

Q. Define wave-particle duality; nature of light.

The concept of wave nature of matter arose from the dual character of the radiation which sometimes behaves as a wave and sometimes as a particle.

In the phenomena where light interacts with itself, such as dispersion, interference, diffraction and polarisation, the wave nature of light dominates. In the phenomena where the light interacts with matter, such as in photoelectric effect, Compton effect, Raman effect, the particle nature of light dominates.

Matter have travel as a wave packet with decreasing amplitude on either side of the present position.

Have packet.

The evidence for the description of light as waves was well established at the turn of the century when the "photoelectric effect" introduced firm evidences of a particle nature as well.

It States that it is impossible to determine at any given instant both the momentum and the position of subatomic particle electrons, simultaneously. $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

Where,

Δx = Uncertainty in position.

Δp = Uncertainty in momentum.

In 1927, Heisenberg proposed a very interesting principle, which is called consequence of the dual nature of matter, known as "Uncertainty principle".

According to Heisenberg's Uncertainty principle "It is impossible to specify precisely and simultaneously the values of both members of particular pair of physical variables that describe the behaviour of atomic system".

prove that $\Delta E \cdot \Delta t \gg h/2\pi$

if Δx is the excursion in position
 $\left(\frac{\Delta x}{\Delta t} - \frac{\Delta k x_n}{\Delta t} \right) = (2n+1)\pi/2$ for n^{th} node,

$$\left(\frac{\Delta x}{\Delta t} - \frac{\Delta k x_n}{\Delta t} \right) = (2n+1)\pi/2 \quad \text{for } (n+1)^{\text{th}} \text{ node,}$$

$$\left(\frac{\Delta x}{\Delta t} - \frac{\Delta k x_{n+1}}{\Delta t} \right) = [2(n+1)+1]\pi/2$$

$$-\frac{\Delta k}{\Delta t} x_{n+1} + \frac{\Delta k}{\Delta t} x_n = (2)\pi/2 \Rightarrow \Delta k (x_n - x_{n+1}) = \pi$$

$$(x_n - x_{n+1}) = \frac{\pi}{\Delta k} \quad \left\{ \begin{array}{l} k = 2\pi = \frac{2\pi P}{h} \\ \Delta k = \frac{2\pi}{h} \end{array} \right.$$

$$\Delta x = \frac{2\pi}{\Delta k} \quad \Delta x = \frac{2\pi}{\Delta P}$$

$$[\Delta x \cdot \Delta P = h] \quad \left[\Delta x \cdot \Delta P \geq h \right]$$

in terms of energy (E) and time (t)

$$c = 1/t, \quad x = ct, \quad \Delta x = c \Delta t \quad (ii)$$

& if P , momentum, $P = mc$

$$P = mc \quad \left\{ \begin{array}{l} E = mc^2 \\ E = mc \end{array} \right.$$

$$P = \frac{E}{c}$$

$$\therefore \Delta P = \frac{\Delta E}{c}$$

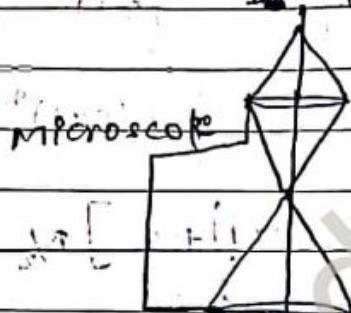
$$\text{if } \Delta t \cdot \Delta E = h \rightarrow \text{momentum}$$

$$\text{and force } \frac{F}{c} \text{ is constant}$$

$$\Delta t \cdot t = t$$

(b) prove that $\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$

$$\Delta x = d \cdot \frac{2\sin\theta}{\lambda}$$



~~uncertainty principle.~~

$$\begin{aligned}\Delta p_x &= p \sin\theta - (-p \sin\theta) \\ &= 2p \sin\theta \\ &= \frac{2h \sin\theta}{d} \quad [\because p = h/d]\end{aligned}$$

from equations (i) and (iv), we have.

$$(i) \Delta x \cdot \Delta p_x = d \cdot \frac{2h \sin\theta}{\lambda} = h > h/2$$

This shows that the product of uncertainties in position and momentum is of the order of Planck's Constant.

$$K = h = 2\pi P$$

Now, the product of Δx and Δk is given as

$$\Delta x \cdot \Delta k \geq \frac{1}{2} \cdot \Delta x \cdot |\Delta k| \geq \frac{1}{2} \frac{\hbar}{\Delta x}$$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} \quad (\text{p}_x = \text{momentum in } x\text{-direction})$$

Heisenberg's Uncertainty principle.

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{4\pi}$$

$$E = \frac{p^2}{2m} = \frac{1}{2}mv^2$$

Uncertainty in Kinetic energy

$$\Delta E = \frac{1}{2} m^2 v^2 \Delta v = m v \Delta v = (m v) \Delta v = \Delta p \Delta v$$

Uncertainty in time $\Delta t = \Delta x / v$

Establish Time Independent Schrödinger equation:

→ Second Order differential equation like the Schrödinger Equation, can be solved by Separation of Variables. These Separated Solutions can be used to solve the problem in general.

Assume that we can factorize the solution between time and space.

$$\psi(n, t) = u(n) T(t)$$

plug this into the Schrödinger Equation,

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2 u(n)}{\partial n^2} + V(n) u(n) \right) T(t) = i\hbar u(n) \frac{d}{dt} T(t)$$

put everything that depends on n on the left and everything that depends on t on the right

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2 u(n)}{\partial n^2} + V(n) u(n) \right) = \frac{i\hbar}{m} \frac{d}{dt} T(t) = (\text{const.}) T(t) = E$$

$$i\hbar \frac{d}{dt} T(t) = E T(t)$$

and an equation in x set equal to a const,

$$-\frac{\hbar^2}{2m} \frac{d^2u(x)}{dx^2} + V(x)u(x) = E u(x)$$

which depends on the problem to be solved.
(through $V(x)$)

The x equation is often called the "time independent Schrödinger equation".

Explain Young's

When sunlight passes through pin hole p_1 , spherical waves are spread out according to Huygen's principle, each point on the wave front is a centre of secondary wavelet. Thus spherical waves also spread out from pin holes p_1 & p_2 and hence they superimpose on each other.

At a point, where crest due to one wave falls on the crest due to another wave, at points where a crest due to one wave falls on the crest due to another wave (or though on a trough) the resultant amplitude is equal to the difference of the amplitude due to each wave separately. As the intensity is proportional to the square of the amplitude and hence decreased, this is called destructive interference. At a point, where a crest due to one wave falls on the trough of the other wave, the resultant amplitude is due to sum of the amplitude due to separate waves and hence the resultant intensity is increased. This is called constructive interference. Hence on the screen MN a number of alternate bright and dark areas of equal width, known

(b). Fringe width \rightarrow

$$\Delta = (S_2 P - S_1 P)$$

$$[D^2 + (y+d)^2]^{1/2} - [D^2 + (y-d)^2]$$

$$\rightarrow \left[1 + \frac{(y+d)^2}{D^2} \right]^{1/2} - \left[1 + \frac{(y-d)^2}{D^2} \right]^{1/2}$$

Binomial Expansion
 $(1+x)^n = (1+nx) + \frac{n(n-1)}{2!} n^2 + \dots$

$$\because (y \pm id) \ll D \Rightarrow$$

$$\therefore \Delta = D \left[1 + \frac{1}{2} \frac{(y+d)^2}{D^2} \right] - D \left[1 + \frac{1}{2} \frac{(y-d)^2}{D^2} \right]$$

$$\therefore \Delta = D \left[D + \frac{(y+d)^2}{2D} \right] - D \left[D + \frac{(y-d)^2}{2D} \right]$$

$$\therefore \Delta = (y+d)^2 - (y-d)^2$$

$$\therefore \Delta = y^2 + d^2 + 2yd - (y^2 + d^2 - 2yd)$$

$$= 4yd$$

$$\therefore \Delta = \left(\frac{4yd}{P} \right)$$

Case 1 :- for maxima $\therefore \Delta = \pi d$

$$\therefore \frac{4yd}{P} = \pi d$$

$$y_n = n \frac{d\Delta}{2d} \quad \text{--- (2)}$$

Case (ii) for minima,

$$\frac{dy_n}{dx} = (n+1) \frac{d\Delta}{2d} = 0 \quad (i.e. n = 0, 1, 2)$$

$$\frac{dy_n}{dx} = (n+1) \frac{d\Delta}{2d} = 0 \quad (n = 0, 1, 2)$$

$$y = (n+1) \frac{d\Delta}{2d}$$

$$y_{n+1} - y_n = (n+1) \frac{d\Delta}{2d} = 1 \frac{d\Delta}{2d} = \Delta$$

Let n corresponds to n^{th} & $(n+1)^{\text{th}}$ bright fringe

$$y_n = n \frac{d\Delta}{2d} \quad ; \quad y_{n+1} = (n+1) \frac{d\Delta}{2d}$$

$$\beta = (y_{n+1} - y_n) = d\Delta (n+1 - n)$$

$$\left\{ \begin{array}{l} \beta = d\Delta \\ d\Delta = \lambda \end{array} \right.$$

$$(L_2) = 1$$

$$LR = 0 \therefore \text{minimum} \rightarrow 1 \text{ min}$$

We have

$$(\operatorname{curl}(\mathbf{A}) \cdot \mathbf{S}_r) = \oint_C \vec{A} \cdot d\vec{l}$$

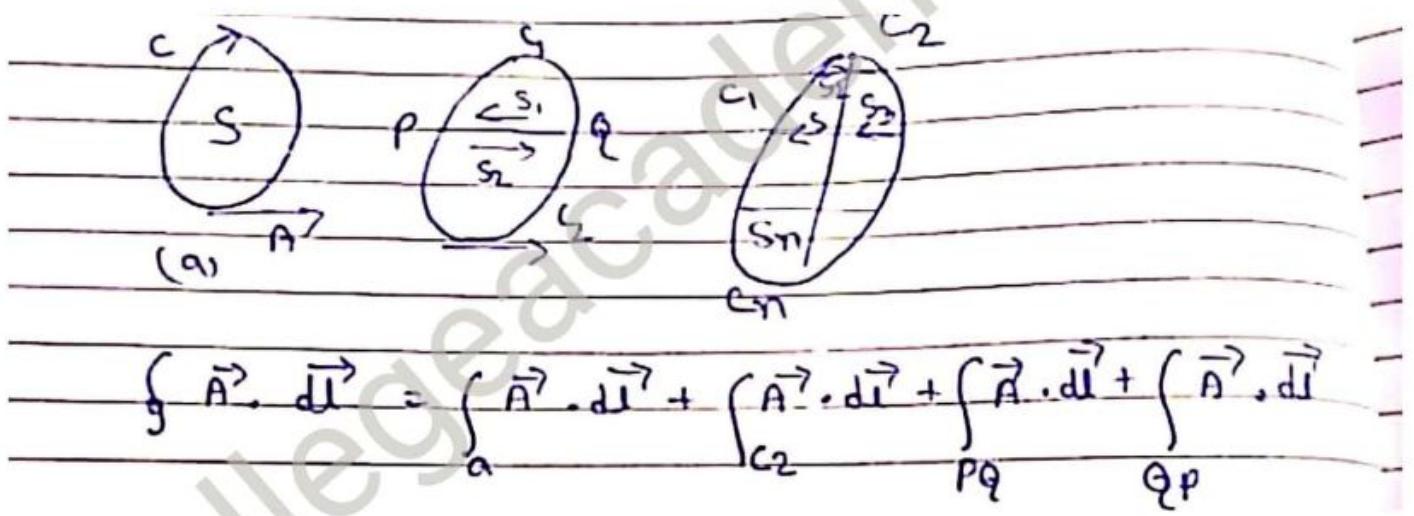
$$\oint_C \vec{A} \cdot d\vec{l} = \sum_{r=1}^n \operatorname{curl} \vec{A} \cdot S_r$$

$$= \operatorname{curl} \vec{A} \cdot \sum_{r=1}^n S_r$$

$$= \operatorname{curl} \vec{A} (\text{S})$$

$$= \operatorname{curl} \vec{A} \iint dS$$

$$\boxed{\oint_C \vec{A} \cdot d\vec{l} = \iint_S \operatorname{curl} \vec{A} \cdot d\vec{S}}$$



$$\oint \vec{A} \cdot d\vec{l} = \int_{C_1} \vec{A} \cdot d\vec{l} + \int_{C_2} \vec{A} \cdot d\vec{l} + \dots + \int_{C_n} \vec{A} \cdot d\vec{l}$$

~~NRK Imp~~ (curl \vec{n}^* = \vec{s}) - Q
So, Stoke's theorem :-

Statement :- The line integral of a vector field (\vec{n}) along the boundary of a closed curve is equal to the surface curl of that vector field over the surface (S) where the surface S is encloses by the curve C . That is

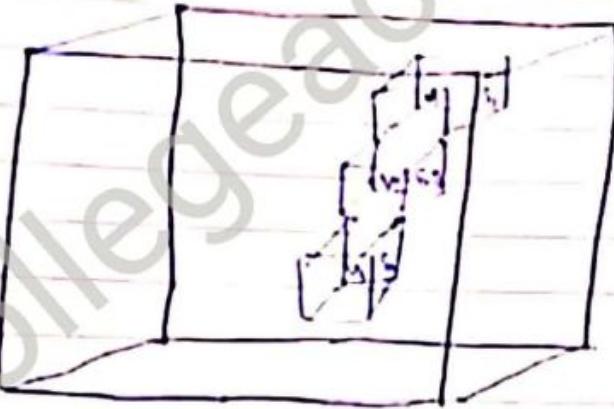
$$\oint_C \vec{A} \cdot d\vec{l} = \iint_S \text{curl } \vec{n}^* \cdot d\vec{s}$$

$$curl \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \rightarrow ①$$

Ans.

$$\operatorname{div} \vec{n} \iiint_V dV = \iint_S \vec{n} \cdot d\vec{s}$$

$$\boxed{\iiint_V \operatorname{div} \vec{n} dV = \iint_S \vec{n} \cdot d\vec{s}} \quad \text{proof}$$



curl of a vector field

$$\vec{A} = (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$$

$$\text{Also; } \vec{\nabla} = (\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z})$$

mathematical definition

$$(\vec{\nabla} \times \vec{A}) = \text{curl}(\vec{A})$$

91st volume element:

$$\operatorname{div} \vec{n} = \iint_{S_r} \vec{n} \cdot d\vec{s}$$

Also we have by the definition of divergence:

$$\operatorname{div} \vec{n} \cdot v_r = \iint_{S_r} \vec{n} \cdot d\vec{s}$$

considering all the volume elements

$$\sum_{r=1}^n \operatorname{div} \vec{n} \cdot v_r = \sum_{r=1}^n \iint_{S_r} \vec{n} \cdot d\vec{s}$$

$$\operatorname{div} \vec{A} \cdot \sum_{r=1}^n (v_r) = \sum_{r=1}^n \iint_{S_r} \vec{n} \cdot d\vec{s}$$

$$\operatorname{div} \vec{A} (V) = \sum_{r=1}^n \left(\iint_{S_r} \vec{n} \cdot d\vec{s} \right)$$

∴ for inner surfaces ($\vec{n}, d\vec{s}$) will be zero.

$$\boxed{\text{volume: } \iint_S \vec{A} \cdot d\vec{s}} \quad \rightarrow (2)$$

$\text{div } \vec{A}$ may be 0, positive and
negative also

~~(say)~~ Gauss's divergence theorem

Statement :- The surface integral of a vector field (\vec{A}) over the surface (S) = The volume integral of divergence of that vector field (A) with in the volume (V) where V is the volume (V) enclosed by the surface (S). That is

$$\iint_S \vec{A} \cdot d\vec{s} = \iiint_V \text{div } \vec{A} \, dv$$

proof :- Let's consider a closed

Application of Newton Rings experiment

Determination of wave length of a monochromatic source of light

Determination of radius of curvature of planoconvex lens.

Determination of refractive index of a liquid

① → We know that the diameter of n^{th} & $(n+p)^{th}$ dark ring are measured using a travelling microscope so this diameter is given by for n^{th}
 $D_n = 4nR\lambda \quad \dots \quad (1)$

$$D_{n+p} = 4(n+p)R\lambda \quad \dots \quad (2)$$

$$\text{eqn } (2) - \text{eqn } (1)$$

$$D_{n+p}^2 - D_n^2 = 4nR\lambda + 4pR\lambda - 4nR\lambda$$

$$\Rightarrow D_{n+p}^2 - D_n^2 = 4pR\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

$$D_n = \lambda n$$

$$D_n = \sqrt{C(2n-1) \frac{\Delta E}{2}}$$

$$D_n = \sqrt{4(2n-1) \frac{\Delta E^2}{2}}$$

$$D_n = \sqrt{(2n-1) 2\Delta E}$$

constant = $2.1 R$

$$D_n \propto \sqrt{C(2n-1)}$$

These form the diameter of bright ring
proportional to the square root of odd natural no.

Ratio of few diameter $n=1$

$$\sqrt{1} : \sqrt{3} : \sqrt{5} : \sqrt{7}$$

$$1 : 1.732 : 2.236 : 2.04$$

$$\approx 1.000 \quad \approx 1.584 \quad \approx 1.414$$

$$2t + \frac{\lambda}{2} = (m+1) \frac{\lambda}{2}$$

This is the condition of Darkening.

\rightarrow Diameter of bright rings,

To evaluate / find the diameter of bright and dark rings consider a plane convex lens placed on a glass plate. Let R be the radius of curvature of plane convex lens. And t thickness of lens.

Let us assume the plane convex lens is the front of its object. Hence..

$$(AC)^2 = ((CD)^2 + (BD))^2$$

$$R^2 = (R-t)^2 + (\delta)^2$$

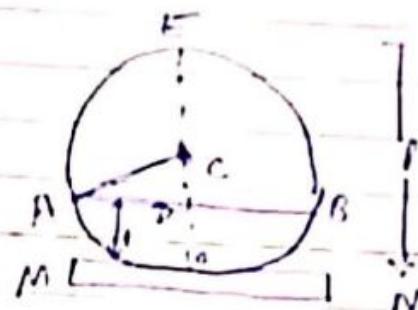
$$R^2 = R^2 + t^2 + 2Rt + \delta^2$$

$$0 = t^2 + 2Rt + \delta^2$$

$$t \ll R$$

$$\text{Or } \rightarrow 2Rt = \delta^2$$

$$\rightarrow 2t = \frac{\delta^2}{R} \quad \text{---(i)}$$



$$\therefore 2t = (m+1) \frac{\lambda}{2}$$

$$\frac{\delta^2}{R} = (m+1) \frac{\lambda}{2}$$

$$\delta^2 = (m+1) \lambda R \quad \delta^2 = R(m+1) \frac{\lambda}{2}$$

Critical angle \Rightarrow thickness of prism
 \Rightarrow angle of refraction
points. $\gamma_2 \Rightarrow$ critical path difference.

$$n = 1 \rightarrow \frac{\sin i}{\sin r} = 1$$
$$r = 0 \rightarrow \left\{ \Delta = \alpha t + \frac{\lambda}{2} \right\} - \textcircled{1}$$
$$i = \lambda - \frac{\lambda}{2}$$

This is a condition of minimum intensity.

Hence, the condition of Newton's ring is obtained.

Maxima Condition

We know that

for maxima $\Delta = n\lambda$

$$\Rightarrow \alpha t + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \alpha t = (2n-1)\frac{\lambda}{2} - \textcircled{2}$$

Minima Condition (condition of darken).
We know that

$$\Delta = (2n+1)\frac{\lambda}{2} - \textcircled{3}$$

When a film of Convex lens, placed on a thin glass plate at the point of contact of film of convex lens is placed between the upper and lower position of lens and glass plate.

The thickness of film is zero at the point of contact and gradually increases as we go up and left side of the film. If monochromatic light is allowed to fall normally on the film then after passing through the alternate thick and bright concentric rings are seen with the help of binocular microscope. These rings are called Newton's rings.

Newton's ring are formed because the interference between the waves from top and bottom surface of two films which are enclosed b/w two convex lens and glass plate.

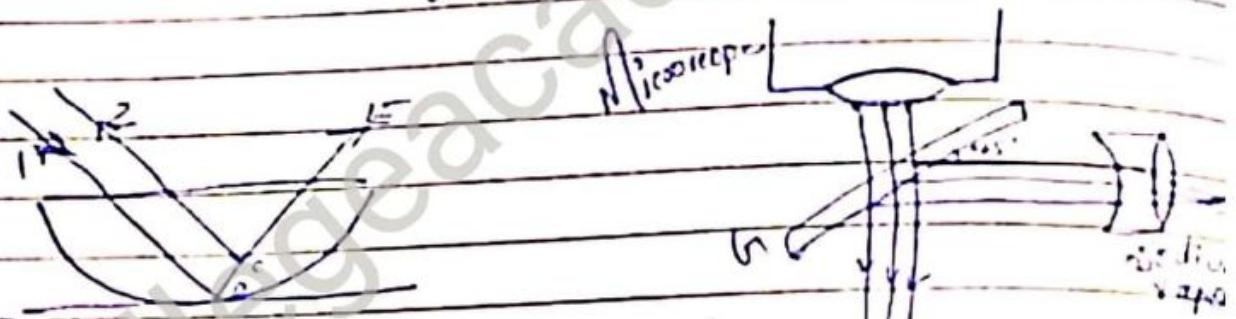
If the rings are objective to reflected light.

Ques the effective path difference b/w interference of two waves is given by $\Delta = \frac{2d}{\lambda} + \frac{1}{2}$

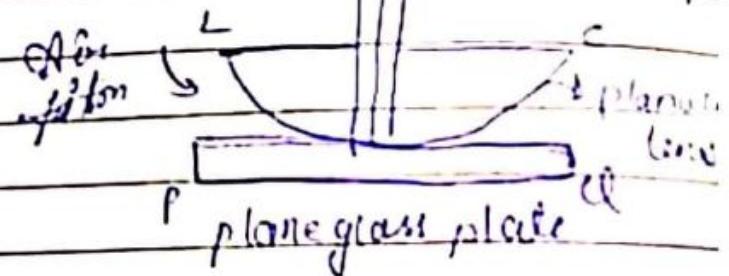
Acc. to Qstn treatment:- (Law),

When a ray of light is reflected of an optically denser medium it suffer a phase change of 180° .

Newton's ring Experiment



fig(b). Ray diagram of
Newton's ring ex.



fig(a).
Experiment setup of
Newton's ring.

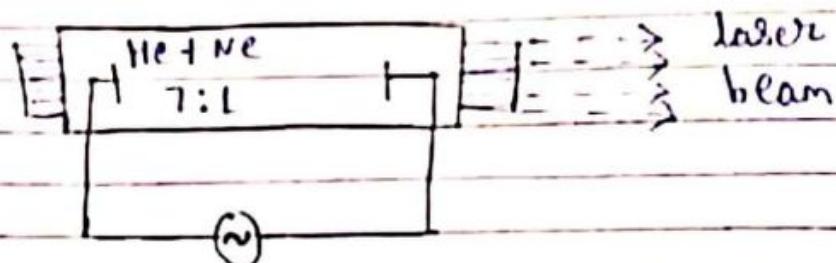


electric discharge. energy is supplied in form of ~~gas~~
~~gas~~

i) Optical Resonator :- both the silver polish ends of quartz tube works as the optical resonator. It is used to increase the Intensity of laser light by laser oscillation.

Construction

SiO_2 (quartz tube).



$4 \times 10^7 \text{ Hz}$ 4.6 (a)

By Maxwell - Boltzmann distribution law;

$$N_n = N_0 \cdot e^{-nh\nu/KT} \quad \textcircled{2}$$

where; h = P. Cancks Constant.

ν = free equation

K = Boltzmann's constant

T = absolute temp.

$$N_n = N_0 e^{-h\nu/KT} \quad \textcircled{3}$$

Eqn ① ③ becomes;

$$BUN_0 = (A + BU) N_0 e^{-h\nu/KT}$$

$$(BU - BN_0 e^{-h\nu/KT}) = A e^{-h\nu/KT}$$

$$BU(1 - e^{-h\nu/KT}) = A e^{-h\nu/KT}$$

$$\frac{U}{e^{-h\nu/KT}} (1 - e^{-h\nu/KT}) = \frac{A}{B}$$

$$U \left(\frac{1}{e^{-h\nu/KT}} - 1 \right) = \frac{A}{B}$$

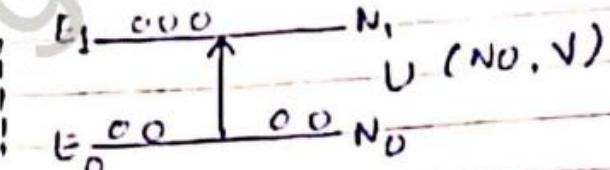
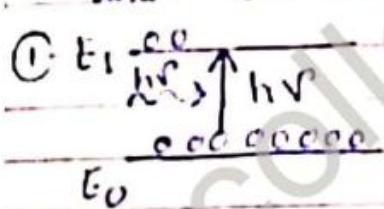
$$U \left(\frac{1}{e^{-h\nu/KT}} - 1 \right) = \frac{A}{B}$$

3) Stimulated Emission also depends on two factors
that is where

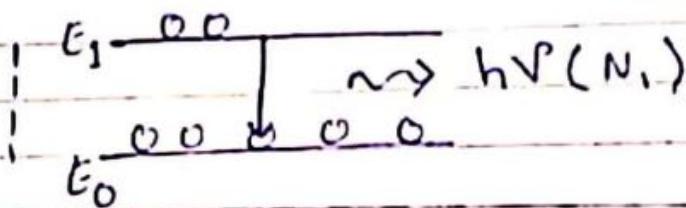
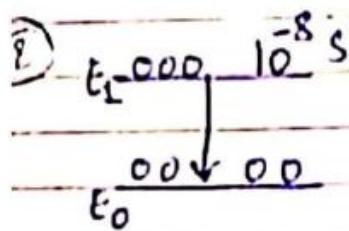
N_e = atoms in excited state
 $U(V)$ = density of incident radiation

Ques: Write Einstein's explanation of relation between emission of radiation & establishment of relation between coefficients A & B.

To explain Absorption & Emission of Radiation let's consider an atomic system having the two energy levels E_0 and E_1 with energy difference ($h\nu$) then we have



(a) absorption

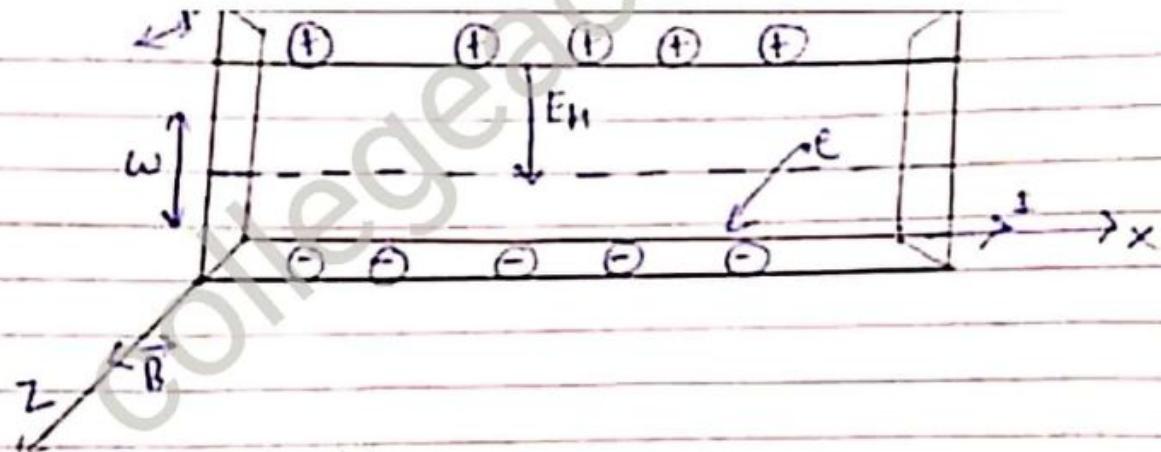


(b) Self emission

\therefore eq. ① becomes

$$\boxed{E_H = \frac{B \times I}{\text{newt}}} \quad \text{--- } ①$$

\therefore This is Hall E.F



In equilibrium
 $eV_B \sin\theta = eE_H$
 $\therefore \theta = 90^\circ$

\therefore eq. (1) becomes

$$\boxed{E_H = \frac{B \cdot I}{\text{newt}}} \quad (4)$$

\therefore This B Hall E. F.

Hall Voltage

$$V_H = E \omega$$

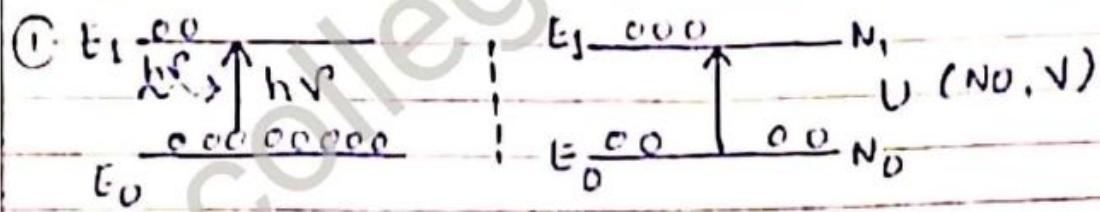
$$= \frac{B I}{\text{newt}} \times \omega$$

$$\boxed{V_H = \frac{B I}{\text{newt}}} \quad (5)$$

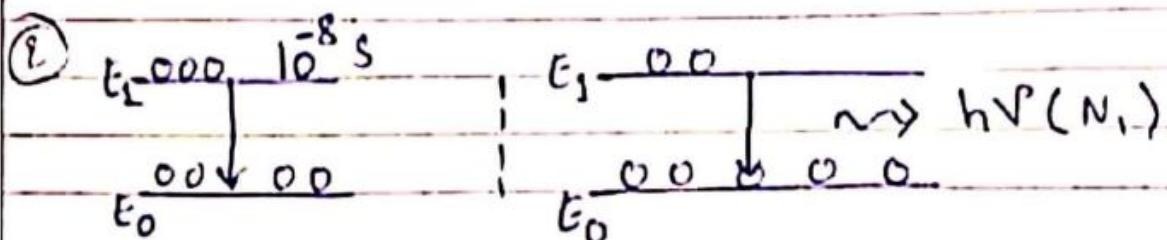
$$\frac{1}{ne} = R_H$$

Hall coeffie.

To explain Absorption emission of Icadien let's consider an atomic system having the two energy level E_0 and E_1 with energy difference ($h\nu$) then we have



(a) absorption



2. As λ increase the width of allowed energy band also increases.

3. (i) If P is large if the free electron has a single energy.
- (ii) If $P = 0$ then electron has continuous values inside the solid

Applying the d. boundary condition we get;

$$y_1(x) = y_2(x)$$

$$\frac{mv_{ob}}{h} \cdot \sin(\alpha) + \cos(\alpha) = \cos(\alpha)$$

$$\left(\frac{mv_{ob}}{h} \right) \left(\sin(\alpha) \right) + \cos(\alpha) = \cos(\alpha)$$

$$P = \left(\frac{mv_{ob}}{h} \right) (\text{say})$$

$$P \left(\frac{\sin(\alpha)}{\alpha} \right) + \cos(\alpha) = \cos(\alpha) = y \text{ (say)}$$

$$y = [P \left(\frac{\sin(\alpha)}{\alpha} \right) + \cos(\alpha)] \quad \text{--- (7)}$$

$$y = \cos(\alpha)$$

$$= \cos\left(\frac{n\pi}{a} \cdot a\right)$$

$$y = \cos(n\pi) \quad \text{--- (8)}$$

In region Sch. wave eqⁿ

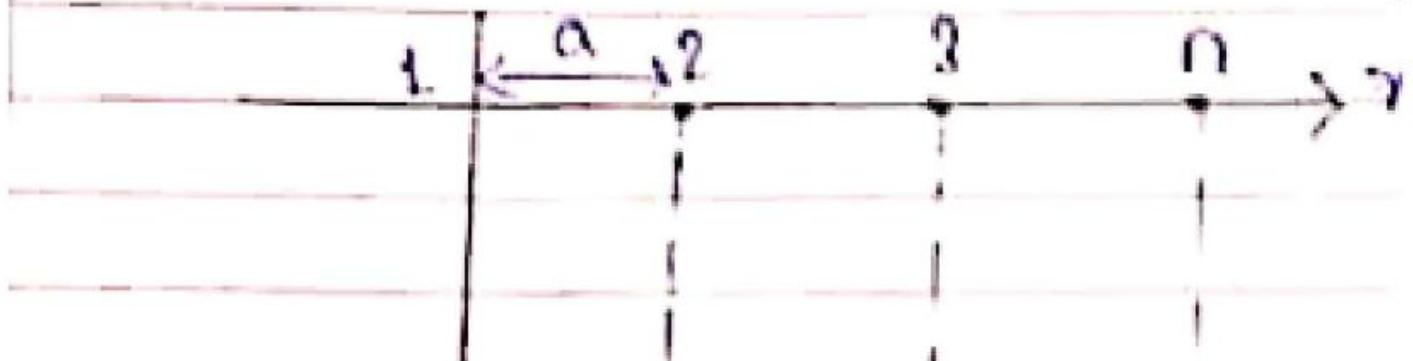
$$\frac{d^2\psi_i}{dx^2} + \left(\frac{E_i - E}{\hbar^2}\right)\psi_i = 0 \quad \text{--- (3)}$$

In IInd region



Fig 1.8 (a) for single atom

b



Let's consider A solid of atomic no Z , then
the potential energy of a free electron
will be $(Z/4) \pi \epsilon_0 E_s^2$.

(b). Derive De-Broglie's Hypothesis

The de-Broglie wavelength is the wave length of a wave associated with an object and is related to its momentum and mass.

De Broglie derived his equation using well established theories through the following process of substituting:

De Broglie first used Einstein's famous equation relating matter and energy.

$$E = mc^2 \quad (1)$$

with,

E = Energy, or basic unit of energy

m = mass, or basic unit of mass

c = speed of light.

Using Planck's theory which states every quantum of a wave has a discrete amount of energy given by Planck's equation

$$E = h\nu \quad (2)$$

with,

E = energy,

h = Planck's Constant, $(6.62607 \times 10^{-34} \text{ J})$

ν = frequency.

Since de Broglie believed particles and waves have the same traits, he hypothesized that the two energies would be equal. In doing so, he substituted $mc^2 = hv$ into the equation $mv^2 = hv$. 3

Because real particles do not travel at the speed of light, De Broglie substituted velocity (v) for the speed of light (c).

$$mv^2 = hv \quad \text{4 (a)}$$

Through the equation a , De Broglie substituted v for c and arrived at the final expression that relates wavelength and particle with speed.

$$mv^2 = hv \quad \text{5 (b)}$$

Hence,

$$\lambda = \frac{h}{mv} = \frac{h}{mc}$$

$$\lambda = \frac{h}{mv} \quad \text{Ans.}$$