Unit-IV

ELECTRODYNAMICS

- **Prerequisite:** *Coulomb's law-force between two point charges, Electric field, electric field due to a point charge, electric field due to a dipole, Gauss's law, Faradays law, Cartesian co-ordinate system, Cylindrical co-ordinate system, Spherical co-ordinate system.*
- Scalar and Vector field,
- Physical significance of gradient, curl and divergence in Cartesian co-ordinate system (Numerical on conversion, gradient, curl and divergence).
- Gausses law for electrostatics, Gausses law for magnetostatics, Faradays Law and Amperes circuital law,(No question this part)
- Divergence theorem, Stokes theorem (Statement and formula) significance.
- Maxwell’s equations (Derivation in Free space and time varying fields), only equation in differential and integral form, Physical significance and Applications of Maxwell’s equations. (Only listing).

ELECTRODYNAMICS

Scalar Field and Vector Field:

In science and technology, we frequently encounter quantities that have magnitude and magnitude only. e.g mass, time, temperature etc. These quantities remain the same no matter what co- ordinates we use. They are called as scalar quantities.

In contrast, many physical quantities have magnitude and direction. These include displacement, velocity, acceleration, force momentum, angular momentum etc. these are called as vector quantities.

The behavior of a physical quantity in a given region is described by its value at each point in that region. A field is a function that describes the behavior of a physical quantity at all points in a given region of space.

The physical quantity described by the field can be either a scalar or vector. Thus a field can also be a scalar field or a vector field.

Scalar Field:

A scalar field is a function that gives us a single field value of some variable for every point in a space.

A scalar field is specified by only magnitude of a physical quantity at each point of the field region.

Ex. Temperature, volume, mass, density, energy, voltage, current etc.

Vector Field:

A vector field is specified by both the magnitude and direction of a physical quantity at each point of the field region.

E.g. velocity, acceleration, force, electric field etc.

Vector Algebra (Optional):

1) $$\vec{A} + \vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) + (B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

2) $$a\vec{A} = a\,(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$$

3) $$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$$

4) $$\hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$$

5) $$\vec{A}\cdot\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = (A_xB_x) + (A_yB_y) + (A_zB_z)$$

6) $$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$
$$\therefore A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

7) $$\hat{x}\times\hat{x} = \hat{y}\times\hat{y} = \hat{z}\times\hat{z} = 0$$
$$\hat{x}\times\hat{y} = -\hat{y}\times\hat{x} = \hat{z}$$
$$\hat{y}\times\hat{z} = -\hat{z}\times\hat{y} = \hat{x}$$
$$\hat{z}\times\hat{x} = -\hat{x}\times\hat{z} = \hat{y}$$

8) $$\vec{A}\times\vec{B} = (A_x\hat{x} + A_y\hat{y} + A_z\hat{z})\times(B_x\hat{x} + B_y\hat{y} + B_z\hat{z})$$
$$= (A_yB_z - A_zB_y)\hat{x} - (A_xB_z - A_zB_x)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$
$$= (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Scalar Triple Product

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Vector Triple Product

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) = \vec{C}(\vec{A}\cdot\vec{B})$$

Position Vector (Optional):

Positon vector for point (x,y,z) is –

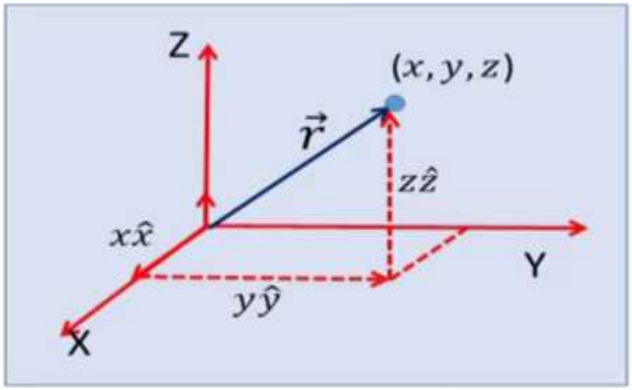
$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

Magnitude of position vector

$$r = \sqrt{x^2+y^2+z^2}$$

Unit vector pointing radially outward is -

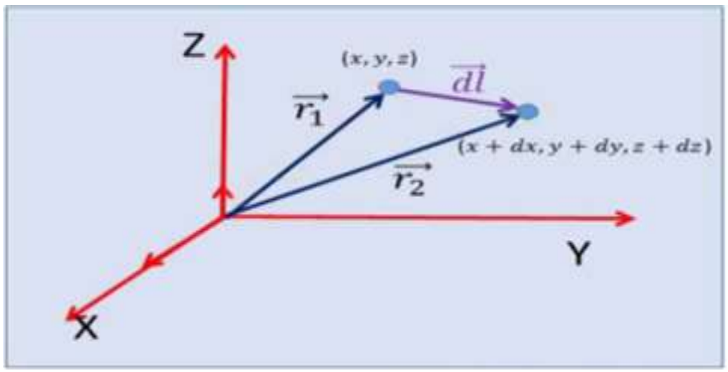
$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\,\hat{x} + y\,\hat{y} + z\,\hat{z}}{\sqrt{x^2+y^2+z^2}}$$



Displacement & Separation Vector (Optional):

The infinitesimal displacement vector from (x,y,z) to (x+dx, y+dy, z+dz) is given by –

$$\overrightarrow{dl} = \overrightarrow{r_2} - \overrightarrow{r_1}$$
$$= dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$



Ex.3 Find the separation vector $\vec{r}$ from the source point (2,8,7) to the field (4,6,8). Determine its magnitude and construct the unit vector $\hat{r}$.

Ans:

Position vector for point (2,8,7) is $\overrightarrow{r_1} = 2\hat{x} + 8\hat{y} + 7\hat{z}$

Position vector for point (4,6,8) is $\overrightarrow{r_2} = 4\hat{x} + 6\hat{y} + 8\hat{z}$

Separation vector $\vec{r} = \overrightarrow{r_2} - \overrightarrow{r_1} = (4-2)\hat{x} + (6-8)\hat{y} + (8-7)\hat{z} = 2\hat{x} - 2\hat{y} + 1\hat{z}$

Its magnitude $|\vec{r}| = r = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$

Unite vector $\hat{r} = \frac{\vec{r}}{r} = \frac{2}{3}\hat{x} - \frac{2}{3}\hat{y} + \frac{1}{3}\hat{z}$

The Operator ∇ (Del Operator):

$\vec{\nabla} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called as **Del operator**.

- This, 'del' is not a vector.
- It does not multiply a function; rather it is an instruction to differentiate what follows.
- It does not have any meaning until we provide it with a function to act upon.
- To be precise, $\vec{\nabla}$ is not a vector that multiplies T. $\vec{\nabla}$ is a vector operator that acts on T.

The operator $\vec{\nabla}$ can act in three ways:

1. On a scalar function T - i.e. $\overrightarrow{\nabla T}$ **(the gradient)**
2. On a vector function $\vec{V}$ via dot product $\vec{\nabla}.\vec{V}$ **(the divergence)**
3. On a vector function $\vec{V}$ via cross product $\nabla \times \vec{V}$ **(the Curl)**

Gradient:

Let us consider a function of three variables T(x,y,z). This function depends on three variables and can vary with different magnitude in different directions. A theorem on partial derivative states that

$$dT = \left(\frac{\partial T}{\partial x}\right)dx + \left(\frac{\partial T}{\partial y}\right)dy + \left(\frac{\partial T}{\partial z}\right)dz$$

This gives how T changes when we alter all three variables by infinitesimal amounts dx, dy and dz. Above equation is reminiscent of a dot product –

$$dT = \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)\cdot(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$$

$$= \overrightarrow{\nabla T}\cdot\overrightarrow{dl}$$

$\overrightarrow{\nabla T} = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$ is a gradient of T. $\overrightarrow{\nabla T}$ is a vector quantity with three components

Significance of gradient:

- Like any vector, gradient has a magnitude and direction.
- The gradient $\overrightarrow{\nabla T}$ points in the direction of maximum increase of the function T. Also, the magnitude $|\overrightarrow{\nabla T}|$ gives us the slope (i.e. rate of increase) along this direction.
- If the value of the function is decreasing along the given direction, the gradient is negative.
- If the value of the function is increasing along the given direction, the gradient is positive.

Ex.1 Find the $\mathbf{r = \sqrt{x^2} + y^2 + z^2}$ gradient of (magnitude of a position vector)

$$\overrightarrow{\nabla r} = \frac{}{\partial x}\hat{x} + \frac{}{\partial y}\hat{y} + \frac{}{\partial z}\hat{z}$$

$$= \frac{\partial\left(\sqrt{x^2+y^2+z^2}\right)}{\partial x}\hat{x} + \frac{\partial\left(\sqrt{x^2+y^2+z^2}\right)}{\partial y}\hat{y} + \frac{\partial\left(\sqrt{x^2+y^2+z^2}\right)}{\partial z}\hat{z}$$

$$= \frac{1}{2}\frac{2x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{1}{2}\frac{2y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{1}{2}\frac{2z}{\sqrt{x^2+y^2+z^2}}\hat{z}$$

$$= \frac{x\,\hat{x} + y\,\hat{y} + z\,\hat{z}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{r} = \hat{r}$$

Ex.2 Find the gradient $\boldsymbol{\phi(x,y,z) = 3x^2y - y^3z^2}$ of at point (1,-2,-1).

Soln:

$$\frac{\partial\phi}{\partial x} = \frac{\partial(3x^2y - y^3z^2)}{\partial x} = 6xy$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial(3x^2y - y^3z^2)}{\partial y} = 3x^2 - 3y^2z^2$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial(3x^2y - y^3z^2)}{\partial z} = -2y^3z$$

$$\overrightarrow{\nabla\phi} = \frac{\partial\phi}{\partial x}\hat{x} + \frac{\partial\phi}{\partial y}\hat{y} + \frac{\partial\phi}{\partial z}\hat{z}$$

$$= (6xy)\,\hat{x} + (3x^2 - 3y^2z^2)\,\hat{y} - (2y^3z)\,\hat{z}$$

$$\overrightarrow{\nabla\phi}\Big|_{(1,-2,-1)} = 6(1)(-2)\,\hat{x} + (3(1)^2 - 3(-2)^2(-1)^2)\,\hat{y} - 2(-2)^3(-1)\,\hat{z}$$

$$= -12\,\hat{x} - 9\,\hat{y} - 16\,\hat{z}$$

Ex.3 Let $\emptyset(\mathbf{x,y,z}) = \mathbf{3x^2z^3 - xy^2}$. Find gradient of a scalar field at point (2, 2,-1).

Ans:

$$\vec{\nabla}.\emptyset(\mathrm{x,y,z}) = \left(\vec{\mathrm{i}}\frac{\partial}{\partial \mathrm{x}} + \vec{\mathrm{j}}\frac{\partial}{\partial \mathrm{y}} + \vec{\mathrm{k}}\frac{\partial}{\partial \mathrm{z}}\right).(3\mathrm{x}^2\mathrm{z}^3 - \mathrm{xy}^2)$$

$$\vec{\nabla}.\emptyset = (6\mathrm{xz}^3 - \mathrm{y}^2)\vec{\mathrm{i}} - (2\mathrm{xy})\vec{\mathrm{j}} + (12\mathrm{x}^2\mathrm{z}^2)\vec{\mathrm{k}}$$

$\vec{\nabla}.\emptyset$ at point $(2,2,-1)$ is

$$\vec{\nabla}.\emptyset = [6(2)(-1)^3 - (2)^2]\vec{\mathrm{i}} - [2(2)(2)]\vec{\mathrm{j}} + [12(2)^2(-1)^2]\vec{\mathrm{k}}$$

$$\vec{\nabla}.\emptyset = -12\vec{\mathrm{i}} - 8\vec{\mathrm{j}} + 47\vec{\mathrm{k}}$$

Ex.4 Find gradient of a scalar field $\emptyset(\mathbf{x,y,z}) = \mathbf{3x^2y - 2y^3z^2}$ at point (1, -2,-1).

Ans:

$$\vec{\nabla}.\emptyset(\mathrm{x,y,z}) = \left(\vec{\mathrm{i}}\frac{\partial}{\partial \mathrm{x}} + \vec{\mathrm{j}}\frac{\partial}{\partial \mathrm{y}} + \vec{\mathrm{k}}\frac{\partial}{\partial \mathrm{z}}\right).(3\mathrm{x}^2\mathrm{y} - 2\mathrm{y}^3\mathrm{z}^2)$$

$$\vec{\nabla}.\emptyset = (6\mathrm{xy})\vec{\mathrm{i}} + (3\mathrm{x}^2 - 6\mathrm{y}^2\mathrm{z}^2)\vec{\mathrm{j}} + (-4\mathrm{y}^3\mathrm{z})\vec{\mathrm{k}}$$

$\vec{\nabla}.\emptyset$ at point $(1,-2,-1)$ is

$$\vec{\nabla}.\emptyset = [6(1)(-2)]\vec{\mathrm{i}} + [3(1)^2 - 6(-2)^2(-1)]\vec{\mathrm{j}} + [-4(-2)^3(-1)]\vec{\mathrm{k}}$$

$$\vec{\nabla}.\emptyset = -12\,\vec{\mathrm{i}} + 27\,\vec{\mathrm{j}}\; - 32\,\vec{\mathrm{z}}$$

Ex.5 Find gradient of a scalar field $\emptyset(\mathbf{x,y,z}) = \mathbf{x^3yz^2 - 2y^3z}$ at point (1, -1, 1).

Ans:

$$\vec{\nabla}.\emptyset(\mathrm{x,y,z}) = \left(\vec{\mathrm{i}}\frac{\partial}{\partial \mathrm{x}} + \vec{\mathrm{j}}\frac{\partial}{\partial \mathrm{y}} + \vec{\mathrm{k}}\frac{\partial}{\partial \mathrm{z}}\right).(\mathrm{x}^3\mathrm{yz}^2 - 2\mathrm{y}^3)$$

$$\vec{\nabla}.\emptyset = (3\mathrm{x}^2\mathrm{yz}^2)\vec{\mathrm{i}} + (\mathrm{x}^3\mathrm{z}^2 - 6\mathrm{y}^2)\vec{\mathrm{j}} + (2\mathrm{x}^3\mathrm{yz})\vec{\mathrm{k}}$$

$\vec{\nabla}.\emptyset$ at point $(1,-1,1)$ is

$$\vec{\nabla}.\emptyset = [3(1)^2(-1)(1)^2]\,\vec{\mathrm{i}} + [(1)^3(1)^2 - 6(-1)^2]\,\vec{\mathrm{j}} + [2(1)^3(-1)(1)]\vec{\mathrm{k}}$$

$$\vec{\nabla}.\emptyset = -3\,\vec{\mathrm{i}} - 5\,\vec{\mathrm{j}}\; - 2\,\vec{\mathrm{z}}$$

Divergence:

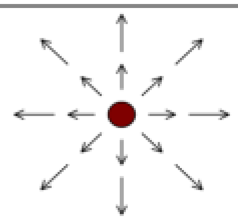
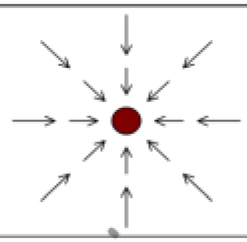
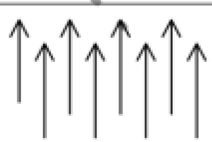
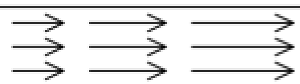
When the operator $\vec{\nabla}$ act on a vector function $\vec{\mathrm{v}}$, via dot product, we get divergence of a vector function $\vec{\mathrm{v}}$.

$$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\mathrm{v}_x\hat{x}+\mathrm{v}_y\hat{y}+\mathrm{v}_z\hat{z}\right)$$

$$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\frac{\partial \mathrm{v}_x}{\partial x}+\frac{\partial \mathrm{v}_y}{\partial y}+\frac{\partial \mathrm{v}_z}{\partial z}\right)$$

Significance of Divergence:

- The divergence of a vector function is a scalar.
- The divergence of a scalar function cannot be written and it is meaningless.
- Divergence $\vec{\nabla}\cdot\vec{\mathrm{v}}$ is a measure of how much the vector $\vec{\nabla}$ spreads out (diverges) from the given point.

	$\vec{\mathrm{v}} = x\,\hat{x}+y\,\hat{y}+z\,\hat{z}$ This vector function has a large positive divergence.	$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(x\hat{x}+y\hat{y}+z\hat{z})$ $\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}\right)$ $= 1+1+1 = 3$
	$\vec{\mathrm{v}} = \vec{\mathrm{v}} = -x\hat{x}-y\hat{y}-z\hat{z}$ This vector function has large negative divergence	$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(-x\hat{x}-y\hat{y}-z\hat{z})$ $\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\frac{\partial(-x)}{\partial x}+\frac{\partial(-y)}{\partial y}+\frac{\partial(-z)}{\partial z}\right)$ $= -1-1-1 = -3$
	$\vec{\mathrm{v}} = K\,\hat{z}$ K is constant. This vector has zero divergence	$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(0\hat{x}+0\hat{y}+K\hat{z})$ $\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\frac{\partial 0}{\partial x}+\frac{\partial 0}{\partial y}+\frac{\partial K}{\partial z}\right)$ $= 0+0+0 = 0$
	$\vec{\mathrm{v}} = y\,\hat{y}$ This vector has positive divergence.	$\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(0\hat{x}+y\hat{y}+0\hat{z})$ $\vec{\nabla}\cdot\vec{\mathrm{v}} = \left(\frac{\partial 0}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial 0}{\partial z}\right)$ $= 0+1+0 = 1$

Ex.1 If $\overrightarrow{\mathbf{v_A}} = x\,\hat{x}+y\,\hat{y}+z\,\hat{z}$ and $\overrightarrow{\mathbf{v_B}} = y\,\hat{y}$ then, calculate their divergence.

Sol^n:

$$\vec{\nabla}\cdot\overrightarrow{\mathrm{v_A}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(x\hat{x}+y\hat{y}+z\hat{z})$$

$$\vec{\nabla}\cdot\overrightarrow{\mathrm{v_A}} = \left(\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}\right) = 1+1+1 = 3$$

$$\vec{\nabla}\cdot\overrightarrow{\mathrm{v_B}} = \left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\cdot(y\hat{y})$$

$$\vec{\nabla}\cdot\overrightarrow{\mathrm{v_B}} = \left(\frac{\partial 0}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial 0}{\partial z}\right) = 0+1+0 = 1$$

Ex.2 If $\vec{A} = 2x^3y\vec{i} + 2y^2z^3\vec{j} - 3xyz^2\vec{k}$. Find the div at point (1, 1,-1)

Sol^n:

$$\vec{\nabla}.\vec{A} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(2x^3y\vec{i} + 2y^2z^3\vec{j} - 3xyz^2\vec{k}\right)$$

$$\vec{\nabla}.\vec{A} = \left(\frac{\partial(2x^3y)}{\partial x} + \frac{\partial(2y^2z^3)}{\partial y} + \frac{\partial(-3xyz^2)}{\partial z}\right)$$

$$\vec{\nabla}.\vec{A} = 6x^2y + 4yz^3 - 6xy^2z$$

$\vec{\nabla}.\vec{A}$ at point $(1, 1, -1)$ is

$$\vec{\nabla}.\vec{A} = 6(1)^2(1) + 4(1)(-1)^3 - 6(1)(1)^2(-1)$$

$$\vec{\nabla}.\vec{A} = 6 - 4 + 6 = 8$$

Ex.3 Find divergence at a point (-1, 3, 2) of a function $\vec{V} = 3x^3y^2\,\vec{i} + y^2z^2\,\vec{j} - 3z^2\vec{k}$

Sol^n:

$$\vec{\nabla}.\vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(3x^3y^2\,\vec{i} + y^2z^2\,\vec{j} - 3z^2\vec{k}\right)$$

$$\vec{\nabla}.\vec{V} = \left(\frac{\partial(3x^3y^2)}{\partial x} + \frac{\partial(y^2z^2)}{\partial y} + \frac{\partial(-3z^2)}{\partial z}\right)$$

$$\vec{\nabla}.\vec{V} = 9x^2y^2 + 2yz^2 - 6z$$

$\vec{\nabla}.\vec{V}$ at point $(-1,3,2)$ is

$$\vec{\nabla}.\vec{V} = 9(-1)^2(3)^2 + 2(3)(2)^2 - 6(2)$$

$$\vec{\nabla}.\vec{V} = 81 + 24 - 12 = 93$$

Ex.4 Find divergence at a point (1,1,2) of a function $\vec{V} = 3xy^3\,\vec{i} + 7z^2\,\vec{j} - 4y^2\vec{k}$

Sol^n:

$$\vec{\nabla}.\vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(3xy^3\,\vec{i} + 7z^2\,\vec{j} - 4y^2\vec{k}\right)$$

$$\vec{\nabla}.\vec{V} = \left(\frac{\partial(3xy^3)}{\partial x} + \frac{\partial(7z^2)}{\partial y} + \frac{\partial(-4y^2)}{\partial z}\right)$$

$$\vec{\nabla}.\vec{V} = 3y^3$$

$\vec{\nabla}.\vec{V}$ at point $(1, 1,2)$ is

$$\vec{\nabla}.\vec{V} = 3(1)^3$$

$$\vec{\nabla}.\vec{V} = 3$$

Ex.5 Find divergence at a point (1,-1, 1) of a function $\vec{V} = x^2z\,\vec{i} - 2y^2z^2\,\vec{j} + xy^2z\,\vec{k}$

Sol^n:

$$\vec{\nabla}.\vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(x^2z\,\vec{i} - 2y^2z^2\,\vec{j} + xy^2z\,\vec{k}\right)$$

$$\vec{\nabla}.\vec{V} = \left(\frac{\partial(x^2z\,)}{\partial x} + \frac{\partial(-2y^2z^2)}{\partial y} + \frac{\partial(xy^2z)}{\partial z}\right)$$

$\vec{\nabla}.\vec{V} = 2xz - 4yz^2 + xy^2$

$\vec{\nabla}.\vec{V}$ at point $(1, -1, 1)$ is

$\vec{\nabla}.\vec{V} = 2(1)(1) - 4(-1)(1)^2 + (1)(-1)^2$

$\vec{\nabla}.\vec{V} = 7$

Curl:

When the operator $\vec{\nabla}$ act on a vector function $\vec{V}$, via cross product, we get curl of a vector function $\vec{V}$.

$$\vec{\nabla} \times \vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[V_x\,\vec{i} + V_y\,\vec{j} + V_z\,\vec{k}\right]$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\vec{\nabla} \times \vec{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\vec{i} - \left[\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right]\vec{j} + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\vec{k}$$

Significance of Curl:

- The curl of a vector function is a vector.
- $\vec{\nabla} \times \vec{v}$ is a measure of how much the vector $\vec{V}$ curls around the given point.
- Zero curl means there is no rotation.

Ex.1 Compute curl $\vec{F}$ for a function, $\vec{F} = 4x^2y\,\vec{i} - 3y^2z\,\vec{j} + z^3xy\,\vec{k}$ at point (1, -1, 2).

Sol^n:

The curl of $\vec{F}$ is given by

$$\vec{\nabla} \times \vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[4x^2y\,\vec{i} - 3y^2z\,\vec{j} + z^3xy\,\vec{k}\right]$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x^2y & -3y^2z & z^3xy \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \left[\frac{\partial(z^3xy)}{\partial y} - \frac{\partial(-3y^2z)}{\partial z}\right]\vec{i} - \left[\frac{\partial(z^3xy)}{\partial x} - \frac{\partial(4x^2y)}{\partial z}\right]\vec{j} + \left[\frac{\partial(-3y^2z)}{\partial x} - \frac{\partial(4x^2y)}{\partial y}\right]\vec{k}$$

$$\vec{\nabla} \times \vec{F} = (z^3x + 3y^2)\vec{i} - (z^3y)\vec{j} - (4x^2)\vec{k}$$

$\vec{\nabla} \times \vec{F}$ at point $(1, -1, 2)$ is

$$\vec{\nabla} \times \vec{F} = [(2)^3(1) + 3(-1)^2]\,\vec{i} - [(2)^3(-1)]\,\vec{j} - [4(1)^2]\,\vec{k}$$

$$\vec{\nabla} \times \vec{F} = 11\,\vec{i} + 8\,\vec{j} - 4\,\vec{k}$$

Ex.2 If $\vec{V} = xy^2\vec{i} + 2yx^2z\,\vec{j} - 3yz^2\,\vec{k}$ is a vector function. Compute curl $\vec{V}$ at a point (1,-1, 1)

Sol^n:

$$\vec{\nabla} \times \vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[xy^2\vec{i} + 2yx^2z\,\vec{j} - 3yz^2\,\vec{k}\right]$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & 2yx^2z & -3yz^2 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{V} = \left[\frac{\partial(-3yz^2)}{\partial y} - \frac{\partial(2yx^2z)}{\partial z}\right]\vec{i} - \left[\frac{\partial(-3yz^2)}{\partial x} - \frac{\partial(xy^2)}{\partial z}\right]\vec{j} + \left[\frac{\partial(2yx^2z\,)}{\partial x} - \frac{\partial(xy^2)}{\partial y}\right]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = (-3z^2 - 2yx^2)\vec{i} + (4yxz - 2xy)\vec{k}$$

$\vec{\nabla} \times \vec{V}$ at a point (1,-1,1) is

$$\vec{\nabla} \times \vec{V} = [-3(1)^2 - 2(-1)(1)^2]\,\vec{i} + [4(-1)(1)(1) - 2(1)(-1)]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = [-3 + 2]\,\vec{i} + [-4 + 2]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = -\,\vec{i} - 2\vec{k}$$

Ex.3 Find curl at a point (2,-1, 1) of a function $\vec{V} = (xy^2z)\vec{i} + 3x^2y\,\vec{j} + (xz^2 - y^2z)\vec{k}$

Sol^n:

$$\vec{\nabla} \times \vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[(xy^2z)\,\vec{i} + 3x^2y\,\vec{j} + (xz^2 - y^2z)\vec{k}\right]$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2z & 3x^2y & (xz^2 - y^2z) \end{vmatrix}$$

$$\vec{\nabla} \times \vec{V} = \left[\frac{\partial(xz^2 - y^2z)}{\partial y} - \frac{\partial(3x^2y\,)}{\partial z}\right]\vec{i} - \left[\frac{\partial(xz^2 - y^2z)}{\partial x} - \frac{\partial(xy^2z)}{\partial z}\right]\vec{j} + \left[\frac{\partial(3x^2y\,)}{\partial x} - \frac{\partial(xy^2z)}{\partial y}\right]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = -2yz\,\vec{i} - (z^2 - xy^2)\vec{j} + (6xy - 2xyz)\vec{k}$$

$\vec{\nabla} \times \vec{V}$ at a point (2,-1,1) is

$$\vec{\nabla} \times \vec{V} = [-2(-1)(1)]\,\vec{i} - [(1) - (2)(-1)^2]\vec{j} + [6(2)(-1) - (2)(-1)(1)]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = 2\,\vec{i} + \,\vec{j} - 10\,\vec{k}$$

Ex.4 If $\vec{F} = x^3y\,\vec{i} - (z^2 - 5x)\,\vec{j} + 7y^2\,\vec{k}$ is a vector function. Calculate the divergence and curl of $\vec{F}$.

Sol^n:

$$\vec{\nabla} \times \vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[x^3y\vec{i} - (z^2 - 5x)\vec{j} + 7y^2\vec{k}\right]$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -(z^2-5x) & 7y^2 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{F} = \vec{i}\left(\frac{\partial(7y^2)}{\partial y} - \frac{\partial(5x-z^2)}{\partial z}\right) - \vec{j}\left(\frac{\partial(7y^2)}{\partial x} - \frac{\partial(x^3y)}{\partial z}\right) + \vec{k}\left(\frac{\partial(5x-z^2)}{\partial x} - \frac{\partial(x^3y)}{\partial y}\right)$$

$$\vec{\nabla}\times\vec{F} = (14y+2z)\vec{i} + (5-x^3)\vec{k}$$

Divergence of $\vec{F}$ is given by

$$\vec{\nabla}.\vec{F} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(x^3y\,\vec{i} - (z^2-5x)\,\vec{j} + 7y^2\,\vec{k}\right)$$

$$\vec{\nabla}.\vec{F} = \left(\frac{\partial(x^3y)}{\partial x} + \frac{\partial-(z^2-5x)}{\partial y} + \frac{\partial(7y^2)}{\partial z}\right)$$

$$\vec{\nabla}.\vec{F} = 3x^2y$$

Divergence of Curl of function $\vec{V}$:

Que: Show that divergence of curl of vector function $\vec{V}$ is zero i.e. $\vec{\nabla}.\left(\vec{\nabla}\times\vec{V}\right) = 0$

Let us consider $\vec{V} = V_x\,\vec{i} + V_y\,\vec{j} + V_z\,\vec{k}$

The curl of $\vec{V}$ is given by

$$\vec{\nabla}\times\vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\times\left[V_x\,\vec{i} + V_y\,\vec{j} + V_z\,\vec{k}\right]$$

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\vec{i} - \left[\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right]\vec{j} + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\vec{k}$$

Divergence of $\vec{V}$ is given by

$$\vec{\nabla}.\left(\vec{\nabla}\times\vec{V}\right) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\vec{i} - \left[\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right]\vec{j} + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\vec{k}$$

$$\vec{\nabla}.\left(\vec{\nabla}\times\vec{V}\right) = \frac{\partial}{\partial x}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\vec{\nabla}.\left(\vec{\nabla}\times\vec{V}\right) = \frac{\partial^2 V_z}{\partial x\,\partial y} - \frac{\partial^2 V_y}{\partial x\,\partial z} - \frac{\partial^2 V_z}{\partial y\,\partial x} + \frac{\partial^2 V_x}{\partial y\,\partial z} + \frac{\partial^2 V_y}{\partial z\,\partial x} - \frac{\partial^2 V_x}{\partial z\,\partial y}$$

$$\vec{\nabla}.\left(\vec{\nabla}\times\vec{V}\right) = 0$$

Thus the divergence of curl of vector function $\vec{V}$ is zero

Ex. If $\vec{V} = (x^2 - y^2)\vec{i} + 2xy\,\vec{j} + (y^2 - xy)\,\vec{k}$ is a vector function. Prove that for given function $\vec{V}$ the div of curl of $\vec{V}$ is zero.

Sol^{n}:

$$\vec{\nabla} \times \vec{V} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[(x^2 - y^2)\vec{i} + 2xy\,\vec{j} + (y^2 - xy)\,\vec{k}\right]$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (x^2 - y^2) & 2xy & (y^2 - xy) \end{vmatrix}$$

$$\vec{\nabla} \times \vec{V} = \left[\frac{\partial(y^2 - xy)}{\partial y} - \frac{\partial(2xy)}{\partial z}\right]\vec{i} - \left[\frac{\partial(y^2 - xy)}{\partial x} - \frac{\partial(x^2 - y^2)}{\partial z}\right]\vec{j} + \left[\frac{\partial(2xy\,)}{\partial x} - \frac{\partial(x^2 - y^2)}{\partial y}\right]\vec{k}$$

$$\vec{\nabla} \times \vec{V} = (2y - x)\vec{i} + y\,\vec{j} + 4y\,\vec{k}$$

Div of Curl of $\vec{V}$ is given by

$$\vec{\nabla}.\left(\vec{\nabla} \times \vec{V}\right) = \frac{\partial}{\partial x}(2y - x) + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}\,4y$$

$$\vec{\nabla}.\left(\vec{\nabla} \times \vec{V}\right) = -1 + 1$$

$$\vec{\nabla}.\left(\vec{\nabla} \times \vec{V}\right) = 0$$

Gauss Theorem (Divergence theorem)

Gauss Theorem States that – If V is the volume bound by the surface S, volume integral of divergence of a function $\vec{v}$ over volume V is equal to surface integral of the function $\vec{v}$ over the surface S that surrounds the given volume.

$$\int_V \vec{\nabla} \cdot \vec{v}\ dv = \oint_S \vec{v} \cdot \overrightarrow{ds}$$

Stoke's Theorem (Fundamental Theorem for Curl)

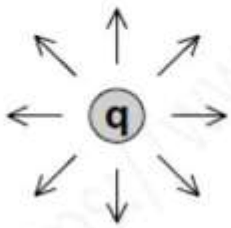
Stokes Theorem States that – If S is the surface area bound by the boundary P, surface integral of curl of a vector function $\vec{v}$ over surface area S is equal to line integral of the vector function $\vec{v}$ over the closed curve P binding that surface.

$$\int_S (\vec{\nabla} \times \vec{v}) \cdot ds = \oint_P \vec{v} \cdot dl$$

Gauss Theorem (Divergence theorem) and Stoke's Theorem (Fundamental Theorem for Curl) are used while writing the Maxwell's equations in differential form.

Coulomb's Law:

The static charge q develops a spherical electric field surrounding it. The line of force of the field are directed radially outwards from q.



The force exerted by this field on a test charge Q is given by Coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\,\hat{r}$$

$\epsilon_0 = 8.85 \times 10^{-12}\dfrac{C^2}{Nm^2}$ is called permittivity of free space

- This force is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.
- The force points along the line from q to Q.
- It is repulsive if charges have same sign and attractive if charges have opposite sign.

Electric Field Intensity:

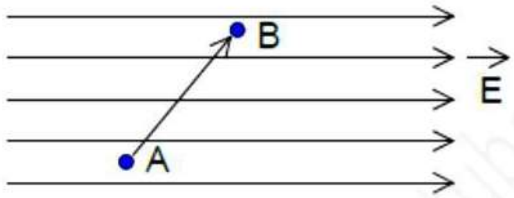
Electric Field Intensity is defined as the force per unit charge at any point. In the field region. It is given by - $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}\,\hat{r}$

Electric flux density :

Electric flux density is defined as the electric charge over unit area of the spherical surface having its centre at charge q. Thus, electric flux density is same as electric charge density and is given by- $\vec{D} = \dfrac{q}{4\pi r^2}\,\hat{r} = \epsilon_0\vec{E}$

Total flux over entire spherical surface is given by - $\phi = \int_S \vec{D}\ ds = \epsilon_0 \int_S \vec{E}\ ds$

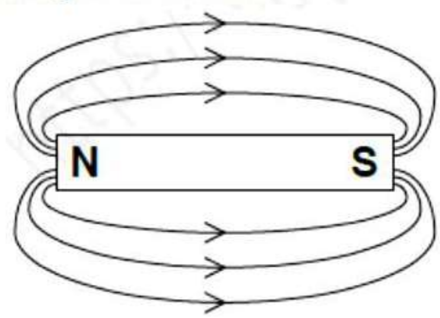
Electric potential:



It is a work done by an external source in moving a charge q from one point to another in an electric field $\vec{E}$. It is given by - $V_{AB} = \dfrac{W}{q} = -\int_A^B \vec{E}\,dl$.

It is called potential difference between point A and B.

Magnetic Field :



- A region around a magnet within which the influence of the magnet can be experienced is called the magnetic field.
- The magnetic lines of force or magnetic flux start from the north pole and end at south pole. An isolated magnetic pole can never exist.

Magnetic Field Intensity ($\vec{H}$):

The magnetic field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole when placed at that point.

Magnetic flux density ($\vec{B}$):

Total magnetic lines of force (magnetic flux) per unit surface area in the perpendicular direction is called as magnetic flux density.

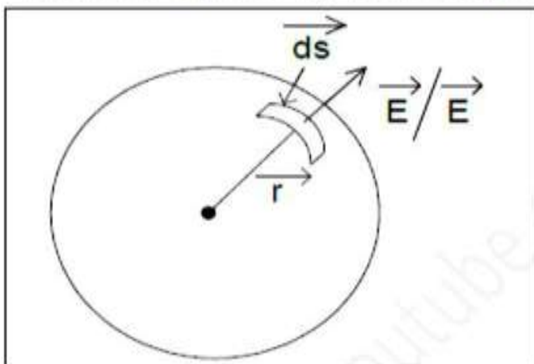
$$\vec{B} = \mu\vec{H}$$

Where μ is permeability, $\mu = \mu_0\mu_r$ where $\mu_0 = 4\pi \times 10^{-7} N/A^2$ is permeability of free space and μ_r is relative permeability.

Maxwell's Equations :

1. First Maxwell's Equation (Gauss's Law for Electric Field):

Gauss law for electric field states that – The total electric flux passing through any closed surface is equal to the total charge enclosed by that surface.



Mathematically, It can be written as -

$$\boldsymbol{\phi} = \oint_S \vec{\mathbf{D}}\ \boldsymbol{ds} = \boldsymbol{\epsilon_0} \oint_S \vec{\boldsymbol{E}}\ \boldsymbol{ds} = \boldsymbol{Q_{enclosed}} \text{ ---- (1)}$$

If ρ_v is the charge density, total charge enclosed by surface can be written as -

$$\boldsymbol{Q_{enclosed}} = \int_V \boldsymbol{\rho_v}\ \boldsymbol{dv} \text{ ------------------(2)}$$

Therefore, Gauss law in integral form can be written as –

$$\oint_S \vec{\boldsymbol{D}}\ \boldsymbol{ds} = \boldsymbol{\epsilon_0} \oint_S \vec{\boldsymbol{E}}\ \boldsymbol{ds} = \int_V \boldsymbol{\rho_v}\ \boldsymbol{dv} \text{ ----------------(3)}$$

Using divergence theorem, we can write -

$$\int_V \vec{\nabla}\cdot\vec{\mathbf{E}}\ \boldsymbol{dv} = \oint_S \vec{\mathbf{E}}\ \boldsymbol{ds} \text{ ---------------- (4)}$$

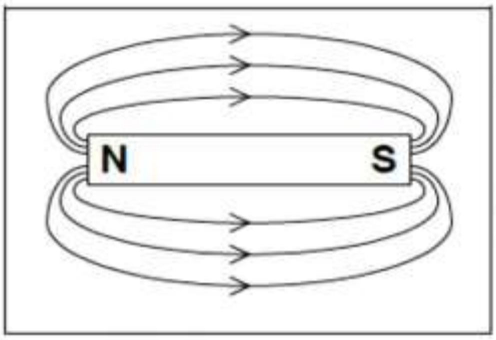
From (3) and (4), we can write – $\boldsymbol{\epsilon_0} \int_V \vec{\nabla}\cdot\vec{\mathbf{E}}\ \boldsymbol{dv} = \int_V \boldsymbol{\rho_v}\ \boldsymbol{dv}$

Therefore,

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \frac{\rho_v}{\epsilon_0}$$ **(This is first Maxwell's Equation in differential form)**

2. Second Maxwell's Equation (Gauss Law for Static Magnetic Field) –

Gauss Law for Static Magnetic Field states that – "In a magnetic field, the magnetic lines force are closed on themselves as shown –



Total outgoing flux is zero. This can be written mathematically as –

$$\oint_S \vec{B}\ ds = 0 \text{ -------- (1)}$$

This is called Gauss Law for magnetic field in integral form.
Using divergence theorem, we can write -

$$\int_V \vec{\nabla} \cdot \vec{B}\ dv \ = \ \oint_S \vec{B}\ ds = 0 \text{ -------- (2)}$$

Thus, $\vec{\nabla} \cdot \vec{B} = 0$ **(This is second Maxwell's Equation in differential form)**

3. Third Maxwell's Equation (Faraday's Law) :

A changing magnetic field induces an electric field.
Faraday's law states that "Whenever there is a change in magnetic flux linked with the circuit, an emf is induced in that circuit. The magnitude of induced emf is equal to the rate of change of the flux."

Therefore, the work done in moving a test charge from one point to the other point is given by –

$$\oint_P \vec{E}\ dl = -\frac{d\phi}{dt} = -\oint_S \frac{\partial \vec{B}}{\partial t}\ ds \quad \text{(This is integral form of the Faraday's law)}$$

Using Stoke's theorem, we can write –

$$\oint_P \vec{E}\ dl = \int_S (\vec{\nabla} \times \vec{E})\ ds = -\oint_S \frac{\partial \vec{B}}{\partial t}\ ds$$

Therefore,

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ **(This is third Maxwell's equation in differential form)**

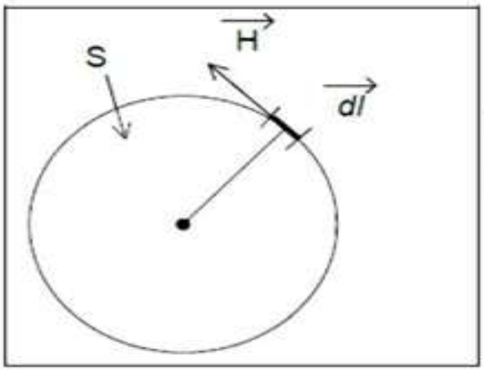
In static field, the work done in moving a test charge around a closed path is zero as ϕ is constant.

$$\oint_P \vec{E}\ dl = 0$$

So, in static field, $\oint_P \vec{E}\ dl = \int_S (\vec{\nabla} \times \vec{E})\ ds = 0$ **and** $\vec{\nabla} \times \vec{E} = 0$

4. Fourth Maxwell's Equation (Ampere's Circuital law) :

It gives the relationship between the current and the magnetic field created by it.
Ampere's circuital law states that- "The line integral of magnetic field intensity $\vec{H}$ around a closed path is equal to the current enclosed by that path."



$$\oint \vec{H}\ dl = I$$

This is integral form of Ampere's law-
As, current density, $J = \frac{I}{ds}$

$$\oint \vec{H}\ dl = I = \int_S \vec{J}\ ds$$

Using Stoke's theorem, we can write –

$$\int_S (\vec{\nabla} \times \vec{H})\, ds = \oint \vec{H}\, dl = \int_S \vec{J}\, ds$$

Therefore, $\vec{\nabla} \times \vec{H} = \vec{J}$

As $\vec{B} = \mu_0 \vec{H}$ in free space, $\quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

This is differential form of Ampere's law (fourth Maxwell equation for static field)

Ampere's law is completely valid for the closed surface through which Electric field doesn't change with time (i.e. for static fields). For closed surfaces through which Electric field changes with time, displacement current density must also be considered. Changing electric field must also produce a magnetic field. Further, since magnetic fields have always been associated with currents, Maxwell postulated that this current was proportional to the rate of change of the electric field and called it displacement current. Displacement current density is defined in terms of the rate of change of electric displacement field. The units of displacement current are same as that of electric current density. Displacement current is not an electric current caused due to moving of charges, but it is caused by a time varying electric field.

So the total current density is given by - $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

And the fourth Maxwell equation for time varying fields becomes

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

Summary of Maxwell's Equations:

Maxwell's Equation No.	Law	Integral Form	Differential Form (Point Form)
1	Gauss's Law for Electric Field	$\epsilon_0 \oint_S \vec{E}\, ds = \int_V \rho_v\, dv$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho_v}{\epsilon_0}$
2	Gauss's Law for Static Magnetic Field	$\oint_S \vec{B}\, ds = 0$	$\vec{\nabla} \cdot \vec{B} = 0$
3	Faraday's Law	$\oint_P \vec{E}\, dl = -\oint_S \frac{\partial \vec{B}}{\partial t}\, ds$	In static field $\vec{\nabla} \times \vec{E} = 0$ In varying field $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4	Ampere's Circuital law	$\oint \vec{H}\, dl = I = \int_S \vec{J}\, ds$	For Static Field $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ For Time varying field $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

Significance of Maxwell's Equations :

Maxwell's Equation No.	Differential Form (Point Form)	
1	$\vec{\nabla}\cdot\vec{E} = \frac{\rho_v}{\epsilon_0}$	➢ Divergence of the electric field is equal to the charge density inside of a closed surface of interest, multiplied by a constant. ➢ The divergence is how much the electric field "spreads out" from a given point. ➢ If there is more charge inside, the divergence is greater. If it's zero, the divergence is zero.
2	$\vec{\nabla}\cdot\vec{B} = 0$	➢ It indicates that there are **no magnetic monopoles**. All magnetic "charge" is found in a dipole, with a North and a South. ➢ The divergence of B is always zero. ➢ As such, there is no "sink" or "source" for B - the field lines have no beginning and no end. ➢ There is no source for them like there is for an electric field (i.e. an electric monopole).
3	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	➢ This equation says that the **curl** of the electric field is equal to the negative of the change in the magnetic field in time. ➢ In other words, if the magnetic field isn't changing, electric field lines are straight. ➢ If it is changing, the electric field "swirls" appropriately, depending on if the field is increasing or decreasing. ➢ A changing magnetic field can induce an electric field (i.e. Faraday induction). The negative sign is called **Lenz's law**
4	$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$	➢ The curl of B, or the "swirliness of B" is equal to the Current Density (the amount of current per unit area) plus any change in the electric field. ➢ The second part is often called the "displacement current. ➢ It helps with dealing with capacitors