CS6046: Multi-Armed Bandits
Worksheet - 2

For questions (1), (2) and (3) let $\Omega = [0, 1]$, and $\mathcal{F}$ be the Borel sigma algebra formed by the open sets, and let $\mathbb{P}((a, b)) = b - a, 0 < a < b < 1$.

(1) Let the random variables X_n be defined as, $X_n(\omega) = \omega^n, \omega \in [0, 1]$. Calculate the following

(i) Find the distribution function $F_{X_n}(x) = \text{Prob}(X_n < x)$. Try to sketch the same.

(ii) Find the density function $f_{X_n}(x) = \frac{dF_{X_n}(x)}{dx}$. Try to sketch the same.

(2) Define random variables X_1 and X_2 which are independent of each other, such that

(i) $\text{Prob}(X_i = -1) = 1/4$, $\text{Prob}(X_i = +1) = 1/4$ and $\text{Prob}(X_i = +2) = 1/2$

(ii) Find the distribution function $F_{X_i}(x)$.

(iii) Find $\mathbb{P}(\{\omega : X_1(\omega) = X_2(\omega)\})$, $\mathbb{P}(\{\omega : X_1(\omega) \neq X_2(\omega)\})$, $\mathbb{P}(\{\omega : X_1(\omega) \geq X_2(\omega)\})$.

(3) Let $X_n(\omega) = n^2\omega$ if $0 < \omega < \frac{1}{n}$, and $X_n(\omega) = 0$ otherwise. Find out if the sequence $\{X_n\}_{n \geq 0}$ converges in almost surely, probability, mean squared or distribution.

(4) Let X be a random variable uniformly distributed between $[0, 1]$. Let $Z = 3X + 2$ be a random variable. Find the distribution $F_Z(z)$, and density $f_Z(z) = \frac{dF_Z(z)}{dz}$

(5) Let X be a random variable uniformly distributed between $[0, 1]$. Let $Z = X^2$ be a random variable. Find the distribution $F_Z(z)$, and density $f_Z(z) = \frac{dF_Z(z)}{dz}$

(6) Let X be a random variable uniformly distributed between $[0, 1]$. Let $Z = e^{-X}$ be a random variable. Find the distribution $F_Z(z)$, and density $f_Z(z) = \frac{dF_Z(z)}{dz}$