INDIAN INSTITUTE OF TECHNOLOGY
KHARAGPUR

DIGITAL IMAGE PROCESSING LABORATORY

A REPORT ON
**Experiment-4**

# Frequency Filtering

**Group No: 10**

**B.Sai Manoj 17EC35007**
**Sanku Yogesh 17EC35020**

Dept of Electronics and Electrical Communication Engineering
Visual Information and Embedded Systems

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# 1 Introduction

**Frequency Filtering:**

Image filtering is useful for many applications, including smoothing, sharpening, removing noise, and edge detection. A filter is defined by a kernel, which is a small array applied to each pixel and its neighbors within an image. In most applications, the centre of the kernel is aligned with the current pixel, and is a square with an odd number (3, 5, 7, etc.) of elements in each dimension. The process used to apply filters to an image is known as convolution, and may be applied in either the spatial or frequency domain.

Within the frequency domain, convolution can be performed by multiplying the FFT(Fast Fourier Transform) of the image by the FFT of the kernel, and then transforming back into the spatial domain. The kernel is padded with zero values to make it of the same size as the image before the forward FFT is applied. Another method could be that filters are usually specified within the frequency domain itself and then it is straightaway multiplied with image pixels element by element and the IFFT is found to get the original image.

Filters are classified into low-pass and high-pass depending on their filtering characteristics. Some common types of filters used in image processing are Ideal, Gaussian, Butterworth, etc. A DFT decomposes a sequence of values into components of different frequencies. A fast Fourier transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and it's inverse.

**Filter Equations:**

1. **Ideal low pass:**

$$H(u,v) = \begin{cases} 0, & if\ D(u,v) \geq D_0 \\ 1, & otherwise \end{cases} \tag{1}$$

2. **Ideal high pass:**

$$H(u,v) = \begin{cases} 0, & if\ D(u,v) \leq D_0 \\ 1, & otherwise \end{cases} \tag{2}$$

3. **Butterworth low pass:**

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)}{D_0}]^{2n}} \tag{3}$$

4. **Butterworth high pass:**

$$H(u,v) = \frac{1}{1 + [\frac{D_0}{D(u,v)}]^{2n}} \tag{4}$$

5. **Gaussian low pass:**

$$H(w_x, w_y) = e^{-\frac{w_x^2 + w_y^2}{2\sigma^2}} \tag{5}$$

6. **Gaussian high pass:**

$$H(w_x, w_y) = 1 - e^{-\frac{w_x^2 + w_y^2}{2\sigma^2}} \tag{6}$$

$$where \ \ D(u,v) = \sqrt{u^2 + v^2}$$

## 2 Algorithm

**FFT:**

Input :Array X

- X_even = FFT(even indices of X)
- X_odd = FFT(odd indices of X)
- for k = 0.....N/2
    1. X[k] = X_even[k] + e$^{-2i*pi*k/n}$ * X_odd[k]
    2. X[N/2+k] = X_even[k] - e$^{-2i*pi*k/n}$ * X_odd[k]

**FFT2D:**

We first perform FFT on each row and then transpose this coefficient matrix, and perform FFT on rows again.

IFFT2D follow the same strategy .

**Filtering:**

1. Calculating FFT of the image.
2. Based on the filter selected, the filter mask in the frequency domain.
3. Then apply this filter to the image FFT in Frequency domain.
4. Take the IFFT and display the filtered image.

# 3 Results

We have added tracker to observe live results as there are many possibilities.

# 4 Analysis

1. Frequency filtering is advantageous compared to spatial filtering in the sense that convolution operator is simplified to the multiplication operator which is easy to handle and implement, thereby reducing the time complexity in some cases.
2. The ideal filters suffer from artifacts arising from the ringing effect.To avoid ringing effects in ideal filters which occur due to discontinuity in filter response, butterworth and Gaussian filters are used as they dont have such discontinuity.
3. Butterworth filter is also referred to as a maximally flat magnitude filter.The frequency response of the Butterworth filter is maximally flat in the passband and rolls off towards zero in the stopband.
4. The gaussian filter allows for smoother transition from the passband to the stopband. However, the passband transfer function is not constant. Thus, we incur a passband attenuation while using a Gaussian low-pass filter and a stopband leakage while using a gaussian high-pass filter.