a wedge shaped thin film. Also explain the applications.

For the wedge shaped film, we have for the n^{th} maximum.

$$2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2}$$

For normal incidence and air film, **r = 0 and μ = 1**

$$\mathbf{2t\cos\theta = (2n-1)\frac{\lambda}{2} \quad \ldots\ldots (1)}$$

Where, t is the thickness corresponding to n^{th} bright band.

Consider Fig. 2.5.1. The n^{th} bright band is produced at a distance x_n from the edge of the wedge.

$$\mathbf{t = x_n \cdot \tan\theta \quad \ldots\ldots (2)}$$

∴ Substituting for t in Equation (1)

we have,

$$2\cdot x_n \tan\theta \cos\theta = (2n-1)\frac{\lambda}{2}$$

$$\text{or} \quad \mathbf{2x_n \cdot \sin\theta = (2n-1)\frac{\lambda}{2} \quad \ldots\ldots (3)}$$

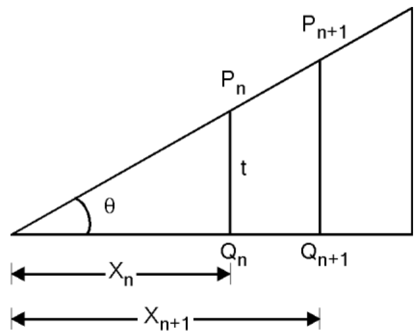


Fig. Fringe width

Let $(n+1)^{th}$ maximum be obtained at a distance x_{n+1} from the thin edge. Then we have

$$2x_{n+1}\sin\theta = [2(n+1)-1]\cdot\frac{\lambda}{2}$$

$$\text{or } \mathbf{2x_{n+1}\cdot\sin\theta = (2n+1)\frac{\lambda}{2} \quad \ldots\ldots (4)}$$

Therefore from Equations (3) and (4) we have

$$\mathbf{2(x_{n+1}-x_n)\cdot\sin\theta = \lambda}$$

Therefore the spacing between two consecutive bright bands is

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\sin\theta}$$

$\sin\theta \to \theta$ if θ is small and measured in radians. β is called fringe width.

$$\beta = \frac{\lambda}{2\theta}$$

For a medium of refractive index 'μ', we have

$$\beta = \frac{\lambda}{2\mu\theta}.$$

as μ, λ and θ are constant, one can say that fringe width in wedge shaped film is constant. Or **wedge shaped fringes are of constant thickness.**

Newton's Ring:

When a plano-convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.

The thickness of the film gradually increases from the point of contact outwards.

When monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film.

These rings were **first studied by Newton and hence known as Newton's rings.**

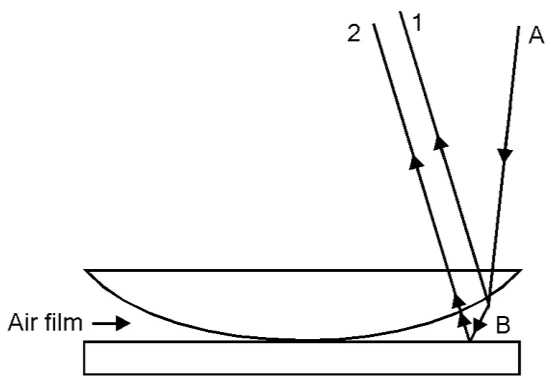


Fig. Newton's Rings

Let 'AB' is a beam of monochromatic light of wavelength 'λ' incident normally on the film. When it reflect from top and bottom faces of the film, rays 1 and 2 are produces which are coherent.

The path difference is given as,

$$\Delta = 2\mu t\cdot\cos(r+\theta) + \frac{\lambda}{2} \text{ ------- (1)}$$

where μ = R.I. of the film

t = Thickness at a point under consideration

r = Angle of refraction

θ = Angle of wedge

For normal incidence r = 0, air film μ = 1 & large radius of curvature, θ = 0

$$\Delta = \mathbf{2t} + \frac{\lambda}{2} \text{ -------- (2)}$$

For Maxima or Constructive interference:

For constructive interference, the path difference is an integral multiple of wavelength 'λ'.

$$2t + \frac{\lambda}{2} = n\lambda$$

$$\mathbf{2t = (2n-1)\frac{\lambda}{2}} \text{ ------- (3)}$$

For Minima or Destructive interference:

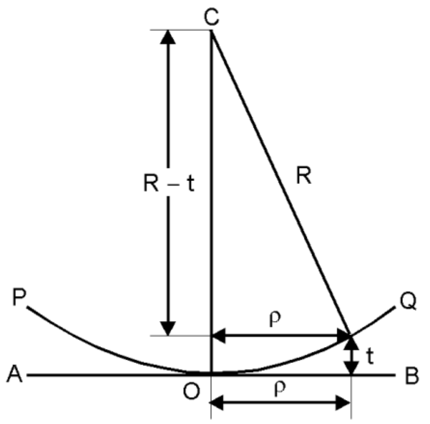
$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\mathbf{2t = n\lambda} \text{ ------- (4)}$$

Diameter of Dark and Bright Rings:

Que: Prove that the diameter of n^{th} dark ring is proportional to square root of ring number for reflected rays in Newton's ring experiment.

Consider POQ be a plano-convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens & ρ be the radius of a Newton's ring corresponding to the constant film thickness't'.



The path difference between the two interfering rays in the reflected system is

$$\Delta = 2\mu t \cos(r+\theta) + \frac{\lambda}{2}.$$

λ = wavelength of incident light.
μ = 1 for air film.
r = 0 for normal incidence
θ = 0 for large R.

$$\therefore \quad \Delta = 2t + \frac{\lambda}{2} \text{ ------- (1)}$$

From Fig. according to Pythagoras prameya we have

$$R^2 = \rho^2 + (R-t)^2$$

$$\rho^2 = R^2 - (R-t)^2$$

$$\therefore \rho^2 = 2Rt - t^2$$

$t \ll R$ and hence t^2 is negliable,

$$\therefore \rho^2 = 2Rt$$

$$\therefore \quad 2t = \frac{\rho^2}{R}$$

$$\therefore \textbf{ Path difference become } (\Delta) = \frac{\boldsymbol{\rho^2}}{\mathbf{R}} + \frac{\boldsymbol{\lambda}}{\mathbf{2}} \text{ ------- } \mathbf{(2)}$$

(A) For dark rings:

The condition for dark rings or minima is,

$$\Delta = (2n+1)\frac{\lambda}{2} \quad (n = 1,2,3,....)$$

$$\therefore \Delta = \frac{\rho^2}{R} + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\frac{\rho^2}{R} = n\lambda$$

But ρ is the radius & D is the diameter, therefore $\rho = \frac{D}{2}$

$$\frac{D_n^2}{4R} = n\lambda$$

where D_n= Diameter of n^{th}dark ring.

$$\therefore \quad \mathbf{D_n^2 = 4nR\lambda} \text{ ------- (3)}$$

$$\mathbf{D_n = \sqrt{4nR\lambda}}$$

$$D_n \propto \sqrt{n}$$

Hence **the diameter of the dark ring is proportional to the square roots of natural numbers.**

(B) For bright rings:

For bright rings or constructive interference the path difference is given as

$$\Delta = n\lambda$$

$$\therefore \frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

$$\frac{\rho^2}{R} = (2n-1)\frac{\lambda}{2}$$

If D is the diameter, then $\rho = \frac{D}{2}$

$$\frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$$

where D_n = Diameter of n^{th} bright ring.

$$\mathbf{D_n^2 = (2n-1) \,.\, 2\lambda R} \text{ ------- } \mathbf{(4)}$$

$$D_n = \sqrt{(2n-1).2\lambda R}$$

$$D_n \propto \sqrt{2n-1} \quad \text{where } n = 1, 2, 3, ...$$

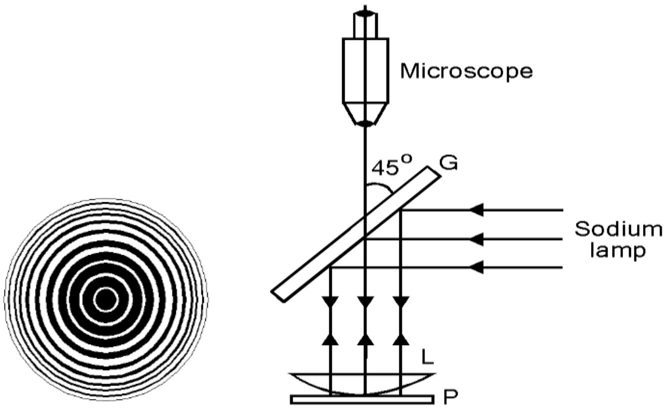
It shows that diameter of bright ring is proportional to the square root of odd natural numbers.

Newton's Ring Experiment:

Determination of Wavelength of Monochromatic light or radius of curvature of Plano-convex lens by Newton's Rings Method.

Que: With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of planoconvex lens.

Figure show experimental setup for Newton's rings method.



In Newton's rings experiment a plano-convex lens 'L' of large radius of curvature is placed on a plane glass plate 'P'.

Another glass plate G is held at a suitable distance above, at an angle 45° (to make the normal incidence on the film).
Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.
The effective path difference between the interfering rays is

$$2\mu t \cos(r + \theta) + \frac{\lambda}{2}$$

where μ = R.I. of film.
θ = Angle of film at any point
λ = Wavelength of light

If D_n is the diameter of n^{th} dark ring then

$$D_n^2 = 4\, n\, R\, \lambda$$

where R = Radius of curvature of lower surface of lens.

$$\text{Let} \quad D_{n+p}^2 = 4\,(n+p)\,R\,\lambda$$

$$\therefore \quad D_{n+p}^2 - D_n^2 = 4\,p\,R\,\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR} \quad \text{-------- (1)}$$

Thus by measuring the diameter of n^{th} and $(n + p)^{th}$ dark rings, the radius of curvature R and the wavelength λ can be calculated.

Determination of Refractive Index of a Liquid by Newton's Rings:

$$\mu = \frac{[D_{n+p}^2 - D_n^2]_{air}}{[D_{n+p}^2 - D_n^2]_{liquid}} \quad \ldots\ldots (7)$$

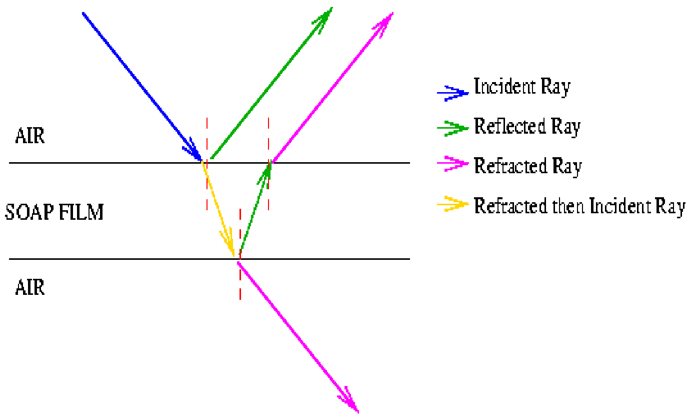
or

$$\mu = \frac{D_n^2\,(air)}{D_n^2\,(liq)}$$

Formation of colours in thin film:

Que: Explain the formation of colours in thin film if illuminated by white light.

- Everyone is familiar with the brilliant colours exhibited by a thin oil film spread on the surface of water and also by a soap bubble.
- When a thin film is exposed to white light from an extended source, it shows beautiful colors in reflected system.
- Light is reflected from top and bottom surface of a thin film forms identical & coherent reflected rays, they reflected rays interfere with each other.
- The path difference between the interfering rays depends on the thickness of the film and the angle of refraction 'r'.
- White light consists of continuous range of wavelength (4000 A^0 to 7000 A^0) associate with different colors.
- At particular point of the films & for particular position of eye (i.e. t & r constant) those wavelength of incident light satisfy the condition of maxima (constructive interference) will shows reflected light of different colors.
- The colors will vary with the thickness of the film and inclination of the ray.



Point Source and Extended source:

- In case of interference in thin films, an extended source is necessary to observed the interference pattern due to whole film while a narrow (point) source limits the validity of the film.
- Consider a thin film and a narrow (point) source of light 'S' as shown in fig. (a). For each incident ray 1, 2 we get a pair of parallel interfering rays.

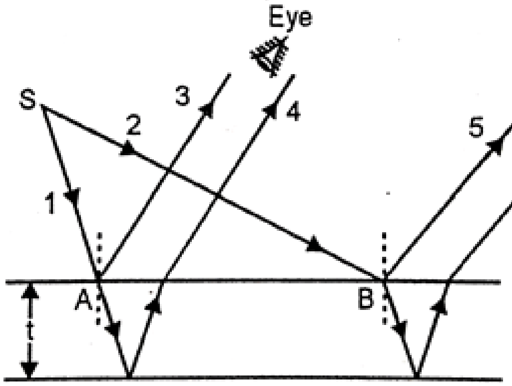


Fig. (a)

- The incident ray 1 produces interference fringes because rays 3 and 4 reaches the eye, where the ray 2 is incident on film at some different angle and is reflected along 5 and 6. Here the rays 5 and 6 do not reach the eye. Hence only the portion A of the film is visible and not the rest.
- If extended source of light as shown in fig. (b) is used (fig) the incident ray 1 after reflection from the upper and the lower surface of the film emerges as 3, 4 which reach the eye. Also incident ray 2 from some other point of the source after reflection from the upper and lower surface of the film emerges as 5, 6 which also reach the eye.

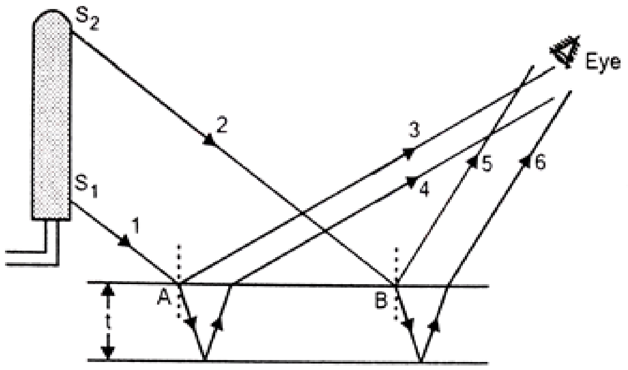


Fig. (b)

- Therefore, in the case of extended source of light, the rays incident at different angles on the film are well adjust by the eye and the field of view is large. Due to this reason, to observe interference phenomenon in this film, a broad source of light is required.

Why centre of Newton's rings is always dark?

Que: Why the central spot appears dark in Newton's rings interference pattern when viewed in reflected light?

At the point of contact the thickness of air film is zero. Consider the case of thin film at this point i.e. condition for bright spot is given by,

$$2\ \mu t \cos (r + \theta) = (2 n + 1)\ \lambda/2 \quad n = 0, 1, 2, \ldots\ldots$$

and condition for dark spot is given by,

$$2\ \mu t \cos (r + \theta) = n\lambda$$

For Newton's ring set up r = 0 (large radius of lens θ = 0), (for normal incidence r = 0)

$$\cos (r + \theta) = \cos 0 = 1$$

∴ Condition for bright

$$\mathbf{2\ \mu t = (2n + 1)\ \lambda/2}$$

Where, n = 0, 1, 2, 3, …..

∴ Condition for dark

$$2\ \mu t = n\lambda$$

Where, n = 0, 1, 2, 3, …..

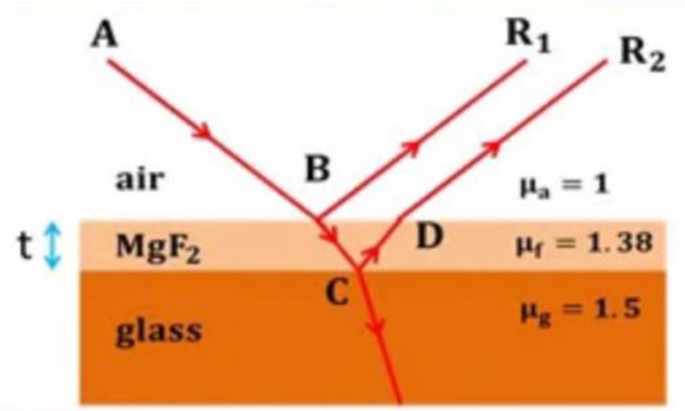
At point of contact, t = 0, one can see that condition for bright does not get satisfied. But condition for dark gets satisfied for t = 0 and n = 0.

At the point of contact the condition for dark gets satisfied central spot appears dark in Newton's rings interference pattern when viewed in reflected light.

Why Newton's rings are circular and wedge shaped films are straight ?

- In both air-wedge film and Newton's ring experiments, each fringe is the locus of points of equal thickness of the film.
- In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of plano-convex lens and the glass plate as centre. So, the fringes are circular in nature and concentric.
- For wedge shape air film, The efffectiv e parallel to the edge of the wedge. So, fringes appear straight and parallel.

Antireflecting coating or Non-reflecting film:



In order to reduce the reflection loss, a transparent film of proper thickness is deposited on the surface. This film is known as "non-reflecting film or Antireflecting coating".
Popular material used for antireflecting coating is MgF_2 because its refractive index is 1.38 i.e. between the refractive index of air (1) and glass (1.5).
Let a ray A is incident upon thin film of MgF_2 coated on glass. This ray is reflected from upper surface as R1 and from lower surface as R2. The incident ray enters from rarer to denser twice i.e. at air to film and film to glass, therefore according to Stoke's law additional path difference λ/2 is added two times.
Hence effective path difference between BR_1 & DR_2 is

$$\mathbf{\Delta = 2\,\mu_f\, t \cos r + \frac{\lambda}{2} + \frac{\lambda}{2}}$$

$$\mathbf{\Delta = 2\,\mu_f\, t \cos r \quad ------ \quad (1)}$$

For normal incidence r = 0

$$\mathbf{\Delta = 2\,\mu_f\, t \quad ------ \quad (2)}$$

For non-reflecting film light ray is interfere destructively after reflection.
For destructive interference

$$\Delta = (2n + 1)\,\lambda/2$$

$$2\,\mu_f\, t = (2n + 1)\,\lambda/2$$

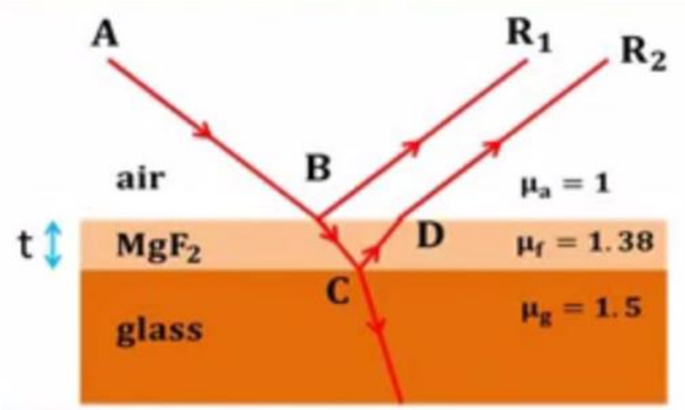
For minimum thickness n = 0

$$2\,\mu_f\, t = \lambda/2$$

$$\mathbf{t} = \frac{\lambda}{4\mu_f} \quad \mathbf{------ \quad (3)}$$

It means in order to have destructive interference or non-reflection a layer of $\mu_f\,\mathbf{t} = \frac{\lambda}{4}$ is coated on the glass plate.

Fully-reflecting coating film:



The material used for fully-reflecting coating is MgF_2 because its refractive index is 1.38 i.e. between the refractive index of air (1) and glass (1.5).
Let a ray A is incident upon thin film of MgF_2 coated on glass. This ray is reflected from upper surface as R1 and from lower surface as R2. The incident ray enters from rarer to denser twice i.e. at air to film and film to glass, therefore according to Stoke's law additional path difference λ/2 is added two times.
Hence effective path difference between BR_1 & DR_2 is

$$\mathbf{\Delta = 2\,\mu_f\, t \cos r + \frac{\lambda}{2} + \frac{\lambda}{2}}$$

$$\mathbf{\Delta = 2\,\mu_f\, t \cos r \quad ------ \quad (1)}$$

For normal incidence r = 0

$$\mathbf{\Delta = 2\,\mu_f\, t \quad ------ \quad (2)}$$

For fully-reflecting film light ray is interfere constructively after reflection.
For constructively interference

$$\Delta = n\,\lambda$$

$$2\,\mu_f\, t = n\,\lambda$$

For minimum thickness, n = 1

$$2\,\mu_f\, t = \lambda$$

$$\mathbf{t} = \frac{\lambda}{2\mu_f} \quad \mathbf{------ \quad (3)}$$

It means in order to have destructive interference or non-reflection a layer of $\mu_f\,\mathbf{t} = \frac{\lambda}{2}$ is coated on the glass plate.

Numerical:

Ex. 1) In Newton's ring experiment the diameter of 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20th ring.

Soln.:

Given : $n = 4, \quad n + p = 12, \quad D_4 = 0.400$ cm,

$D_{12} = 0.700$ cm

$$D_n^2 = 4\,n\,R\lambda,\ D_{n+p}^2 = 4\,(n+p)\,R\lambda$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \qquad \ldots(1)$$

Similarly, $$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \qquad \ldots(2)$$

Divide Equation (1) by Equation (2), we get,

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4 \times 8}{4 \times 16} = \frac{1}{2}$$

$$D_{20}^2 = 2\,D_{12}^2 - D_4^2$$

$$= 2\,(0.700)^2 - (0.400)^2$$

$$= 0.98 - 0.16 = 0.82$$

Diameter of 20^{th} ring $= \sqrt{0.82} = \mathbf{0.906}$ **cm**

Ex. 2) A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 A°.

Soln.:

Given : $i = 30°, \ \mu = 1.46, \ n = 8,$

$\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : For dark band,

$$2\mu t \cos r = n\lambda \qquad n = 0, 1, 2, 3, ...$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin r = \frac{\sin i}{\mu} = \frac{\sin 30°}{1.46} = 0.3424$$

$$\therefore \cos r = \sqrt{1 - \sin^2 r} = 0.9395$$

$$\therefore t = \frac{n\lambda}{2\mu \cdot \cos r} = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times 0.9395}$$

$$\therefore t = 1.7176 \times 10^{-4} \text{ cm.}$$

Ex. 3) White light falls at an angle of 45° on a parallel soap film of refractive index 1.33. At what minimum thickness of the film will it appear bright yellow of wavelength 5896 A° in the reflected light?

Soln.:

Given : $i = 45°, \mu = 1.33,$

$\lambda = 5896$ A° $= 5896 \times 10^{-10}$ m A°

Formula: $2\,\mu t \cos r = (2n - 1)\dfrac{\lambda}{2}$,

$n = 1, 2, 3, \ldots$ For bright fringe

For minimum thickness, n is minimum i.e. n = 1

$$\therefore t = \frac{\lambda}{2 \times 2\mu \cos r} = \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times \cos r}$$

Now $$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{1.33} = 0.5316$$

$$\therefore \cos r = \sqrt{1 - \sin^2 r} = 0.8469$$

$$\therefore t = \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times 0.8469}$$

$$t = 1304.5 \text{ A°}$$

Ex. 4) Light of wavelength 5880 Å is incident on a thin film of glass of μ = 1.5 such that the angle of refraction in the plate is 60 °. Calculate the smallest thickness of the plate which will make it dark by reflection.

Soln. :

$\lambda = 5880 \times 10^{-8}$ cm, $\mu = 1.5$, $r = 60°$, $t = ?$

Condition for film to appear dark is,

$$2\,\mu t \cos r = n\lambda$$

The smallest thickness will be for n = 1.

$$2 \times 1.5 \times t \times \cos 60 = 1 \times 5880 \times 10^{-8}$$

$$t = \frac{5880 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$t = 3920 \times 10^{-8} \text{ cm}$$

Ex. 5) An oil drop of volume 0.2 c.c. is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the surface and white light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band coincides with wavelength 5.5 x 10^{-5} cm in air. Find the refractive index of oil.

Soln.:

The oil drop of volume 0.2 c.c. spreads uniformly over 1 m^2; hence the thickness of the film so formed is given by,

$$t = \frac{0.2}{(100)^2} = 2 \times 10^{-5} \text{ cm.}$$

The film appears dark by reflected light.

Hence, $2\mu t \cos r = n\lambda$

For normal incidence

$$r = 0$$

$$\therefore \cos r = 1$$

$$n = 1 \text{ and } \lambda = 5.5 \times 10^{-5} \text{ cm.}$$

Refractive index of oil is

$$\mu = \frac{n\lambda}{2t\cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1} = \frac{5.5}{4}$$

$$\mu = 1.375$$

Ex. 6) Light of wavelength 5893 A° is reflected at nearly normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear (i) Black (ii) Bright?

Soln.:

i) In reflected system, the condition of dark is

$$2\mu t \cos r = n\lambda$$

For normal incidence, r = 0, cos r = 1

$$\text{So,} \quad 2\mu t = n\lambda, \quad t = \frac{n\lambda}{2\mu}$$

For minimum thickness of the film, n = 1

$$\text{Hence,} \quad t = \frac{\lambda}{2\mu} = \frac{5893 \times 10^{-8}}{2 \times 1.42} \text{ cm} = 2075 \text{ Å}$$

ii) In reflected system, the condition of bright is

$$2\mu t \cos r = (2n - 1)\lambda/2$$

For normal incidence, r = 0, cos r = 1

For minimum thickness of the film, n = 1

$$\text{So,} \quad t = \frac{\lambda}{4\mu} = \frac{5893 \times 10^{-8}}{4 \times 1.42} \text{ cm} = 1037.5 \text{ Å}$$

Ex. 7) A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used.

Soln.:

Given : θ = 40 seconds = $\frac{40}{3600}$ degrees = $\frac{40}{3600} \times \frac{\pi}{180}$ radians, β = 0.12 cm.

Formula : Spacing between the consecutive bright fringes is,

$$\beta = \frac{\lambda}{2\theta}$$

The wavelength is, $\lambda = 2\beta \cdot \theta = 2 \times 0.12 \times \frac{40 \times \pi}{3600 \times 180}$

$$\lambda = 4654 \times 10^{-8} \text{ cm}$$

$$\lambda = 4654 \text{ Å}$$

Ex. 8) Light of wavelength 5500 A° falls normally on a thin wedge shaped film of refractive index 1.4 forming fringes that are 2.5 mm apart. Find the angle of wedge in seconds.

Soln.: We have fringe width,

$$\beta = \frac{\lambda}{2\mu\theta}; \qquad \theta = \frac{\lambda}{2\mu\beta}$$

Given : $\lambda = 5500 \times 10^{-8}$ cm, μ = 1.4, β = 0.25 cm

$$\theta = \frac{5500 \times 10^{-8}}{2 \times 1.4 \times 0.25} = 7.86 \times 10^{-5} \text{ radian}$$

$$\theta = 7.86 \times 10^{-5} \times \frac{180°}{\pi} = 0.0045°$$

$$\theta = 0.0045° \times 3600$$

$$\theta = 16.2 \text{ sec.}$$

Ex. 9) Newton's rings are obtained with reflected light of wavelength 5500 Å. The diameter of 10th dark ring is 5 mm. Now the space between the lens and the plate is filled with a liquid of refractive index 1.25. What is the diameter of the 10th ring now?

Soln.:

Given : λ = 5500 Å = 5500×10^{-8} cm,

D_{10} = 5 mm = 0.5 cm, μ = 1.25

Formula : Diameter of nth dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For the air film, μ = 1

Hence diameter of 10th dark ring is,

$$(0.5)^2 = D_{10}^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1}$$

For the liquid film, the diameter of the 10th dark ring is,

$$D_{10}'^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1.25}$$

$$\frac{D_{10}'^2}{D_{10}^2} = \frac{1}{1.25} \qquad \therefore \quad D_{10}'^2 = \frac{D_{10}^2}{1.25}$$

$$\therefore \quad D_{10}' = \frac{D_{10}}{\sqrt{1.25}} = \frac{D_{10}}{1.1180} = \frac{0.5}{1.118}$$

$$D_{10}' = 0.4472 \text{ cm.}$$

Diameter of 10th dark ring for the liquid film = 4.472 mm.

Ex. 10) Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring?

Soln.:

Let the diameter of the nth dark ring be double the diameter of 40th dark ring.

$$\therefore \quad D_n = 2 D_{40}$$

Now the diameter of n^{th} dark ring is given by the expression

$$\mathbf{D_n^2 = 4\, n\, R\, \lambda \; ...... (1)}$$

where R = Radius of curvature of lens.

λ = Wavelength of light.

Hence for the 40^{th} dark ring.

$$\mathbf{D_{40}^2 = 4 \times 40 \times R\lambda \; .... (2)}$$

From Equations (1) and (2) we have

$$D_n^2 = 4 \times D_{40}^2$$

$$\therefore \; 4\, n\, R\, \lambda = 4 \times 4 \times 40 \times R\lambda$$

$$\mathbf{n = 160}$$

Ex. 11) The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring.

Soln.:

$$\mathbf{D_n^2 = 4\, nR\lambda}$$

As Diameter of 5^{th} dark ring = 0.42 cm

Now diameter of 10^{th} dark ring = ?

$$\therefore \; \frac{D_5^2}{D_{10}^2} = \frac{4(5)\, R\lambda}{4(10)\, R\lambda}$$

$$\therefore \; 2\left(D_5^2\right) = D_{10}^2$$

$$D_{10} = \sqrt{2}\,(D_5) = \sqrt{2}\,(0.42)$$

$$\mathbf{D_{10} = 0.594\ cm}$$

$\therefore$ **Diameter of 10th dark ring = 0.594 cm**

Ex. 12) A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm. It is found that the nth dark ring due to λ_1 coincides with (n + 1)th dark ring for λ_2. If the radius of the curved surface is 90 cm, find the diameter of 3rd dark ring for λ_1.

Soln.:

Given : $\lambda_1 = 6 \times 10^{-5}$ cm, $\lambda_2 = 4.5 \times 10^{-5}$ cm, R = 90 cm

$$[D_n]_{\lambda_2} = [D_{n+1}]_{\lambda_1}$$

Formula : Diameter of n^{th} dark ring is,

$$\mathbf{D_n^2 = 4\, n\, R\, \lambda} \quad (\mu = 1)$$

$\therefore$ For the n^{th} dark ring λ_1

$$\mathbf{\left[D_n^2\right]_{\lambda_1} = 4\, n\, R\, \lambda_1 \; ...... (1)}$$

and for the $(n + 1)^{th}$ dark ring λ_2

$$\mathbf{\left[D_{n+1}^2\right]_{\lambda_2} = 4\,(n + 1) \cdot R \cdot \lambda_2 \; ...... (2)}$$

$$\therefore \; 4\, n\, R\, \lambda_1 = 4\,(n + 1)\, R \cdot \lambda_2 \quad \therefore n\lambda_1 = (n + 1)\,\lambda_2$$

$$\therefore \; n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{0.45 \times 10^{-5}}{(6 - 0.45) \times 10^{-5}}$$

$$\therefore \; \mathbf{n = 3}$$

$\therefore$ Using Equation (1), the diameter of 3^{rd} dark ring for λ_1

$$\left[D_3^2\right] = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$\therefore \; \mathbf{[D_3]_{\lambda_1} = \sqrt{4 \times 3 \times 90 \times 6 \times 10^{-5}} = 0.2545\ cm}.$$

$\therefore$ **Diameter of third dark ring for λ_1 is 0.2545 cm**

Ex. 13) In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and that of 15th ring was 0.59 cm. If the radius of curvature of the plano - convex lens 100 cm. Calculate the wavelength of light.

Soln.:

Given : R = 100 cm, D_{15} = 0.59 cm, D_5 = 0.336 cm

Formula : $$\lambda = \frac{D_{15}^2 - D_5^2}{4 \times n \times R} = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$\mathbf{\lambda = \frac{0.2352}{4000} = 5.88 \times 10^{-5}\ cm}$$

Ex. 14) Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th bright ring is 0.336 cm, what is the wavelength of light used?

Soln.:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$\mathbf{\lambda = 5.880 \times 10^{-5}\ cm = 5880\ Å}$$

Ex. 15) In a Newton's ring experiment the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Soln.:

$$D_n\,(\text{air}) = 1.4\ \text{cm}$$

$$D_n\,(\text{liquid}) = 1.27\ \text{cm}$$

$$\therefore \; \mathbf{\mu = \frac{D_n^2\,(air)}{D_n^2\,(liquid)} = \frac{1.4^2}{1.27^2} = 1.215}$$

Ex. 16) Newton's rings are observed in reflected light of wavelength 6000 A°. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the corresponding air film.

Soln.:

We have, the diameter of dark ring,

$$D_n^2 = 4nR\lambda\ , \quad R = \frac{D_n^2}{4n\lambda}$$

Given : $n = 10$, $D_n = 0.5 \times 10^{-2}$ m, $\lambda = 6 \times 10^{-7}$ m

R = Radius of curvature

$$\therefore \quad R = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 6 \times 10^{-7}} = \frac{25}{24} = 1.04 \text{ m} = \mathbf{104\ cm}$$

Thickness of air film,

$$t = \frac{D^2}{8R}$$

$D = 0.5 \times 10^{-2}$ m, $R = 1.04$ m

$$t = \frac{(0.5 \times 10^{-2})^2}{8 \times 1.04} = 3.0 \times 10^{-6} \text{ m}$$

$$\mathbf{t = 3.0\ \mu m}$$

Ex. 17) Light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33. Which wavelength in the visible region will be reflected most strongly?

Soln.:

The condition of maxima is given by,

$$2\,\mu t \cos r = (2n - 1)\,\lambda/2 \text{ where } n = 1, 2, 3, ...$$

Given : $t = 5 \times 10^{-5}$ cm $\quad \mu = 1.33$

$r = 0°$ $\quad$ i.e. $\cos r = 1$

$$\text{Now,}\quad \lambda = \frac{4\,\mu t \cos r}{(2n-1)} = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n-1)}$$

By substituting the values of n = 1, 2, ... we get a series of wavelengths which shall be predominantly reflected by the film.

$$\text{For } n = 1,\ \lambda_1 = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1}$$

$$\boldsymbol{\lambda_1 = 26.66 \times 10^{-5}\ \text{cm}}$$

Similarly,

For n = 2, $\boldsymbol{\lambda_2 = 8.866 \times 10^{-5}}$ **cm**

For n = 3, $\boldsymbol{\lambda_3 = 5.32 \times 10^{-5}}$ **cm**

For n = 4, $\boldsymbol{\lambda_4 = 3.8 \times 10^{-5}}$ **cm**

Out of these wavelengths 5.320×10^{-5} cm lies in the visible region (4000 Å to 7500 Å).

Hence 5320 Å is the most strongly reflected wavelength.

Diffraction

Definition:

The phenomenon of **bending of light** round the corner of slit or obstacle and spreading of light into the region of geometrical shadow is called diffraction.

The bending of light depends on size of obstacle. For diffraction the size or dimension of obstacle are very small or nearly equal to wavelength of light.

Difference between Interference and Diffraction:

Interference	Diffraction
i) The modification of intensity due to superposition of two or more light waves is called interference.	i) The phenomenon of bending of light round the corners of the obstacles and spreading of light in the geometrical region of the shadow of the obstacle is called diffraction.
ii) It is the result of the interaction of light coming from two different wave fronts originating from the same source.	ii) It is the result of the interaction of light coming from two different parts of the same wave fronts.
iii) Interference fringes may or may not be of same width.	iii) Diffraction fringes are not of same width.
iv) Points of minimum intensity are perfectly dark.	iv) Points of minimum intensity are not perfectly dark
v) All bright bands are of uniform intensity.	v) All bright bands are not of uniform intensity

Types of diffraction:

There are two types of diffraction.

1) Fresnel's diffraction, 2) Fraunhofer diffraction

Fresnel's diffraction:

The diffraction phenomenon in which light **source and screen are placed at finite distance from obstacle** (slit) is termed as Fresnel diffraction.

The incident and diffracted wave front are either spherical or cylindrical.

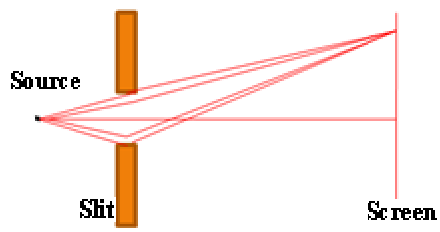


Fig. Fresnel diffraction

Fraunhofer diffraction:

The diffraction phenomenon in which light **source and screen are placed at infinite distance from obstacle** (slit) is termed as Fraunhofer diffraction.

The incident beam is making parallel by using biconvex lens. Therefore to perform this experiment in laboratory we used two biconvex lenses

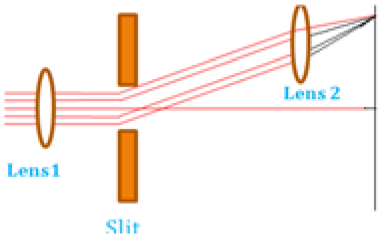


Fig. Fraunhofer diffraction

The incident and diffracted wave front is plane.

Difference between Fresnel and Fraunhofer diffraction:

Fresnel's diffraction	Fraunhofer diffraction
i) The distance of source and screen from obstacle are finite.	i) The distance of source and screen from obstacle are infinite.
ii) No lenses are used.	ii) Two bio-convex lenses are used.
iii) Wave front incident on obstacle is either spherical or cylindrical.	iii) Wave front incident on obstacle is plane.
iv) The diffracted wave front is either spherical or cylindrical.	iv) The diffracted wave front incident on obstacle is plane.
v) Initial phase of secondary wavelets is different at different point in the plane of obstacle.	v) Initial phase of secondary wavelets is same at all points in the plane of obstacle.
vi) It has no importance in optical instrument.	vi) It is highly important in optical instrument.
vii) The mathematical treatment of Fresnel's diffraction is complicated.	vii) The mathematical treatment of Fraunhofer diffraction is comparatively simple.

Fraunhofer diffraction at single slit:

$$\therefore E_\theta = E_m \frac{\sin\alpha}{\alpha} \quad \ldots\ldots (1)$$

Where, $\alpha = \frac{\pi}{\lambda} a \sin\theta$ ……. (2)

If I_θ is resultant intensity of light at 'P' and I_m is maximum intensity at 'P_o'

$$\therefore I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \ldots\ldots (2)$$

Principal maximum:

For E_θ to be maximum: all the phasors must be in phase i.e. α = 0

$$\alpha = \frac{\pi}{\lambda} a \sin\theta = 0$$

⇒ sin θ = 0 or θ = 0

Minimum:

The intensity at point P will be zero when sin α = 0 and α ≠ 0

The values of α which satisfies this condition are

α = ± m π   where m = 1, 2, 3 ..

$$\alpha = \frac{\pi}{\lambda} a \sin\theta = \pm m\pi$$

a sin θ = mλ ……. (3)

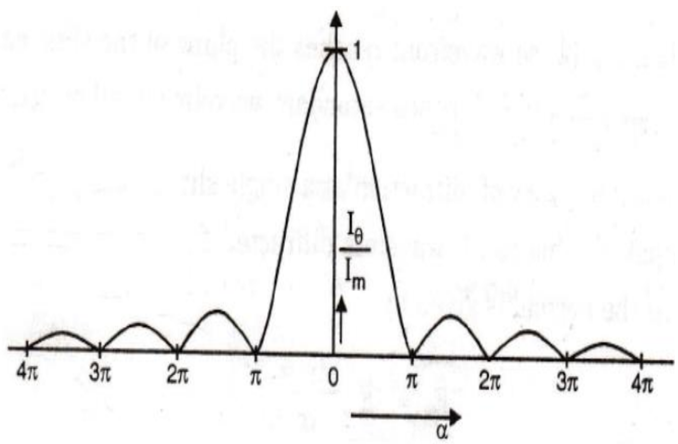
Secondary maximum:

The secondary maximum is present between two minimum, and can be occur when

$$\alpha = \pm\left(n + \frac{1}{2}\right)\pi$$

a sin θ = (2n+1) λ/2 ……. (4)

Where a = Width of Slit, θ = Angle by which light gets diffracted, λ = Wavelength of Incident Light, n = Order of maxima (Value of n = 1, 2, 3,_ _ _)



Fraunhofer diffraction at a double slit:

The resulting amplitude from single slit is given by,

$$\therefore E_\theta = E_m \frac{\sin\alpha}{\alpha} \quad \ldots\ldots (1)$$

Where, $\alpha = \frac{\pi}{\lambda} a \sin\theta \quad \ldots\ldots (2)$

The resulting Intensity from double slit is given by,

$$I_\theta = 4E_m^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta \quad \ldots\ldots (3)$$

Where, $\beta = \frac{\pi}{\lambda}(\mathrm{a}+\mathrm{b}) \sin\theta \quad \ldots\ldots (4)$

The first factor $4E_m^2 \frac{\sin^2\alpha}{\alpha^2}$ gives intensity distribution in the diffraction due to a single slit.

The second factor $\cos^2\beta$ gives the interference pattern due to light waves of same amplitude from the two slits.

Maxima:

The resultant intensity of light at point P is given by equation

$$I_\theta = 4E_m^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

The intensity term due to interference is maximum when $\cos^2\beta = 1$

i.e. $\beta = \pm m\pi$

Where m= 0, 1, 2, 3, ---- and $\beta = \frac{\pi}{\lambda}(\mathrm{a}+\mathrm{b}) \sin\theta$

$$\therefore \frac{\pi}{\lambda}(\mathrm{a}+\mathrm{b}) \sin = m\pi$$

$$\therefore \mathbf{(a+b)\sin\theta = m\lambda} \quad \ldots\ldots (5)$$

Where m is called order of diffraction.
When m= 0, corresponds to zeroth order maximum i.e. central maximum or principle maximum.
For m = 1, 2, 3, we get first order, second order principle maximum called secondary maximum.

For Minima:

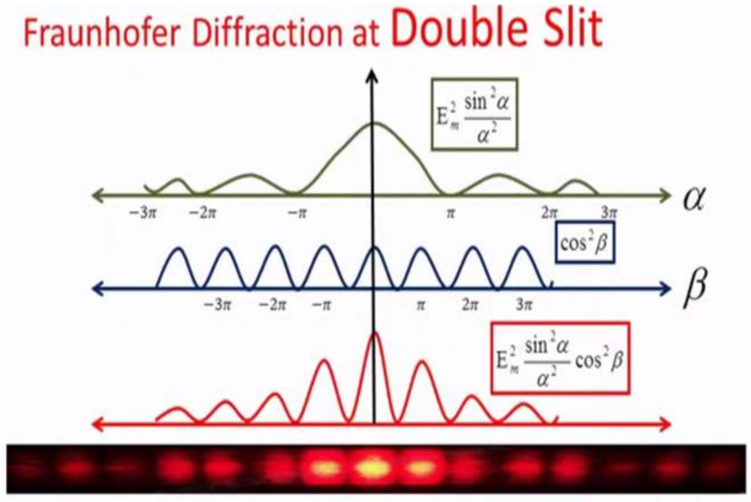
The intensity of light is minimum when, $I_\theta = 0$, i.e. $\cos^2\beta = 0$,

i.e. $\beta = \pm\left(n + \frac{1}{2}\right)\pi$ Where n = 0,1, 2, 3, ---

but, $\beta = \frac{\pi}{\lambda}(\mathrm{a}+\mathrm{b}) \sin\theta$

$$\therefore \frac{\pi}{\lambda}(\mathrm{a}+\mathrm{b}) \sin\theta = \pm\left(n + \frac{1}{2}\right)\pi$$

$$\therefore (\mathrm{a}+\mathrm{b}) \sin\theta = \pm(2n+1)\frac{\lambda}{2} \quad \ldots\ldots (7)$$



Diffraction Grating:

It is an arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating.

It is obtained by ruling equidistant parallel lines on a transparent material, such as glass plate by means of a fine diamond point. The lines act as opaque surface and the space between any two lines is transparent to light and acts as a slit. The arrangement so obtained is known as plane transmission grating. The number of lines in a plane transmission grating is of the order of 15000 to 20,000 lines per inch.

Grating Element:

The distance between center of two adjacent slit is called grating element.

Let, a = width of slit

b = width of each opaque surface

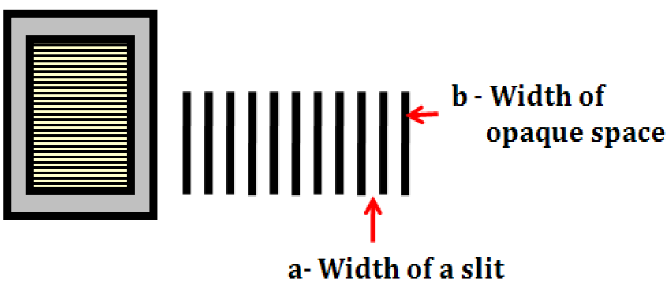
N = number of lines/cm.

Then, d = (a + b) = Grating Element

$$d = (a+b) = \frac{1}{N}$$

$$(a+b) = \frac{1}{N/cm} = \frac{2.54}{N/inch}$$

$$N = \frac{1}{(a+b)} lines/cm$$



Derivation for diffraction grating:

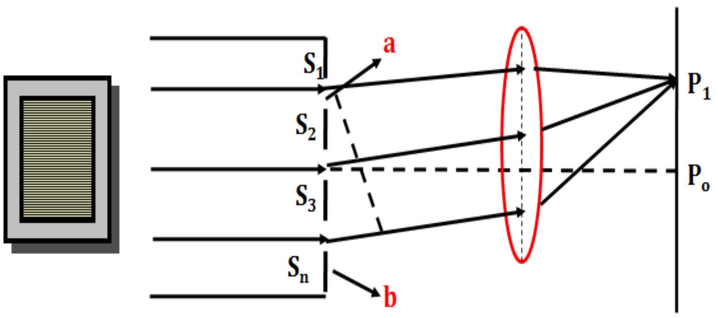
When a monochromatic source of light of wavelength 'λ' incident normally on a grating, then each slit act as a source of secondary wavelet and give out disturbance at point P_1 and P_0.

The amplitude at point 'P_1' is nothing but resultant of amplitude generated by all slits.

P_1 is called first order diffraction pattern and it observed on both side of central maximum (P_0).

P_0 is called central maximum or beam without diffraction.

The resulting amplitude from single slit is given by,



$$\therefore E_\theta = E_m \frac{\sin\alpha}{\alpha} \quad \ldots\ldots (1)$$

Where, $\alpha = \frac{\pi}{\lambda} a \sin\theta$

E_m = maximum amplitude at point (P_o)

The path difference between two slit S1 and S2 is

Path difference = (a+b) sinθ

And the corresponding phase difference between successive slit is given by,

$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \text{ (say)} \quad \text{------ } (2)$$

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

Hence for N number of parallel slit, the resultant amplitude of these N diffracted waves can be found out by vector addition method and is given by,

$$E_\theta = E_m\left(\frac{\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta} \quad \text{------ } (3)$$

As the intensity is proportional to square of amplitude i.e. $E_\theta^2 = I_\theta$, therefore the resultant intensity is given

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \quad \text{------ } (4)$$

The first factor $I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$ is gives intensity distribution in the diffraction due to a single slit.

The second factor $\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$, gives the interference pattern due to N slits.

Principal Maxima:

The resultant intensity of light at point P_1 is given by equation

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$$

The intensity term due to interference is maximum when $\sin\beta = 0$

i.e. $\beta = \pm m\pi$

Where m= 0, 1, 2, 3, ----

but, $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

$$\therefore \frac{\pi}{\lambda}(a+b)\sin = m\pi$$

$\therefore$ **(a+b) sinθ = mλ** ……. (5)

Where m is called order of diffraction.

When m= 0, corresponds to zeroth order maximum i.e. central maximum or principle maximum occurs at point P_0.

For m= 1, 2, 3, we get first order, second order principle maximum called secondary maximum.

For Minima:

The intensity of light is minimum when, $I_\theta = 0$, i.e. $Sin\, N\beta = 0$, but $\sin\beta \neq 0$

i.e. $N\beta = \pm n\pi$ Where n = 1, 2, 3, -----

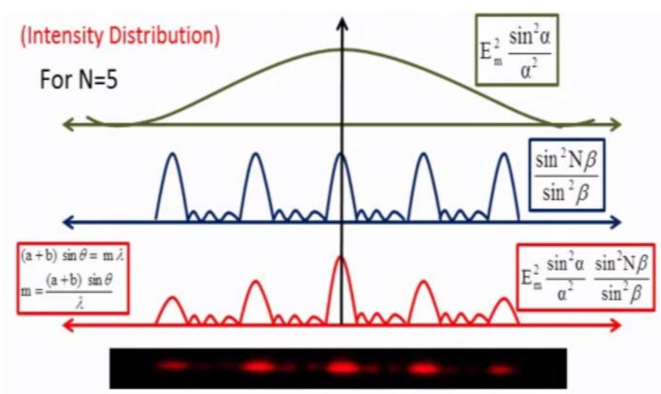
but, $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

$$\therefore N\frac{\pi}{\lambda}(a+b)\sin = \pm n\pi$$

$\therefore$ **N (a+b) sinθ = ± nλ** ……. (6)

For Secondary Minima:

As there are (N-1) minima between two adjacent principal maxima, there must be (N-2) other maxima between two principal maxima. These are known as secondary maxima.



Condition for Absent Spectra:

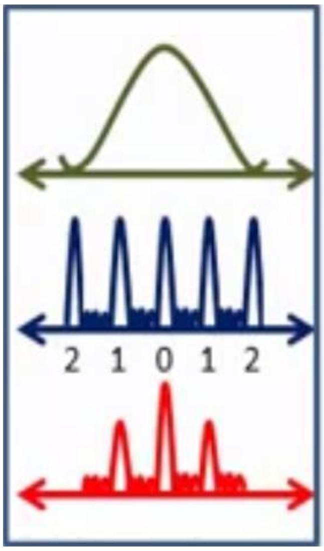
The intensity of light due to a diffraction grating in a direction making an angle θ with normal to the grating is given by,

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$$

Where, $\alpha = \frac{\pi}{\lambda} a \sin\theta$ and

$\beta = \frac{\pi}{\lambda}(\mathrm{a} + \mathrm{b}) \sin\ \theta$

The factor $I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$ is gives intensity distribution due to a single slit while the factor $\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$ gives that due to combine effect or N slits of the grating.



The principal maxima in case of a grating are obtain in the directions given by,

(a+b) sinθ = mλ ……. (1)

'm' being the order of diffraction.

Also the minima in case of a single slit are obtain in the directions given by,

a sin θ = nλ …… (2) n = 1, 2, 3, ……

If both the condition (1) and (2) are satisfied simultaneously, a particular maximum of order m will be missing in the grating spectrum.

Hence dividing equation (1) by (2) we have,

$$\frac{a+b}{a} = \frac{m}{n} \quad \text{…… (3)}$$

Which the condition for absent spectra.

Case 1: If **a = b**, then we have from equation (3)

$$\frac{m}{n} = 2 \text{ or } m = 2n$$

If n = 1, 2, 3, …., then m = 2, 4, 6, ….

i.e. **2nd, 4th, 6th,** …. etc order maxima are missing in the diffraction pattern, because these maxima coincide with 1st, 2nd, 3rd, ….., diffraction minima. i.e. even order spectra will be absent.

Case 2: If **2a = b**, then we have from equation (3)

$$\frac{m}{n} = 3 \textbf{ or } m = 3n$$

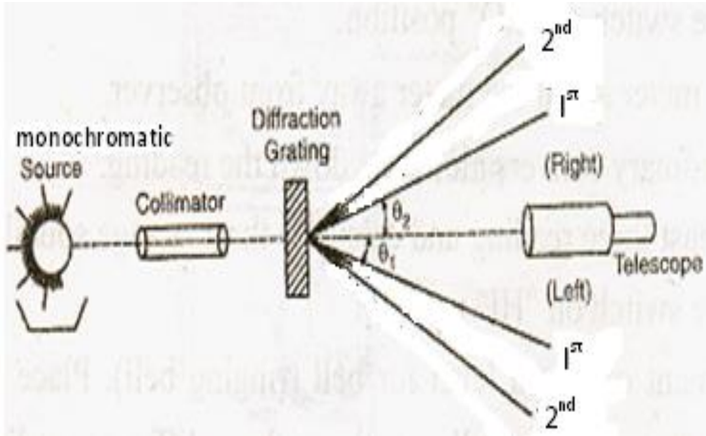
If n = 1, 2, 3, …., then m = 3, 6, 9, ….

i.e. **3rd, 6th, 9th,** …. etc order maxima are missing in the diffraction pattern, because these maxima coincide with 1st, 2nd, 3rd, ….., diffraction minima.

Determination of wavelength of light using diffraction grating:

Diffraction grating is often used in laboratories for determination of determination of unknown wavelength of light using spectrometer. The grating spectrum is observed by using a spectrometer.

It consists of monochromatic source of light, grating, & spectrometer consists of collimeter, telescope as shown in fig.



Adjust the telescope and collimator for parallel rays by Schuster's method, then diffraction grating is placed on the prism table such its plane is perpendicular to the prism table.

The diffraction pattern is observed on both side of central maximum. The diffraction pattern consists of different order maxima as shown in fig. The angle of diffraction for particular order is measured by using telescope

The number of lines per cm of a grating is given as,

$$(a+b) = \frac{1}{N/cm} = \frac{2.54}{N/inch}$$

The value of number of lines is written on grating**.**

The wavelength is calculated by using equation of maximum.

$$(\mathrm{a} + \mathrm{b}) \sin\theta = m\lambda$$

$$\lambda = \frac{(\mathrm{a}+\mathrm{b})\sin\theta}{m}$$

Thus the wavelength 'λ' is calculated from value of 'θ' & Number of lines 'N'.

Despressive power of a Grating:

The despressive power of grating is define as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in the wavelength between the two spectral lines.

It can also be define as the rate of variation of angle of diffraction with wavelength. It is expressed as $\frac{d\theta}{d\lambda}$

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} \quad .... (2)$$

The despressive power is directly proportional to order m i.e. higher the order greater is the despressive power.

The despressive power is inversely proportional to grating element. Hence smaller the grating element more widely spread is the spectrum.

The despressive power is inversely proportional to cos θ. Larger the value of θ, smaller the value of cos θ and higher the despressive power.

Resolving Power of Grating:

"It is defined as ability of grating to separate two spectral lines which have nearly same wavelength". It is also defined as the ratio of the wavelength (λ) of any spectral line in the spectrum to the least difference in the wavelength (dλ) between two adjacent or neighboring next lines such that two lines appear to be just resolved.

$$\therefore R.P = \frac{\lambda}{d\lambda} = \text{mN}$$

Where,

λ = wavelength of line

dλ = difference between two wavelengths λ & λ+ dλ.

m = order of diffraction

N= total number of lines in grating.

N = (Number of lines per cm) x (width of grating)

The above equation shows that resolving power of grating

(i) Increases with order of spectrum

(ii) Increases with increase in total number of lines

(iii) Is independent of grating element.