Roll No. 2102020304 Total Pages : 05

July-22-00247

B. Tech. EXAMINATION, 2022

Semester IV (CBCS)

OPTIMIZATION AND CALCULUS OF VARIATIONS

MA-401

(Common for B. Tech. All Branches)

Time : 3 *Hours* *Maximum Marks* : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Section A, B, C and D. Q. No. 9 is compulsory.

Section A

1. Explain the concept of duality in linear programming. Point out various useful aspects of the concept. **10**

2. Define the following : **10**

   (i) Feasible solution

(ii) Optimal solution

(iii) Slack variable

(iv) Surplus variable

Section B

3. Solve the following problem by Simplex method

Maximize $Z = 6x_1 + 8x_2$

Subject to constraints

$$30x_1 + 20x_2 \leq 300$$

$$5x_1 + 10x_2 \leq 110$$

where $x_1, x_2 \geq 0$ 10

4. (a) Solve the following transportation problem by NWCM. Find difference between the cost obtained [illegible] 5

	W1	W2	W3	W4	Supply
F1	20	25	40	20	100
F2	29	26	35	40	250
F3	31	33	37	30	150
Demand	90	160	200	50	500/500

(b) Solve the transportation by VOGEL's approximation method 5

	P	Q	R	S	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Section C

5. Use the method of Lagrangian multipliers to solve the following

Minimize $Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to constraints

$$x_1 + x_2 + x_3 = 20$$

where $x_1, x_2, x_3 \geq 0$ 10

6. What are Kuhn-Tucker conditions for non-linear programming problems ? Formulate these conditions for the problem

Maximize $f(x) = \log (x_1 + x_2)$

Subject to constraints :

$$x_1 + x_2 \leq 5,$$

where $x_1, x_2 \geq 0.$ **10**

Section D

7. (a) Find extremals for $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$. **5**

(b) Find the extremals of the functional

$$J[y(x)] = \int_1^2 \frac{x^3}{y'^2} dx, \text{ where } y(1) = 0,\ y(2) = 3.$$

5

8. (a) Find the extremals of the functional

$$J[y(x)] = \int_1^2 yy''^3 dx, \quad \text{where} \quad y(0) = 0,$$

$y(1) = 1$. **5**

(b) Derive the Euler's equation $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0,$

for functional containing first order derivatives and one independent variable. **5**

(Compulsory Question)

9. (i) Define PERT.

(ii) Define an unbalanced assignment problem.

(iii) When a transportation problem is called unbalanced ?

(iv) Explain Initial Basic feasible solution.

(v) What is degeneracy in transportation problem ?

(vi) What is the full form of CPM ?

(vii) Define Feasible solution.

(viii) Define extremals.

(ix) Write Euler's equation for extremals when F is independent of x.

(x) Define convex function. **10×2=20**