Project 3

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Problem 6: Hidden Markov Models

(a) What is the maximum number of parameters to define the HMM?

Taking into account that,

1. each hidden variable Z can take on K different values,
2. each observed variable X can each take on M different values,
3. and that the HMM has L different states,

we have the following parameters in the model:

- Initial state probabilities: $I_k = P(Z_1 = k)$
- Transition probabilities: $T_{kk'} = P(Z_n = k' \mid Z_{n-1} = k)$
- Emission probabilities: $E_{km} = P(X_n = m \mid Z_n = k)$

Since the $\sum_k I_k = 1$ we only need to define $K - 1$ initial state probabilities.

Given that $Z_{n-1} = k$, there are K transition probabilities into state Z_n, one for each value that Z_n can take. Since $\sum_{k'=1}^{K} P(Z_n = k' \mid Z_{n-1} = k) = 1$ we only need to define $K - 1$ transition probabilities for each possible imputation of $Z_{n-1} = k$. Hence we need to define $K * (K - 1)$ transition probabilities per (Z_n, Z_{n-1}) pair. Because we have $L - 1$ such pairs we then need overall $K * (K - 1) * (L - 1)$ transition probabilities.

Similarly because $\sum_{m=1}^{M} P(X_n = m \mid Z_n = k) = 1$, and Z_n can take on K different values, we need to define overall $K * (M - 1)$ emission probabilities per state. Because we L of these we therefore need to define $K * (M - 1) * L$ emission probabilities overall.

In sum we need to define $(K - 1) + K(K - 1)(L - 1) + K(M - 1)L$ parameters. This is equal to $K^2(L - 1) + K(2 - 2L + ML) - 1$ parameters.