# UNIT – III

# FRICTION

# PART – I: GENERAL FRICTION

## INTRODUCTION

It has been established since long that all surfaces of the bodies are never perfectly smooth. It has been observed that whenever, even a very smooth surface is viewed under a microscope, it is found to have some roughness and irregularities, which may not be detected by an ordinary touch.

It will be interesting to know that if a block of one substance is placed over the level surface of the same or different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one of the blocks moves or tends to move tangentially with respect to the surface, on which it rests, the interlocking property of the projecting particles opposes the motion.

*This opposing force, which acts in the opposite direction of the movement of the block, is called force of friction or simply friction. Frictional force always acts tangentially at points of contact.*

### 1.1. Classification of Friction

The friction may be classified in two main categories:

*1. Statics Friction* *and* *2. Kinetic Friction (Dynamic Friction)*

### 1.1.1. Statics Friction

It is the friction experienced by a body when it is ***at rest***. Or in other words, it is the friction when the body ***tends to move***.

### 1.1.2. Kinetic Friction (Dynamic Friction)

It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types:

**i) Sliding friction:** It is the friction, experienced by a body when it ***slides*** over another body.

**ii) Rolling friction:** It is the friction, experienced by a body when it ***rolls*** over another body.

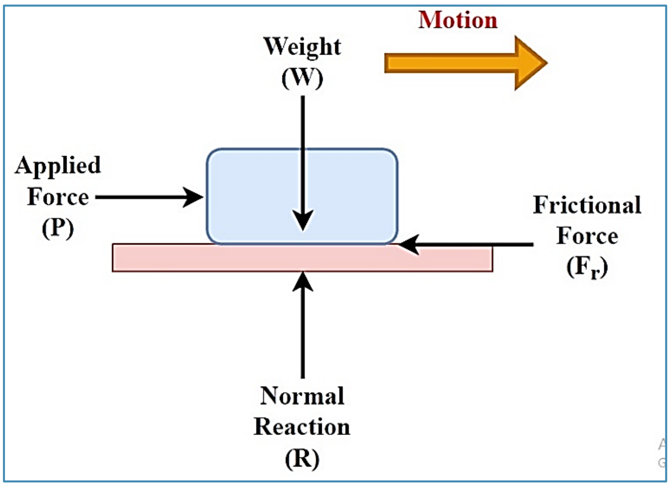
### 1.2. Theory of Friction

To understand concept of theory of friction let us consider a block of weight W pulled horizontally by a force P on a rough horizontal surface.

When a force P is increasing from zero, the frictional force F also goes on increasing to maintain equilibrium.

At one stage, block is on the verge of sliding and frictional force F attains its maximum value. (Limiting Friction)

Any further increase in applied force P, causes motion and value of F decreases rapidly to a kinetic value.




**1.3. LIMITING FRICTION**

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite of the applied force. Thus the force of friction has a remarkable property of adjusting its magnitude, so as to become exactly equal and opposite to the applied force, which tends to produce motion.

There is, however, *a limit* beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move, in the direction of the applied force. *This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction.* It may be noted that:

i) When the applied force is less than the limiting friction, *the body remains at rest*, and the friction is called static friction, which may have any value between zero and limiting friction.

**Applied Force (P) < Frictional Force ($F_r$) ………. (Stable Equilibrium)**

ii) When the applied force is equal to the limiting friction, *the body is on the verge of motion (tends to move),* and the friction is called limiting friction. Body is said to have *impending motion.*

**Applied Force (P) ≈ Frictional Force ($F_r$) ………. (Limiting Equilibrium)**

iii) When the applied force is greater than the limiting friction, *the body starts moving*, and the friction is called kinetic friction (dynamic friction)

**Applied Force (P) > Frictional Force ($F_r$) ………. (Unstable Equilibrium)**

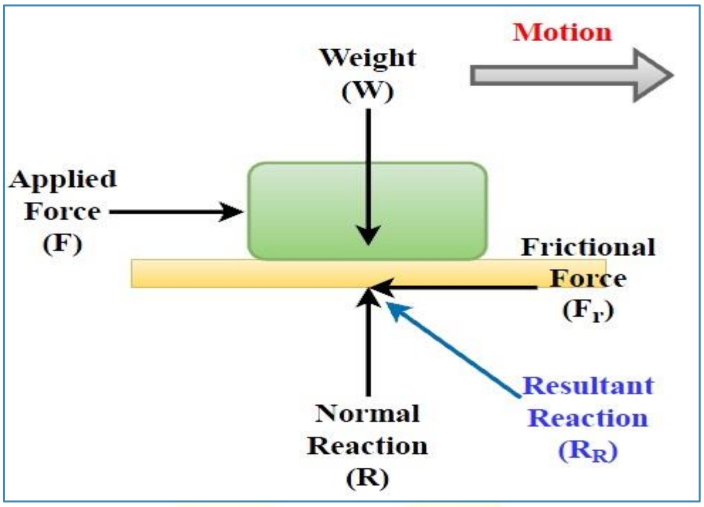
**1.4. Normal Reaction**

It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its center of gravity. The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called *normal reaction* and is, generally, denoted by *R*. It will be interesting to know that the term 'normal reaction' is very important in the field of friction, as the force of friction is directly proportional to it.

### 1.5. A Block on Rough Surface

As we know that when a block is placed on a smooth surface, the normal reaction (N) or (R) is perpendicular to surface but if it is placed on a rough surface, the reaction is not acting normal to the surface. The reaction which is actually inclined to the surface can be resolved into two components:

*i) Along the surface* *ii) Normal to the surface*




The component acting *along the surface (tangential) is frictional force ($F_r$)* and the *component normal to the surface is called normal reaction (N) or (R).*

### 1.6. Coefficient of Friction (μ)

#### 1.6.1. Coefficient of Static Friction ($\mu_s$)

The magnitude of limiting static frictional force ($F_s$) is directly proportional to the magnitude of normal reaction (R).

The relation can be expressed as,

$$F_s \propto R$$

$$\therefore F_s = \mu_s \times R$$

*Where constant of proportionality is called coefficient of static friction and is denoted by "$\mu_s$"*

$$\therefore \mu_s = \frac{F_s}{R} \qquad \text{....... For Impending motion}$$

#### 1.6.2. Coefficient of Kinetic Friction ($\mu_k$)

When magnitude of P acting on a block is increased and if it becomes greater than maximum friction force, the motion starts and magnitude of 'F' drops to a smaller value $F_k$ known as kinetic friction force.

The magnitude of this kinetic frictional force ($F_k$) is directly proportional to the magnitude of normal reaction (R).

The relation can be expressed as,

$$F_k \propto R$$

$$\therefore F_k = \mu_k \times R$$

*Where constant of proportionality is called coefficient of kinetic friction and is denoted by "$\mu_k$"*

$$\therefore \mu_k = \frac{F_k}{R} \qquad \text{....... For Actual motion}$$

### 1.7. Angle of Friction (Φ)

#### 1.7.1. Angle of Static Friction ($\Phi_s$)

For impending motion the angle between normal reaction and resultant reaction is called as angle of static friction.

By geometry, we have

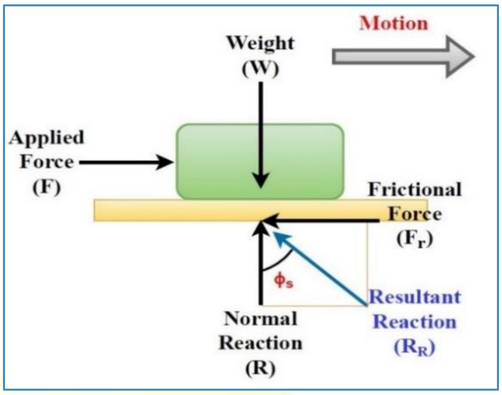
$$\tan \Phi_s = \frac{F_s}{R} \quad \text{....... (refer fig.)}$$

But We know,

$$\frac{F_s}{R} = \mu_s$$

So we can relate

$$\mu_s = \tan \Phi_s$$




**∴ Angle of Static Friction ($\Phi_s$) = $\tan^{-1}(\mu_s)$**

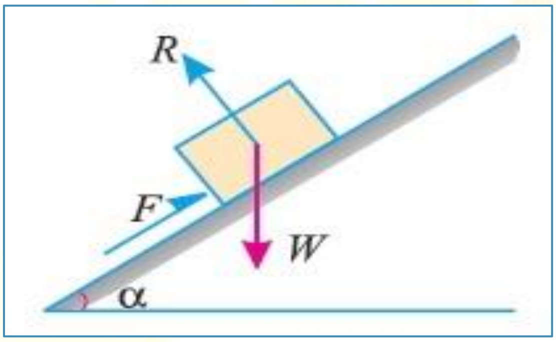
#### 1.7.2. Angle of Kinetic Friction ($\Phi_k$)

For actual motion when P = $F_k$, the angle between normal reaction and resultant reaction is called as angle of kinetic friction.

**∴ Angle of Kinetic Friction ($\Phi_k$) = $\tan^{-1}(\mu_k)$**

### 1.8. Angle of Repose (α)

It is defined as angle of inclined plane with horizontal at which body is just on *verge of sliding (impending motion)* and so Fs = $\mu_s \times R$. Angle of Repose is denoted by 'α'




### 1.9. Relation between Angle of Friction (Φ) and Angle of Repose (α)

To derive this relation consider F.B.D. of block of weight *W* on an inclined plane.

**$\Sigma F_X = 0$**

$\mu_s \cdot R - W \sin\alpha = 0$

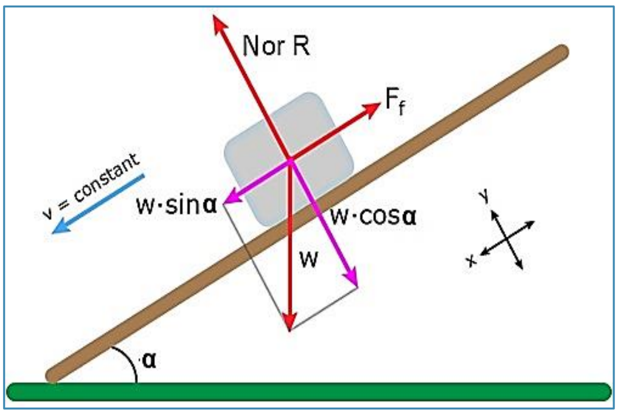
$\therefore \mu_s \cdot R = W \sin\alpha$ ----eq$^n$ (i)

**$\Sigma F_y = 0$**

$R - W\cos\alpha = 0$

$\therefore R = W\cos\alpha$ ----eq$^n$ (ii)

Put value of R in eq$^n$ (i)



We get,

$$\mu_s \,.\, W \cos \alpha = W \sin \alpha$$

$$\therefore \mu_s = \tan \alpha$$

But we know that,

$$\mu_s = \tan \Phi_s$$

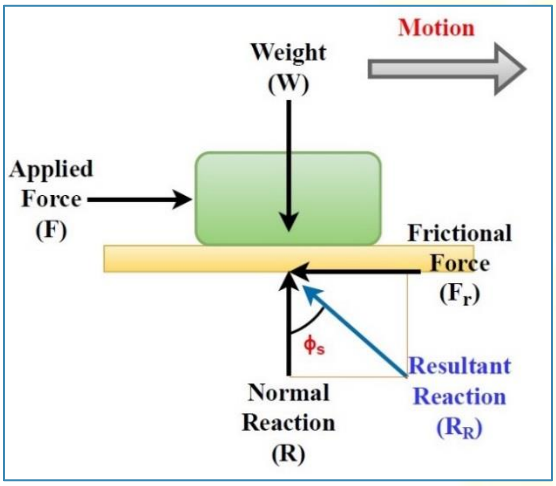
$$\therefore \tan \Phi_s = \tan \alpha$$

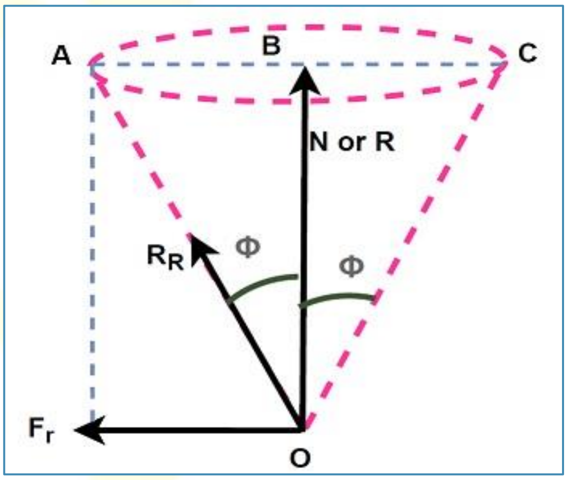
$$\therefore \mathbf{\Phi_s = \alpha}$$

Therefore, ***Angle of Static Friction (Φs) and Angle of Repose (α) are numerically equal.***

### 1.10. Cone of Friction

It is an imaginary cone generated by revolving resultant reaction ($R_R$) about the normal reaction (R)




### 1.10.1. Properties of Cone of Friction

i) The radius of this cone is represented by frictional force ($F_r$)
ii) The semi-cone angle represents angle of friction (Φ)
iii) For co-planer forces, in order for motion not to occur the reaction R must be within angle AOC
iv) For non-coplanar forces it must be within the cone of friction.

### 1.11. Laws of Friction (Coulomb's Laws of Dry Friction)

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads:

*a) Laws of static friction, and*
*b) Laws of kinetic or dynamic friction.*

***a) Laws of Static Friction***

*Following are the laws of static friction:*

i) The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.

ii) The magnitude of the force of friction is exactly equal to the force, which tends to move the body.

iii) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically:

$$\frac{Fmax}{R} = \text{constant (i.e. } \boldsymbol{\mu_s}\text{)}$$

Where, $F_{max}$ = Limiting friction, and

R = Normal reaction.

iv) The force of friction is independent of the area of contact between the two surfaces.

v) The force of friction depends upon the roughness of the surfaces.

***b) Laws of Kinetic Friction or Dynamic Friction***

*Following are the laws of kinetic or dynamic friction:*

i) The force of friction always acts in a direction, opposite to that in which the body is moving.

ii) The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.

$$\boldsymbol{\frac{Fk}{R} = \text{constant (i.e. } \mu_k\text{)}}$$

*** but the value of $\boldsymbol{\mu_k}$ **is always less than value of** $\boldsymbol{\mu_s}$**.**

$$\boldsymbol{\therefore \mu_k < \mu_s}$$

iii) For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

# PART – II: BELT FRICTION (BELT DRIVES)

## INTRODUCTION

The belts are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

It may be noted that:

a) The shafts should be properly in line to insure uniform tension across the belt section.

b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

d) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

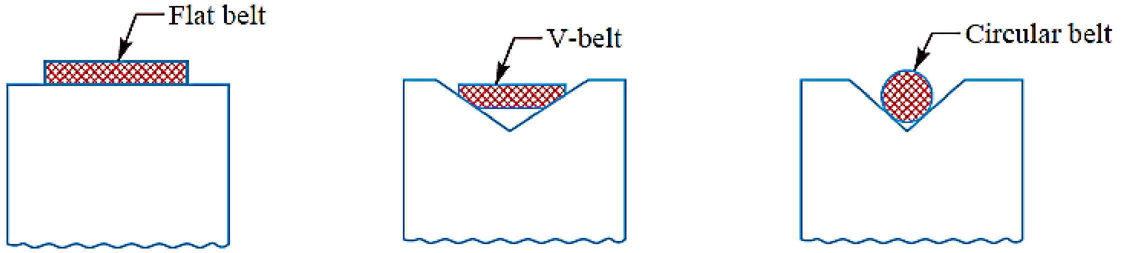
e) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

f) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley.

### 1.1. Types of Belts

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat Belt:*** The flat belt as shown in Fig. (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 meters apart.
2. ***V- Belt:*** The V-belt as shown in Fig. (b), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.
3. ***Circular Belt or Rope:*** The circular belt or rope as shown in Fig. (c) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.



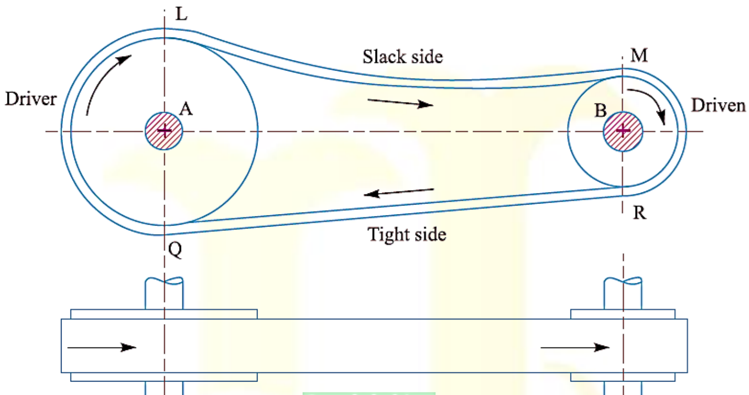

(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

### 1.2. Types of Flat Belt Drives

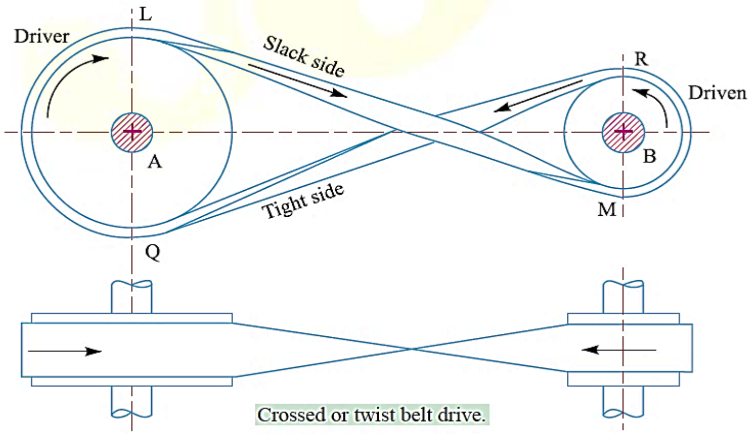
The power from one pulley to another may be transmitted by any of the following types of belt drives.

1. **Open Belt Drive:** The open belt drive, as shown in Fig., is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ) and delivers it to the other side (i.e. upper side LM). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side, as shown in Fig.



Open belt drive.

2. **Crossed Belt Drive:** Crossed or twist belt drive. The crossed or twist belt drive, as shown in Fig., is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. LM). Thus, the tension in the belt RQ will be more than that in the belt LM. The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side, as shown in Fig.



Crossed or twist belt drive.

A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of belt and the speed of the belt should be less than 15 m/s.

**1.3. Velocity Ratio of a Belt Drive**

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let, $d_1$ = Diameter of the driver,
$d_2$ = Diameter of the follower,
$N_1$ = Speed of the driver in r.p.m.,
$N_2$ = Speed of the follower in r.p.m.,

$\therefore$ Length of the belt that passes over the driver, in one minute

$= \pi\, d_1\, N_1$

Similarly, length of the belt that passes over the follower, in one minute

$= \pi\, d_2\, N_2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \pi\, d_1\, N_1 = \pi\, d_2\, N_2$$

As velocity ration is $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

***The above relation shows that speed of the pulley is inversely proportional to the diameter of the pulley.***

Also, When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes: **1. The velocity ratio of a belt drive may also be obtained as discussed below**:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 N_1}{60}\ \mathrm{m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60}\ \mathrm{m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

**2. In case of a compound belt drive as shown in Fig., the velocity ratio is given by**

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}} \rightarrow \frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

**1.4. Length of the Belt Drives**

Though there are many types of belts used these days, yet the following are important from the subject point of view:

**i)** ***Length of Open-Belt Drive:*** Total length of the belt is the sum of three parts,

1. The length of belt in contact with the driver (larger) pulley (APB),
2. The length of belt in contact with the driven (smaller) pulley (CQD), and
3. The length of belt NOT in contact with the driver and driven pulley (BC + AD)

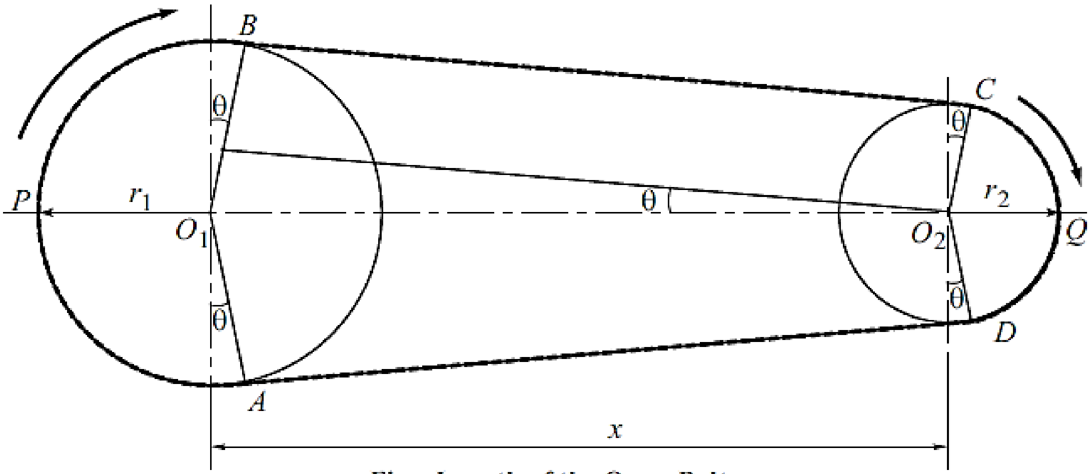



**Fig.: Length of the Open-Belt**

Let $x$ = Distance between the centres of the two pulleys ($O_1$ and $O_2$)

$r_1$ = Radius of the driver (larger) pulley

$r_2$ = Radius of the driven (smaller) pulley

$L$ = Total length of the belt ($APB + BC + CQD + DA$)

From $O_2$ draw a line parallel to $BC$ which will be perpendicular to $O_1B$ at $M$

$$\therefore O_1M = r_1 - r_2$$

Consider the right angled $\Delta O_1MO_2$, we have

$$BC = MO_2 = \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

$$BC = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

By binomial theorem, we have

$$BC = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots..\right] \quad \text{Further terms are negligible.}$$

$$BC = x - \frac{(r_1 - r_2)^2}{2x}$$

Total length, $L$ = Arc $APB$ + Arc $CQD$ + $BC$ + $AD$ (But $BC = AD$)

$$L = 2(\text{Arc } PB + \text{Arc } CQ + BC)$$

$$L = 2\left[r_1\left(\frac{\pi}{2}+\theta\right) + r_2\left(\frac{\pi}{2}-\theta\right) + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_2\frac{\pi}{2} + r_1\theta - r_2\theta + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\theta(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

But $\sin\theta = \dfrac{r_1-r_2}{x}$ ; since $\theta$ is very small we can assume $\sin\theta = \theta$

$$\therefore \theta = \frac{r_1-r_2}{x}$$

$$\therefore L = \pi(r_1+r_2) + 2\frac{(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\therefore L = \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x$$

**ii) *Length of Cross-Belt Drives:*** Total length of the belt is sum of three parts :

1. The length of belt in contact with the driver (larger) pulley (APB),
2. The length of belt in contact with the driven (smaller) pulley (CQD) and
3. The length of belt NOT in contact with the driver and driven pulley (BD + AC).

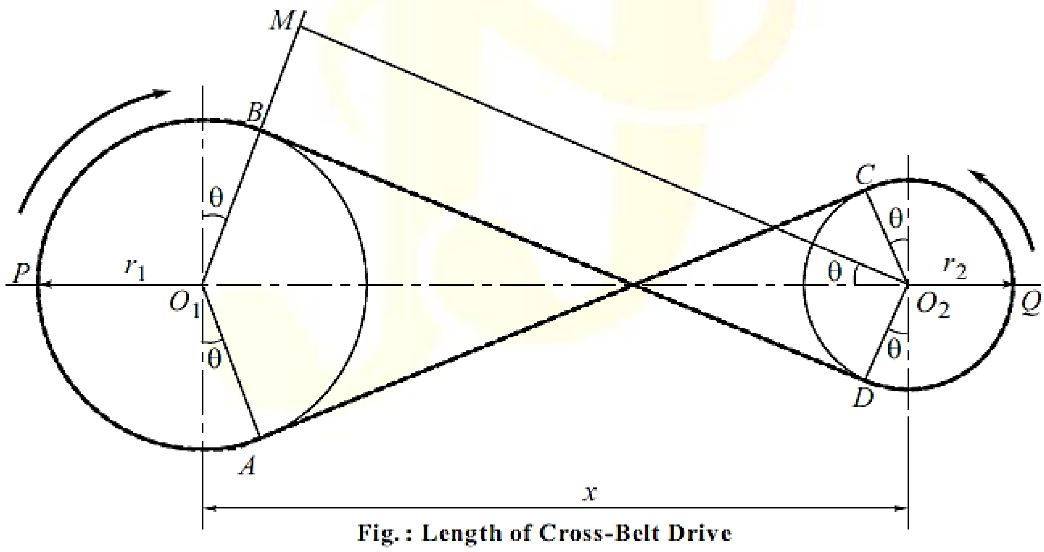



**Fig. : Length of Cross-Belt Drive**

Let $x$ = Distance between the centres of the two pulleys ($O_1$ and $O_2$)

$r_1$ = Radius of the driver (larger) pulley

$r_2$ = Radius of the driven (smaller) pulley

$L$ = Total length of the belt ($APB + BD + CQD + CA$)

From $O_2$ draw a line parallel to $BC$ which will be perpendicular to $O_1B$ at $M$

$$\therefore O_1M = (r_1 + r_2)$$

Consider the right angled $\Delta O_1MO_2$, we have

$$BD = MO_2 = \sqrt{x^2 - (r_1 + r_2)^2} = x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

$$BD = x\left[1 - \left(\frac{r_1 - r_2}{x}\right)^2\right]^{1/2}$$

By binomial theorem, we have

$$BD = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + .....\right] \quad \text{Further terms are negligible.}$$

$$BD = x - \frac{(r_1 + r_2)^2}{2x}$$

Total length, $L = \text{Arc } APB + \text{Arc } CQD + BD + AC \quad (\text{But } BC = AD)$

$$L = 2(\text{Arc } PB + \text{Arc } CQ + BD)$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \theta\right) + r_2\left(\frac{\pi}{2} + \theta\right) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_2\frac{\pi}{2} + r_1\theta + r_2\theta + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\theta(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

But $\sin\theta = \dfrac{r_1 + r_2}{x}$; since $\theta$ is very small, we can assume $\sin\theta = \theta$

$$\therefore \theta = \frac{r_1 + r_2}{x}$$

$$\therefore L = \pi(r_1 + r_2) + 2\frac{(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$\therefore L = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

**1.5. Derivation for relation between the Tight side, Slack Side, Lap Angle and Coefficient of Friction for a Flat Belt (Ratio of Driving Tensions for Flat Belt Drive)**

Consider a flat belt passing over a fixed drum and belt is just about to slide towards right (impending motion).

Let, $T_1$ = Tension on Tight Side ***(Greater Tension)***

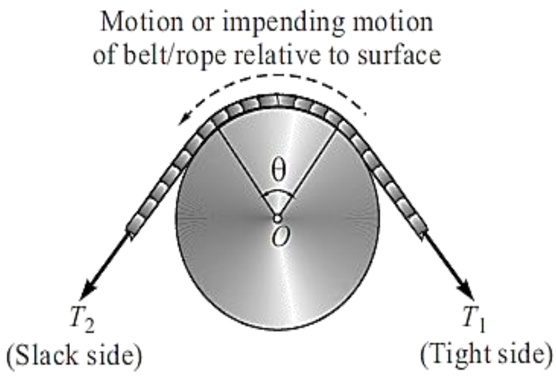
$T_2$ = Tension on Slack Side ***(Smaller Tension)***

$\theta$= Lap Angle OR Angle of contact in ***Radians***

$\mu_s$ = Co-efficient of Static Friction.

Relation between $T_1$ and $T_2$ can be given as:

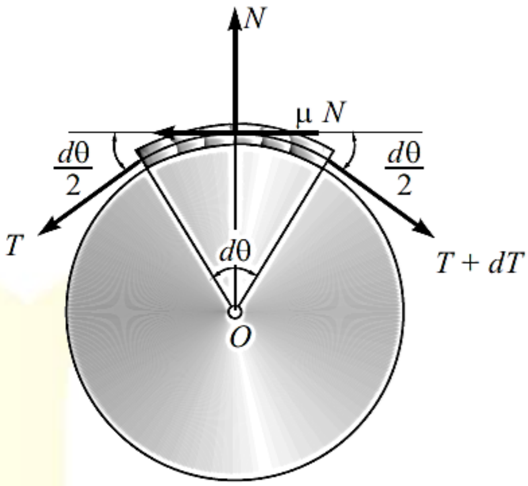
$$\frac{T_1}{T_2} = e^{\mu_s \cdot \theta}$$



***Derivation:**

Consider a belt wrapped around a fixed pulley with friction. The slack side tension $T_2$ increases along the surface of contact to $T_1$ tight side tension because of friction.

Consider the FBD of an element of belt subtending an angle dθ at the centre. Let T be the tension on one face of element which increases to *(T + dT)* on the other face.




$$\Sigma F_x = 0$$

$$(T + dT)\cos\left(\frac{d\theta}{2}\right) - \mu N - T\cos\left(\frac{d\theta}{2}\right) = 0$$

$$dT\cos\left(\frac{d\theta}{2}\right) = \mu N$$

Now,

$$\cos\left(\frac{d\theta}{2}\right) = 1 \qquad \{\because\ d\theta \text{ is very small}\}$$

$$\therefore dT = \mu N \qquad \dots \text{(I)}$$

$$\Sigma F_y = 0$$

$$N - T\sin\left(\frac{d\theta}{2}\right) - (T + dT)\sin\left(\frac{d\theta}{2}\right) = 0$$

Now,

$$\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2} \qquad \{\because\ d\theta \text{ is very small}\}$$

$$T\left(\frac{d\theta}{2}\right) + (T + dT)\left(\frac{d\theta}{2}\right) = N$$

$$\therefore T\,d\theta = N \quad \{\text{neglecting product of } dT \text{ and } d\theta\} \qquad \dots \text{(II)}$$

From Eqs. (I) and (II), we get

$$dT = \mu T\,d\theta$$

$$\frac{dT}{T} = \mu\,d\theta$$

Integrating both sides

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^{\theta} d\theta$$

$$\left[\log T\right]_{T_2}^{T_1} = \mu\left[\theta\right]_0^{\theta}$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\,\theta$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \textbf{In general,} \qquad \frac{Tight\ Side}{Slack\ Side} = e^{\mu_s \cdot \theta}$$

*The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.*

***** Points to be Remembered:**

1. The above relation is used only when the belt is about to slip.
2. If belt is actually slipping, then use $\frac{Tight\ Side}{Slack\ Side} = e^{\mu_k \cdot \theta}$
3. The lap angle or the angle of contact $\beta$ must be in radians.
4. The tight side represents the larger tension in part of belt which pulls and slack side represents the smaller tension in part which resists.

**1.6. Centrifugal Tension**

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides.

The tension caused by centrifugal force is called centrifugal tension. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

We know if a particle of mass **M** is rotated in a circular path of radius **r** at a uniform velocity **v**, a centrifugal force acts radially outwards and its magnitude is equal to

$$F_C = \frac{Mv^2}{r}$$

In pulley belt arrangement also as the belt material is moving with certain velocity on a pulley, it develops force which acts outwards from the centre of the pulley. This force is called the ***centrifugal force (Fc)***.

Let us consider an elemental length of belt which subtends an elemental angle $d\theta$ at the centre of the pulley as shown in Fig.

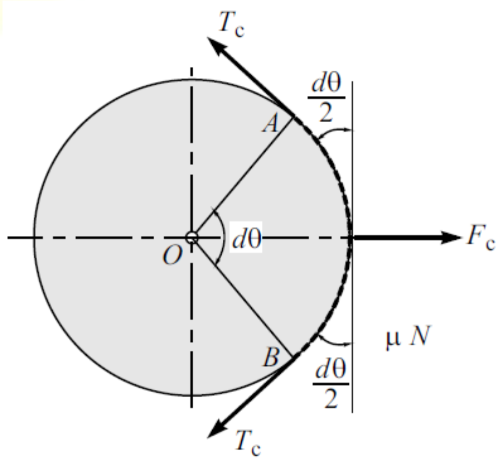
Let v = Velocity of the belt in m/s

r = Radius of pulley over which the belt runs

m = Mass of the belt per metre length

$T_c$ = Centrifugal tension acting on belt

$F_c$ = Centrifugal force acting radially outward




Elemental length of the belt AB = r dθ

Mass of the belt AB = m . r . dθ

Centrifugal force $F_C = \frac{Mv^2}{r} = m \cdot r \cdot d\theta \frac{v^2}{r}$

Resolving the forces horizontally and applying limiting equilibrium condition, we have

$$F_C = 2\, T_c \sin \frac{d\theta}{2}$$

Equating both equations

$$\frac{mr\,d\theta\,v^2}{r} = 2T_c \sin\frac{d\theta}{2} \qquad \left[\begin{array}{l}\text{Since } \dfrac{d\theta}{2} \text{ is very small} \\ \therefore\ \sin\dfrac{d\theta}{2} = \dfrac{d\theta}{2}\end{array}\right]$$

$$m\,d\theta\,v^2 = T_c\,d\theta$$

$$T_c = mv^2$$

**Note :** When we consider the centrifugal tension, we have

Tension on tight side $= T_1 + T_c$

Tension on slack side $= T_2 + T_c$

$\therefore$ The maximum tension in the belt $T_{max} = T_1 + T_c$

For safety purpose $T_{max}$ should not exceed the maximum permissible limit of tension otherwise belt will break.

**1.7. Initial Tension in the Belt**

Initially the belt is mounted such that a firm grip develops between the belt and pulley. Therefore, in stationary condition some amount of tension is developed in the belt. This tension is known as initial tension. This initial tension helps to prevent slipping of the belt on the pulley, which would result in loss of power and excessive wear. The initial tension is equal in both parts of the belt.

Under working condition, tension in the two side of the belt will change. The tight side of the belt stretches until the pull is increased from $T_0$ to $T_1$ and slack side shortens until the pull is decreased from $T_0$ to $T_2$.

$$\text{Increase of tension on tight side} = T_1 - T_0$$

$$\text{Decrease of tension on slack side} = T_0 - T_2$$

$$\text{Increase in length of tight side} = \text{Decrease in length of slack side}$$

$$\varepsilon(T_1 - T_0) = \varepsilon(T_0 - T_2)$$

$$T_1 - T_0 = T_0 - T_2$$

$$T_0 = \frac{T_1 + T_2}{2}$$

When centrifugal tension is also considered, we have

$$\text{Initial Tension} = \frac{T_1 + T_2 + 2T_c}{2}$$

**1.8. Power Transmitted and Condition for Maximum Power Transmitted**

Let $T_1$ = Tension on tight side

$T_2$ = Tension on slack side

$v$ = Linear velocity of belt

Power = Force ´ Velocity

$\therefore$ Power transmitted by the belt is given by

$$P = (T_1 - T_2)\,v$$

$$P = (T_1 - T_2)v$$

But we know that $\dfrac{T_1}{T_2} = e^{\mu\theta} \quad \therefore \; T_2 = \dfrac{T_1}{e^{\mu\theta}}$

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)$$

$$P = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) v$$

$$P = T_1 k v \qquad \left(\text{where } k = 1 - \frac{1}{e^{\mu\theta}} = \text{constant}\right)$$

We know, maximum tension ($T_{max}$) on tight side will have a relation

$$T_{max} = T_1 + T_c$$

$$P = (T_{max} - T_c)\, vk$$

$$P = (T_{max} - mv^2)\, vk \qquad \left(\because T_c = mv^2\right)$$

$$P = (T_{max}\, v - mv^3)\, k$$

For power to be maximum, apply the maxima condition

$$\frac{dP}{dv} = 0$$

$$\frac{dP}{dv} = (T_{max} - 3mv^2) = 0$$

$$T_{max} - 3mv^2 = 0$$

$$T_{max} = 3mv^2$$

$$T_{max} = 3T_c \qquad \left(\because T_c = mv^2\right)$$

**1.9. Slip of Belt**

We have assumed the motion of belts and pulleys has a firm frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let $s_1$ % = Slip between the driver and the belt, and

$s_2$ % = Slip between the belt and follower,

∴ Velocity of the belt passing over the driver per second,

$$v = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{s_1}{100}$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad \ldots(i)$$

and velocity of the belt passing over the follower per second

$$\frac{\pi d_2 N_2}{60} = v - v\left(\frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (*i*), we have

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \ldots\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left[1 - \left(\frac{s_1 + s_2}{100}\right)\right] = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

...(where $s = s_1 + s_2$ *i.e.* total percentage of slip)

If thickness of the belt ($t$) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$