Implement Gradient Descent in Python

What is gradient descent?

It is an optimization algorithm to find the minimum of a function. We start with a random point on the function and move in the **negative direction** of the **gradient of the function** to reach the **local/global minima**.

Example by hand:

Question: Find the local minima of the function $y=(x+3)^2$ starting from the point x=2

Solution : We know the answer just by looking at the graph. $y = (x+3)^2$ reaches it's minimum value when x = -3 (i.e when x=-3, y=0). Hence x=-3 is the local and global minima of the function.

Now, let's see how to obtain the same numerically using gradient descent.

Step 1: Initialize x = 2. Then, find the gradient of the function, dy/dx = 2*(x+3).

Step 2: Move in the direction of the negative of the gradient. Stopping condition: how much to move? For that, we require a learning rate. Let us assume the **learning** $rate \rightarrow 0.01$

Step 3: Let's perform 2 iterations of gradient descent

Initialize Parameters:

$$x_0 = 2$$

Learning rate = 0.01

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$$\frac{dy}{dx} = \frac{d}{dx}(x+3)^2 = 2 \times (x+3)$$

Iteration 1:

$$x_1 = x_0 - (learning\ rate) \times \left(\frac{dy}{dx}\right)$$

$$x_1 = 2 - (0.01) \times (2 \times (2 + 3)) = 1.9$$

Iteration 2:

$$x_2 = x_1 - (learning\ rate) \times \left(\frac{dy}{dx}\right)$$

$$x_2 = 1.9 - (0.01) \times (2 \times (1.9 + 3)) = 1.802$$

Step 4: We can observe that the X value is slowly decreasing and should converge to -3 (the local minima). However, how many iterations should we perform?

Let us set a precision variable in our algorithm which calculates the difference between two consecutive "x" values . If the difference between x values from 2 consecutive iterations is lesser than the precision we set, stop the algorithm!

Gradient descent in Python:

Step 1: Initialize parameters

```
cur_x = 2 # The algorithm starts at x=2
rate = 0.01 # Learning rate
precision = 0.000001 #This tells us when to stop the algorithm
previous_step_size = 1 #
max_iters = 10000 # maximum number of iterations
iters = 0 #iteration counter
df = lambda x: 2*(x+3) #Gradient of our function
```

Step 2: Run a loop to perform gradient descent:

i. Stop loop when difference between x values from 2 consecutive iterations is less than 0.000001 or when number of iterations exceeds 10,000

```
while previous_step_size > precision and iters < max_iters:
    prev_x = cur_x #Store current x value in prev_x
    cur_x = cur_x - rate * df(prev_x) #Grad descent
    previous_step_size = abs(cur_x - prev_x) #Change in x
    iters = iters+1 #iteration count
    print("Iteration",iters,"\nX value is",cur_x) #Print iterations
print("The local minimum occurs at", cur_x)</pre>
```

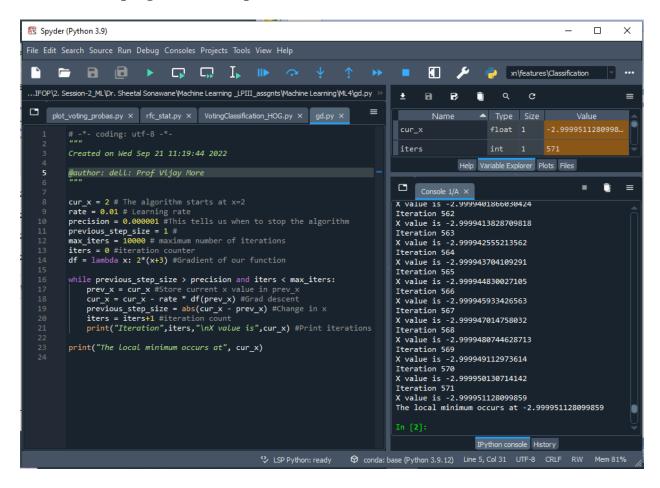
Output: From the output below, we can observe the x values for the first 10 iterations- which can be cross checked with our calculation above. The algorithm runs for 571 iterations before it terminates. The code and solution is embedded below for reference.

X value is -2.9999364493143186 Iteration 559 X value is -2.9999377203280324 Iteration 560 X value is -2.999938965921472 Iteration 561 X value is -2.9999401866030424 Iteration 562 X value is -2.9999413828709818 Iteration 563 X value is -2.999942555213562 Iteration 564 X value is -2.999943704109291 Iteration 565 X value is -2.999944830027105 Iteration 566 X value is -2.999945933426563 Iteration 567 X value is -2.999947014758032

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Iteration 568
X value is -2.9999480744628713
Iteration 569
X value is -2.999949112973614
Iteration 570
X value is -2.999950130714142
Iteration 571
X value is -2.999951128099859
The local minimum occurs at -2.999951128099859

Screenshot of program and output



Source: https://towardsdatascience.com/implement-gradient-descent-in-python-9b93ed7108d1