Poisson Simulation of Cardiac Arrests

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1 Introduction

Cardiac arrests are critical medical emergencies that require immediate response. Modeling their occurrence can significantly improve **Emergency Medical Service (EMS)** planning and readiness.

In this project, we analyze how cardiac arrests can be modeled as a **Poisson process**, a type of stochastic process commonly used to represent the occurrence of random, rare events over time.

This report focuses on two core components:

- The theoretical formulation of the Poisson process and its relationship to the exponential distribution.
- A simulation in `R` that generates synthetic cardiac arrest data based on known statistical rates.

Stochastic Process Concepts Used:

- **Stochastic process:** A collection of random variables indexed by time (or space), representing systems that evolve randomly over time.
- **Counting process:** A stochastic process that counts the number of events occurring up to time t.
- **Poisson process:** A specific type of counting process characterized by independent increments and a constant average rate of occurrence.
- **Exponential distribution:** Describes the time between consecutive events in a Poisson process.

We aim to show how well the real-world data aligns with the Poisson model and what implications this has for emergency healthcare logistics.

2 Background and Literature Review

The modeling of real-life random events, such as cardiac arrests, is a central concern in stochastic processes. In particular, events that occur independently and unpredictably over time are often modeled using the Poisson process, first formalized by Siméon Denis Poisson in the 19th century.

One of the earliest formal medical applications of the Poisson process was in epidemiology and call center modeling. Its strength lies in its simplicity: it requires only one parameter, the average rate λ, to fully describe the distribution of both event counts and time between events.

Relevant Studies

- **Skogvoll and Lindqvist (1999):** Analyzed over 800 cardiac arrests (in-hospital and out-of-hospital) over five years, finding that event timings closely matched a Poisson model.

- **Przybysz and Bunch (2017):** Extended the model spatially using Poisson kriging to map arrest hotspots in Toronto.
- **BMJ Open (2015):** Employed Poisson regression to analyze time trends in survival rates after cardiac arrest, treating counts as Poisson-distributed outcomes.

These studies validate the assumptions behind using Poisson and exponential distributions to model medical emergencies. Our project builds on this foundation by conducting a small-scale simulation and connecting it to the empirical parameters from the literature.

3 Poisson Process: Theoretical Framework

3.1 Definition

A Poisson process is a stochastic process $N(t), t \geq 0$, where N(t) represents the number of events in time interval $(0, t]$. The process is characterized by:

- Independent increments
- Stationary increments
- The number of events in interval (0,t] follows $P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$

3.2 Exponential Inter-arrival Times

The time between two events, denoted by T, follows an exponential distribution:

$$P(T \leq t) = 1 - e^{-\lambda t}$$

3.3 Properties and Interpretation

Key properties of the Poisson process include:

- **Independent increments:** The number of events occurring in disjoint time intervals are independent.
- **Stationary increments:** The probability of a given number of events in an interval depends only on its length, not its position.
- **Poisson distribution of counts:** For a fixed interval of length t, the number of events follows $P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$.
- **Exponential inter-arrival times:** The time T between consecutive events follows an exponential distribution with parameter λ, i.e., $P(T > t) = e^{-\lambda t}$.
- **Mean and variance:** For Poisson, $\mathbb{E}[N(t)] = \lambda t$ and $\text{Var}[N(t)] = \lambda t$. For exponential, $\mathbb{E}[T] = 1/\lambda$.
- **Memorylessness:** The exponential distribution satisfies $P(T > s + t \mid T > s) = P(T > t)$.

4 Methodology

We assume cardiac arrests occur independently at a constant average rate λ. Using this assumption:

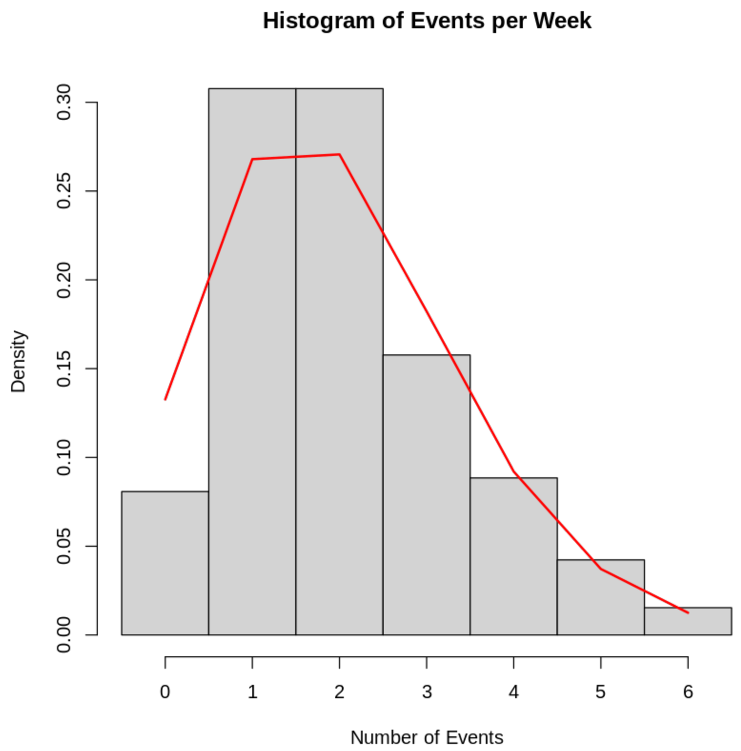
- We estimate λ from literature.
- Simulate event times using exponential inter-arrival times.
- Analyze and visualize the process using R.

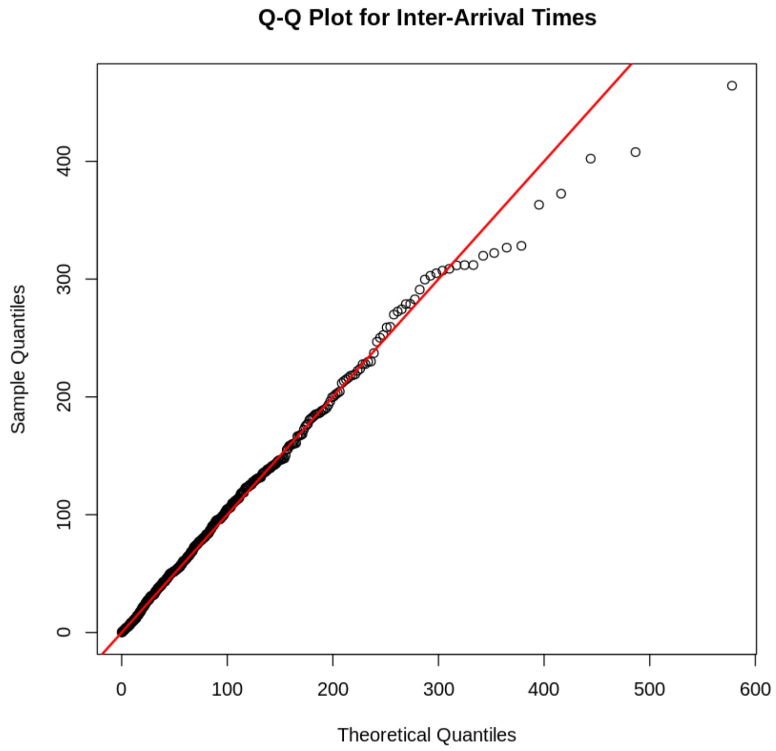
5 Simulation in R

The simulation was implemented in `R`. Full source code is provided in the Appendix.

5.1 Results

- The cumulative number of events increases in a stepwise manner.
- Histogram of inter-arrival times matches exponential distribution.





6 Applications and Practical Implications

The ability to model cardiac arrests as a Poisson process provides several practical benefits for emergency medical service (EMS) planning:

- **Staffing decisions:** Understanding the likelihood of multiple arrests within a short period can help allocate backup teams during peak hours.
- **Resource deployment:** Estimating call volumes helps with ambulance and defibrillator stationing in both urban and rural settings.
- **Predictive planning:** Simulation can support training drills and contingency planning by estimating worst-case or rare but possible clusters of events.
- **Policy-making:** Quantified risk estimates can be useful in health policy and public funding decisions for EMS infrastructure.

7 Data Summary and Statistical Analysis

7.1 Study Parameters and Population Data

- County population: 155,000
- Hospital beds: 850

- Study duration: 5 years (260 weeks)
- Annual hospital admissions: 37,500 patients

7.2 Cardiac Arrest Counts and Estimated Rates

- Out-of-hospital arrests: 525 events over 5 years
- Incidence rate (out-of-hospital): 67.7 per 100,000/year
- Poisson parameter estimate (weekly, out-of-hospital): 2.02
- Mean monthly rate (out-of-hospital): 8.75 events/month
- In-hospital arrests: 284 events over 5 years
- Incidence rate (in-hospital): 1.5 per 1,000 admissions/year
- Poisson parameter estimate (weekly, in-hospital): 1.09

7.3 Mean Time Intervals Between Events

- Out-of-hospital: 83 hours between events
- In-hospital: 154 hours between events

7.4 Probability and Frequency Calculations

- $P(X = 5)$ for out-of-hospital events (weekly, λ = 2.02): 0.0366 (3.66%)
- Expected number of such weeks in 5 years: 9.52
- Observed number of such weeks: 10
- $P(T \leq 1 \text{ hr})$ for in-hospital events: 0.0065 (0.65%)
- Expected intervals (2+ events in 1 hr) in 5 years: 1.85
- Observed intervals: 4
- Probability of 4 or more such episodes: 0.12

7.5 Statistical Properties and Approximations

- 95% of Poisson observations lie within 2 standard deviations from the mean
- Poisson $\rightarrow$ Normal approximation valid for mean > 20
- Critical value for 95% confidence: 1.96
- Significance level used: $\alpha = 0.05$

7.6 Confidence Intervals

- Out-of-hospital (yearly rate): 95% CI = (96, 114)
- Population-scaled CI: 62 to 74 per 100,000/year
- In-hospital (weekly rate): 95% CI = (0.96, 1.22)
- Valid for event counts > 50

7.7 Goodness-of-Fit Testing

To validate the Poisson assumption, the observed cardiac arrest counts were compared to the expected frequencies using the chi-squared (χ^2) test and the Poisson dispersion statistic.

- The χ^2 goodness-of-fit tests yielded high P-values (> 0.5), indicating that the observed frequencies closely align with a Poisson model.
- The Poisson dispersion statistic compares the sample variance to the mean. For a perfect Poisson model, variance and mean are equal. The test yielded:
  - Out-of-hospital: $\chi^2 = 285.7$ (df = 259), $P = 0.12$
  - In-hospital: $\chi^2 = 258.1$ (df = 259), $P = 0.51$
- The results support the use of Poisson and exponential models for this dataset.

7.8 Clinical Trial Simulation Example

- Trial goal: 300 patients, 2 patients/week
- Remaining: 50 patients
- Expected in 20 weeks: 40 patients
- $P(X \geq 50)$ in 20 weeks: 0.06 (6%), $Z = 1.58$
- $P(X \geq 50)$ in 30 weeks: 0.90 (90%), $Z = -1.29$
- To be 99% certain: trial duration $\approx$ 35 weeks ($Z = -2.32$)

8 Discussion

Our simulation supports the hypothesis that cardiac arrests can be modeled using a Poisson process. While our rate was hypothetical, literature-based estimates (e.g., Skogvoll: 2.02/week) showed good empirical fit. The simplicity of the Poisson process makes it attractive for modeling rare and independent medical events.

9 Limitations

Although the Poisson model provides a good fit to the data, several limitations should be acknowledged:

- **Assumption of constant rate:** Real-world event rates may vary by time of day, season, or population health trends.
- **Independence assumption:** Certain events may be correlated, e.g., multiple patients from a single incident.
- **No geographic component:** The current model ignores spatial distribution of arrests.
- **Simplified simulation:** The R simulation assumes a constant λ and does not incorporate hospital-specific workflows or patient histories.

10 Future Work

To enhance the model and bring it closer to real-world complexity, future work may include:

- **Non-homogeneous Poisson process:** Incorporate time-varying $\lambda(t)$ to model diurnal or seasonal trends.
- **Spatial modeling:** Use spatial Poisson processes or point processes to map events across a region.
- **Poisson regression:** Introduce covariates such as age, time of day, and location to predict event rates.
- **Comparative studies:** Compare Poisson with other count models like negative binomial to handle overdispersion or repeated events.
- **Integration with real EMS data:** Validate and tune the model using real-time EMS logs or hospital datasets.

11 Conclusion

We presented a stochastic model for cardiac arrest occurrence using the Poisson process framework. Both theory and simulation support this model's applicability. This approach can inform EMS planning and resource allocation.

12 References

1. Skogvoll E, Lindqvist BH. Modeling the occurrence of cardiac arrest as a Poisson process. *Ann Emerg Med*. 1999;33(4):409-417.
2. Przybysz R, Bunch M. Exploring Spatial Patterns of Sudden Cardiac Arrests Using Poisson Kriging. *PLOS ONE*. 2017.

3. BMJ Open. Temporal Trends in Out-of-Hospital Cardiac Arrest Survival Outcomes. 2015.

A Appendix: Full R Code

```
# Set parameters for out-of-hospital cardiac arrests
lambda_weekly <- 2.02  # mean number of events per week
num_weeks <- 260       # 5 years * 52 weeks/year

# Simulate number of events per week
set.seed(123)  # for reproducibility
events_per_week <- rpois(num_weeks, lambda = lambda_weekly)

# Calculate mean and variance
mean_events <- mean(events_per_week)
var_events <- var(events_per_week)
print(paste("Simulated mean number of events per week:", round(mean_events, 2)))
print(paste("Simulated variance of number of events per week:", round(var_events, 2)))
print(paste("Theoretical mean and variance:", lambda_weekly))

# Plot histogram with Poisson distribution
max_events <- max(events_per_week)
hist(events_per_week,
    breaks = seq(-0.5, max_events + 0.5, by = 1),
    probability = TRUE,
    main = "Histogram of Events per Week",
    xlab = "Number of Events")
x <- 0:max_events
lines(x, dpois(x, lambda = lambda_weekly), col = "red", lwd = 2)

# Set parameters for inter-arrival times
hours_per_week <- 168
lambda_hourly <- lambda_weekly / hours_per_week  # rate per hour
total_time_hours <- 5 * 365.25 * 24  # approximately 43830 hours

# Simulate inter-arrival times
inter_arrival_times <- rexp(10000, rate = lambda_hourly)
event_times <- cumsum(inter_arrival_times)
num_events <- sum(event_times <= total_time_hours)
simulated_inter_arrival_times <- inter_arrival_times[1:num_events]

# Calculate mean inter-arrival time
mean_inter_arrival <- mean(simulated_inter_arrival_times)
theoretical_mean <- 1 / lambda_hourly
print(paste("Simulated mean inter-arrival time:", round(mean_inter_arrival, 2), "hours"))
print(paste("Theoretical mean inter-arrival time:", round(theoretical_mean, 2), "hours"))

# Total number of events
print(paste("Simulated total number of events:", num_events))
print(paste("Expected total number of events:", round(lambda_weekly * num_weeks)))
```

```
# Optional: Q-Q plot to check exponential distribution
qqplot(qexp(ppoints(num_events),
        rate = lambda_hourly),
        simulated_inter_arrival_times,
        main = "Q-Q Plot for Inter-Arrival Times",
        xlab = "Theoretical Quantiles",
        ylab = "Sample Quantiles"
    )
abline(0, 1, col = "red", lwd = 2)
```

B Appendix: Additional Tables

You may also include a table here comparing:

- Simulated vs observed frequencies of weekly cardiac arrests
- Probability values vs observed counts for multiple-event intervals