



A.Y. 2022-23-Year-2 /Semester-IV

Program: FYBTECH (COMMON TO ALL)
 Course: **Engineering Mathematics-I (22BSFY1010T)**
 Date: 29/05/2023

Max Marks: 65
 Time: 9 Am-11Noon
 Duration: 2 Hrs

REEND SEMESTER EXAMINATION ODD SEM- I- MAY-JUNE- 2023

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.

- (1) This question paper contains 02 pages.
- (2) All Questions are Compulsory.
- (3) Answer to each new question is to be started on a fresh page.
- (4) Figures in the brackets on the right indicate full marks.
- (5) Assume suitable data wherever required, but justify it.
- (6) Support your answers with neat labelled diagrams, wherever necessary.

Q. 1-A (i) Find the rank of the following matrix by reducing to echelon (canonical) 05

form
$$\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

04

(ii) Prove that the matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is unitary.

Q. 1-A Or

Examine whether the following vectors are linearly independent or dependent, find relation if dependent. $[1, 0, 2, 1], [3, 1, 2, 1], [4, 6, 2, 4], [-6, 0, -3, 0]$ 09

Q. 1-B Show that $\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ 06

Q. 1-B Or

Solve the equation $z^3 = (z + 1)^3$ and show that the real part of all roots is $-\frac{1}{2}$. 06

Q. 1-C If $u = \tan^{-1} \left(\frac{x^4 + y^4}{x^2 + y^2} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ 05

Q. 2-A Find the n^{th} derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ 07

Or

Q. 2-A If $y = e^{m \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ 07

Q. 2-B If $f(u, v) = 0$ where $u = lx + my + nz$, $v = x^2 + y^2 + z^2$, prove that 08

$$(ly - mx) + (ny - mz) \frac{\partial z}{\partial x} + (lz - nx) \frac{\partial z}{\partial y} = 0$$

Q. 3-A In calculating the volume of a right circular cone, errors of 2% and 1% are made in height and radius of base respectively. Find the % error in volume. 07

Or

Q. 3-A Find the maxima and minima of $xy(a - x - y)$. 07,

Q. 3-B If $\tan(\alpha + i\beta) = \sin(x + iy)$, prove that $\frac{\tan x}{\tanh y} = \frac{\sin 2\alpha}{\sinh 2\beta}$ 08

Q. 4-A Fit a curve $y = a + bx + cx^2$ to the following data 06

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Or

Q. 4-A Find the smallest root of an equation $x - e^{-x} = 0$ correct to three significant digits using Regula Falsi Method. 06

Q. 4-B Use Taylor's theorem to evaluate $\sqrt{25.25}$ 03

Or

Q. 4-B Find the Maclaurin's series expansion of $\tan x$ 03

Q. 4-C Solve the following system of equations using Gauss-Siedel method (3 Iterations) 06

$$5x + y - z = 10$$

$$2x + 4y + z = 14$$

$$x + y + 8z = 20$$