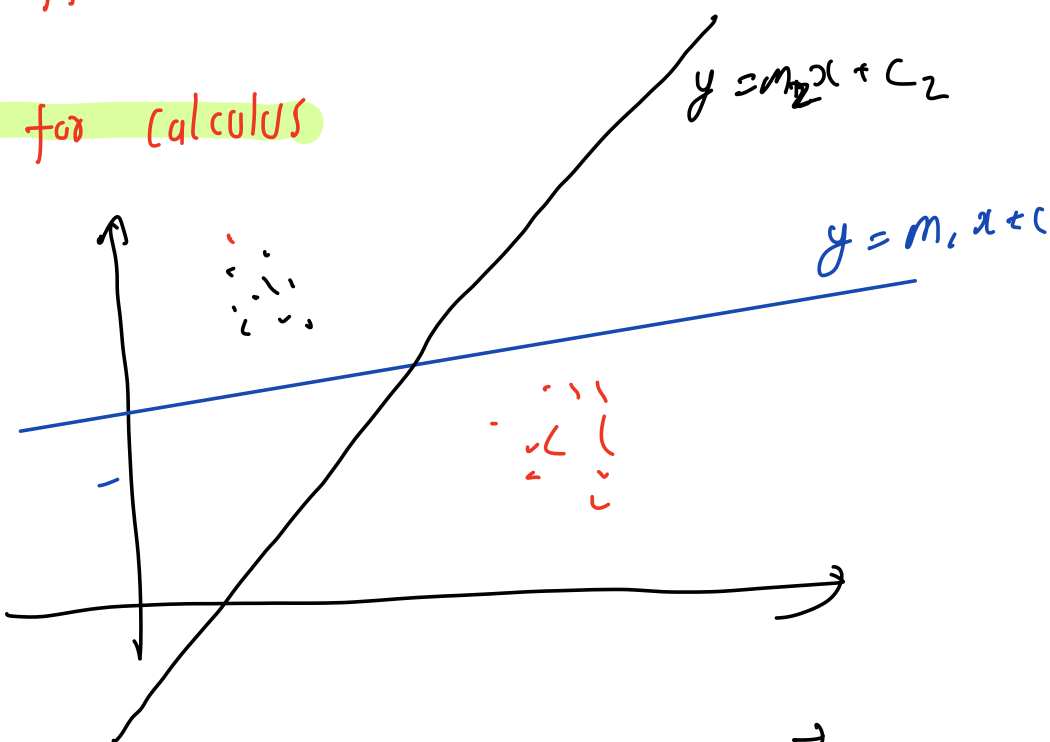
Introduction to Calculus
[optimization]

- Motivation
- Limits
- Functions
- Differentiation

Need for calculus



$$
\begin{array}{c|cccc}
 & f_1 & f_2 & f_2 & f_n \\
\hline
\vec{x}_1 & x_{11} & x_{12} & \cdots & x_{1n} \\
\vec{x}_2 & x_{21} & x_{22} & \cdots & x_{2n}
\end{array}
\qquad
\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}
$$

vectors

Features Target

Mathmetical defination for a binary classifier

Given : $D = \{ (\vec{x}_i, \vec{y}_i) \}$, $\vec{x}_i \in R$
$\vec{y}_i \in \{-1, 1\}$

Find f s.t.

$$f(x_i) = y_i$$

$$\underbrace{f}_{\text{model}} \; (\underbrace{\vec{x}_i}_{\text{input}}) = \underbrace{\hat{y}_i}_{\text{Predict}} \sim \underbrace{y_i}_{\text{target}}$$

$$\Rightarrow \quad f(x_i) = \text{sign}(w^T x_i + w_0) \approx \hat{y}_i$$

-3

$+5$

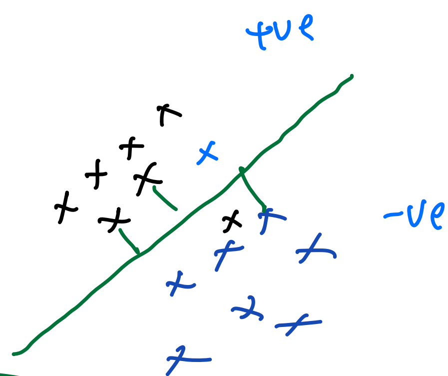
$y_i = +1$
$= -1$

$\hat{y}_i = y_i \Rightarrow$ most of the time

Best Fit line

Distance of all points from the line should be max

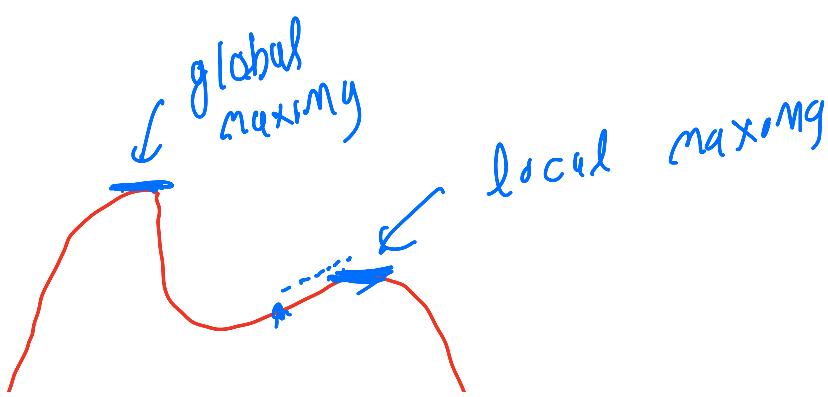
$$\text{distance} = \frac{w^T x_i + w_0}{\|w\|}$$

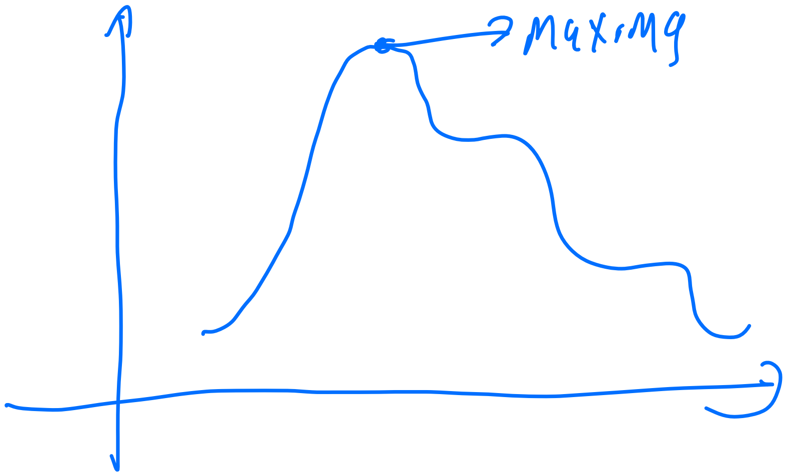


$$\boxed{\max_{\vec{w},\, w_0} \underbrace{\sum_{i=1}^{n} y_i \cdot \frac{w^T x_i + w_0}{\|w\|}}_{\text{optimisation target / loss function}}}$$

$\vec{w}, w_0$ → optimisations param.

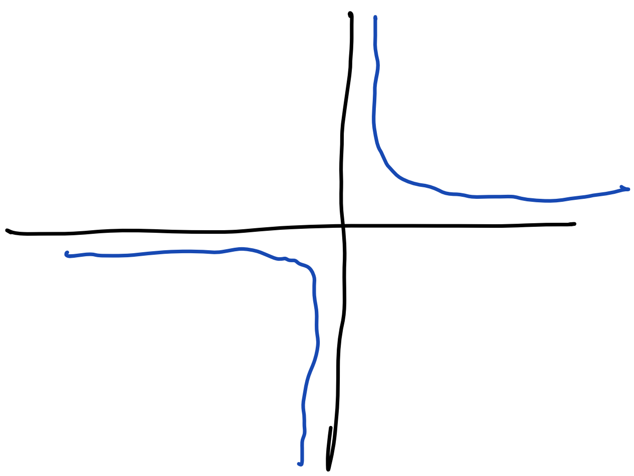
optimisation target / loss function





Limits

$f(x) = \frac{1}{x}$



$f(3) = \frac{1}{3}$

$f(\infty) = \frac{1}{\infty} = 0$

$f(-\infty) = \frac{1}{-\infty} = 0$

$f(0) = ?$

$f(0^+) = \infty \Rightarrow \lim_{x \to 0^+} f(x) = \infty$

$f(0^-) = -\infty \quad \lim_{x \to 0^-} f(x) = -\infty$

Continuous Func

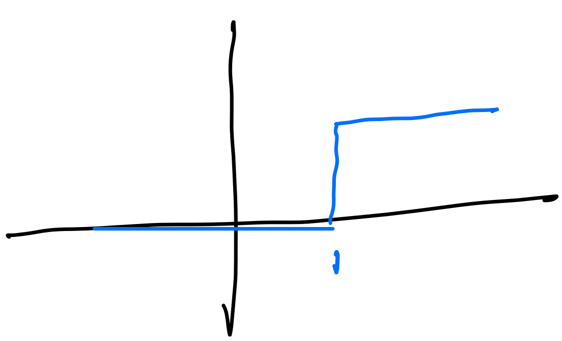
A func is said to be continous
if
LHS Limit = RHS Limit

$f(x) = \frac{1}{x} \Rightarrow$ not continous

$\Rightarrow$

$\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1^-} f(x) = 0$



Functions

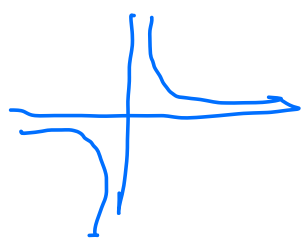
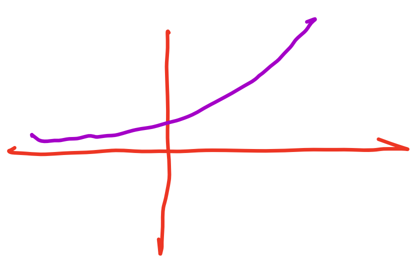
$x \rightarrow f(x) \rightarrow y \Rightarrow$ univariate function

$x, y \rightarrow \underbrace{f(x, y)}_{\text{func for } x, y} \rightarrow z \Rightarrow$ Bivariate function

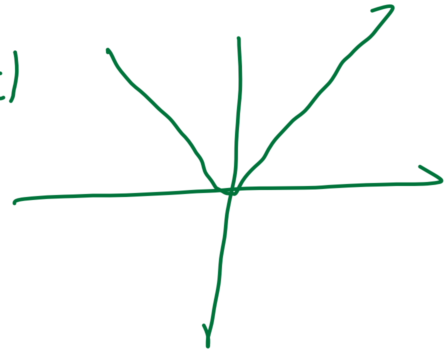
domain (i/p values): x, y

Range: z

Some imp func of MV

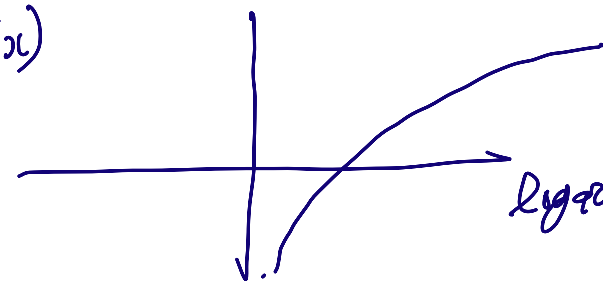
Domain	Range	Func	Plot	Name
$\mathbb{R} - \{0\}$	$\mathbb{R}$	$f(x) = \frac{1}{x}$		Hyperbola
$\mathbb{R}$	$\mathbb{R}^+$	$f(x) = e^x$		Exponent

$\mathbb{R}$ $\mathbb{R}^+$ $f(x) = |x|$



modulus

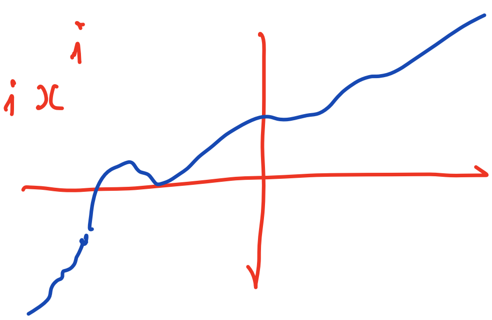
$\mathbb{R}^+$ $\mathbb{R}$ $f(x) = \log(x)$



logarithm

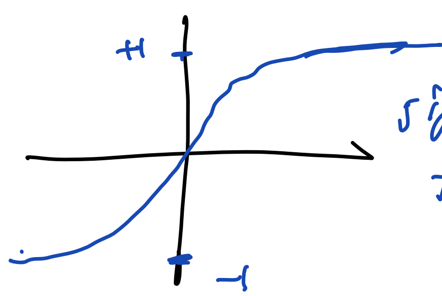
$\lim_{x \to 0^+}$

$\mathbb{R}$ $\mathbb{R}$ $f(x) = \sum a_i x^i$



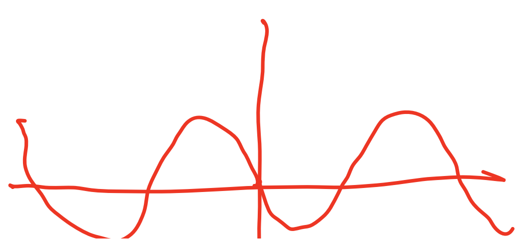
polinomial

$\mathbb{R}$ $(-1, 1)$ $f(x) = \dfrac{1}{1+e^{-x}}$



sigmoid funct

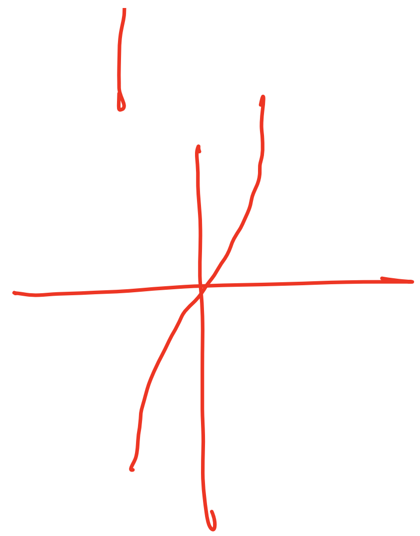
$\mathbb{R}$ $(-1, 1)$ $f(x) = \sin x$

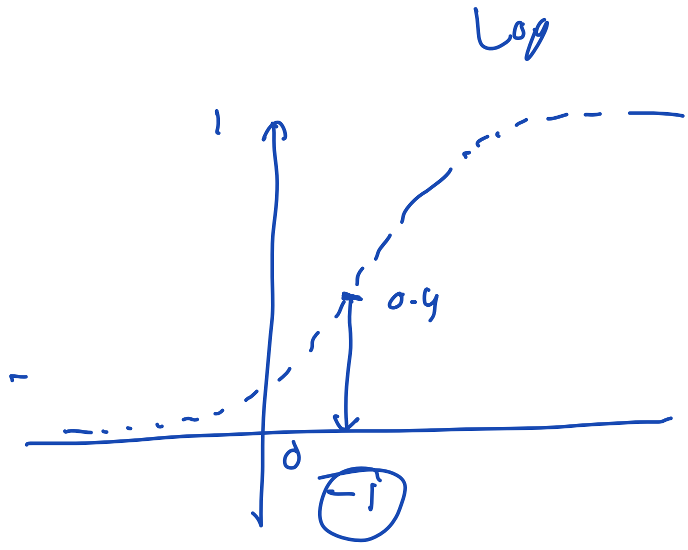


sin

$\mathbb{R} - (2n+1)\frac{\pi}{2} \quad \mathbb{R}$

$f(x) = \tan(x)$



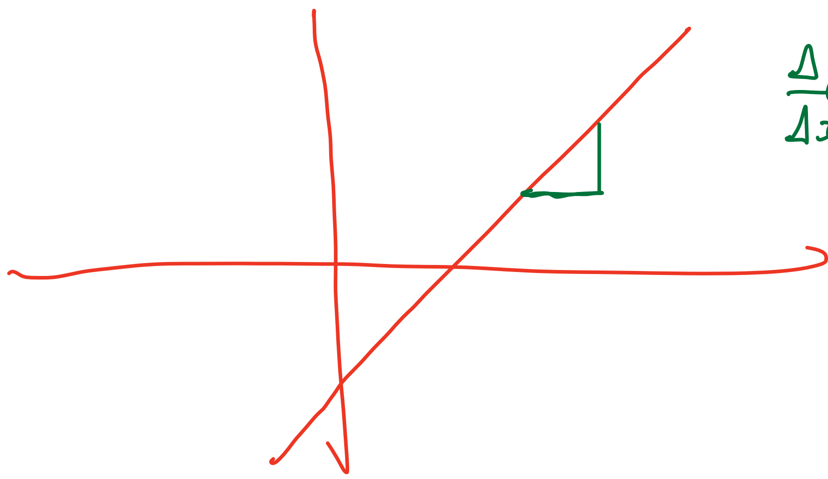


$0 \downarrow \{-1, 1\}$

Break: 8:35

Diffrentiation

$y = mx + c$

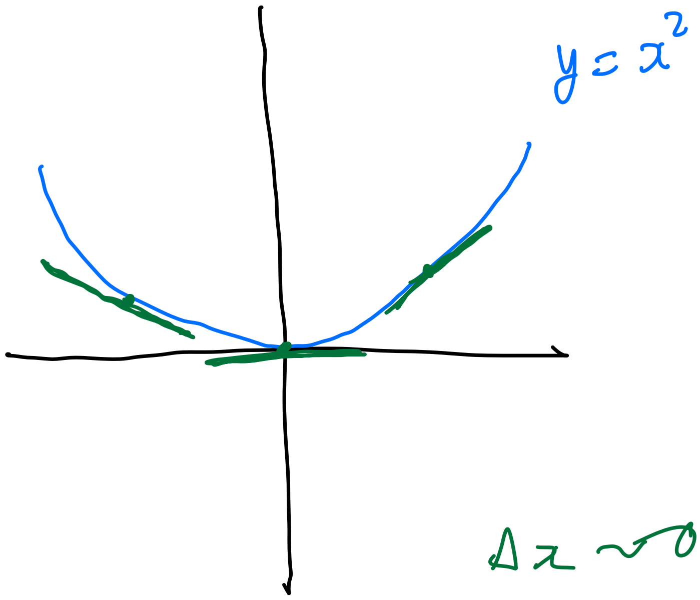


$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

diffrentiation $\Rightarrow$ rate of change
$\Rightarrow$ slope

$\frac{d f(x)}{dx}$ = diffrention of $f(x)$ with respect to x



$\Delta x \to 0$

$\frac{dy}{dx}$

$\frac{d\,c}{dx} = 0$

$\frac{d\,x^n}{dx} = n\,x^{n-1}$

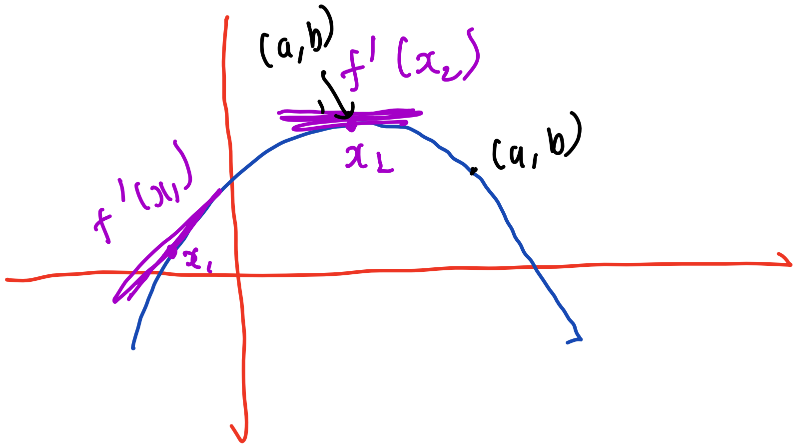
$\frac{d\,\sin(x)}{dx} = \cos(x)$

$\frac{d\,\cos(x)}{dx} = -\sin(x)$

$$\frac{d}{dx} b^x = b^x \ln(b)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

Find optimal using calculus



$f'(x_1) > 0$

$f'(x_2) = 0 \Rightarrow$ maxima or minima

$f(x) = a - 4(x-b)^2$

$\underline{f'(x)} = 0 - 4 \cdot 2(x-b)$

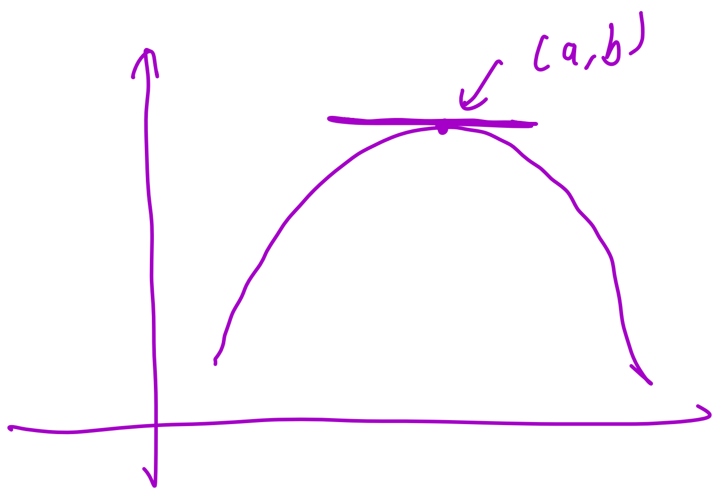
$0 = -8x - 8b$

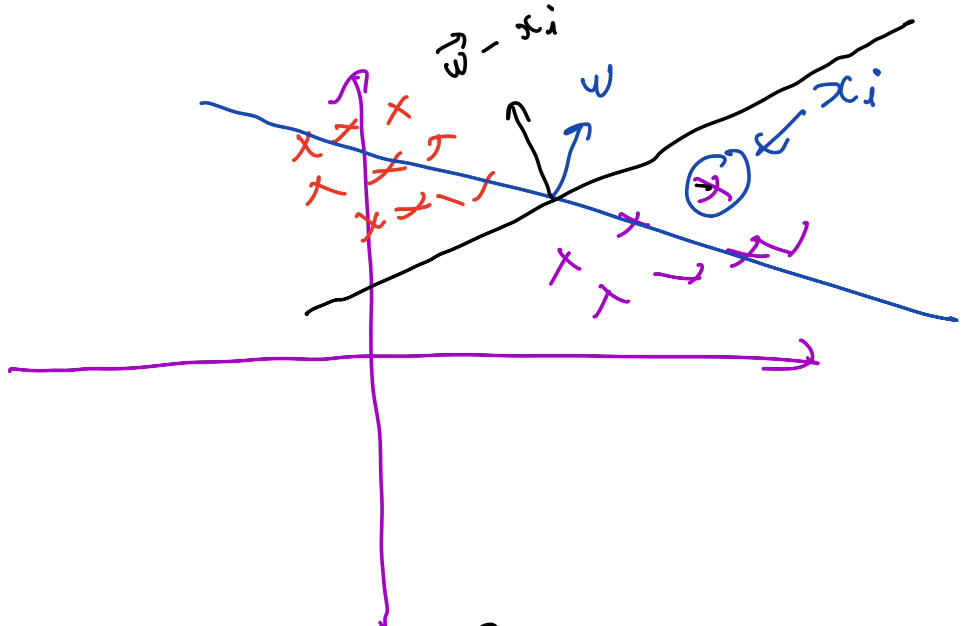
$x_{max} = \underline{b}$

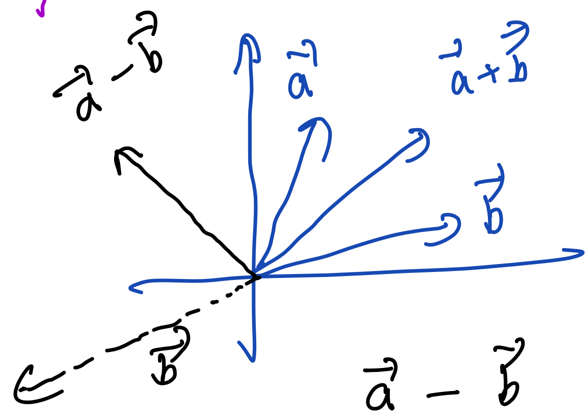
$f(x)_{max} = a$

maxima $\rightarrow (b, a)$

$\frac{df(x)}{dx} \approx 0$




$\vec{w} - x_i$
w
x_i


$\vec{a} - \vec{b}$
$\vec{a}$
$\vec{a} + \vec{b}$
$\vec{b}$
$\vec{b}$
$\vec{a} - \vec{b}$