Shirpur Education Society's

R. C. PATEL INSTITUTE OF TECHNOLOGY, SHIRPUR

An Autonomous Institute

(Affiliated to Dr. Babasaheb Ambedkar Technological University, Lonere)

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(स्वायत्त महाविद्यालय)

Programme: B.TECH (AIML)/B.TECH (COMP)/B.TECH (DATA SCI.)/B.TECH (EXTC)/B.TECH (ELECT)/B.TECH (MECH)/B.TECH (CIVIL)

Year: I / Semester I (Exam Year: 2023-2024)

Subject: Mathematics- I (RCP23FCBS101) Max Marks: 60

Date: 04 Jan 2024 **Time:** 02:30 pm - 04:30 pm (02:00 Hrs.)

END SEMESTER EXAMINATION-ODD SEM-I (AY: **2023-2024**)

Instructions:

1. This question paper contains 3 pages
2. Answer to each new question to be started on a fresh page.
3. Figure in right hand side indicates full marks.
4. All Questions are Compulsory.
5. Assume suitable data wherever required but justify it.
6. Support your answers with neat, labelled diagrams, wherever necessary.

1. 1 **15**

a. **7**

i. Determine the values of λ for which the following equations are consistent. Also solve the system for these values of λ. **7**

$$x + 2y + z = 3$$
$$x + y + z = \lambda$$
$$3x + y + 3z = \lambda^2.$$

---------- OR ----------

ii. Show that every square matrix can be uniquely expressed as the sum of Hermitian and Skew-Hermitian matrix. **7**

b. **8**

i. Reduce the following matrix to echelon form and hence find it's rank. **4**

$$\begin{bmatrix} 3 & 2 & 1 & 4 \\ -1 & 3 & 2 & 2 \\ 2 & 5 & 3 & 6 \end{bmatrix}$$

ii. If $x^4 + y^4 = 5a^2xy$, find $\frac{dy}{dx}$. **4**

2. 2 **15**

a. **7**

i. Solve by using Demoivre's theorem, $x^4 - x^3 + x^2 - x + 1 = 0$. **7**

---------- OR ----------

ii. *Using De Moivre's Theorem, prove that* $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$. **7**

b. Prove that $sin^{-1} x = -i\, log\left(ix + \sqrt{1 - x^2}\right)$ and hence prove that $sin^{-1}(sinh\, x) = tan^{-1}(tanh\, x) - \frac{i}{4}\, log(cosh^2 x + sinh^2 x)$ **8**

3. 3 **15**

a. **7**

i. If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that **7**

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$

---------- OR ----------

ii. If $u = f(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$, prove that **7**

$u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r}f'(r)$.

b. **8**

i. Fit a parabola $y = a + bx + cx^2$ to the following data **4**

x	0	1	2	3	4
y	1	2	1	3	8

ii. Using Newton–Raphson method, find the root of the equation $e^x - 4x = 0$ by taking initial condition $x_0 = 2.1$. Perform three iterations. **4**

4. 4 **15**

a. Expand $\tan^{-1}(x)$ in powers of $(x - 1)$. **7**

b. **8**

8

If $u = \text{cosec}^{-1}\sqrt{\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)}$

Using Euler's theorem for homogeneous functions, prove that

i. i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\frac{1}{12}\tan u$

ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial x^2}=\frac{\tan u}{144}(13+\tan^2 u)$

---------- OR ----------

ii. Find the extreme values of $x^3+3xy^2-3x^2-3y^2+7$. **8**