# Implicit generative models

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DeepBayes

# Outline

- Adversarial objective discussion
- Training tricks for GANs
- Applications

# Problem formulation

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$G_\theta$ is a mapping with the same codomain as $p(x)$

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$-$Binary cross entropy for classifier
$f$ : probability of data being “real”

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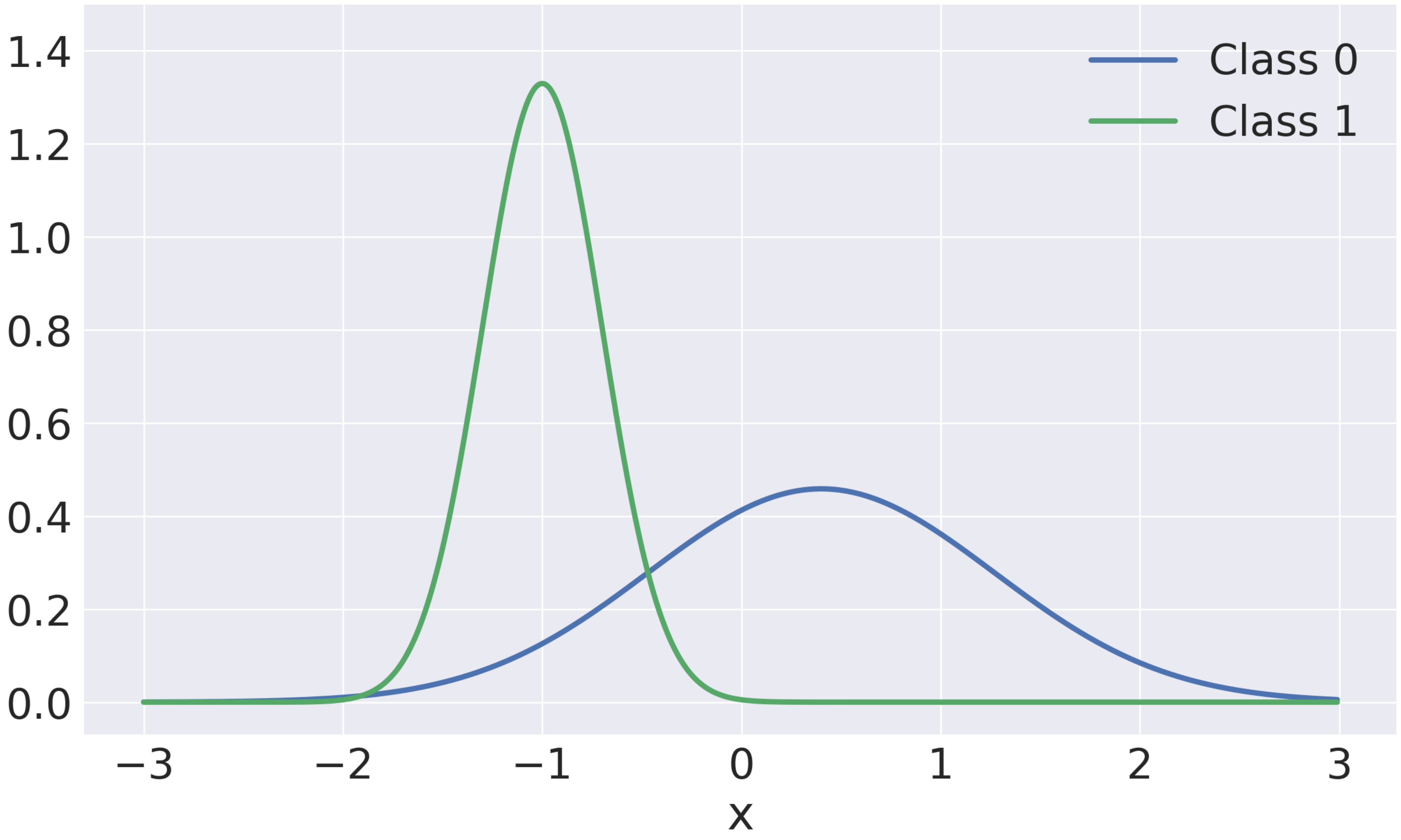
Optimization problem for $G_\theta$ :

$$\boxed{\begin{aligned} &\min_\theta \ \mathcal{D}(P||Q) \\ &\text{s.t. } f^*(x) = \arg\max_f \ \mathcal{D}(P||Q) = \frac{p(x)}{p(x) + q(x)} \end{aligned}}$$

# Example



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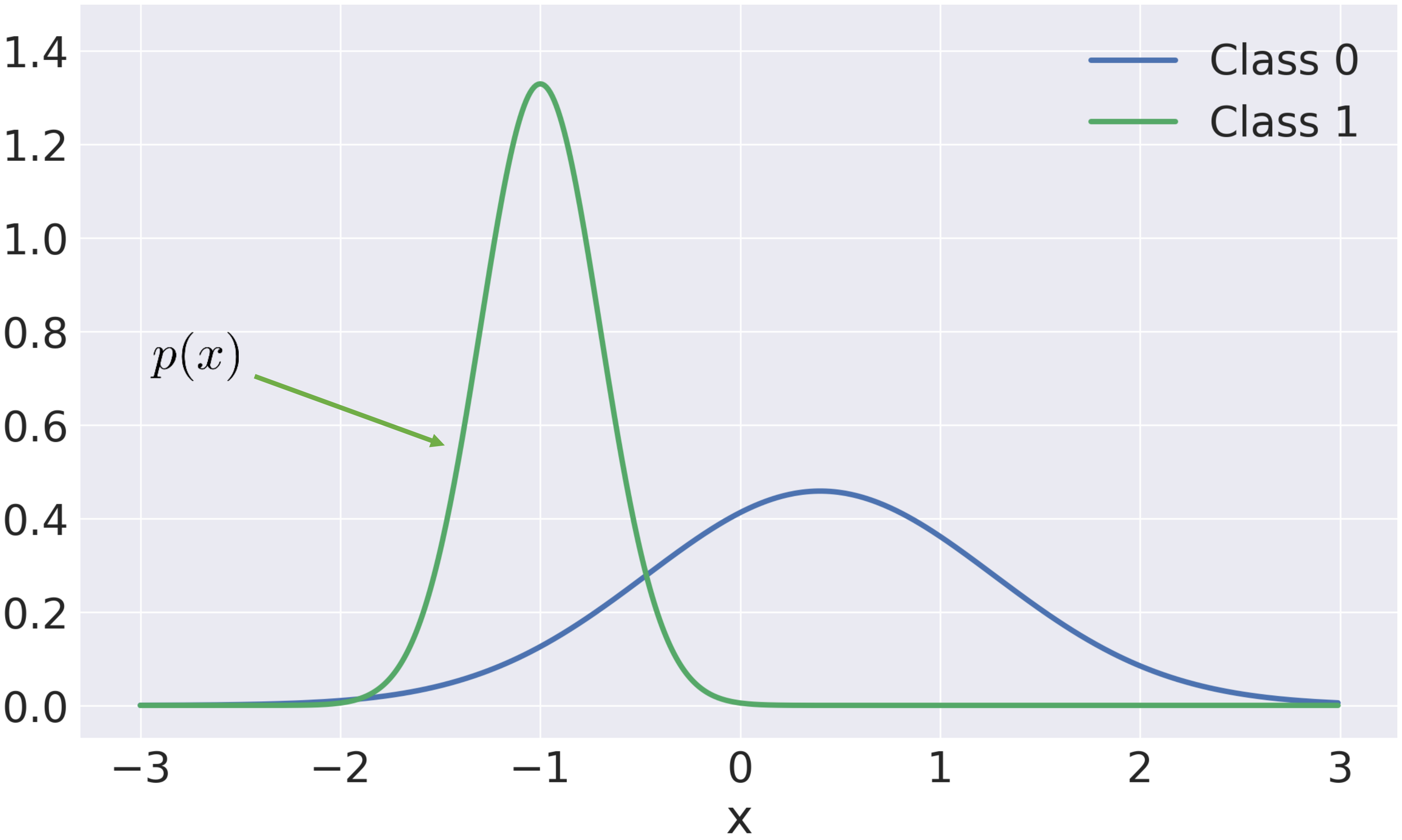

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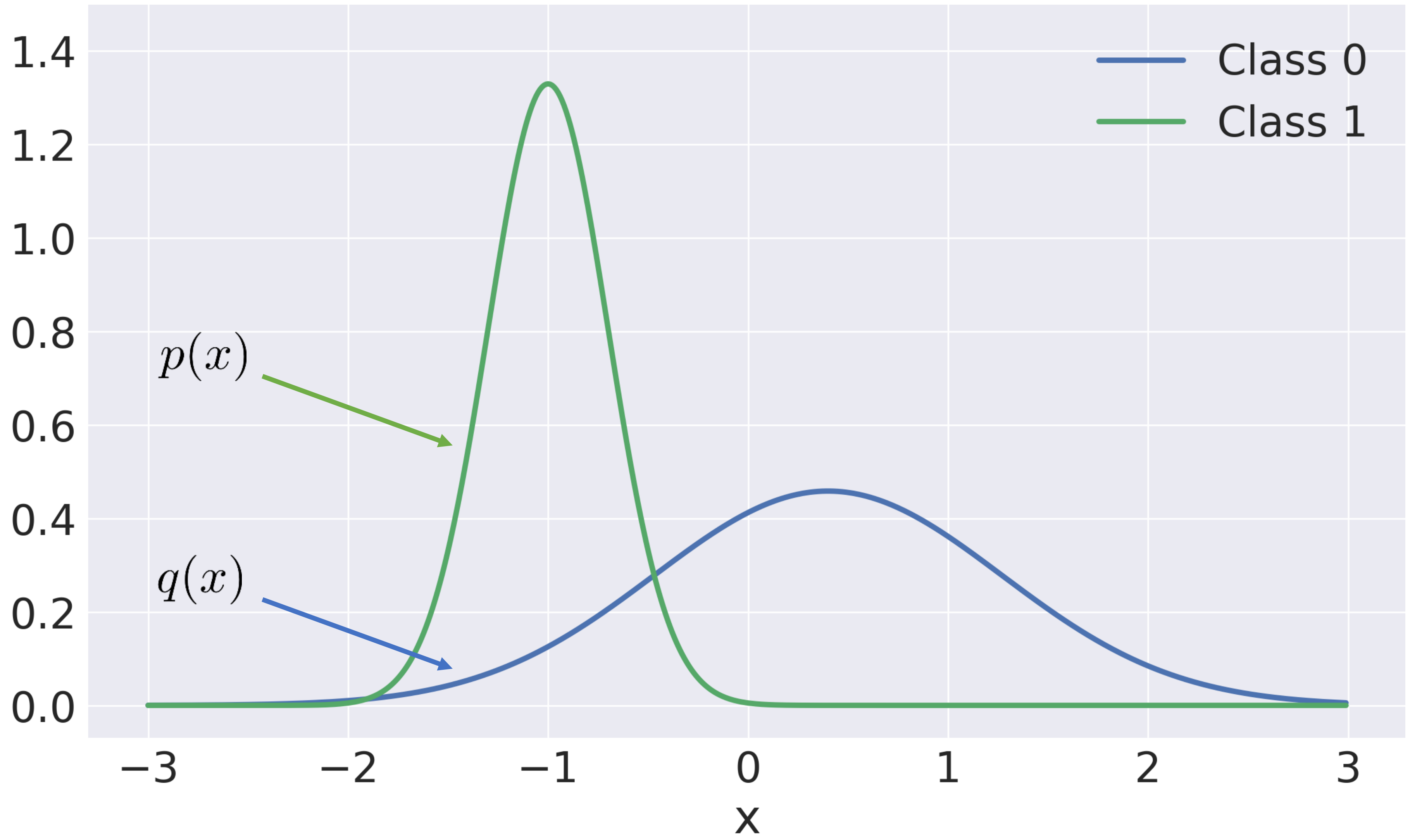

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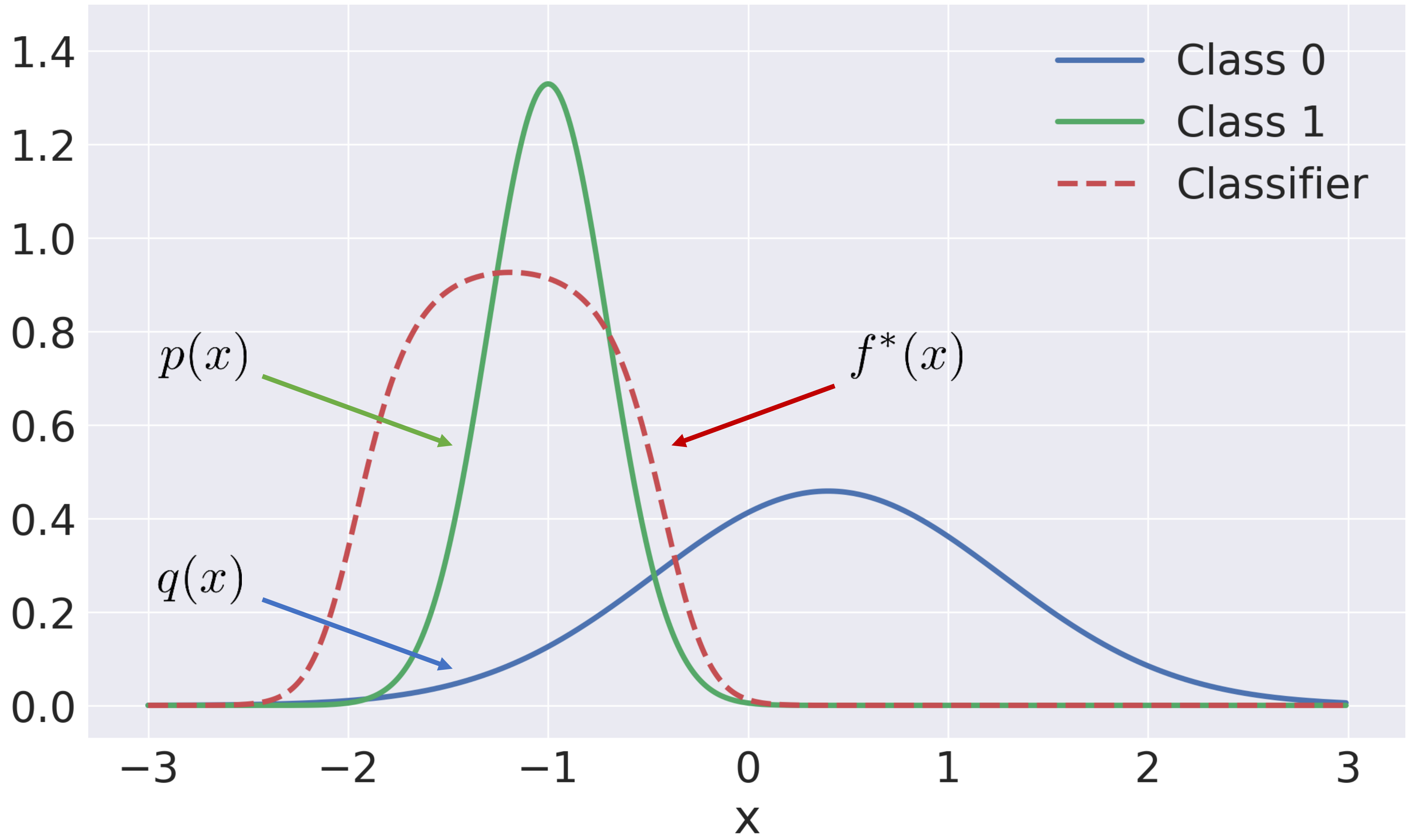

Class 0
Class 1
Classifier
$p(x)$
$f^*(x)$
$q(x)$
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1.0
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0.4
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0.0
−3
−2
−1
0
1
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3
x

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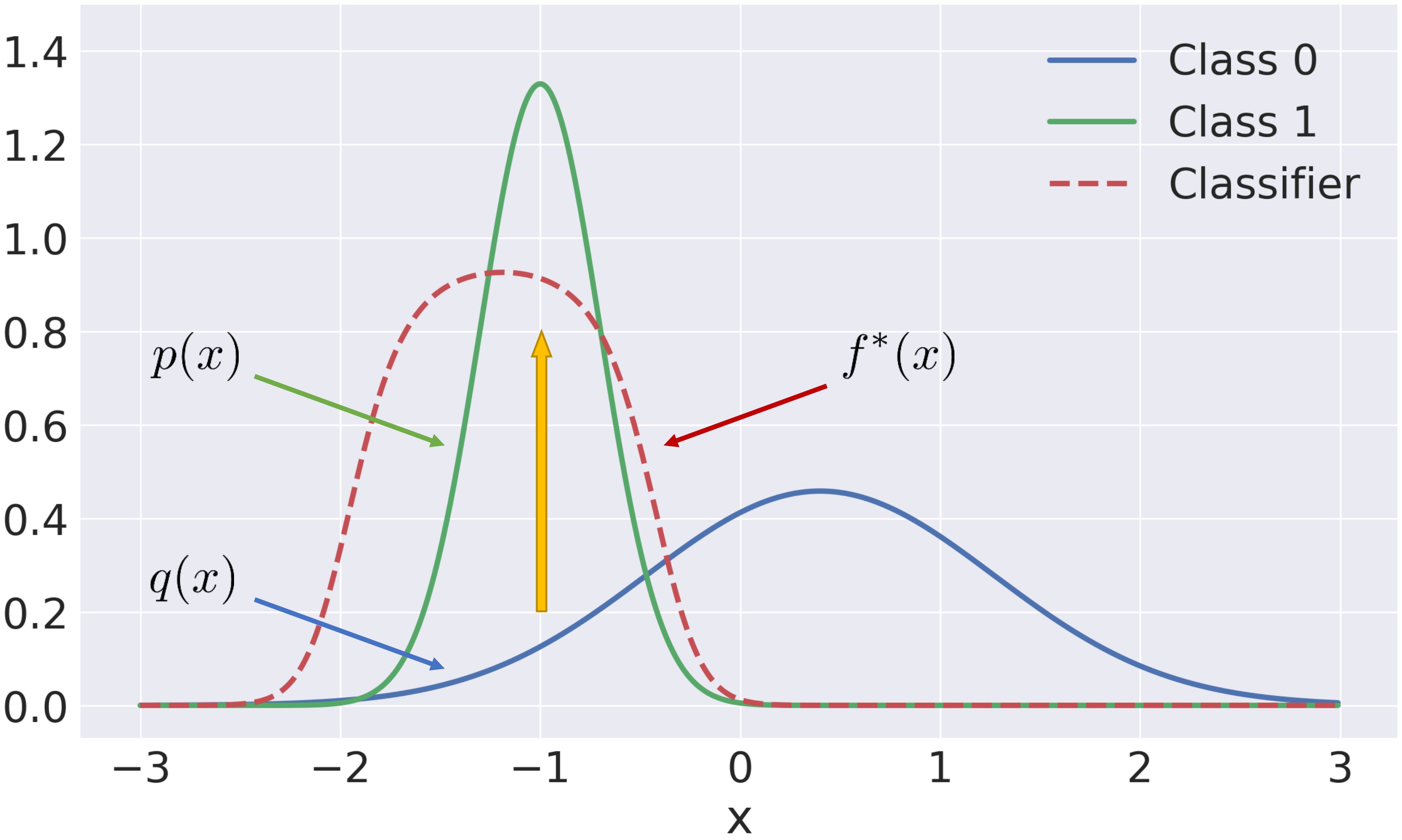

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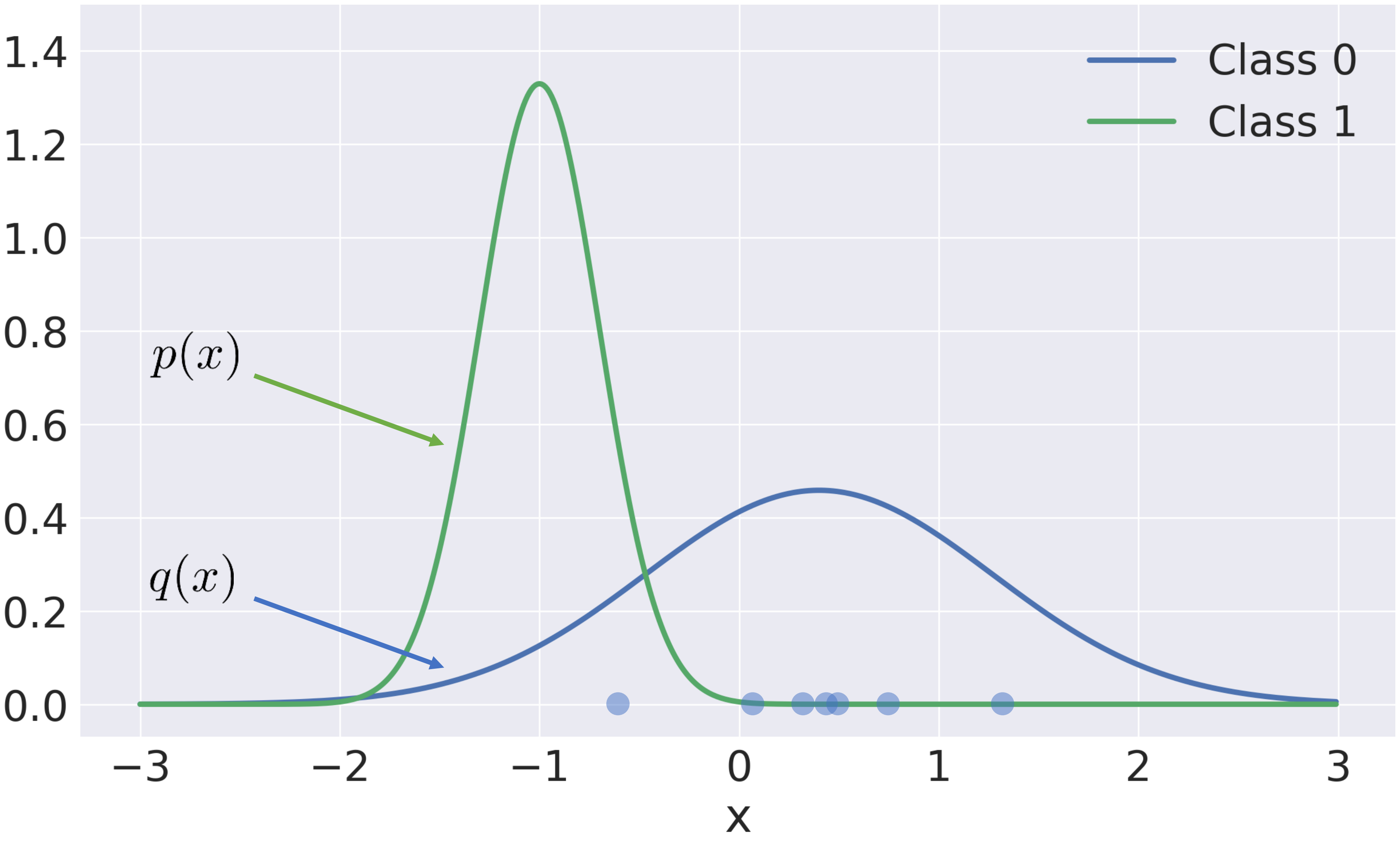

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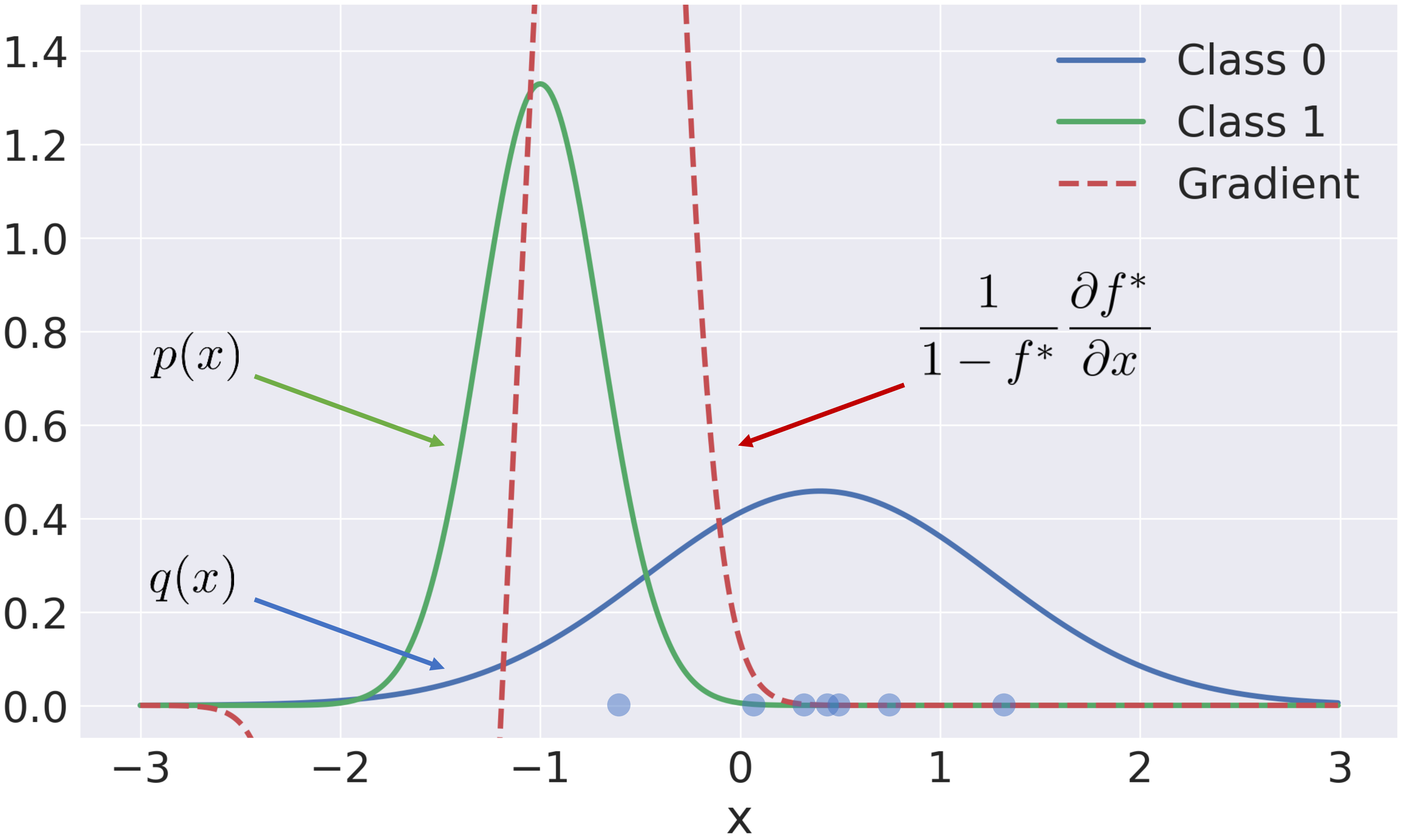

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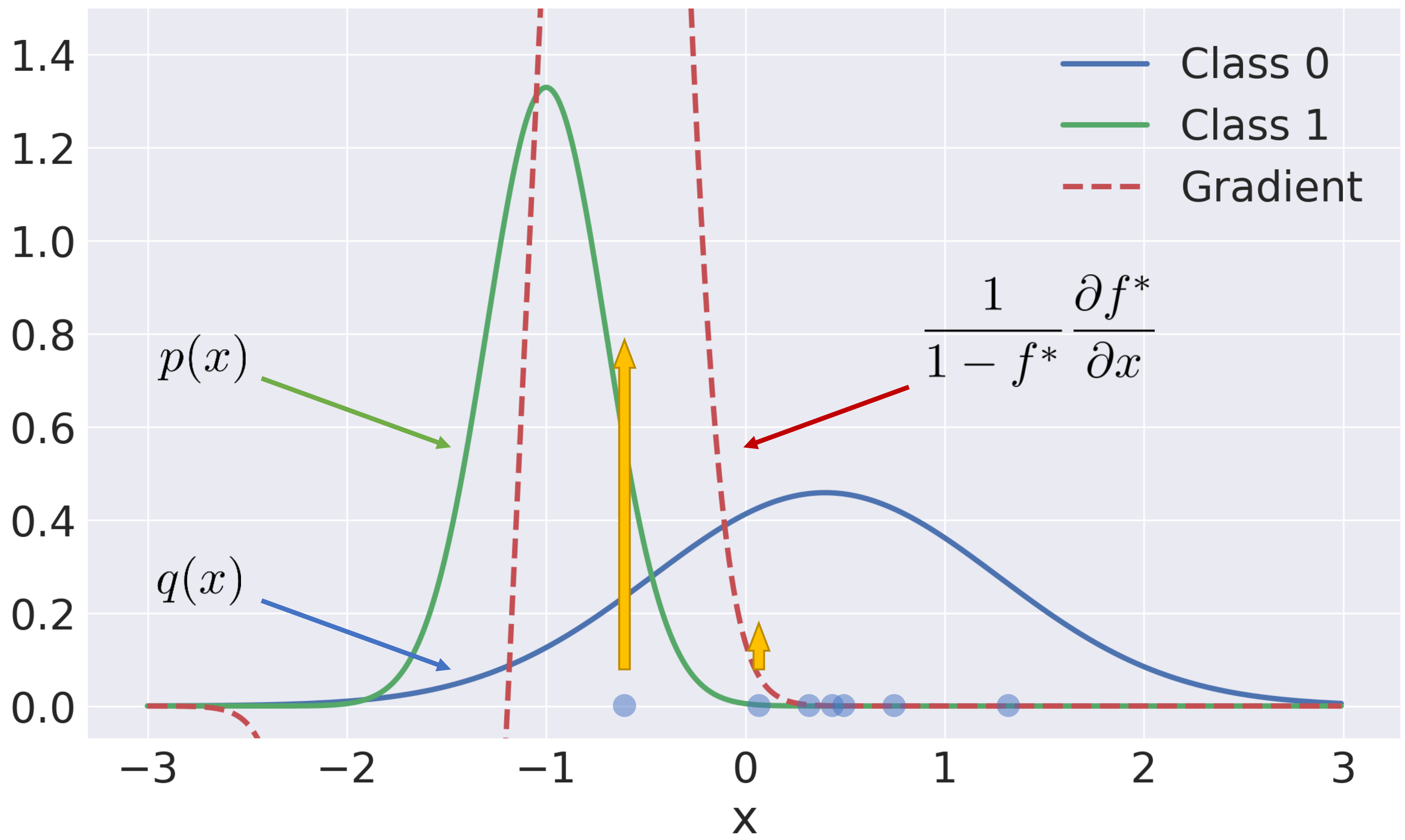
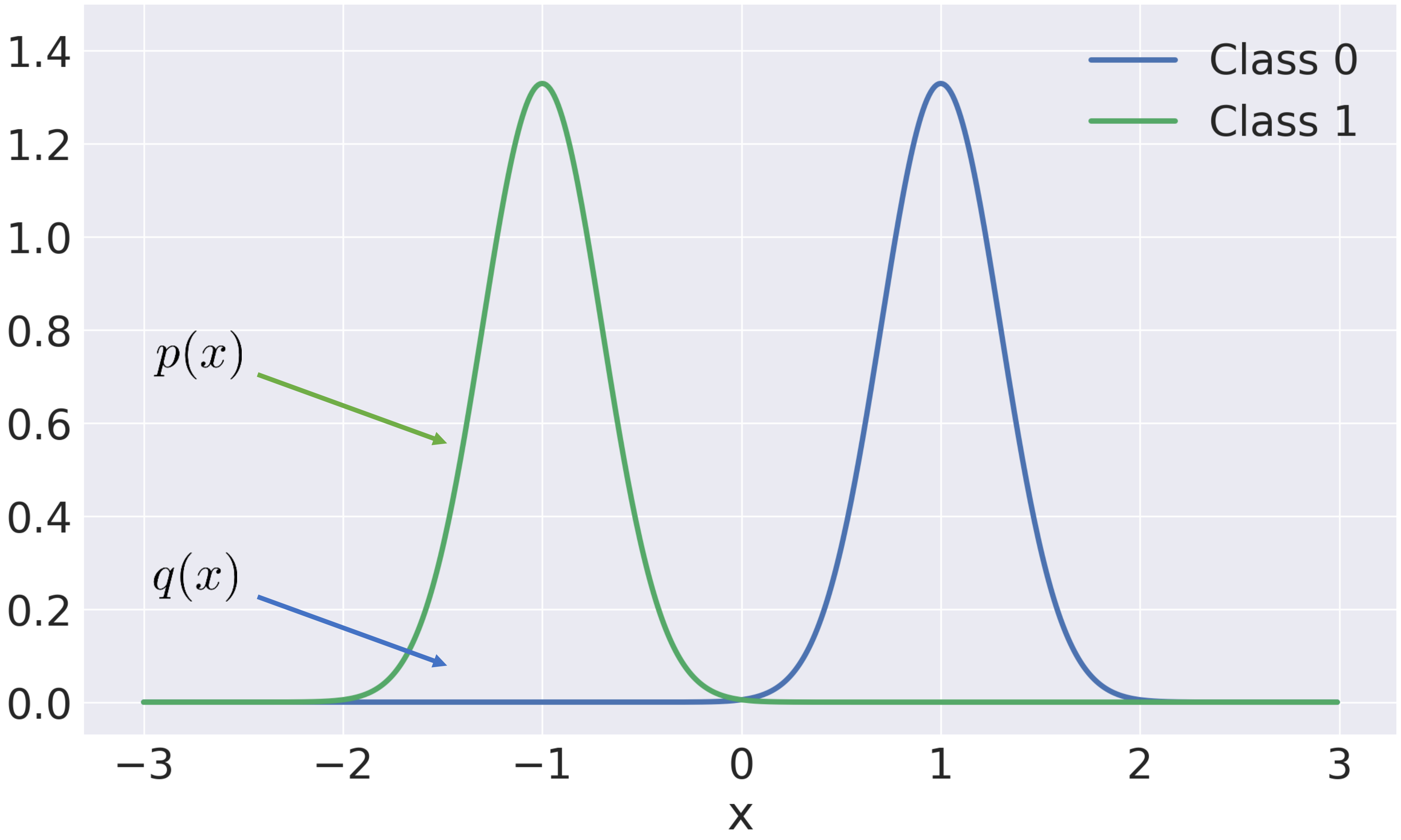


# Failure case



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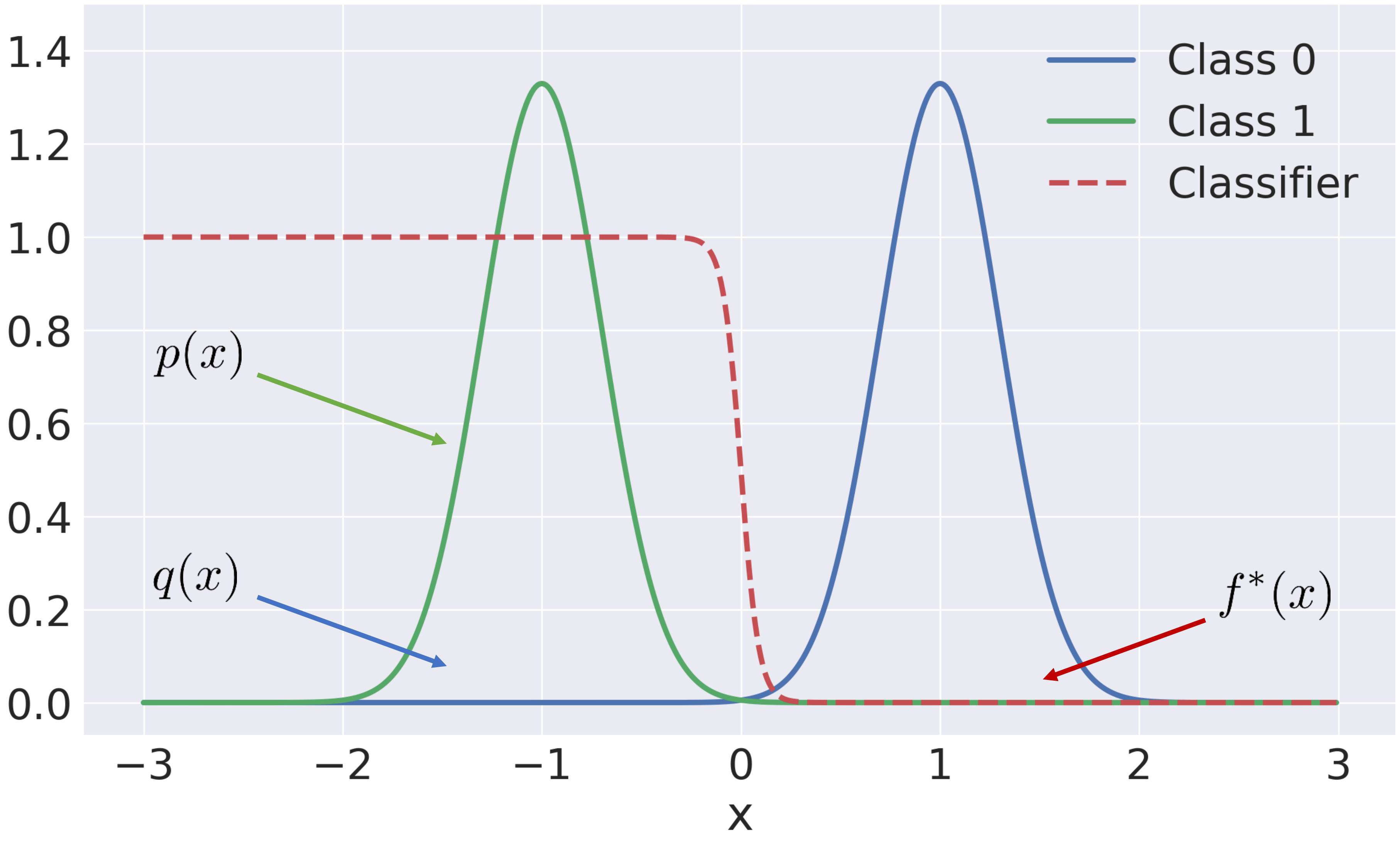

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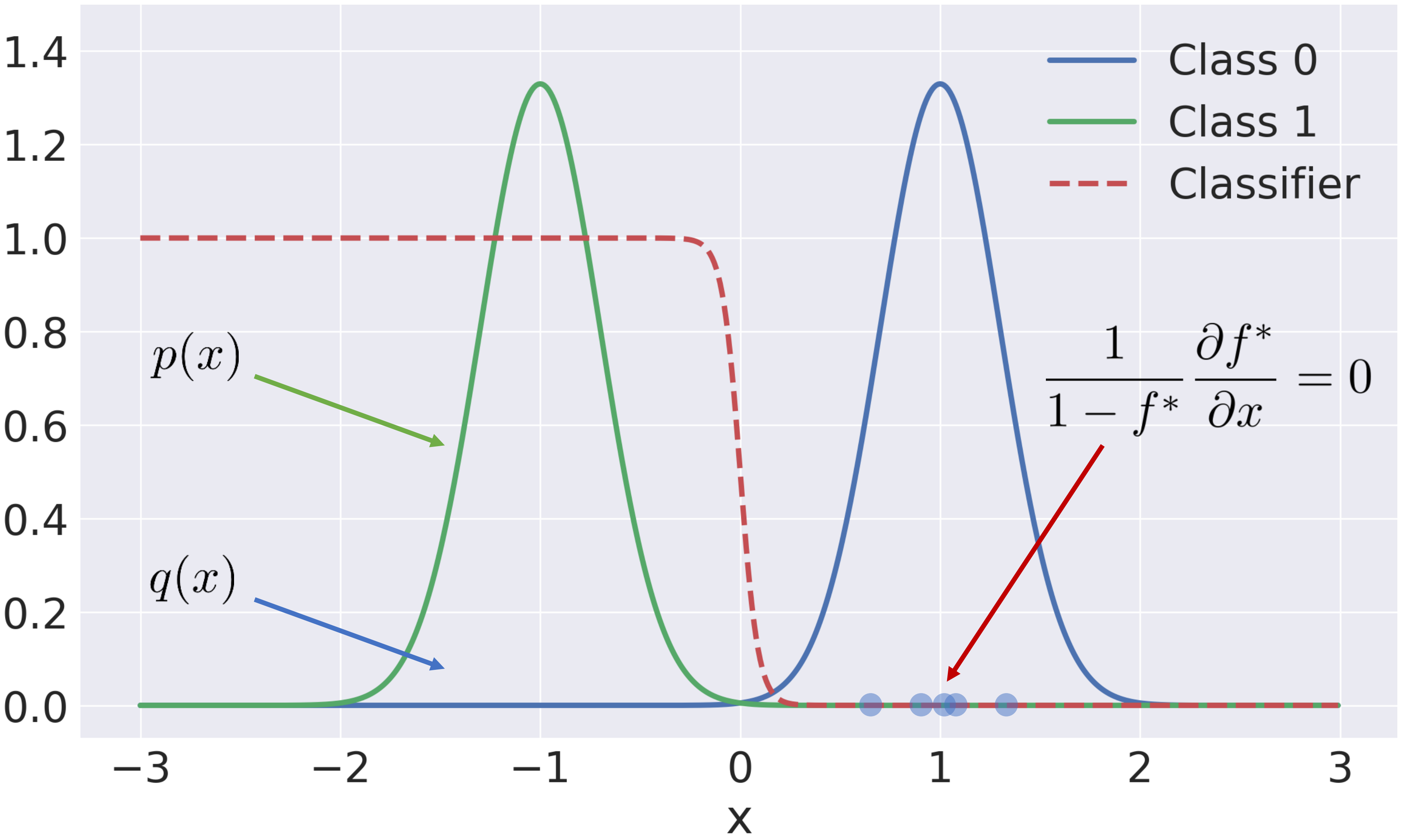

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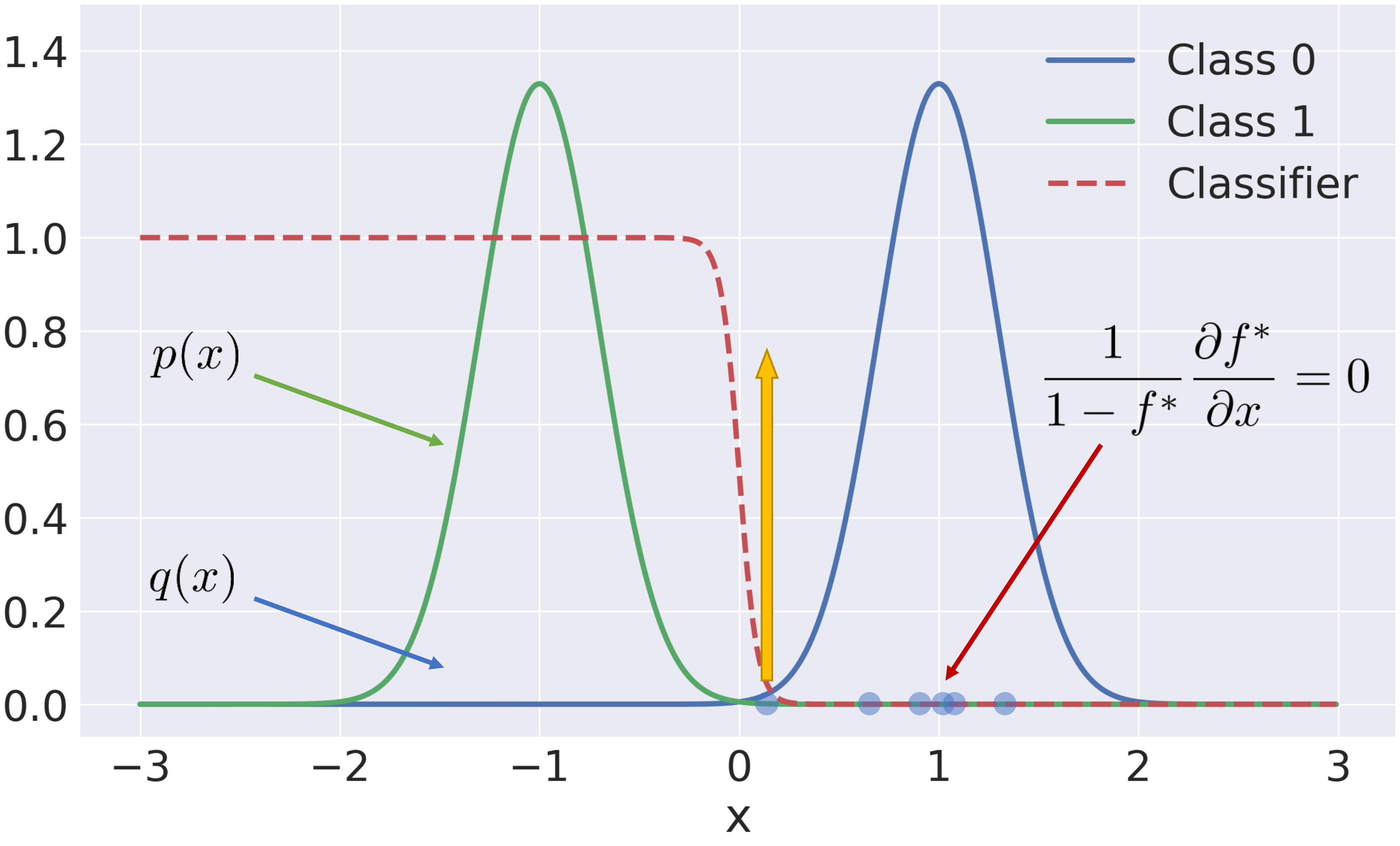

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We will use gradient ascend until convergence

# Example

# Alternating gradients

$$\min_{\theta} \max_{\phi} \mathbb{E}_{x \sim p(x)}[\log D_{\phi}(x)] + \mathbb{E}_{z \sim p(z)}[\log(1 - D_{\phi}(G_{\theta}(z)))]$$

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Minimization of lower bound for divergence

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- GANs are inspired by the optimization of divergence, but we minimize a very loose lower bound to it

- They are optimized using alternating gradient descend, convergence to saddle point is not guaranteed

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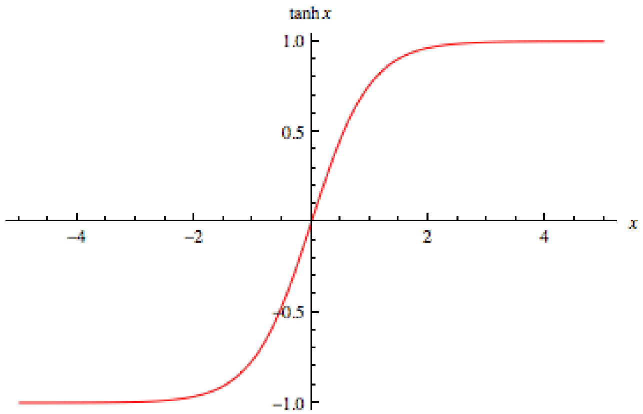

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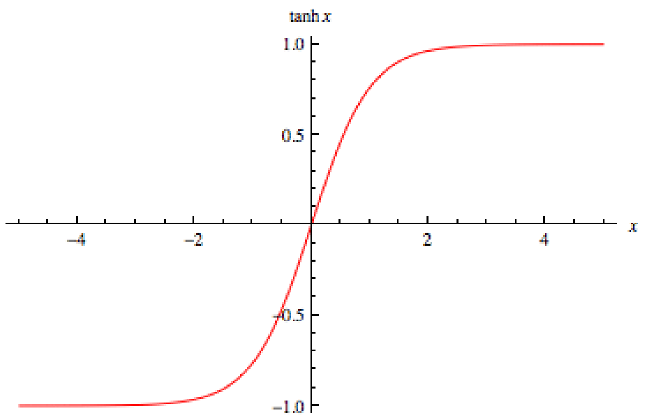


2. Avoid sparse gradients in discriminator

   Replace ReLUs with LeakyReLUs and MaxPool with AvgPool

   Has to do with intrinsically smooth domains (1d signals, images)

# Use prior knowledge

https://distill.pub/2016/deconv-checkerboard/

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Answer: no

Example?

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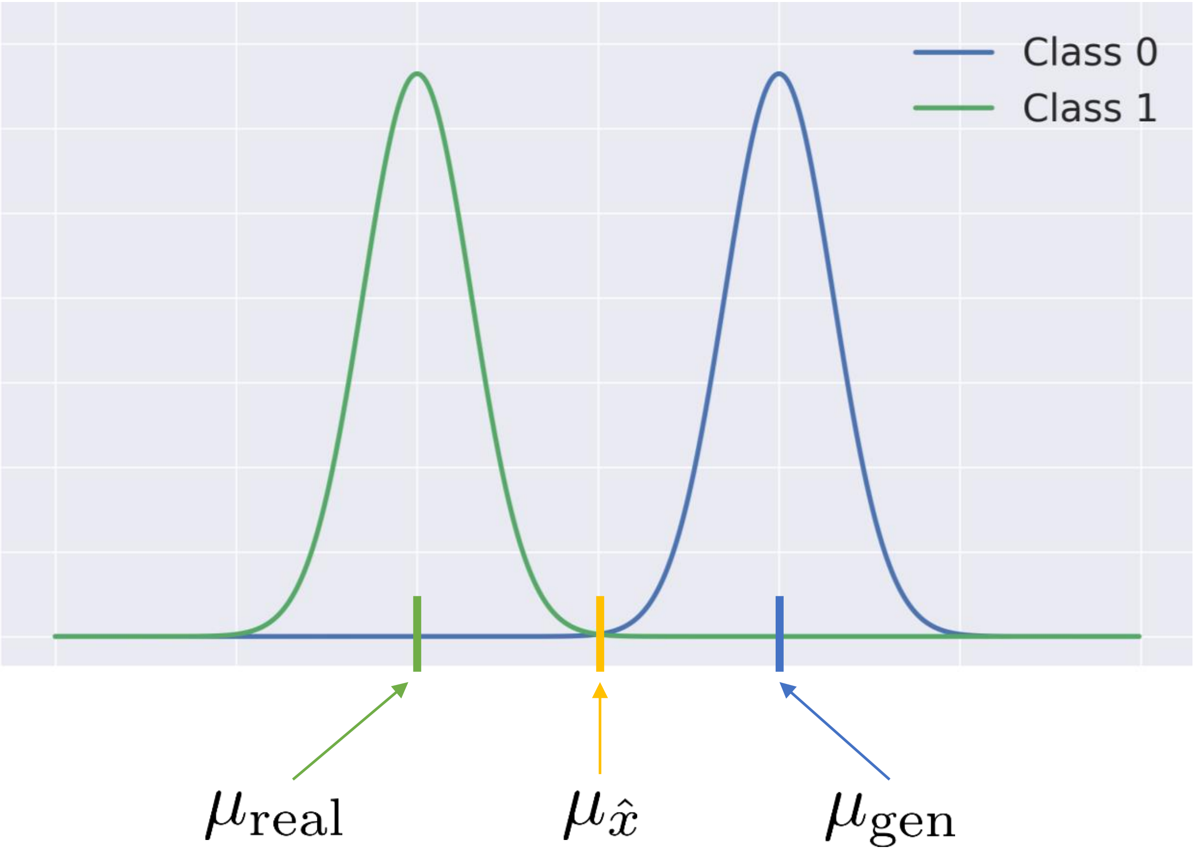
Ex.: BatchNorm

# Merged batches for real and generated data

Distribution of activations for real and generated data



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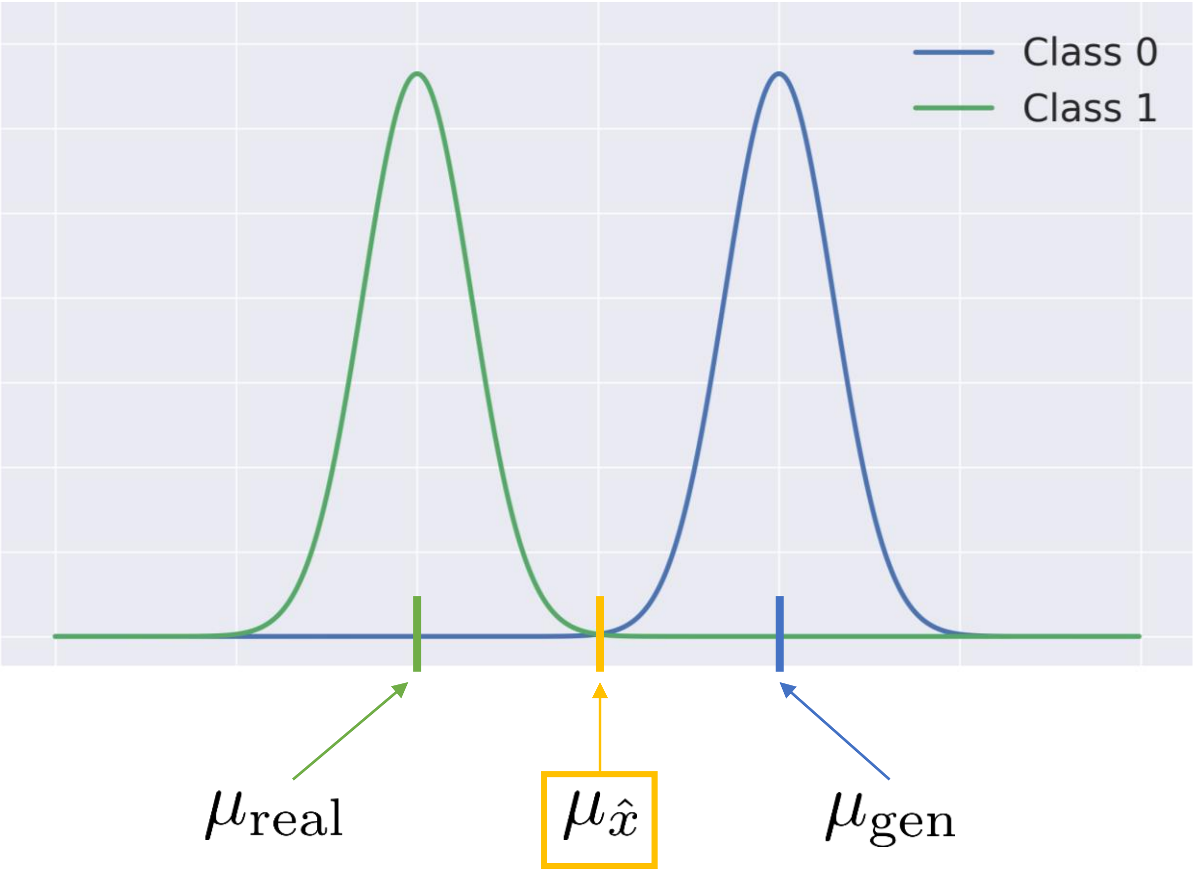
$$\nabla_\phi \mathcal{L}(\theta, \phi) \;=\; \mathbb{E}_{\hat{x},y}\left[\mathbb{I}[y=1]\frac{1}{D_\phi^{\mu_{\hat{x}}}}\frac{\partial D_\phi^{\mu_{\hat{x}}}}{\partial \phi} + \mathbb{I}[y=0]\frac{1}{1-D_\phi^{\mu_{\hat{x}}}}\frac{\partial D_\phi^{\mu_{\hat{x}}}}{\partial \phi}\right]$$



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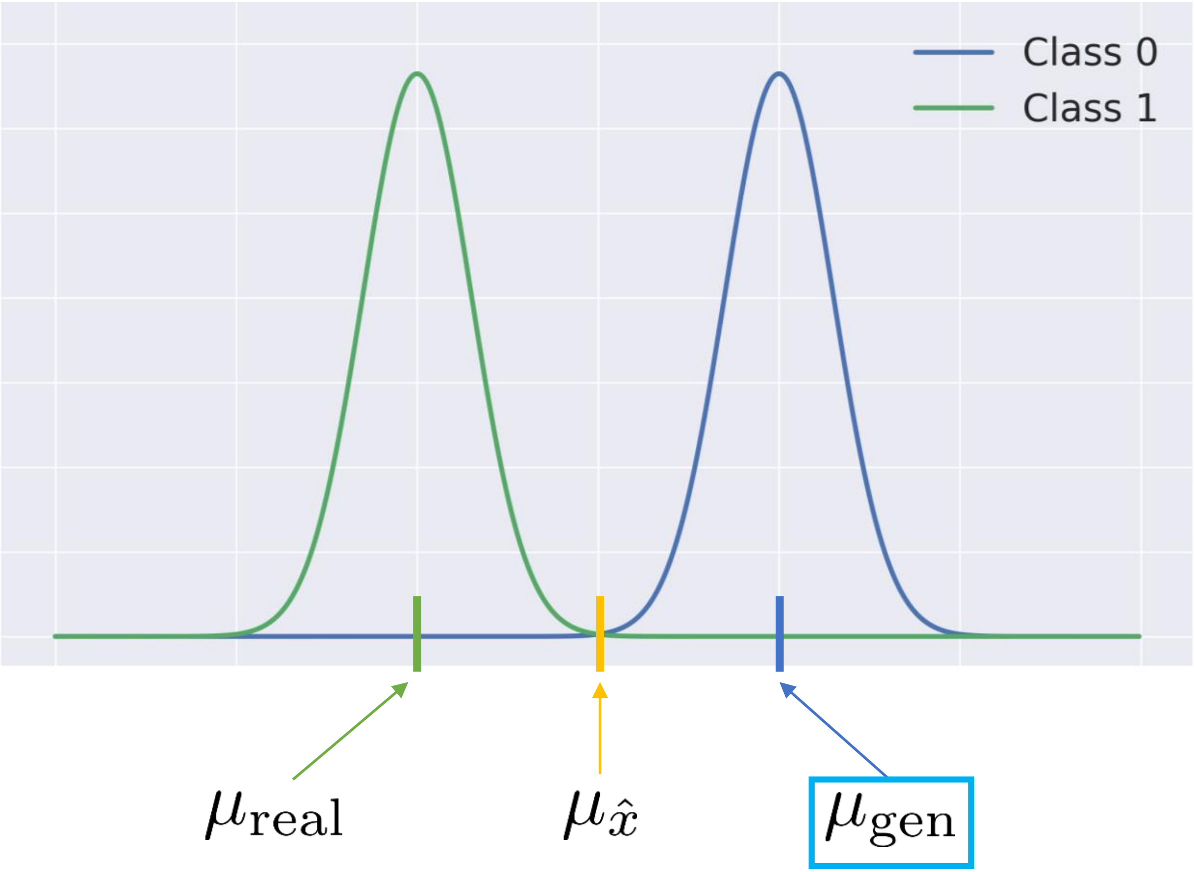
$$\nabla_\theta \mathcal{L}(\theta, \phi) \;=\; \mathbb{E}_z\left[\frac{1}{1 - D_\phi^{\mu_{\text{gen}}}} \frac{\partial D_\phi^{\mu_{\text{gen}}}}{\partial G_\theta} \frac{\partial G_\theta}{\partial \theta}\right]$$



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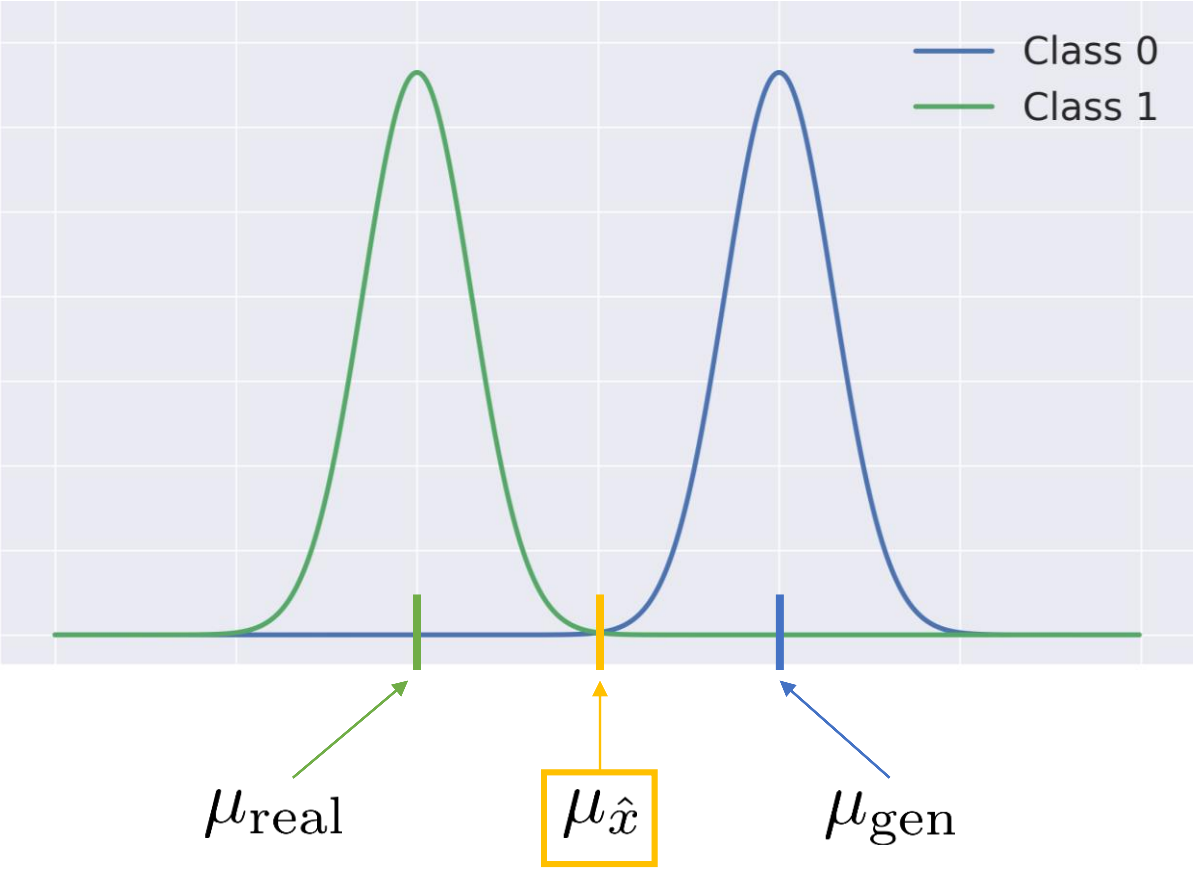


We optimize slightly different objectives during discriminator backward pass

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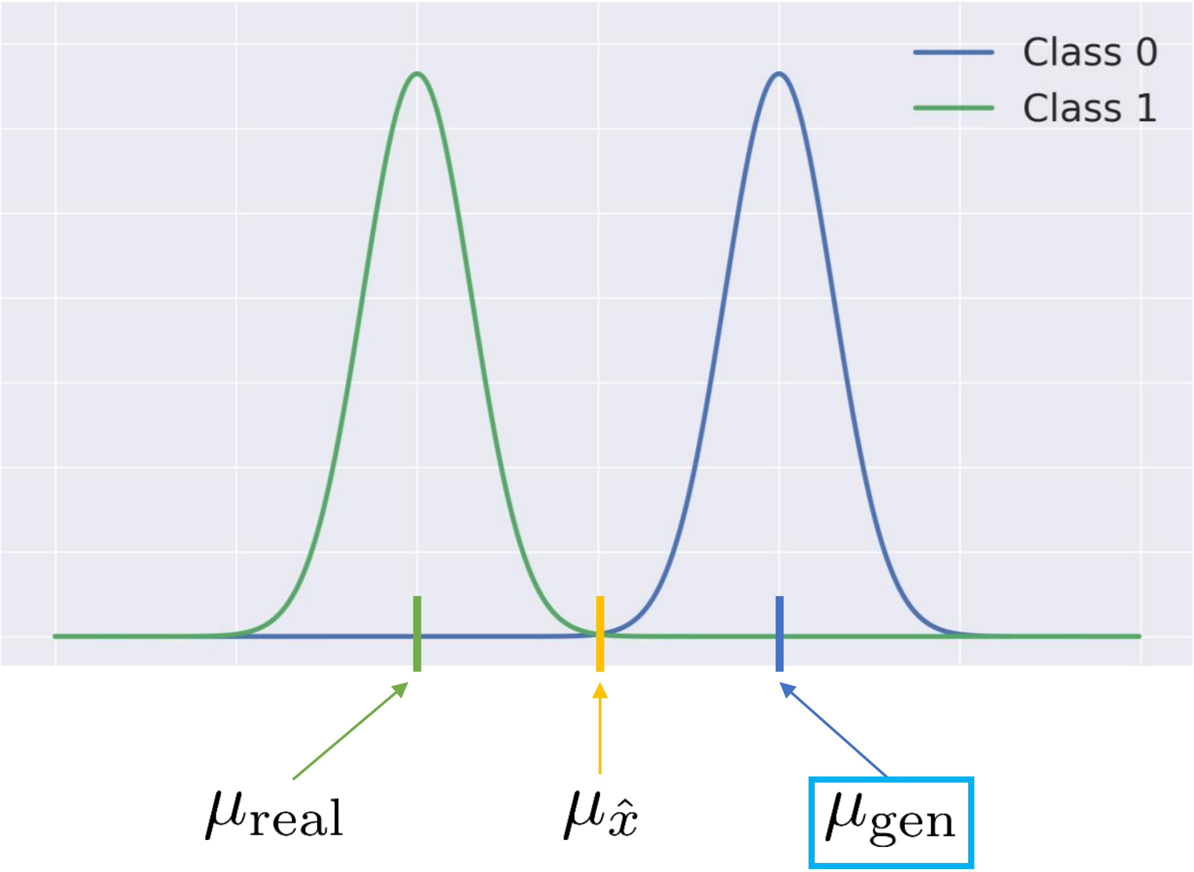


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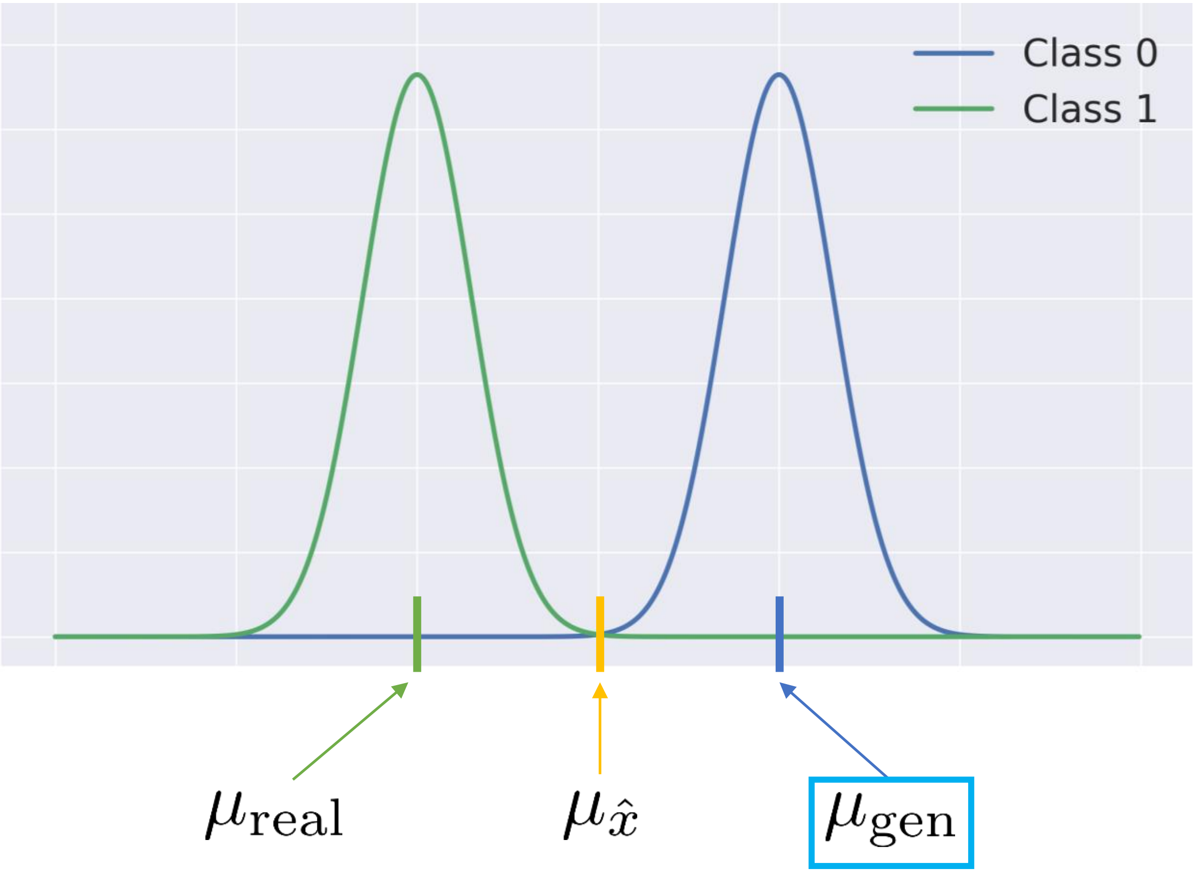


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$$\mathbb{E}(\mu_{\hat{x}} - \mu_{\text{gen}})^2 = (\mathbb{E}[\mu_{\text{gen}} - \mu_{\hat{x}}])^2 + \sigma^2_{\mu_{\text{gen}}} + \sigma^2_{\mu_{\hat{x}}}$$

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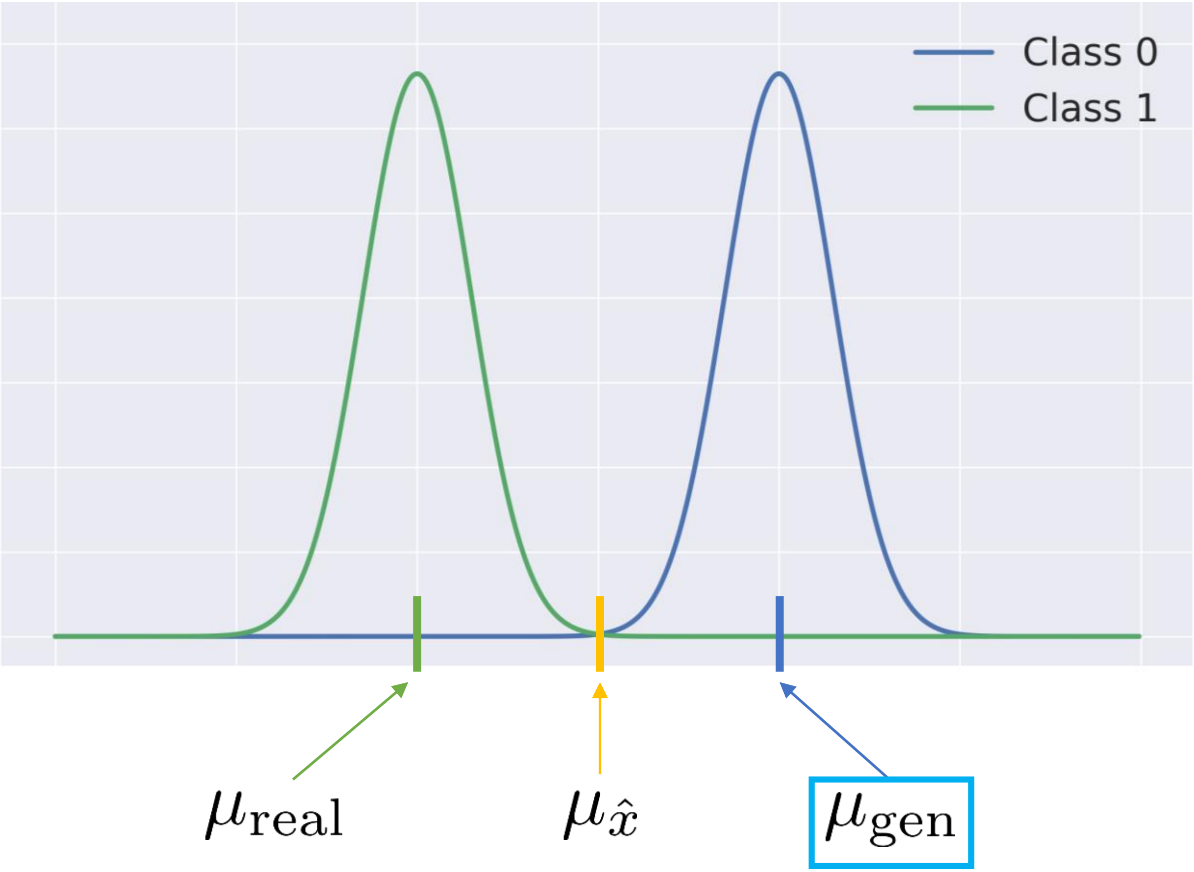


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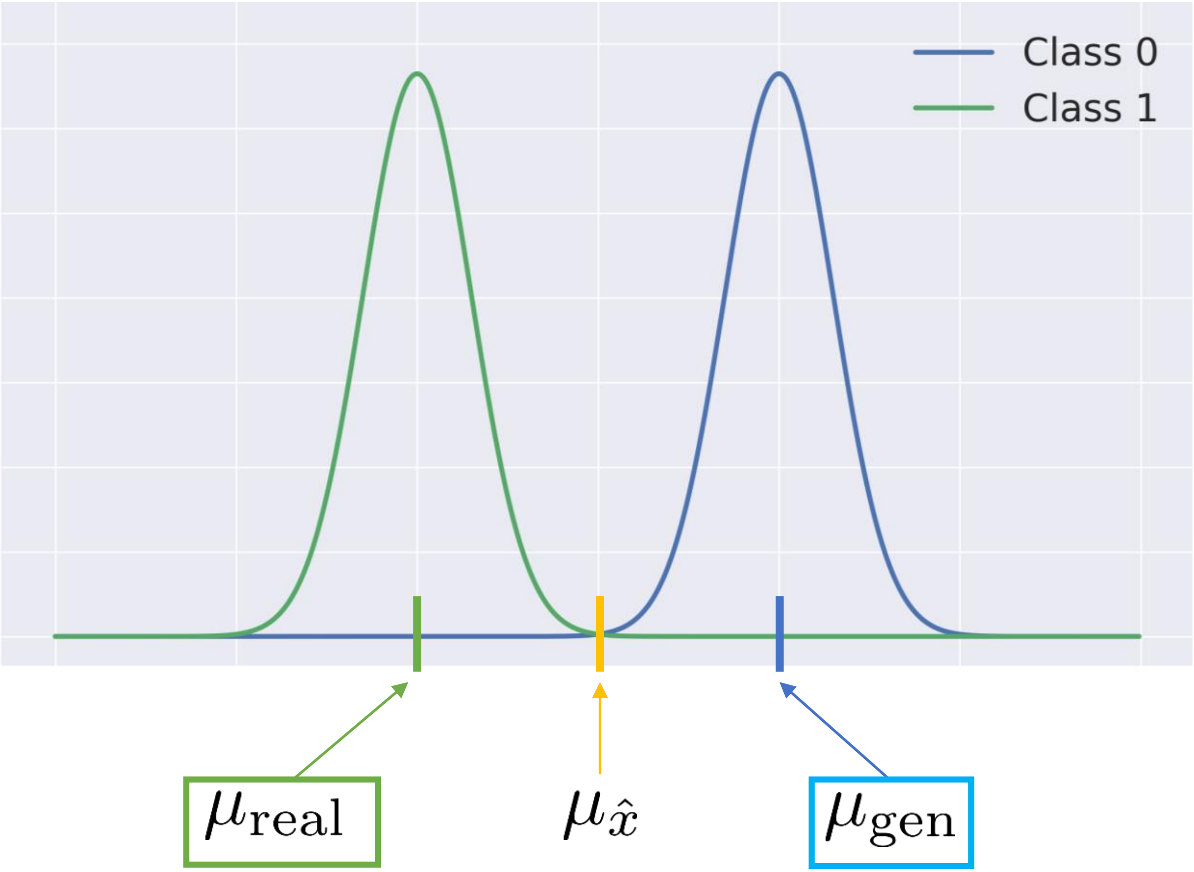
True when bias term is dominating (depends on the objective and architecture!)

# Merged batches for real and generated data

$$\nabla_\phi \mathcal{L}(\theta,\phi) = \mathbb{E}_{x\sim p(x)}\boxed{\left[\frac{1}{D_\phi^{\mu_{\text{real}}}(x)}\frac{\partial D_\phi^{\mu_{\text{real}}}(x)}{\partial \phi}\right]} + \mathbb{E}_{z\sim p(z)}\boxed{\left[\frac{1}{1-D_\phi^{\mu_{\text{gen}}}(G_\theta(z))}\frac{\partial D_\phi^{\mu_{\text{gen}}}(G_\theta(z))}{\partial \phi}\right]}$$




Recommended: separate batches of real and generated data

# GANs as a trainable objective

Another way to apply GANs:

- Pick a well defined problem where regular objectives can be used
- Use GAN as another unsupervised objectiv

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Example: super-resolution

- Given low resolution image, obtain its higher resolution version
- Objective:

$$\min_{\theta} ||G_{\theta}(x_{\mathrm{LR}}) - x_{\mathrm{HR}}||$$

# Super-resolution

Groundtruth

https://adeshpande3.github.io/Deep-Learning-Research-Review-Week-1-Generative-Adversarial-Nets

# Super-resolution

Groundtruth MSE

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# Super-resolution

Groundtruth MSE MSE + GAN

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- Use separate batches for real and fake data when using BatchNorm

- Combine GAN with other supervised objectives

# Applications


StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks

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Problem: given initial data for the particle, produce high dimensional response of the detector

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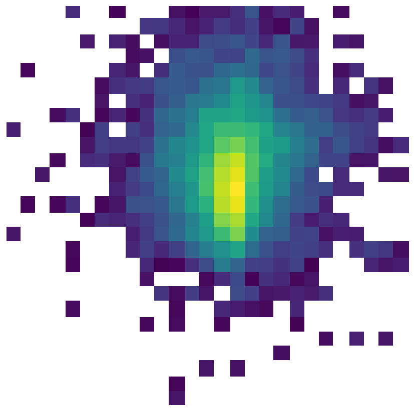
$$p_x, p_y, p_z, \ldots \qquad \xrightarrow{\quad G \quad}$$

$$z - \text{latent variables}$$



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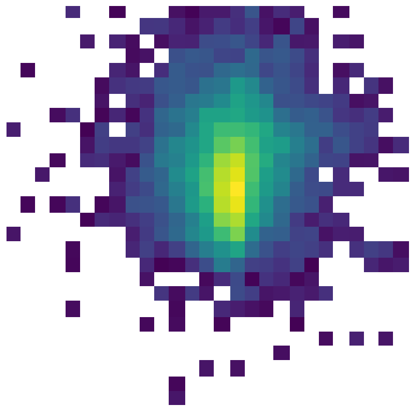

Physical model

- Real physical equations are simulated
- Sampling is slow
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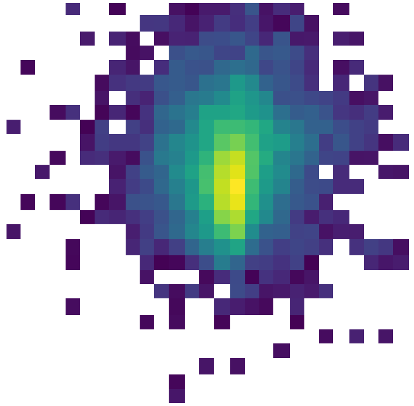
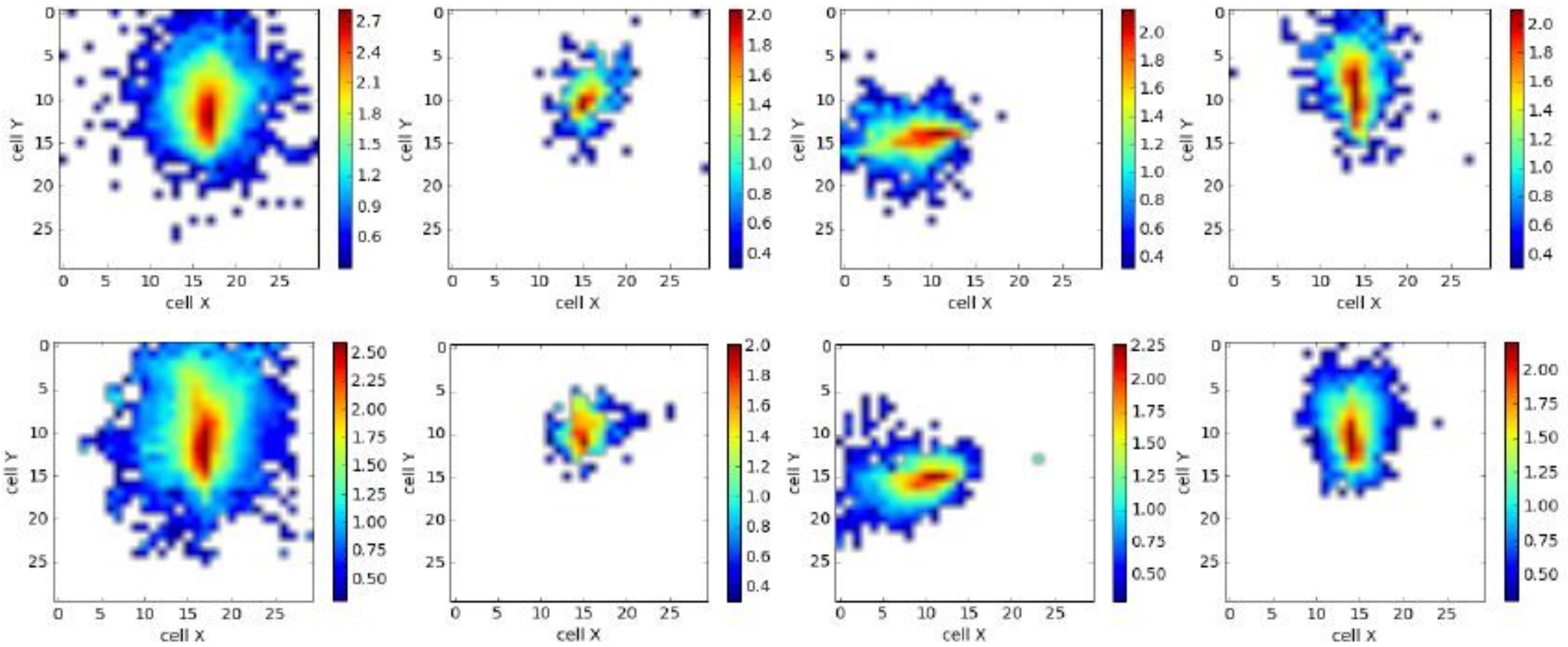


| Physical model | GANs |
|---|---|
| • Real physical equations are simulated | • Sampling function is approximated |
| • Sampling is slow | • Training takes days, sampling is free |
| • Accurate results | • Approximation quality needs to be verified |

# Applications




First row: simulated data, second row: data generated using GAN

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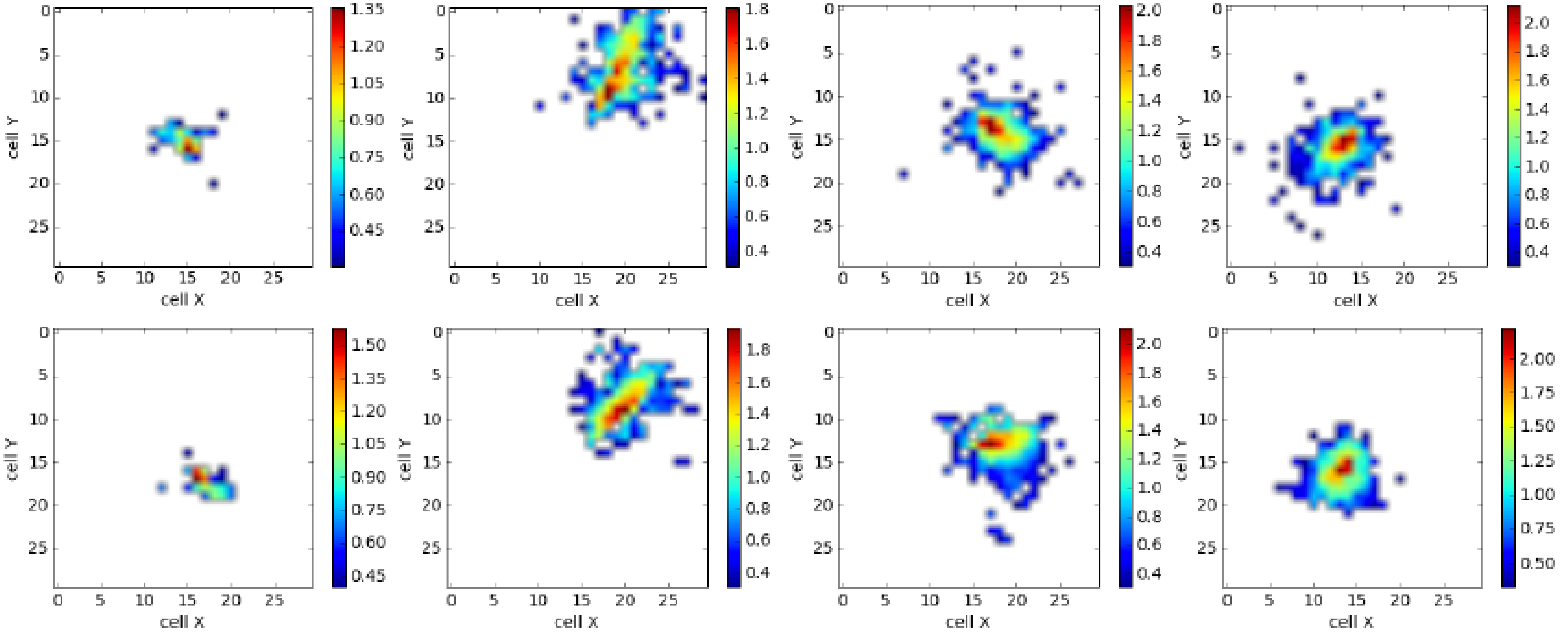


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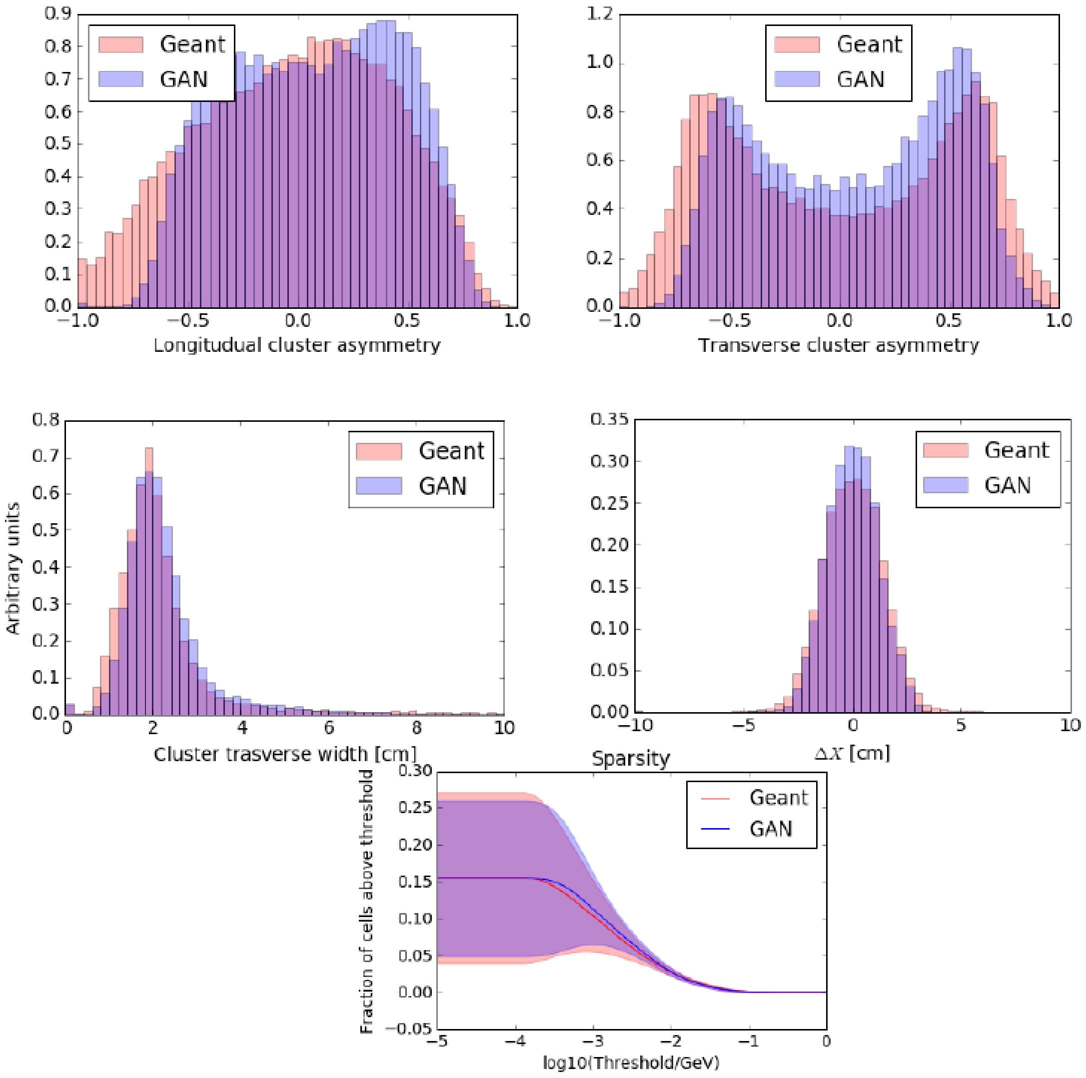


<u>Theorem (informal)</u>

Two distributions match if and only if all their statistics match

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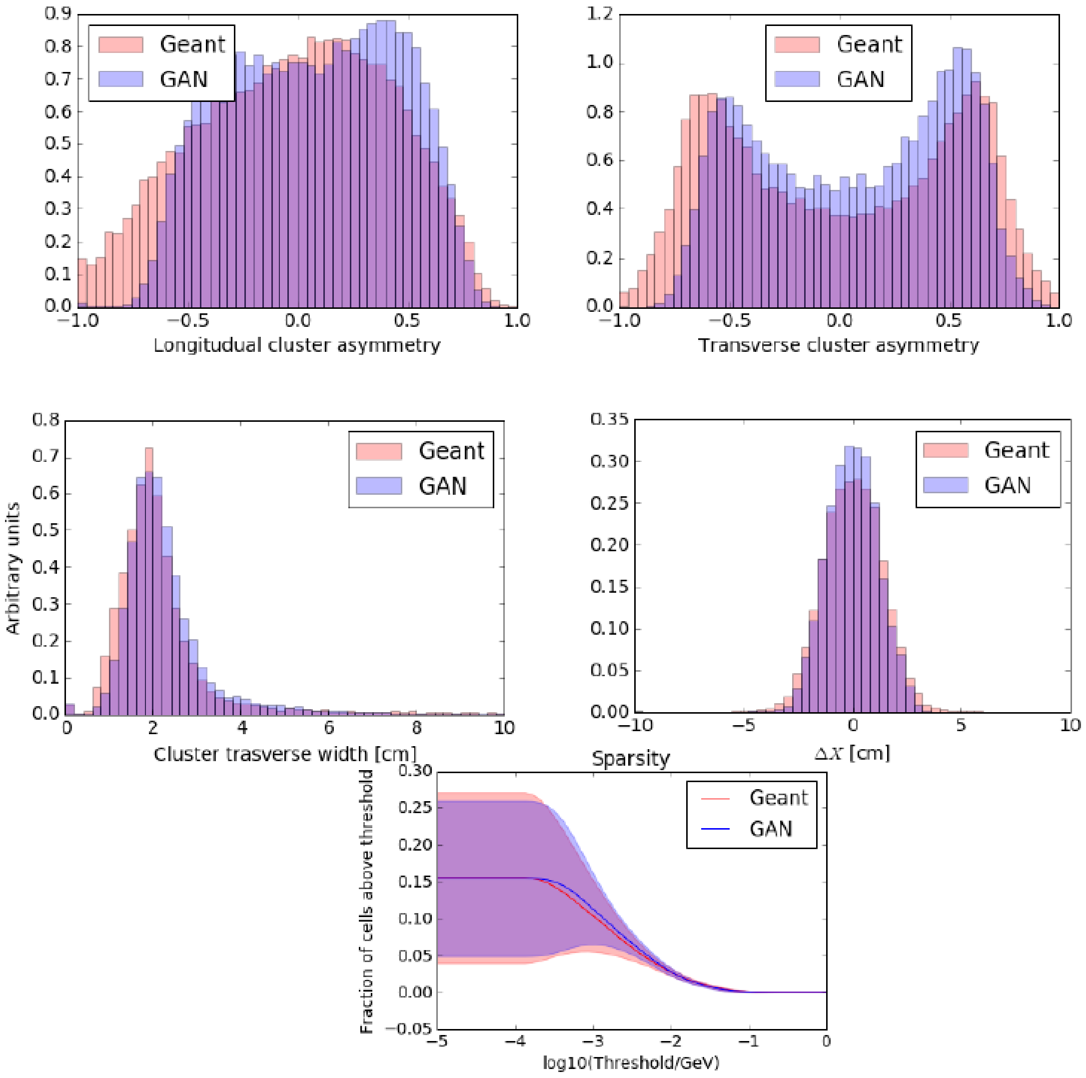


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Validation method: compare statistics between real data from test set (unseen during training) and generated data

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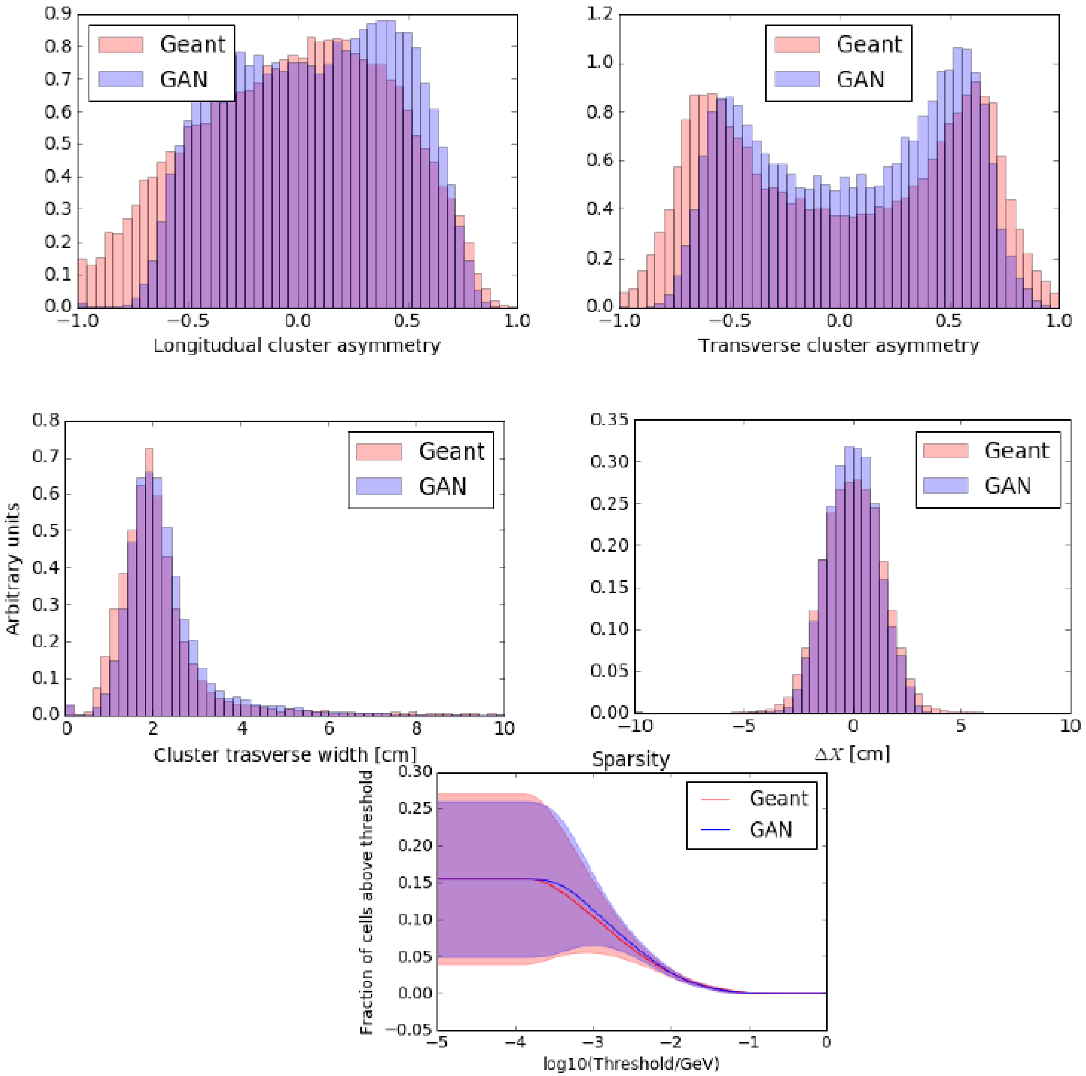


<u>Theorem (informal)</u>

Two distributions match if and only if all their statistics match

Validation method: compare statistics between real data from test set (unseen during training) and generated data

If statistics match, this is an indication that p(x) matches q(x)

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- GANs cannot be explained only by optimization of divergence
- Training requires finetuning of many hyperparameters, use other peoples' work instead of doing it yourself
- Produce solid practical results as an auxiliary objective for the existing problem
- Good application of GANs as a generative model is still work in progress