Inventory Management for a Retail Store

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1 Introduction

1.1 What is Optimization?

Optimization is the process of finding the most effective or efficient solution among a set of possible options. It involves improving an objective — such as minimizing cost, maximizing profit, or saving time — while satisfying any constraints or limitations that apply. Optimization techniques are widely used across disciplines, including engineering, business, operations, and everyday personal decisions.

1.2 Optimization in Daily Life

In our daily lives, we constantly make decisions that involve limited resources — whether it be time, money, energy, or materials. Optimization helps us make the most of these resources by enabling smarter, more informed choices.

1. **Better Resource Utilization** – Optimization helps avoid waste and ensures that resources like time, money, or fuel are used in the most effective way. For example, by planning your errands to minimize travel distance, you save both time and fuel.

2. **Time Management** – From scheduling study time to planning meals for the week, optimization helps individuals make the best use of their limited hours each day. Prioritizing important tasks and assigning the right amount of time improves productivity.

3. **Financial Efficiency** – Budgeting is a form of optimization where people aim to spend money in a way that meets their needs while staying within income limits. Choosing the most affordable yet effective products or services is a daily optimization task.

4. **Decision Making** – Optimization allows for structured decision-making by evaluating various options and selecting the one that leads to the best result. Whether it's selecting the best data plan, booking the cheapest flight, or preparing healthy meals within a calorie limit, optimization supports better outcomes.

5. **Technology and Smart Tools** – Many apps and digital tools we use today are built on optimization algorithms. GPS systems, for example, use route optimization to find the shortest or fastest path. Food delivery apps optimize order assignments to reduce wait times and fuel costs.

6. **Energy and Environmental Benefits** – By optimizing daily habits — such as turning off unused appliances or carpooling — individuals can reduce their energy consumption and carbon footprint. Optimization contributes to sustainability.

1.3 Examples of Daily Optimization

- **Route Planning:** Using navigation apps to find the fastest or most fuel-efficient path.

- **Meal Planning:** Creating a meal plan that balances cost, nutrition, and preparation time.
- **Study Scheduling:** Organizing study time around exam schedules and personal productivity patterns.
- **Fitness Goals:** Designing workout routines that maximize health benefits in the least amount of time.

1.4 Optimization in the Retail Industry

Retail businesses operate in highly competitive environments and must constantly make decisions that impact costs, profits, and customer satisfaction. Optimization tools and techniques help retailers streamline operations and make smarter business decisions.

1. **Inventory Management** – Effective inventory control ensures that the right products are available at the right time. Optimization helps:
    - Forecast product demand based on trends and sales history.
    - Determine the optimal quantity to order and when to reorder.
    - Minimize excess stock and reduce storage costs while avoiding stockouts.
2. **Pricing and Promotions** – Optimization algorithms assist retailers in setting prices that balance profitability with customer appeal. By analyzing competition, demand, and seasonality, businesses can:
    - Maximize sales during high-demand periods.
    - Offer discounts strategically to clear inventory without hurting profit margins.
3. **Store Layout and Product Placement** – Retailers use optimization models to design store layouts that improve traffic flow and product visibility. This encourages more purchases by strategically placing high-demand or complementary items in prime locations.
4. **Supply Chain and Logistics** – Optimization ensures goods move from warehouses to stores in the most cost-effective and timely manner. This includes:
    - Route optimization for delivery trucks.
    - Efficient warehouse layout planning.
    - Optimal placement of distribution centers.
5. **Demand Fulfillment** – Meeting customer demand without overstocking is a key challenge. Optimization supports:
    - Demand forecasting using historical and real-time data.
    - Dynamic stock allocation to various store locations based on sales trends.

2 Understanding Linear Programming (LPP)

2.1 Introduction

Linear Programming (LP) is a mathematical optimization technique used to determine the best possible outcome—such as maximum profit or minimum cost—under a given set of constraints. It is widely used in operations research, business, engineering, and economics.

2.2 Definition and Structure of a Linear Programming Problem (LPP)

A Linear Programming Problem (LPP) involves:

- **Objective Function:** A linear expression to be maximized or minimized, such as profit or cost.
  Example:
  $$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$
- **Constraints:** A system of linear inequalities or equalities that define the limitations on the decision variables.
  Example:
  $$\begin{aligned} a_{11}x_1 + a_{12}x_2 &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 &\leq b_2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.3 Types of Problems Solved by LPP

Linear Programming is applicable in various real-world scenarios such as:

- **Production and Manufacturing:** Determining the optimal product mix under limited resources.
- **Transportation and Logistics:** Minimizing the cost of shipping or routing.
- **Diet/Nutrition Planning:** Meeting dietary requirements at minimum cost.
- **Workforce Scheduling:** Assigning shifts to workers efficiently.
- **Marketing and Advertising:** Allocating budget across platforms to maximize outreach.

2.4 Example: Advertising Mix Problem (Graphical Method)

Problem Statement:
A company wants to decide how many TV ads and social media ads to run. Each TV ad costs \$1000 and yields 40 exposure units, while each social media ad costs \$500 and yields 30 exposure units. The total advertising budget is \$10,000 and there are 20 hours available for airing ads. A TV ad requires 2 hours, and a social media ad requires 1 hour. Determine the optimal number of ads to maximize exposure.

1. **Decision Variables:**
   Let x = number of TV ads, y = number of social media ads.

2. **Objective Function:**
   Maximize
   $$Z = 40x + 30y$$

3. **Constraints:**
   $$\begin{gathered} 1000x + 500y \leq 10000 \quad \text{(Budget)} \\ 2x + y \leq 20 \quad \text{(Time)} \\ x, y \geq 0 \end{gathered}$$

   Divide the budget constraint by 500 to simplify:
   $$2x + y \leq 20$$

   So both constraints are the same: $2x + y \leq 20$

4. **Graphical Solution:**
   Plot the line $2x + y = 20$.

   - $x = 0 \Rightarrow y = 20$
   - $y = 0 \Rightarrow x = 10$

   Feasible region is the area under this line in the first quadrant.

5. **Corner Points of Feasible Region:**

   - $A = (0, 0)$
   - $B = (0, 20)$
   - $C = (10, 0)$

6. **Evaluate Objective Function:**
   $$\begin{aligned} Z_A &= 40(0) + 30(0) = 0 \\ Z_B &= 40(0) + 30(20) = 600 \\ Z_C &= 40(10) + 30(0) = 400 \end{aligned}$$

7. **Conclusion:**
   The maximum exposure of 600 occurs at point $B = (0, 20)$. Therefore, the optimal solution is to run 20 social media ads and 0 TV ads.

2.5 Solution Methods Overview

Method	When Used
Graphical Method	For 2-variable problems
Simplex Method	For 3 or more variables
Big M Method	When $\geq$ or $=$ constraints are present (introduces artificial variables)
Two-Phase Method	Alternative to Big M; solves in two stages
Dual Simplex Method	When the initial solution is infeasible but optimality conditions are satisfied

3 Problem Formulation

3.1 Decision Variables

Let there be 10 products sold in the retail store. We define the decision variables as:

- x_i = Number of units to be stocked for Product i, for $i = 1, 2, ..., 10$

3.2 Objective Function

The goal is to minimize the total procurement cost for stocking the products.

Let c_i = Cost price per unit of Product i, for $i = 1, 2, ..., 10$.

The objective function is:

$$\text{Minimize } Z = \sum_{i=1}^{10} c_i \cdot x_i$$

3.3 Constraints

(i) **Budget Constraint:** Let B = Total available budget.

$$\sum_{i=1}^{10} c_i \cdot x_i \leq B$$

(ii) **Storage Constraint:** Let t_i = Storage space required per unit of Product i, and S = Total available storage space.

$$\sum_{i=1}^{10} t_i \cdot x_i \leq S$$

(iii) **Demand Satisfaction:** Let d_i = Minimum required demand (units) for Product i.

$$x_i \geq d_i \quad \text{for } i = 1, 2, ..., 10$$

(iv) **Non-Negativity:**

$$x_i \geq 0 \quad \text{for all } i = 1, 2, ..., 10$$

3.4 Assumptions Made

- All cost prices, demands, and storage capacities are known in advance and remain constant.
- Partial units cannot be stocked; however, we solve the problem as a continuous linear program (ignoring integer constraints for simplicity).
- No constraints from suppliers or availability beyond budget and storage.
- All stocked units will eventually be sold (no spoilage or obsolescence considered).

4 Graphical Solution (for Two-Variable Case)

4.1 Problem Statement

Consider a simplified case of the inventory management problem with only two products in the form of an example:

- x_1 = number of units of Product 1 to stock
- x_2 = number of units of Product 2 to stock

The objective is to minimize total cost:

$$\text{Minimize } Z = 5x_1 + 4x_2$$

Subject to the following constraints:

$$\begin{aligned}
5x_1 + 4x_2 &\leq 100 && \text{(Budget Constraint)} \\
2x_1 + 3x_2 &\leq 60 && \text{(Storage Constraint)} \\
x_1 &\geq 10 && \text{(Minimum demand for Product 1)} \\
x_2 &\geq 5 && \text{(Minimum demand for Product 2)} \\
x_1, x_2 &\geq 0 && \text{(Non-negativity)}
\end{aligned}$$

4.2 Graphical Representation

To solve the above problem graphically, we plot the constraint lines and shade the feasible region. The intersection of all inequalities forms a convex polygon, and the optimal solution lies at one of the corner (extreme) points.

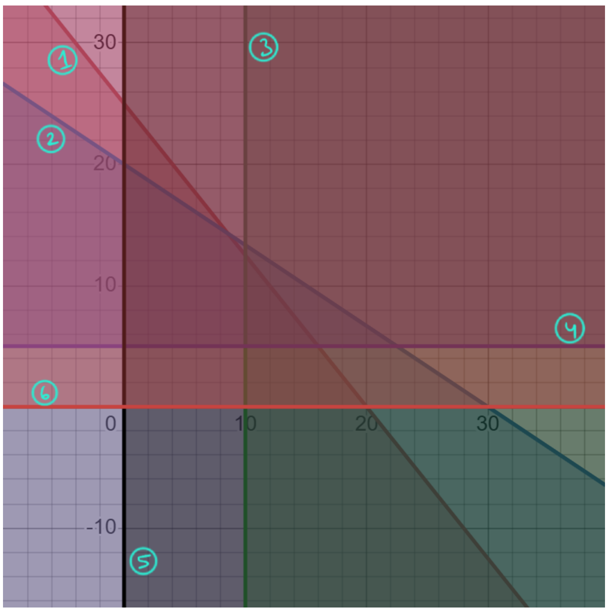

Figure 1: Feasible Region for Two-Product Case

Line Annotations:

- Line 1: $5x_1 + 4x_2 \leq 100$ (Budget Constraint)
- Line 2: $2x_1 + 3x_2 \leq 60$ (Storage Constraint)
- Line 3: $x_1 \geq 10$ (Minimum Demand for Product 1)
- Line 4: $x_2 \geq 5$ (Minimum Demand for Product 2)
- Line 5: $x_1 \geq 0$ (Non-negativity constraint)
- Line 6: $x_2 \geq 0$ (Non-negativity constraint)

4.3 Corner Points and Objective Function Evaluation

We identify the corner points of the feasible region and evaluate the objective function $Z = 5x_1 + 4x_2$ at each.

Point	(x_1, x_2)	$Z = 5x_1 + 4x_2$
A	$(10, 12.5)$	100
B	$(10, 5)$	70
C	$(16, 5)$	100

Table 1: Objective Function Values at Corner Points

4.4 Optimal Solution

The minimum value of Z occurs at the corner point where Z is lowest. Hence, the optimal solution is:

$$(x_1^*, x_2^*) = (10,5), \quad Z_{\text{min}} = 70$$

5 Solving Higher-Dimensional Problem

5.1 3-Variable Problem Setup

In this section, we extend our problem to include three products, making the problem suitable for higher-dimensional solution techniques such as the simplex method.

Objective: Minimize the total cost of stocking 3 products.

Decision Variables:

- x_1, x_2, x_3 are the number of units stocked for products 1, 2, and 3 respectively.

Given Constraints:

$$\begin{aligned} \text{(i) Budget Constraint:} \quad & x_1 + 2x_2 + x_3 \leq 40 \\ \text{(ii) Storage Constraint:} \quad & 2x_1 + x_2 + 2x_3 \leq 50 \\ \text{(iii) Minimum Demand:} \quad & x_1 \geq 5, \quad x_2 \geq 4, \quad x_3 \geq 3 \\ \text{(iv) Non-Negativity:} \quad & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Cost Function to Minimize:

$$Z = 3x_1 + 2x_2 + 4x_3$$

To handle the minimum demand constraints without artificial variables, we define shifted variables:

$$y_1 = x_1 - 5, \quad y_2 = x_2 - 4, \quad y_3 = x_3 - 3 \quad \Rightarrow \quad y_1, y_2, y_3 \geq 0$$

Reformulated Problem:

Minimize:

$$Z = 3y_1 + 2y_2 + 4y_3 + 35$$

Subject to:

$$\begin{aligned} y_1 + 2y_2 + y_3 &\leq 24 \\ 2y_1 + y_2 + 2y_3 &\leq 20 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

Note on the Objective Function Constant: During the substitution step, we introduced new variables to eliminate the lower bounds:

$$x_1 = y_1 + 5, \quad x_2 = y_2 + 4, \quad x_3 = y_3 + 3$$

Substituting into the original objective function:

$$Z = 3x_1 + 2x_2 + 4x_3 = 3(y_1 + 5) + 2(y_2 + 4) + 4(y_3 + 3) = 3y_1 + 2y_2 + 4y_3 + 35$$

The constant term 35 is independent of the decision variables and does not affect the optimization process. Therefore, it is omitted during the simplex iterations.

After obtaining the optimal value of the simplified objective function, we simply add the constant back:

$$Z = Z' + 35$$

where Z' is the final value obtained from the simplex method.

5.2 Solution using Simplex Method

We convert the inequalities into equalities by introducing slack variables s_1 and s_2:

$$y_1 + 2y_2 + y_3 + s_1 = 24$$
$$2y_1 + y_2 + 2y_3 + s_2 = 20$$

The objective is to minimize:

$$Z = 3y_1 + 2y_2 + 4y_3 + 35$$

Initial simplex tableau:

$\mathbf{C_B}$	**Basis**	y_1	y_2	y_3	s_1	s_2	**RHS** (b_i)	b_i/a_{ij}^*
0	s_1	1	2	1	1	0	24	12
0	s_2	2	1	2	0	1	20	10
$C_j - Z_j$		3	2	4	0	0		

Table 2: Initial Simplex Tableau

Since all values in the $C_j - Z_j$ row are non-negative, the optimality condition is already satisfied. Therefore, no further iterations are required.

Optimal Solution:

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 0$$
$$\Rightarrow x_1 = y_1 + 5 = 5, \quad x_2 = y_2 + 4 = 4, \quad x_3 = y_3 + 3 = 3$$
$$Z_{\min} = Z' + 35 = 0 + 35 = 35$$

Hence, the optimal stocking quantities are 5 units of Product 1, 4 units of Product 2, and 3 units of Product 3 with a total cost of 35.

5.3 Alternate Method: Two-Phase or Big M (Optional)

In this case, we avoided the use of artificial variables. However, if constraints such as $x_i \geq$ demand could not be handled through variable shifting, then artificial variables would be required. In such scenarios, Big M or Two-Phase methods are preferred.

5.4 Comparison of Methods

- **Graphical Method:** Useful for 2-variable problems. Visual and intuitive but not scalable.
- **Simplex Method:** Systematic and applicable for higher dimensions. Iterative but efficient.
- **Big M / Two-Phase:** Necessary for constraints with equality or $\geq$ type, especially when artificial variables are unavoidable.
- **Dual Simplex Method:** Used when the initial solution is infeasible but the optimality conditions are met; useful for post-optimality analysis.

6 Conclusion

6.1 Project Summary

This project focused on applying Linear Programming Problem (LPP) techniques to optimize inventory management for a retail store. The primary objective was to minimize the total procurement cost while meeting demand constraints and staying within budget and storage limits. We formulated the problem using decision variables for product quantities, established a cost-minimizing objective function, and imposed realistic constraints. Both graphical and simplex methods were applied to demonstrate solution techniques for low- and higher-dimensional cases respectively.

6.2 Real-Life Applications of the Model

Inventory optimization is a practical and vital task in retail operations. By using LP models like the one in this project:

- Retailers can determine optimal stock quantities to reduce costs and avoid overstocking.
- The model supports better warehouse and shelf-space planning.
- It assists in meeting customer demand consistently, improving satisfaction and sales.
- Seasonal trends and variable demands can be modeled with updated constraints, making the approach adaptable.

6.3 Reflections and Learnings

Through this project, we gained hands-on experience in:

- Translating a real-world problem into a mathematical formulation.
- Solving LP problems graphically and using the simplex algorithm.
- Understanding how optimization contributes to business efficiency.
- Interpreting results within the practical context of inventory decisions.

We also encountered challenges, such as deciding between different solution techniques and managing demand constraints, which helped deepen our understanding of operations research tools.

6.4 Scope for Future Work

The current model can be extended in several ways:

- **Integer Programming:** Introduce integer constraints to reflect that product units cannot be fractional.

- **Multi-Objective Optimization:** Include other goals such as maximizing customer satisfaction or minimizing delivery time.
- **Uncertainty Handling:** Incorporate probabilistic or fuzzy demand to reflect real-world fluctuations.
- **Dynamic Inventory Models:** Adapt the static model to a time-based one for managing inventory over multiple periods.

This project demonstrates the power and versatility of linear programming in solving practical optimization problems in retail and beyond.

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