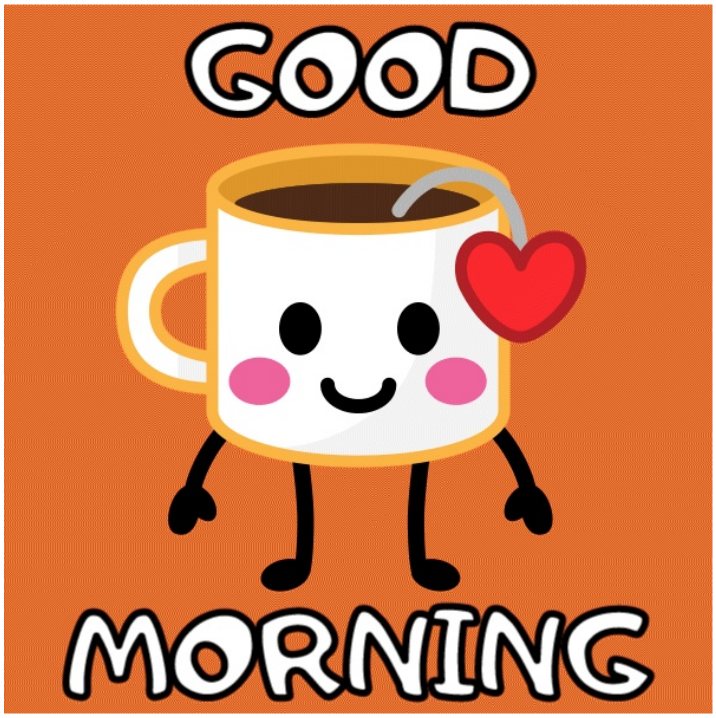


will start at 7:35am

Constraint optimization

$$\min_{\vec{w},\, w_0} -\sum y_i \cdot \frac{w^T x_i + w_0}{||w||}$$

$$\boxed{\theta_i^{t+1} = \theta_i^{t} - \lambda \frac{\partial f}{\partial \theta_i}}$$

$$\frac{\partial f}{\partial w_1}, \frac{\partial f}{\partial w_2}, - - - , \frac{\partial f}{\partial w_0}$$

where

$$f(w_0, w_1, \dots, w_n) = -\sum_{i=1}^{n} y_i \cdot \frac{w^T x_i + w_0}{||w||}$$

1^{st} Principal

$$\frac{\partial f}{\partial w_1} = \frac{f(w_1+\Delta, w_2, \dots, w_n) - f(w_1, w_2 \dots w_n)}{\Delta}$$

$$\Downarrow$$

computationaly expensive

2^{nd} way

$$\frac{\partial f}{\partial w_1} \Rightarrow -\frac{\sum y_i (w_1 x_1 + w_2 x_2 + \dots + w_n)}{\sqrt{w_1^2 + w_2^2 + \dots w_n^2}}$$

$$\Rightarrow -\sum y_i \frac{f(w_1)}{g(w_1)}$$

$$\Downarrow$$

very computational expensive

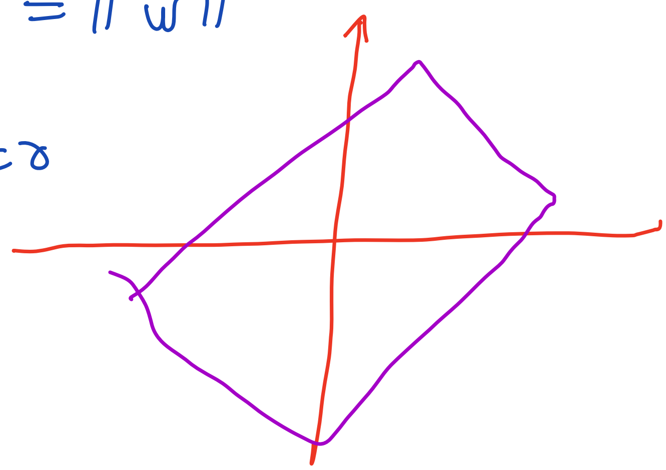
Idea

Eq of Hyperplane

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$d = \sqrt{w_1^2 + w_2^2} = \|\vec{w}\|$$

$$w_1' x_1 + w_2' x_2 + w_0' = 0$$



$$w_1' = \frac{w_1}{\|\vec{w}\|}$$

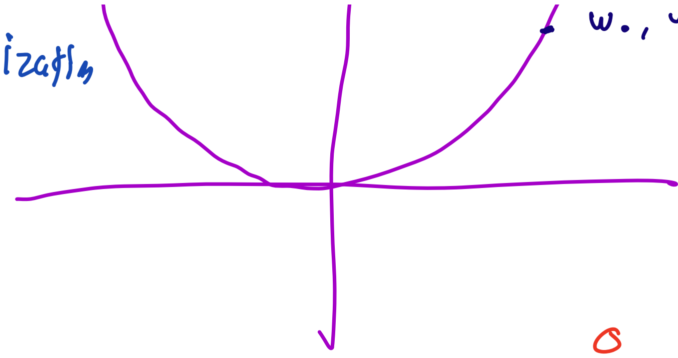
$$w_2' = \frac{w_2}{\|\vec{w}\|}$$

$$- \sum_{i=1}^{n} y_i \cdot \frac{w^T x_i + w_0}{\|w\|}$$

$$\min_{\vec{w}, w_0} \; - \sum y_i \left(w^T x + w_0\right)$$

s.t. $\|w\| = 1$

Constrained optimization



$$\frac{\partial f}{\partial w_1} = -\sum y_i \left[\underbrace{w_1 x_1}_{x_1} + \overbrace{\cancel{w_2 x_2}}^{0} + \cdots + \overbrace{\cancel{x_d w_d}}^{0} \right]$$

$$\Rightarrow \quad -\sum y_i x_1$$

$$w_j^{(t+1)} = w_j^{(t)} - \alpha \left(-\sum_{i=1}^{n} y_i x_{ij} \right)$$

$$\frac{\partial f}{\partial w_0} = -\sum y_i \left(\overbrace{\cancel{w_1 x_1}}^{0} + \overbrace{\cancel{w_2 x_2}}^{0} + \cdots \underbrace{w_0}_{1} \right)$$

$$= -\sum y_i$$

$$w_0^{(t+1)} = w_0^{t} - \alpha \left(-\sum y_i \right)$$

General form of optimization problem

$\underset{\theta}{\text{max / min}}$ operation   optimization fun $\rightarrow f(\theta)$

s.t.

constraint func $\rightarrow g_1(\theta), g_2(\theta), \ldots$

Ex:

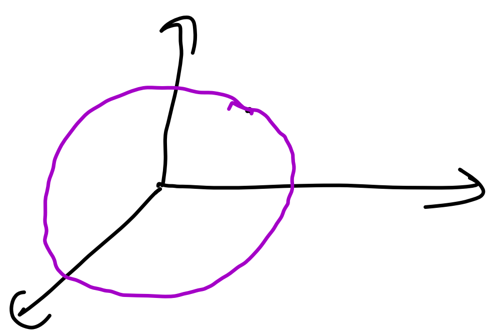
$$\min_{\theta_1, \theta_2} \quad f(\theta_1, \theta_2)$$

s.t. $\quad g_1(\theta_1), g_2(\theta_2), g_3(\theta_1, \theta_2) \ldots$

We wanted to solve

$$\min_{\vec{w}, w_0} \quad - \Sigma\, y_i (w^T x_i + w_0)$$

s.t. $\quad \|w\| - 1 = 0$



⇒ $\boxed{\text{NLTK , CU 2}}$

Experation design

⇒ Interview :

$$w_{new} = w_{old} - \eta \nabla_{\vec{w}} f$$

$\|w\| = 1$

$$\nabla = \begin{bmatrix} \frac{\partial f(w_1)}{\partial w_1} \\ \frac{\partial f(w_2)}{\partial w_2} \\ \vdots \end{bmatrix}$$

$\|w_{new}\| = 1$

$\Downarrow$

no guarante

Constraint prob ⇒ Unconstraint prob

$\Downarrow$

$\boxed{\text{Lagrange Multiplier}}$

$$\min_{\theta} \quad f(\theta) \quad s.t. \quad \begin{aligned} g_1(\theta) &= 0 \\ g_2(\theta) &= 0 \\ &\vdots \\ g_n(\theta) &= 0 \end{aligned}$$

$$\Downarrow$$

$$\min_{\theta} \quad f(\theta) + \underset{\substack{\downarrow \\ \text{Lagrange} \\ \text{multiplier}}}{\lambda_1} g_1(\theta) + \lambda_2 g_2(\theta) + \cdots$$

Example

$$\min_{x,y} \quad x^2 + y^2$$

$$s.t. \quad x + 2y - 1 = 0$$

$$L \Rightarrow$$

$$\min_{x,y,\lambda} \quad x^2 + y^2 + \lambda(x + 2y - 1) = 0$$

$$\frac{\partial L}{\partial x} = 2x + \lambda$$

$$\frac{\partial L}{\partial y} = 2y + 2\lambda$$

$$\frac{\partial L}{\partial \lambda} = x + 2y - 1$$

$$2x + \lambda = 0 \quad \Rightarrow \quad \lambda = -2x$$

$$2y + 2\lambda = 0 \quad \Rightarrow \quad \lambda = -y$$

$$y = 2x$$

$$x + 2y - 1 = 0$$

$$x + 2(2x) - 1 = 0$$

$$5x - 1 = 0$$

$$x = \frac{1}{5}$$

$$y = 2/5$$

$$\lambda = -2/5$$

Intution (very high level)

$$\min_{x,y,\lambda} \; L = f(x,y) + \lambda\,(g(x,y))$$

If $\lambda \to$ large

$g(x,y) \to$ large ≥ 0

$$(y_i - \hat{y})^2$$

$$(y_i - (w_0 + w_1 x_1 + w_2 x_2))^2 + \lambda \|w\|^2$$

$$w_{i_n} = w - \alpha \frac{\partial L}{\partial w_i}$$

$$\lambda = \lambda - \alpha \frac{\partial L}{\partial \lambda}$$

$$\frac{f(x)}{g(x)} \rightarrow 1$$

$$L \Rightarrow \quad f(x) + \lambda g(x) \rightarrow 2$$

λx

Dhruv Jawali