Kaggle

Gradient Descent

$$\max_{\vec{w}, w_0} \sum_{i=1}^{n} y_i \frac{w^T x_i + w_0}{\|w\|}$$ } Gain fun

Maximisation problem

$\Rightarrow +\beta$

$1, 3, 6, \beta$ $\quad * -1 * \beta$

$-1, -3, -6, -\beta$ ✓ $\quad -\beta$

$$\min_{\vec{w}, w_0} -\sum_{i=1}^{n} y_i \cdot \frac{w^T x_i + w_0}{\|w\|}$$ } Loss function

$$\min_{\vec{w}, w_0} f(w, w_0)$$

$$\min_{w_0, w_1, \ldots, w_n} f(w_0, w_1, \ldots, w_n)$$

$\Downarrow$

perform multivariate optimisation

Multivariate Differentiation

$y = f_1(x) \rightarrow$ single var

$y = f_2(x_1, x_2, \ldots, x_n) \rightarrow$ multivariate

$$\frac{d f(x)}{dx} = f'(x)$$

$$\frac{d f(x_1, x_2, \ldots, x_n)}{dx_1} = ??$$

$z = f(x, y)$ find $\frac{dz}{dx}$

$y = g(x)$

$z = f(x, g(x))$

Example

$$z = f(x,y) = x^2 + y^2$$

$$z = x^2 + [g(x)]^2$$

$$y = g(x)$$

$$\frac{dz}{dx} = 2x + 2\,g(x)\cdot g'(x)$$

$$z' = 2x + 2y \cdot y'$$

$$\frac{dz}{dx} = 0 \qquad \frac{dz}{dy} = 0$$

Partial derivative

$$\frac{\partial z}{\partial x} = \frac{df(x,\ y = \text{const})}{dx}$$

$$\frac{\partial z}{\partial y} = \frac{df(y,\ x = \text{const})}{dy}$$

Ex

$$f(x,y) = 2x^2y + 3y^3x^2 + 3y$$

$$\frac{\partial f}{\partial x} = 4xy + 6y^3x + 0$$

$$\frac{\partial f}{\partial y} = 2x^2 + 9y^2x^2 + 3$$

Gradient

Goal:

$$\min_{w_0, w_1, w_2, \ldots w_n} f(w_0, w_1, \ldots, w_n)$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial w_0} \\ \frac{\partial f}{\partial w_1} \\ \frac{\partial f}{\partial w_2} \\ \vdots \end{bmatrix}$$

Gradient Descent

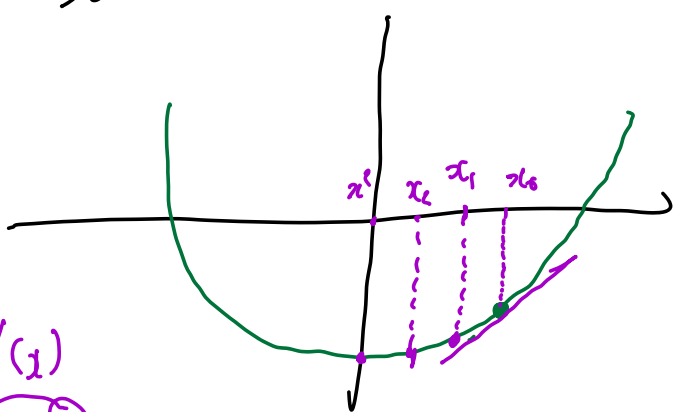
$$y = x^2 - 30$$

$$\frac{\partial y}{\partial x} = 2x$$

Step1

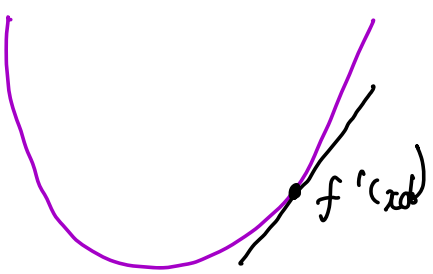
$$f(x_0) = x_0^2 - 30$$

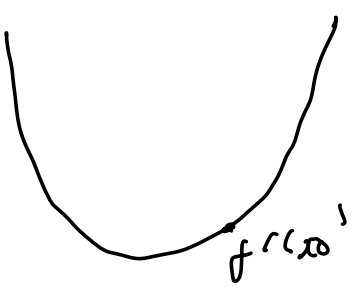
$$f'(x_0) = 2x_0$$

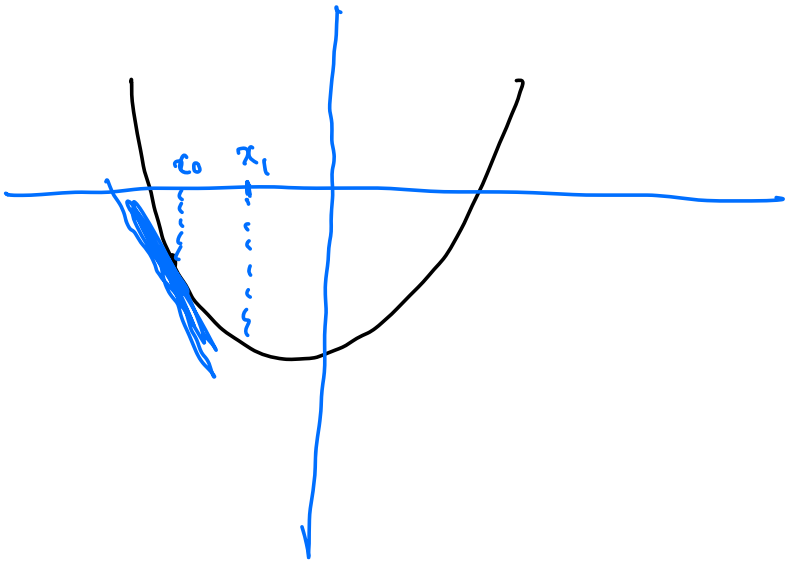


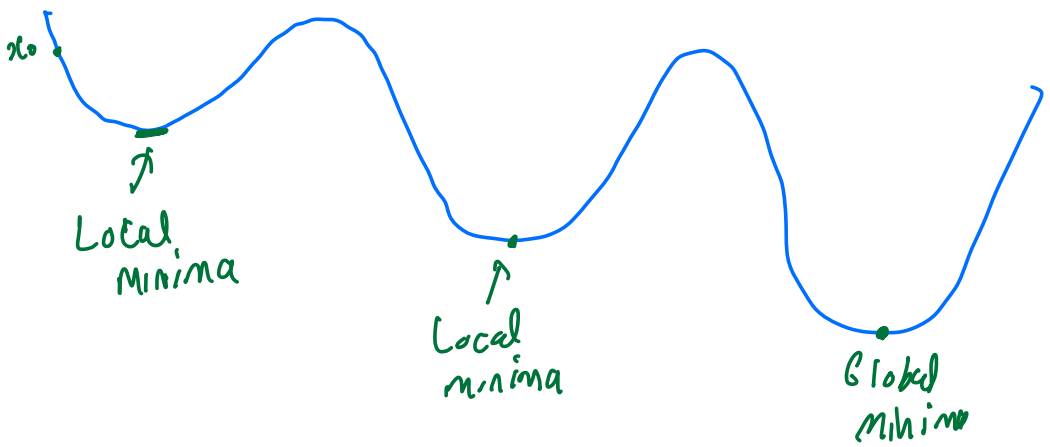
$$x_1 = x_0 - \eta f'(x)$$

$2x_0$ → +ve





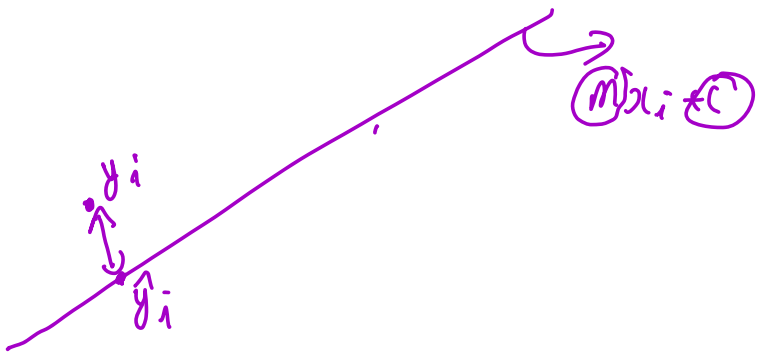




$$w_1[\text{next}] = w_1[\text{current}] - \eta \frac{\partial f}{\partial w_1}$$

$$\vdots$$

$$w_n[\text{next}] = w_n[\text{current}] - \eta \frac{\partial f}{\partial w_n}$$



$$J(m,c) = \sum_{i=1}^{n} (y_i - (mx_i + c))^2$$

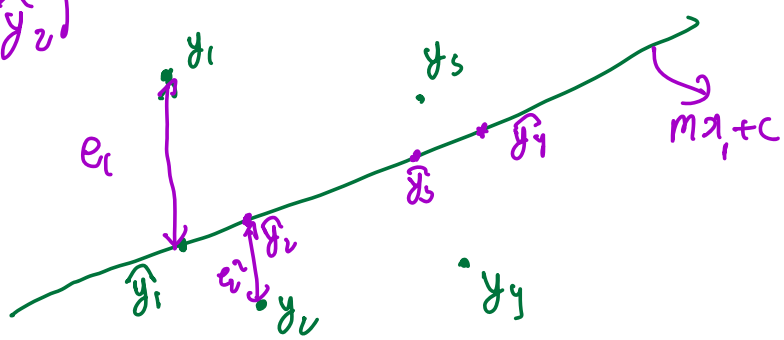
$$\frac{\partial J}{\partial m} = 2 \sum_{i=1}^{n} (y_i - (mx_i + c))(-x_i)$$

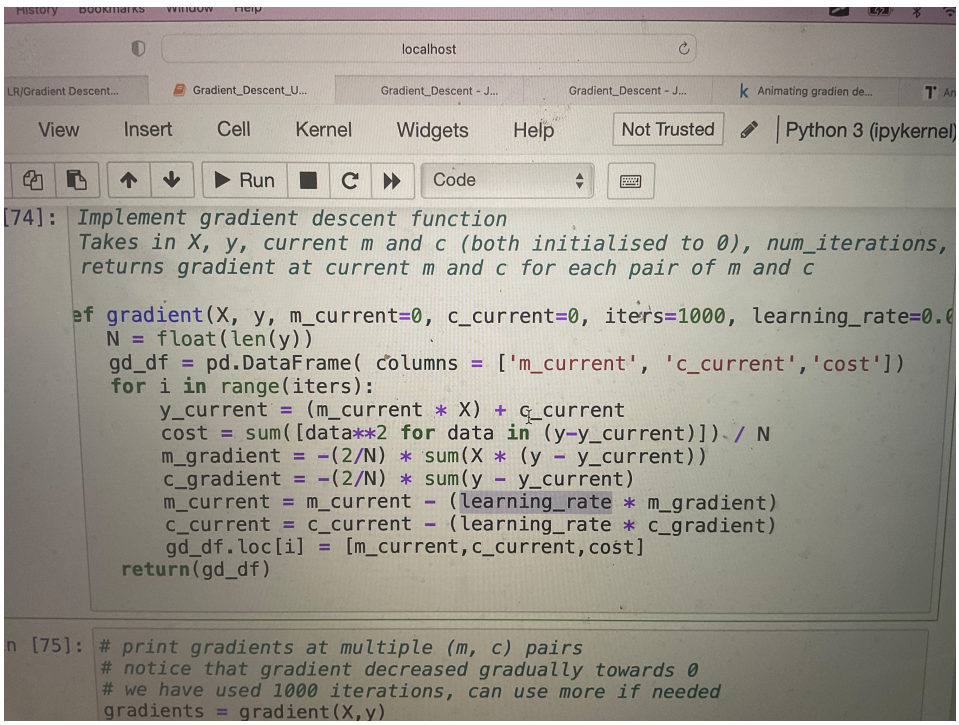
$$\frac{\partial J}{\partial c} = 2 \sum_{i=1}^{n} (y_i - (mx_i + c))(-1)$$

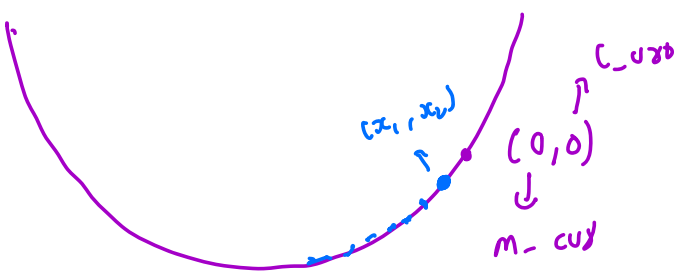
$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2$$

$$\sum (y_i - \hat{y}_i)^2$$

$$\sum (y_i - (mx_i + c))^2$$







$$\frac{\partial J}{\partial m} = -2x_i \sum_{i=1}^{N} (y_i - (mx_i + c))$$

$\prod \bar{y}_i$

$\frac{1}{\text{[illegible]}}$

3, 5, 8

$\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{8}$

−3, −5, −8