Coordinate systems and transforms

Ivan Marković Matko Orsag Damjan Miklić

Automation and Robotics
Robot Programming and Simulation

2020

UNIVERSITY OF ZAGREB
Faculty of Electrical Engineering and Computing

Overview

1. Introduction to coordinate systems
2. The ROS TF tree
3. Transformation matrices
4. rospy TF2 API and PyKDL

- Install ROS TF2 tools

```
$ sudo apt install ros-noetic-tf2-tools
```

- Install Python symbolic math library

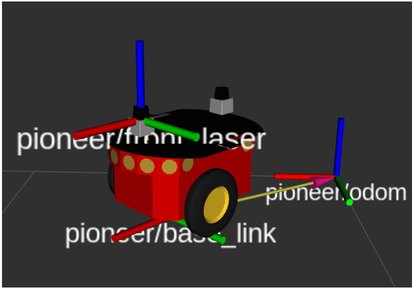
```
$ sudo apt install isympy3
```

- Clone the PSR Stage Worlds and build your workspace

```
$ roscd && cd ../src
$ git clone https://github.com/pftros/rps_stage_worlds.gi
$ cd ..
$ catkin_make
$ source devel/setup.bash
```

Coordinate systems

We assign coordinate systems (frames) to localize (parts of) objects relative to each other.

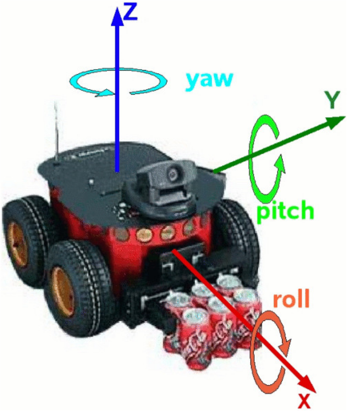


By convention, we use right-handed Cartesian coordinate systems with principal axes x, y and z.

Representing rotations in 3D

A minimal description (parametrization) of rotation in 3D is given by Euler angles. In mobile robotics we use the convention:

- roll - rotation around x-axis
- pitch - rotation around y-axis
- yaw -rotation around z-axis
- Positive rotation is counterclockwise



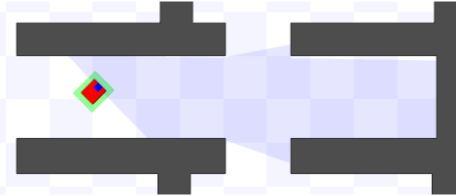
Rotation conventions

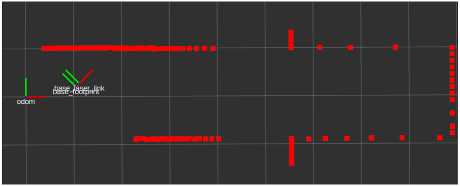
There are many possible Euler angle definitions, e.g., for manipulators, rotation around the end-effector z-axis is typically called roll. When assigning angles, be careful to check the convention!

Coordinate systems in 2D

For simplicity, we will consider a 2D (planar) world, ignoring z, roll and pitch:

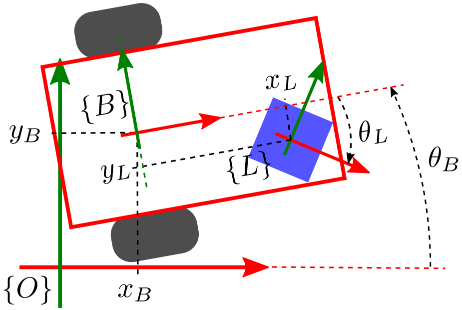
```
$ rosrun stage_ros stageros \
`rospack find rps_stage_worlds`/worlds/simple_rps.world
$rviz -d `rospack find rps_stage_worlds`/rviz/simple_rps.rviz
```





Coordinates: a naive representation

- A 2D pose consists of position (x, y) and orientation (yaw) θ
- Every pose defines a frame, and every frame has a pose
- A transform links the poses of two frames
- A pose is always expressed relative to a frame of reference
- `base_footprint` pose in the `odom` frame: $p_{OB} = (x_{OB}, y_{OB}, \theta_{OB})$
- Laser pose in the `base_footprint` frame: $p_{BL} = (x_{BL}, y_{BL}, \theta_{BL})$



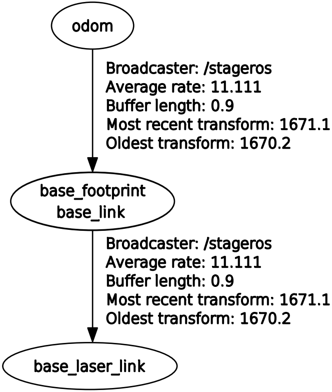
Notation

For brevity and aestethics, we will often simplify indexing by omitting the frame of reference when we are referring to coordinates in the parent frame, e.g. $x_{OB} \rightarrow x_B$.

Frame relationships

- Parent-child relationships
- By ROS convention, represented by a (poly)tree (directed acyclic graph)
- A node may have only one parent!

```
$ rosrun rqt_tf_tree rqt_tf_tree
```



Note

Because `base_footprint` and `base_link` frames coincide in 2D, we will treat them as one frame in this lecture. We will also refer to `base_laser_link` as `laser_link`.

Querying frame reationships using tf2_tools

The TF2 echo.py script can provide the relative pose of any frame in the TF tree:

```
$ rosrun tf2_tools echo.py odom base_laser_link
At time 9698.0, (current time 9698.0)
- Translation: [0.071, 0.071, 0.420]
- Rotation: in Quaternion [0.000, 0.000, 0.383, 0.924]
            in RPY (radian) [0.000, -0.000, 0.785]
            in RPY (degree) [0.000, -0.000, 45.000]
```

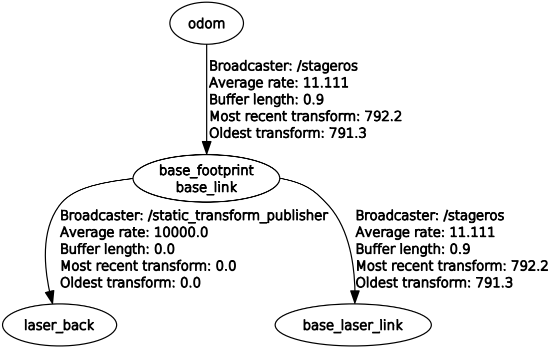
TF and TF2

TF2 is a new implementation of the TF library. TF is still available for backwards compatibility, but it is deprecated. Always use TF2!

Publishing transforms to the TF tree

The TF2 `static_transform_publisher` can publish static transforms to the TF tree:

```
$ rosrun tf2_ros static_transform_publisher \
-0.1 0.0 0.0 3.14 0 0 base_link laser_back
```

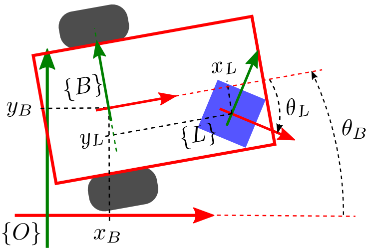



```
$ rosrun tf2_tools echo.py base_laser_link laser_back
```

Assignment: Transforming a pose

Assuming we can directly measure robot pose p_B in the `odom` frame and laser pose p_L in the `base_link` frame, find the equations of the laser pose p_{OL} expressed in the `odom` frame.

$$x_{OL} = \dots$$
$$y_{OL} = \dots$$
$$\theta_{OL} = \dots$$

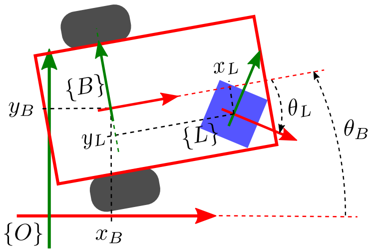


Transforming coordinates by hand

Assignment: Transforming a pose

Assuming we can directly measure robot pose p_B in the `odom` frame and laser pose p_L in the `base_link` frame, find the equations of the laser pose p_{OL} expressed in the `odom` frame.

$$x_{OL} = x_B + x_L \cos(\theta_B) - y_L \sin(\theta_B)$$
$$y_{OL} = y_B + x_L \sin(\theta_B) + y_L \cos(\theta_B)$$
$$\theta_{OL} = \theta_B + \theta_L$$



Frame Transforms

We say that by the equations above, we have transformed the laser pose from the `base_link` frame to the `odom` frame.

Inverting the coordinate transform (by hand)

Assignment: Inverting a transform

Find the equations for the pose of the `odom` frame expressed in the `laser_link` frame, p_{LO}. You can verify your solution for specific values in the Stage simulation using `echo.py` from `tf2_tools`.
Warning: Doing this by hand is tricky, it is ok if you are unable to come up with a solution, we will show a more elegant way of finding this transform.

Inverting the coordinate transform (by hand)

Assignment: Inverting a transform

Find the equations for the pose of the `odom` frame expressed in the `laser_link` frame, p_{LO}. You can verify your solution for specific values in the Stage simulation using `echo.py` from `tf2_tools`.
Warning: Doing this by hand is tricky, it is ok if you are unable to come up with a solution, we will show a more elegant way of finding this transform.

$$
\begin{aligned}
x_{LO} &= -x_B \cos(\theta_B + \theta_L) - y_B \sin(\theta_B + \theta_L) - x_L \cos(\theta_L) - y_L \sin(\theta_L) \\
y_{LO} &= x_B \sin(\theta_B + \theta_L) - y_B \cos(\theta_B + \theta_L) + x_L \sin(\theta_L) - y_L \cos(\theta_L) \\
\theta_{LO} &= -\theta_B - \theta_L
\end{aligned}
$$

```
$ rosrun tf2_tools echo.py base_laser_link odom
```

Problems with manual handling of coordinate transforms:

- Can be difficult to visualize
- Complex algebraic expressions
- Error prone
- Becomes a mental health hazard in 3D

The solution

We need some more powerful mathematical tools for dealing with coordinate transforms. We would like to have well-defined mathematical objects that we can (efficiently) do algebra with.

Rotation matrix

A 2D rotation by angle θ can be written down in matrix form as

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Describing transforms with matrices

Rotation matrix

A 2D rotation by angle θ can be written down in matrix form as

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Transformation matrix

A 2D coordinate transformation from child frame C to parent frame P, consisting of a rotation by angle θ and translation by vector $\mathbf{t} = [x, y]^T$ can be written as

$$\mathbf{T}_{PC} = \begin{bmatrix} \mathbf{R}(\theta) & \mathbf{t} \\ \mathbf{0}_{[1x2]} & 1 \end{bmatrix}$$

Transformation matrix examples

Robot pose in odom:

$$\mathbf{T}_{OB} = \begin{bmatrix} \cos(\theta_B) & -\sin(\theta_B) & x_B \\ \sin(\theta_B) & \cos(\theta_B) & y_B \\ 0 & 0 & 1 \end{bmatrix}$$

Laser pose in base link:

$$\mathbf{T}_{BL} = \begin{bmatrix} \cos(\theta_L) & -\sin(\theta_L) & x_L \\ \sin(\theta_L) & \cos(\theta_L) & y_L \\ 0 & 0 & 1 \end{bmatrix}$$

Transforming coordinates

Written down in matrix form, transforms become matrix multiplications, e.g. laser pose in `odom` frame:

$$
\begin{aligned}
\mathbf{T}_{OL} &= \mathbf{T}_{OB} \cdot \mathbf{T}_{BL} \\
&= \begin{bmatrix} \cos(\theta_B + \theta_L) & -\sin(\theta_B + \theta_L) & x_B + x_L \cos(\theta_B) - y_L \sin(\theta_B) \\ \sin(\theta_B + \theta_L) & \cos(\theta_B + \theta_L) & y_r + x_L \sin(\theta_B) + y_L \cos(\theta_B) \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Transforming coordinates by matrix multiplication

Transforming coordinates

Written down in matrix form, transforms become matrix multiplications, e.g. laser pose in `odom` frame:

$$\begin{aligned}\mathbf{T}_{OL} &= \mathbf{T}_{OB} \cdot \mathbf{T}_{BL} \\ &= \begin{bmatrix} \cos(\theta_B + \theta_L) & -\sin(\theta_B + \theta_L) & x_B + x_L \cos(\theta_B) - y_L \sin(\theta_B) \\ \sin(\theta_B + \theta_L) & \cos(\theta_B + \theta_L) & y_r + x_L \sin(\theta_B) + y_L \cos(\theta_B) \\ 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

Retrieving the angle from the rotation matrix

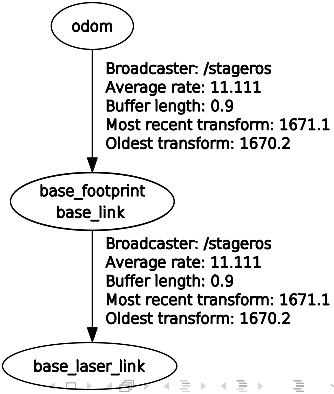
$$\theta_{OL} = \mathrm{atan2}(r_{21}, r_{11}) = \mathrm{atan2}\left(\sin(\theta_B + \theta_L), \cos(\theta_B + \theta_L)\right) = \theta_B + \theta_L$$

Inverting transforms

Transform inversion

Transforms are inverted by inverting matrices. By multiplying transform matrices and/or their inverses, we can "walk" up and down the transform tree, finding any transform that we need.

$$\mathbf{T}_{LO} = (\mathbf{T}_{OL})^{-1} = (\mathbf{T}_{OB} \cdot \mathbf{T}_{BL})^{-1}$$
$$= (\mathbf{T}_{BL})^{-1} \cdot (\mathbf{T}_{OB})^{-1} = \mathbf{T}_{LB} \cdot \mathbf{T}_{BO}$$



Sympy is a Python library for symbolic computation:

```
$ isympy3
```

```
In [1]: xB, yB, thetaB = symbols('x_B y_B theta_B')
In [2]: T_OB = Matrix([[cos(thetaB),-sin(thetaB), xB],\
   ...: sin(thetaB), cos(thetaB), yB],[0,0,1]])
In [3]: xL, yL, thetaL = symbols('x_L y_L theta_L')
In [4]: T_BL = Matrix([[cos(thetaL),-sin(thetaL), xL],\
   ...: sin(thetaL), cos(thetaL), yL],[0,0,1]])
In [5]: T_OL = T_OB*T_BL
In [6]: T_OL.simplify()
In [7]: T_OL
Out[7]: ...
In [8]: T_LO = T_OL.inv()
```

Recap (theory)

- Right-handed Cartesian coordinate systems for spatial relationships
- Object pose consists of position and orientation
- Euler angles are a minimal description of orientation
- We use the roll-pitch-yaw convention
- Poses, coordinate frames and transforms are equivalent
- Poses are always relative
- Matrices are a mathematically convenient pose representation

Recap (ROS tools)

ROS provides convenience tools for handling transforms:

- `rqt_tf_tree` for visualizing transforms
- `tf2_tools echo.py` for getting pose info
- `tf2_ros static_transform_publisher` for publishing

Task

Write a node that is going to compute the coordinates of the closest obstacle point detected by the laser scanner and broadcast its pose to the TF tree, in the `odom` frame.

Solution outline:

1. Write the program structure (the action will be taking place in the `LaserScan` callback)
2. Implement a TF2 broadcaster, sending a dummy pose in front of the laser
3. Compute the closest obstacle coordinates and broadcast them in the `laser_link` frame
4. Transform the obstacle coordinates and broadcast them in the `odom` frame

Broadcasting a transform (initialization)

```python
import tf2_ros
from geometry_msgs.msg import TransformStamped
from tf_conversions import transformations

def __init__(self):
  # ...
  self.tf_bcaster = tf2_ros.TransformBroadcaster()
  self.tf_laser_obst = TransformStamped()
  # Initialize the constant transform fields
  self.tf_laser_obst.header.frame_id = 'base_laser_link'
  self.tf_laser_obst.child_frame_id = 'obstacle'
  self.tf_laser_obst.transform.translation.z = 0.0
```

Broadcasting a transform (broadcasting)

```python
def scan_callback(self, scan):
  """ This is where all the action happens! """
  self.tf_laser_obst.header.stamp = rospy.Time.now()
  self.tf_laser_obst.transform.translation.x = 1
  self.tf_laser_obst.transform.translation.y = 0.5
  q = transformations.quaternion_from_euler(0, 0, 0.707)
  self.tf_laser_obst.transform.rotation.x = q[0]
  self.tf_laser_obst.transform.rotation.y = q[1]
  self.tf_laser_obst.transform.rotation.z = q[2]
  self.tf_laser_obst.transform.rotation.w = q[3]
  self.tf_bcaster.sendTransform(self.tf_laser_obst)
```

Quaternions can be regarded as a 3D extension of complex numbers:

$$\mathbf{q} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} + w, \;\; \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

Unit quaternions provide an alternative representation of 3D rotations, which is

- More powerful than Euler angles
- More compact than rotation matrices

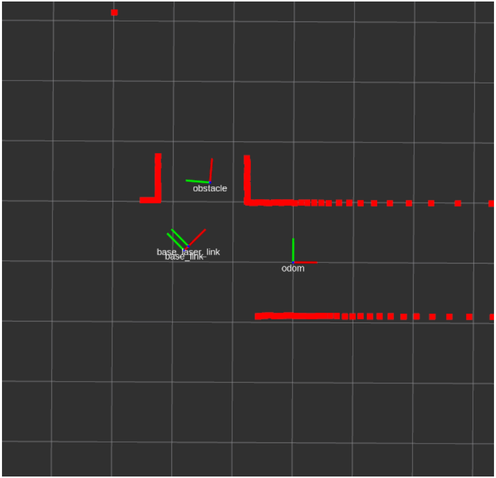
Rotation in the plane as a quaternion

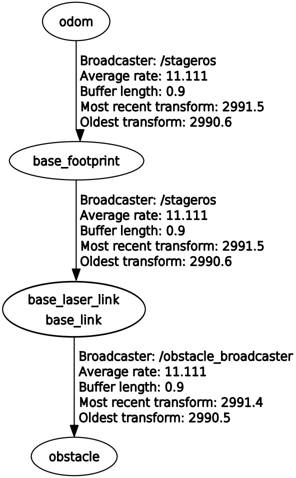
Rotation by θ around the z-axis:

$$\mathbf{q}(\theta) = 0\mathbf{i} + 0\mathbf{j} + \sin\left(\frac{\theta}{2}\right)\mathbf{k} + \cos\left(\frac{\theta}{2}\right) \tag{1}$$

Broadcasting a transform (RViz and TF tree)

```
rosrun rps_tf2_tutorial closest_obstacle.py scan:=base_scan
```

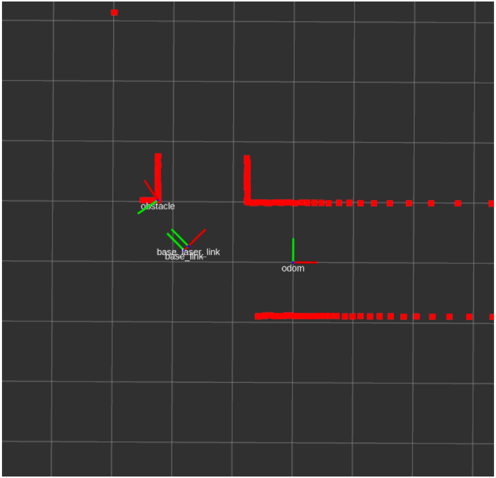


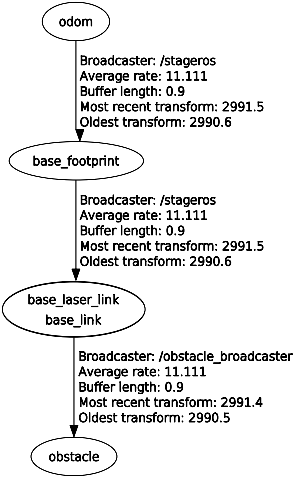


Broadcasting the closest point in a laser scan

```python
# ...
from geometry_msgs.msg import Quaternion
# ...
def scan_callback(self, scan):
  """ This is where all the action happens! """
  self.tf_laser_obst.header.stamp = rospy.Time.now()
  range_obs = min(scan.ranges)
  idx_obs = scan.ranges.index(range_obs)
  angle_obs = scan.angle_min + idx_obs*scan.angle_increment
  trans = self.tf_laser_obst.transform.translation
  trans.x = range_obs*cos(angle_obs)
  trans.y = range_obs*sin(angle_obs)
  q = transformations.quaternion_from_euler(0, 0, angle_obs)
  self.tf_laser_obst.transform.rotation = Quaternion(*q)
  self.tf_bcaster.sendTransform(self.tf_laser_obst)
```

Broadcasting the closest point (RViz and TF tree)

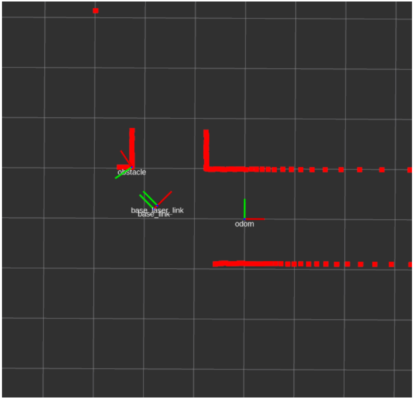


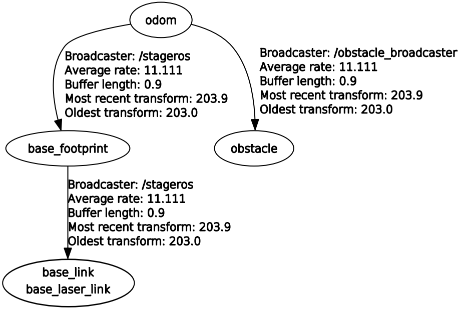


```
# ...
from geometry_msgs.msg import PoseStamped
# ...
def __init__(self):
  # ...
  self.tf_buffer = tf2_ros.Buffer()
  self.tf_listener = tf2_ros.TransformListener(self.tf_buffer)
  # ...
  self.tf_odom_obst.header.frame_id = 'odom'
  # ...
def scan_callback(self, scan):
  pose_obs = PoseStamped()
  pose_obs.pose.position.x = range_obs*cos(angle_obs)
  pose_obs.pose.position.y = range_obs*sin(angle_obs)
  q = transformations.quaternion_from_euler(0, 0, angle_obs)
  pose_obs.pose.orientation = Quaternion(*q)
```

Listening to transforms and transforming a pose

```
try:
  buf = self.tf_buffer
  tf_odom_laser = buf.lookup_transform('odom',
                                       'base_laser_link',
                                       rospy.Time())
  pose_odom_obs = do_transform_pose(pose_obs, tf_odom_laser)
  pose_odom_obs = pose_odom_obs.pose
  tf = self.tf_odom_obst.transform
  tf.translation = pose_odom_obs.position
  tf.rotation = pose_odom_obs.orientation
  self.tf_bcaster.sendTransform(self.tf_odom_obst)
  except (tf2_ros.LookupException,
          tf2_ros.ConnectivityException,
          tf2_ros.ExtrapolationException) as ex:
    rospy.logwarn(ex)
```





The ROS package with the example code is available at the pftros/rps_tf2_tutorial GitHub repo.

TF2 behind the scenes

How is TF2 actually implemented?

- The `/tf` and `/tf_static` topics
- The `geometry_msgs/TransformStamped` message
- The TF2 buffers in Python and C++ which perform the lookups and computations

Recap: TF2 Python API

- TF2 provides tools for working with transforms in Python (and C++)
- Analogy with topics:
  - Publishing <-> Broadcasting
  - Subscribing <-> Listening
- The TF tree is global, frame names are not scoped

Useful links

- Official TF2 package doc
- Official TF2 tutorials
- TF Quaternion tutorial (a bit outdated)
- A nice visualization of Euler angle shortcomings (Gimbal lock)
- A brief introduction to the PyKDL library