CS6046: Multi-Armed Bandits
Worksheet - 1

(1) Construct a probability space with the following random variables (i) a die with $\mathbb{P}(1) = 0.5, \mathbb{P}(2) = \mathbb{P}(3) = \mathbb{P}(4) = \mathbb{P}(5) = \mathbb{P}(6) = 0.1$, and (ii) coin_1 with $\mathbb{P}(H) = 0.6$ and (iii) coin_2 with $\mathbb{P}(H) = 0.5$. The die and coin_1 are independent of each other.

(2) Consider a bandit problem with 2 arms namely $\text{Arm}_1, \text{Arm}_2$. Each arm has a stack of rewards. At round t, when an arm $a_t \in \{\text{Arm}_1, \text{Arm}_2\}$ is chosen (i) the reward obtained is equal to the entry in the top of the stack, and (ii) once the reward is obtained it is also removed/deleted from the top of the stack. The diagram below shows three different bandit problems namely (I), (II) and (III). We will pull the arms for rounds $t = 1, \ldots, T$, and the total reward in T rounds is given by

$$R_{\text{total}}(T) = \sum_{t=1}^{T} r(a_t), \tag{1}$$

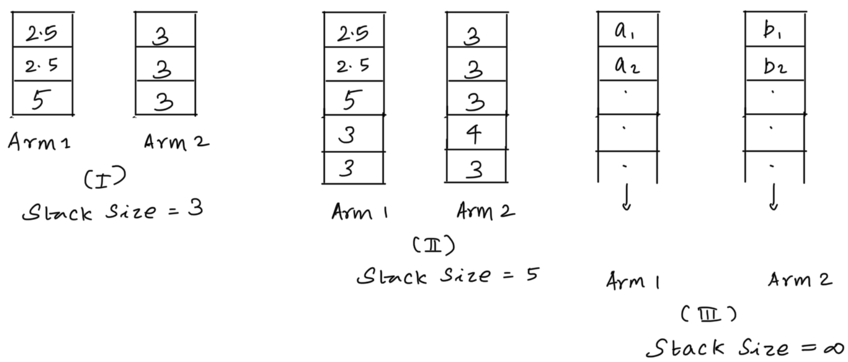
where $r(a_t)$ is the reward on the top of the stack in arm a_t.

i. For instance (I) find the maximum value of $R_{\text{total}}(T)$ (also the sequence of arms that need to be chosen) for the following values of $T = 1, 2, 3, 4, 5, 6$.

ii. For instance (II) find the maximum value of $R_{\text{total}}(T)$ (also the sequence of arms that need to be chosen) that can be obtained for $T = 7$.

iii. For instance (III) explain how you will maximise the value of $R_{\text{total}}(T)$ (you can use the notation $a_1, a_2, \ldots$ and $b_1, b_2 \ldots$ to explain your answer) as a function of T.

(3) An unprepared student has one day to prepare for the exam, and has two actions $\mathcal{A} = \{a^1 = \text{study}, a^2 = \text{play}\}$. The reward for playing is 1 and the reward for studying is 0. If the student studies then with probability p the student gets prepared. If the student decides to play then the student is unprepared with probability 1. On the



day of exam, if prepared student passes the exam which is equivalent to reward of 10, and if the student is unprepared then the student fails the exam which is equivalent to reward of 0. Find p for which the optimal action for the student is to study.

(4) Let X_1 and X_2 be two independent and identically distributed random variables with $\text{Prob}(X_1 = -2) = \text{Prob}(X_1 = -1) = \text{Prob}(X_1 = 0) = \text{Prob}(X_1 = 1) = \text{Prob}(X_1 = 1) = 0.2$. What will be the distribution of the random variable $Y = X_1 + X_2$, i.e., find $\text{Prob}(Y = y)$ for all possible values of Y.

(5) Let X_1 and X_2 be two independent and identically distributed random variables which are uniformly distributed between $[0, 1]$. What will be the probability density of the random variable $Y = X_1 + X_2$?

(6) Let $\{X_1(t)\}$ be a sequence of i.i.d random variable such that $\mathbb{P}(X_1(t) = -1) = 1/2$ and $\mathbb{P}(X_1(t) = +1) = 1/2$ and $\{X_2(t)\}$ be a sequence of i.i.d random variable such that $\mathbb{P}(X_2(t) = -2) = 1/2$ and $\mathbb{P}(X_2(t) = +2) = 1/2$. Let $C(t)$ be a i.i.d sequence of fair coin tosses. Let $Y(t)$ be defined as

$$Y(t) = \left\{ \begin{array}{ll} \frac{X_1(t)}{X_2(t)}, & \text{if } C(t) \text{ toss is head} \\ \frac{X_2(t)}{X_1(t)}, & \text{if } C(t) \text{ toss is tail} \end{array} \right\} \tag{2}$$

What is $\mu = \mathbb{E}Y(t)$?

(7) Let Ω be a set. Let $\mathcal{F}$ be a collection of subsets of Ω. $\mathcal{F}$ is a sigma algebra for iff,
(1) $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$.
(2) $A_1, \ldots, A_i, \ldots \in \mathcal{F}$ then $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$.
(3) $\Omega \in \mathcal{F}$.
Identify (yes/no) if the following are sigma algebras.
(i) $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{F} = \{\Omega, \emptyset\}$.
(ii) $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{F} = \{\Omega, \emptyset, \{1\}, \{2, 3, 4, 5, 6\}\}$.
(iii) $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{F} = \{\Omega, \emptyset, \{1\}, , \{2\}, \{2, 3, 4, 5, 6\}\}$.
If the answer is 'no', what will be the minimum number of sets that you can add to make $\mathcal{F}$ to be a sigma algebra (also write down the $\mathcal{F}$ after adding these sets).

(8) A real valued random variable X is called measurable with respect to $(\Omega, \mathcal{F}, \mathcal{P})$, if $X^{-1}((a, b)) \in \mathcal{F}$, where $X^{-1}((a, b))\{\omega \in \Omega | a < X(\omega) < b\}$. With $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{F} = \{\Omega, \emptyset, \{1\}, \{2, 3, 4, 5, 6\}\}$, define two random variables X and Y such that X is measurable and Y is not measurable.