CS6046: Multi-Armed Bandits
Worksheet - 3

(1) Let X_1 and X_2 be σ_1 and σ_2 subgaussian respectively. Find an upper bound for $\mathbb{P}(X_1 \geq \epsilon_1, X_2 \geq \epsilon_2)$ when (i) X_1 and X_2 are independent, and (ii) X_1 and X_2 are not independent.

(2) Consider a random variable X such that $\mathbb{P}(X = -1) = 1/2$ and $\mathbb{P}(X = +1) = 1/2$. What is the sub-gaussian constant of this random variable.

(3) Consider two random variables X_1 and X_2 with sub-gaussianity constants σ_1 and σ_2. Let us toss a fair coin and let Y be a random variable defined as follows:

$$Y = \left\{ \begin{array}{ll} X_1, & \text{if coin toss is head} \\ X_2, & \text{if coin toss is tail} \end{array} \right\} \tag{1}$$

How will you attempt to bound $\mathbb{P}(Y \geq \epsilon)$?

(4) Let $\{X_1(t)\}$ be a sequence of i.i.d random variable such that $\mathbb{P}(X_1(t) = -1) = 1/2$ and $\mathbb{P}(X_1(t) = +1) = 1/2$ and $\{X_2(t)\}$ be a sequence of i.i.d random variable such that $\mathbb{P}(X_2(t) = -2) = 1/2$ and $\mathbb{P}(X_2(t) = +2) = 1/2$. Let $C(t)$ be a i.i.d sequence of fair coin tosses. Let $Y(t)$ be defined as

$$Y(t) = \left\{ \begin{array}{ll} \frac{X_1(t)}{X_2(t)}, & \text{if } C(t) \text{ toss is head} \\ \frac{X_2(t)}{X_1(t)}, & \text{if } C(t) \text{ toss is tail} \end{array} \right\} \tag{2}$$

What is $\mu = \mathbb{E}Y(t)$? and Let $\hat{\mu}(t) = \frac{\sum_{s=1}^{t} Y(t)}{t}$. How will you upper bound $\mathbb{P}(|\mu(t) - \mu| \geq \epsilon)$?

(5) Consider a bandit problem with k arms. Derive an upper bound for the regret for the UCB algorithm which plays the arm

$$A_t = \arg \max_{i=1,\ldots,k} \hat{\mu}_i(t-1) + \sqrt{\frac{6 \log(t)}{T_i(t-1)}}$$

(6) Consider a bandit problem with two arms with true means μ_1 and μ_2. We know that $\mu_1 > 0$ and $\mu_2 > 0$ and that $\mu_1 + \mu_2 = 1$. How will you modify the UCB algorithm? If possible provide an upper bound for the regret in terms of the gap $\Delta = |\mu_1 - \mu_2|$.