Project 1. Solution

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Problem 1: Conditional independence and BNs

a)

The condition $A \perp B|C$ holds for this Bayesian network a). Proof:

$$P(A,B,C) = P(A|C)P(B|C)P(C)$$

Consider

$$\begin{aligned} P(A,B|C) &= \frac{P(A,B,C)}{P(C)} \\ &= P(A|C)P(B|C) \\ &\implies A \perp B|C \end{aligned}$$

Hence Proved.

b)

The condition $A \perp B$ holds for this Bayesian network b). Proof:

$$P(A,B,C) = P(A)P(B)P(C|A,B)$$

$$P(C|A,B) = \frac{P(A,B,C)}{P(A,B)}$$

Substituting

$$P(A,B) = P(A)P(B)$$

$$\implies A \perp B$$

Hence Proved.

Problem 2: Markov blanket

Problem 3: Learning Bayesian networks from protein data

a)

```
## Set seed
set.seed(2022)

## Read and index data
data <- vroom(file = "sachs.data.txt", show_col_types = FALSE)
data = data %>% rowid_to_column(var = 'row_id')

## Visualize data dimensions.
dim(data)
```

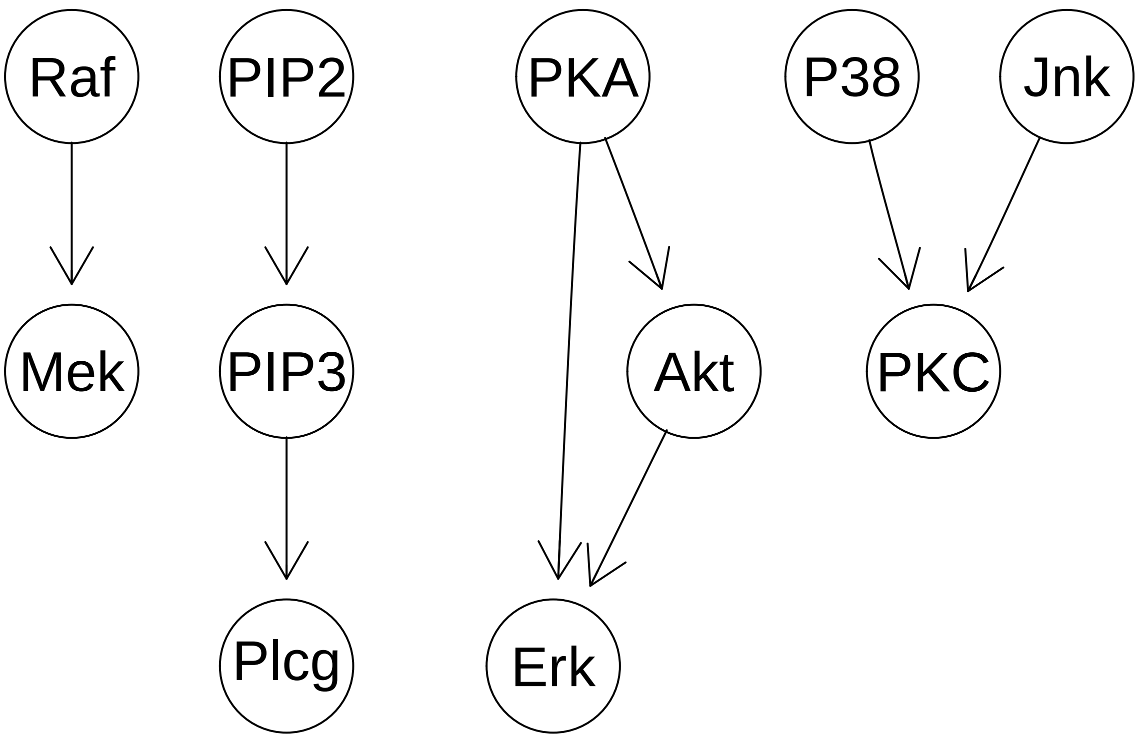
```
## [1] 853  12
```

- $N = 853$
- $n = 11$

```r
## Split data
train = data %>% sample_frac(0.8, replace = FALSE)
test = anti_join(data, train, by = "row_id")

## Remove index column used for splitting
train = train %>% select(-row_id)
test = test %>% select(-row_id)

## Initializing score objects
score_object_train = scoreparameters(scoretype = 'bge', train)
score_object_test = scoreparameters(scoretype = 'bge', test)
```

b)

```r
## Infer network on train data
infered_network = orderMCMC(score_object_train)

## Construct graph object with the inferred network
graph_object = graphAM(adjMat = infered_network$DAG,
                       edgemode='directed',
                       values=NA)
## Plot graph
plot(graph_object)
```




```r
## Evaluate the log BGe score of the test data against the estimated DAG.
DAGscore(score_object_test, infered_network$DAG)
```

```
## [1] -9456.304
```

c)

```r
## Define function for average BGe score and average number of edges
avg_BGe_score_and_n_edges <- function(am) {

  ## Have to load libraries inside the function otherwise it can not find other
  ## functions even if they are alredy loaded.
  library(BiDAG)
  library(dplyr)

  ## Init objects to keep track of BGe score and number of edges
  n_edges = c()
  BGe_scores = c()

  for (i in 1:10) {
    ## Split data
    train = data %>% sample_frac(0.8, replace = FALSE)
    test = anti_join(data, train, by = "row_id")

    ## Remove id column used for splitting
    train = train %>% select(-row_id)
    test = test %>% select(-row_id)

    ## Initializing score objects
    score_object_train = scoreparameters(scoretype = 'bge',
                                         train,
                                         bgepar = list(am = am, aw = NULL))
    score_object_test = scoreparameters(scoretype = 'bge',
                                        test,
                                        bgepar = list(am = am, aw = NULL))

    ## Infer network on train data
    infered_network = orderMCMC(score_object_train)

    ## Append number of edges of the inferred network
    n_edges = c(n_edges, sum(infered_network$DAG))

    ## Append BGe score
    BGe_scores = c(BGe_scores, DAGscore(score_object_test,
                                        infered_network$DAG))
  }

  ## Return mean number of edges and mean BGe score
  return(list("am_value" = am,
              "mean_n_edges" = mean(n_edges),
              "mean_BGe_score" = mean(BGe_scores)))
}

## Create list of alpha mu values (am values)
am_values = list(10e-5, 10e-3, 10e-1, 10, 10e2)

## Run the each average pair (for each am value) on parallel
```

```r
## You might have to change this if you computer doesn't have 5 cores!!
results = mclapply(X = am_values,
                   FUN = avg_BGe_score_and_n_edges,
                   mc.cores = 5,
                   mc.preschedule = FALSE)

## Print results
do.call(rbind, results)
```

```
##      am_value mean_n_edges mean_BGe_score
## [1,] 1e-04    5.5          -9587.819
## [2,] 0.01     6.4          -9433.594
## [3,] 1        7            -9245.559
## [4,] 10       7.3          -9662.856
## [5,] 1000     17.8         -36812.82
```

```r
## The closer x is to 0 the more negative the log(x) is. Therefore the least neg
## average log(BGe_score) corresponds to the largest BGe score (hence am = 1).

## Plot relearned DAG with whole data set
## Initializing score objects
score_object_data = scoreparameters(scoretype = 'bge',
                                    data %>% select(-row_id),
                                    bgepar = list(am = 1, aw = NULL))

## Infer network on train data
infered_network = orderMCMC(score_object_data)

## Construct graph object with the inferred network
graph_object = graphAM(adjMat = infered_network$DAG,
                       edgemode='directed',
                       values=NA)
## Plot graph
plot(graph_object)
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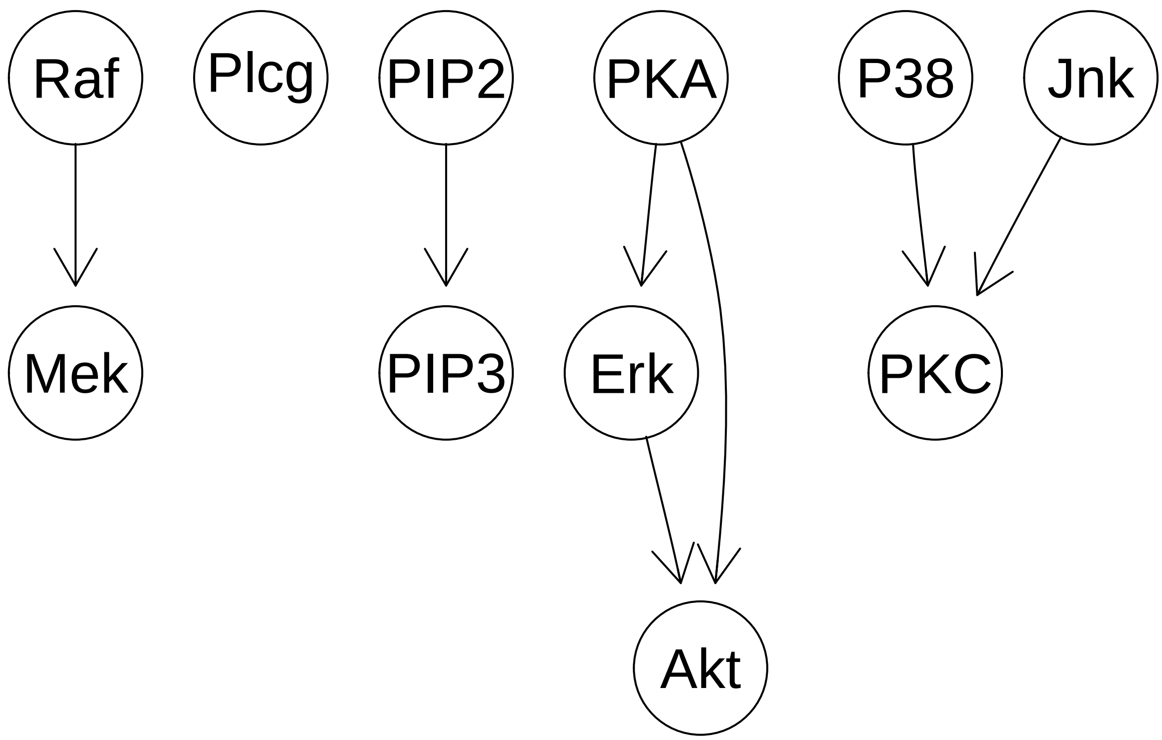
Raf
Plcg
PIP2
PKA
P38
Jnk
Mek
PIP3
Erk
PKC
Akt