$\theta \Rightarrow$

G M M M M M
G M M M M M
G M M M M M
G M M M M M
G M M M M M

F M M M M M
F M M M M M
F M M M M M
F M M M M M
F M M M M M

$$P(F \mid PHD) = \frac{3}{15} = \frac{3}{3+12}$$

$$P(F \mid PHD) = \frac{P(PHD \mid F) * P(F)}{P(PHD)}$$

$$= \frac{\quad}{\quad + \quad}$$

$\Rightarrow P(PHD \mid F) * P(F)$

$\Rightarrow P(PHD \mid M) * P(M)$

$$P(PHD) = P(PHD \mid F) * P(F) + P(PHD \mid M) * P(M)$$

$$P(A) = P(A \mid B_1) * P(B_1) + P(A \mid B_2) * P(B_2) +$$

$$P(A|B_3) * P(B_3)$$

Recap

Sample space

Outcomes

Events

Conditional prob :

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow \frac{P(B|A) \cdot P(A)}{P(B)}$$

Independence

$$P(A|B) = P(A)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Mutual Exclusive

$$A \cap B = \phi$$

Q ⇒ A family has 2 children. Given that at least one child is a girl. What is the probability that both are girls

$$S = \{bb, bg, gb, gg\}$$

$$B = \{bg, gb, gg\}$$

$$A = \{gg\}$$

$$P[A|B] = \frac{P(A\cap B)}{P(B)} = \frac{P\{gg\}}{P\{bg, gb, gg\}}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

Q) In MCQ with 4 options, let 0.8 be the probab that student knows the answers & 0.2 the prob of guessing. What is the conditional prob that the student knew the ans to a quiz given that it was answered correctly.

$C \rightarrow$ correct

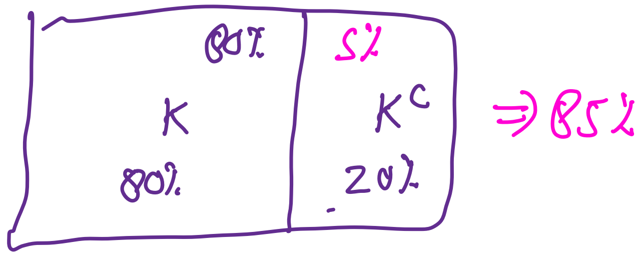
$P[K] = 0.8$ $\qquad$ $P[K^c] = 0.$

$P[C|K^c] \Rightarrow \frac{1}{4}$

$P[C|K] = 1$

$$P[K|C] = \frac{P[C|K] \cdot P[K]}{P[C]}$$

$$= \frac{(1) \cdot (0.8)}{0.85} = \frac{16}{17}$$



$P[C] = P[C \mid K] \cdot P[K] + P[C \mid K^c] \cdot P[K^c]$

$= (1) * 0.8 + 1/4 * 0.2$

$= 0.85$

Q ⇒ $P[E] = 0.6$
what can we say about $P[E \mid F]$
when E & F are mutually Exclusive

$P[E] = 0.6$

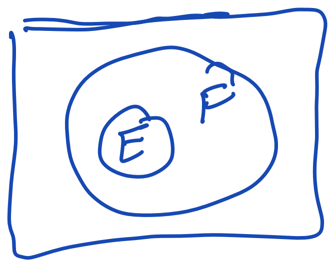
$$P[E \mid F] = \frac{P(E \cap F)}{P(F)} = \frac{0}{P[F]}$$

$= 0$

Q ⇒ $P[E) = 0.6$
What can we say abt $P[E \mid F]$
when E is subset of F

$$P[E|F] = \frac{P[E \cap F]}{P[F]} = \frac{P[E]}{P[F]}$$

$$= \frac{0.6}{\leq 1}$$



$$\frac{0.6}{0.8} \Rightarrow \boxed{\geq 0.6}$$

$\not< 0.6$

$$\frac{\cancel{6}^{3}}{\cancel{8}_{4}} = .75$$

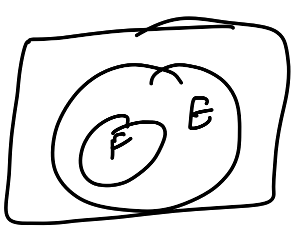
$$\frac{0.6}{1} = 0.6$$

Q $\Rightarrow$ Let $P[E] = 0.6$

What can you say abt $P[E|F]$

when F is subset of E

$$P[E|F] \Rightarrow \frac{P[E \cap F]}{P[F]} = \frac{P(F)}{P(F)} = 1$$



Q ⇒ A & B toss a coin alternatively till one of them gets a Head. The prob of heads is "p". Game starts with A tossing first. What is the prob that A wins the game?

$S = \{ "H", TH, TTH, TTTTH, TTTTTTH, \ldots$

$E \rightarrow$ "A" wins

$\Rightarrow \{ H, TTH, TTTTH, \ldots \}$

$P[E] = p + (1-p)^2 \cdot p + (1-p)^4 \cdot p + \ldots$

$r = (1-p)^2$

$P[E] = p + r \cdot p + r^2 \cdot p + \ldots$

$P[E] \cdot r = rp + r^2 p + r^3 p + r^4 p + \ldots$

$P[E] - r \cdot P[E] = p$

$P[E] * (1-r) = p$

$P(E) = p/(1-r)$

$$= \frac{p}{(1 - (1-p)^2)}$$

$$= \frac{p}{1 - [1 + p^2 - 2p]}$$

$$= \frac{p}{2p - p^2} = \frac{1}{2-p}$$

Break : 10 : 15 PM

SA ~~, 6 Oranges~~

Q ⇒ A gambler has in his pocket a fair coin & 2 - headed coin. He selects one coin at random & when he flips it, it shows head. What is the prob that it is fair coin ?

F → Fair $\quad F^c$ → 2 headed coin

$P(F | H) = ?$

$P(F) = 1/2. \quad P(F^c) = 1/2$

$$P(F \mid H) = \frac{P(H \mid F) * P(F)}{P(H)}$$

$$= \frac{1/2 * \frac{1}{2}}{3/4} = \frac{1}{3}$$

$$P(H) = P(H \mid F) * P(F) + P(H \mid F^c] * P[F^c)$$

$$= (1/2) * (1/2) + (1) * 1/2$$

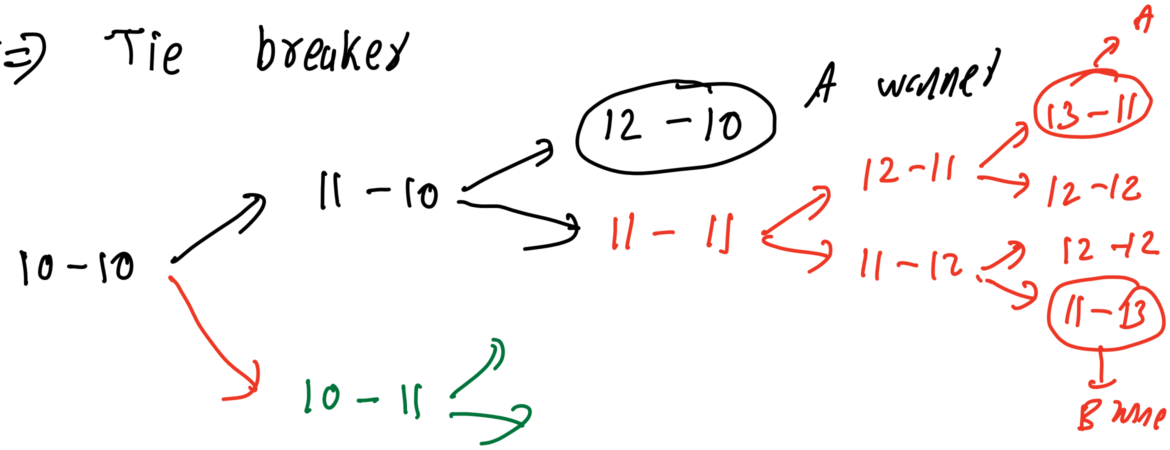
$$= 3/4$$

Q⇒ A gambler has in his pocket a fair coin & 2 headed coin. He selects one coin at random, & he flips it twice. It shows head both times. Whats the prob its a fair coin.

$$P(F \mid HH) = \frac{P(HH \mid F) * P(F)}{P(HH \mid F) * P(F) + P(HH \mid F^c) * P(F^c)}$$

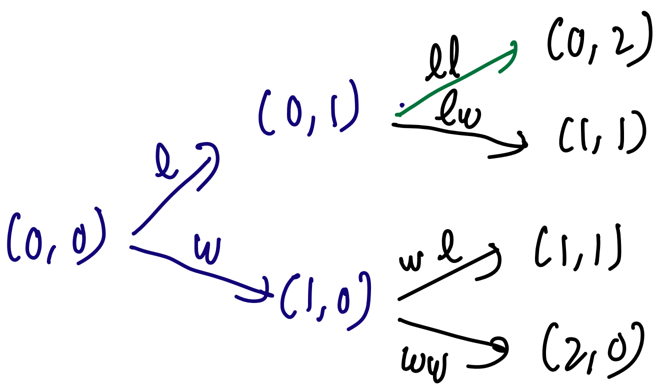
$$= \frac{\frac{1}{4} * \frac{1}{2}}{\frac{1}{4} * \frac{1}{2} + 1 * \frac{1}{2}} = \frac{1}{5}$$

Q ⇒ Tie breaker



Prob of "A" winning a point is "p"

Find prob of "A" winning this game



$$P[A] = P[A \mid wl] \cdot P[wl] + P[A \mid ww] \cdot P[ww] + P[A \mid lw] \cdot P[lw] + P[A \mid ll] \cdot P[ll]$$

$$P[A] = P[A] \cdot p \cdot (1-p) + 1 \cdot p^2 + P[A] \cdot p(1-p) + 0$$

$$P[A] = \frac{p^2}{1 - 2p + p^2}$$

$$P[\underline{w\,l}] = p(1-p)$$

$A \rightarrow p \Rightarrow w$

$\rightarrow (1-p) \Rightarrow l$

$$= p \neq (1-p)$$