

Causal survival analysis

Treatment Effect Estimation in Causal Survival Analysis: Practical Recommendations

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Estimators

Unadjusted Kaplan-Meier

IPCW Kaplan-Meier

This alternative approach for IPCW Kaplan-Meier would also be valid for IPTW and IPTW-IPCW Kaplan-Meier.

Buckley-James based estimator

IPTW Kaplan-Meier

G-formula

IPTW-IPCW Kaplan-Meier

IPTW-BJ estimator

AIPTW-AIPCW

Simulations

[1] "Descriptive statistics for group A=0: 1036"

X1	X2	X3	X4
Min. : -1.8280	Min. : -2.7455	Min. : -4.7270	Min. : -2.3782
1st Qu.: 0.3708	1st Qu.: 0.3043	1st Qu.: -1.6797	1st Qu.: 0.2556
Median : 0.9881	Median : 1.0281	Median : -1.0109	Median : 0.9224
Mean : 1.0037	Mean : 1.0030	Mean : -1.0107	Mean : 0.9642
3rd Qu.: 1.6713	3rd Qu.: 1.6749	3rd Qu.: -0.3314	3rd Qu.: 1.7201
Max. : 4.4024	Max. : 4.1660	Max. : 1.9269	Max. : 3.8427

C	T1	T0	status
Min. : 0.00614	Min. : 10.01	Min. : 0.0084	Min. : 0.0000
1st Qu.: 9.82432	1st Qu.: 12.87	1st Qu.: 2.8682	1st Qu.: 0.0000
Median : 21.89198	Median : 18.49	Median : 8.4852	Median : 1.0000
Mean : 33.40304	Mean : 33.85	Mean : 23.8514	Mean : 0.6728
3rd Qu.: 44.30954	3rd Qu.: 33.90	3rd Qu.: 23.8957	3rd Qu.: 1.0000
Max. : 234.30127	Max. : 721.46	Max. : 711.4606	Max. : 1.0000

T_tild

Min. : 0.00614
1st Qu.: 2.20440
Median : 6.00186
Mean : 10.64019
3rd Qu.: 13.53585
Max. : 121.75436

[1] "Descriptive statistics for group A=1: 964"

X1	X2	X3	X4
Min. : -2.2160	Min. : -2.5397	Min. : -4.1076	Min. : -1.9142
1st Qu.: 0.3330	1st Qu.: 0.3035	1st Qu.: -1.7821	1st Qu.: 0.2981
Median : 0.9758	Median : 0.9291	Median : -1.0706	Median : 0.9905
Mean : 1.0001	Mean : 0.9535	Mean : -1.0479	Mean : 1.0024
3rd Qu.: 1.6681	3rd Qu.: 1.6244	3rd Qu.: -0.3053	3rd Qu.: 1.7180
Max. : 3.9925	Max. : 4.1498	Max. : 2.3626	Max. : 4.5355

C	T1	T0	status
Min. : 0.04002	Min. : 10.00	Min. : 0.0042	Min. : 0.0000
1st Qu.: 10.60459	1st Qu.: 12.85	1st Qu.: 2.8517	1st Qu.: 0.0000
Median : 22.78997	Median : 18.33	Median : 8.3318	Median : 1.0000
Mean : 33.27502	Mean : 33.43	Mean : 23.4282	Mean : 0.5124
3rd Qu.: 47.94150	3rd Qu.: 33.65	3rd Qu.: 23.6506	3rd Qu.: 1.0000
Max. : 237.66851	Max. : 597.85	Max. : 587.8488	Max. : 1.0000

T_tild

Min. : 0.04002
1st Qu.: 10.11489
Median : 13.31504

Mean : 16.72029
 3rd Qu.: 19.90145
 Max. :102.71160

[1] "Descriptive statistics for group A=0: 1046"

X1	X2	X3	X4
Min. :-1.7973	Min. :-3.1829	Min. :-3.7832	Min. :-1.9452
1st Qu.: 0.3725	1st Qu.: 0.3473	1st Qu.: -1.6630	1st Qu.: 0.4295
Median : 0.9830	Median : 1.0252	Median : -1.0018	Median : 1.0631
Mean : 1.0207	Mean : 1.0067	Mean : -0.9917	Mean : 1.0673
3rd Qu.: 1.6365	3rd Qu.: 1.6466	3rd Qu.: -0.3146	3rd Qu.: 1.7064
Max. : 4.5930	Max. : 4.3551	Max. : 2.7585	Max. : 4.0848

C	T1	T0	status
Min. : 0.00811	Min. : 10.01	Min. : 0.0051	Min. :0.0000
1st Qu.: 2.58789	1st Qu.: 12.77	1st Qu.: 2.7656	1st Qu.:0.0000
Median : 6.92190	Median : 18.08	Median : 8.0819	Median :0.0000
Mean : 15.39003	Mean : 30.44	Mean : 20.4417	Mean :0.4532
3rd Qu.: 17.61904	3rd Qu.: 30.91	3rd Qu.: 20.9072	3rd Qu.:1.0000
Max. :251.74937	Max. :670.46	Max. :660.4618	Max. :1.0000

status_tau	T_tild
Min. :0.0000	Min. : 0.00507
1st Qu.:0.0000	1st Qu.: 1.31826
Median :0.0000	Median : 3.83315
Mean :0.4971	Mean : 8.07986
3rd Qu.:1.0000	3rd Qu.: 8.82089
Max. :1.0000	Max. :189.88932

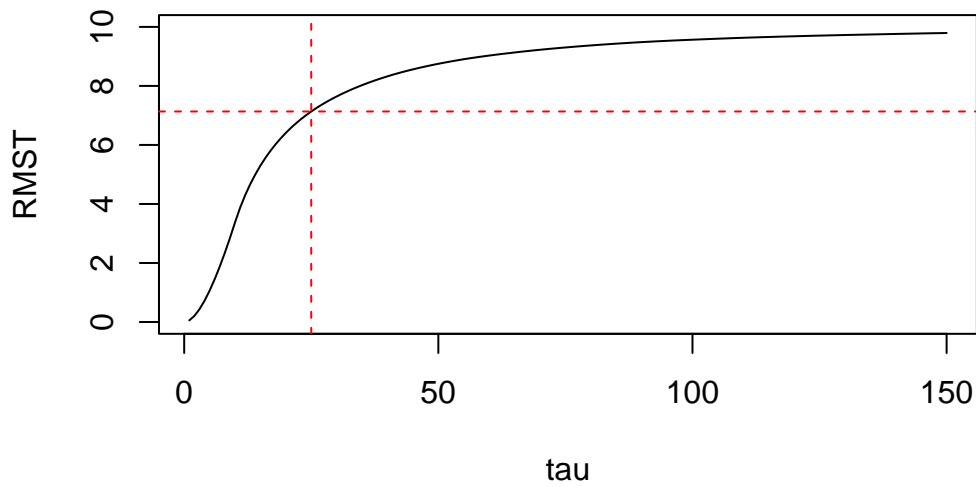
[1] "Descriptive statistics for group A=1: 954"

X1	X2	X3	X4
Min. :-2.5947	Min. :-1.9559	Min. :-3.8388	Min. :-2.4342
1st Qu.: 0.3991	1st Qu.: 0.3235	1st Qu.: -1.6904	1st Qu.: 0.3912
Median : 1.0544	Median : 0.9827	Median : -1.0161	Median : 1.0531
Mean : 1.0267	Mean : 0.9933	Mean : -1.0425	Mean : 1.0265
3rd Qu.: 1.6175	3rd Qu.: 1.6559	3rd Qu.: -0.4135	3rd Qu.: 1.6762
Max. : 3.9187	Max. : 3.8936	Max. : 2.5556	Max. : 4.0068

C	T1	T0	status
Min. : 0.01643	Min. : 10.00	Min. : 0.0031	Min. :0.0000
1st Qu.: 3.16905	1st Qu.: 12.88	1st Qu.: 2.8820	1st Qu.:0.0000
Median : 8.86728	Median : 18.47	Median : 8.4719	Median :0.0000

Mean : 18.16227	Mean : 28.67	Mean : 18.6707	Mean : 0.2484
3rd Qu.: 20.63316	3rd Qu.: 31.03	3rd Qu.: 21.0298	3rd Qu.: 0.0000
Max. : 294.72368	Max. : 361.27	Max. : 351.2701	Max. : 1.0000
status_tau		T_tild	
Min. : 0.0000	Min. : 0.01643		
1st Qu.: 0.0000	1st Qu.: 3.16905		
Median : 0.0000	Median : 8.86728		
Mean : 0.3092	Mean : 11.87564		
3rd Qu.: 1.0000	3rd Qu.: 14.43205		
Max. : 1.0000	Max. : 148.67175		

True difference in RMST for RCT scenario 1



[1] "The ground truth for RCT scenario 1 and 2 at time 25 is 7.1"

[1] "Descriptive statistics for group A=0: 1114"

X1	X2	X3	X4
Min. : -1.9206	Min. : -1.9086	Min. : -2.879596	Min. : -2.004
1st Qu.: 0.5302	1st Qu.: 0.4798	1st Qu.: -1.100883	1st Qu.: 0.558
Median : 1.1392	Median : 1.1775	Median : -0.547608	Median : 1.189
Mean : 1.1626	Mean : 1.1706	Mean : -0.500471	Mean : 1.193
3rd Qu.: 1.8109	3rd Qu.: 1.8394	3rd Qu.: 0.002337	3rd Qu.: 1.850
Max. : 4.5383	Max. : 3.9480	Max. : 2.539992	Max. : 4.323
C	T1	T0	status
Min. : 0.00287	Min. : 10.00	Min. : 0.0003	Min. : 0.0000

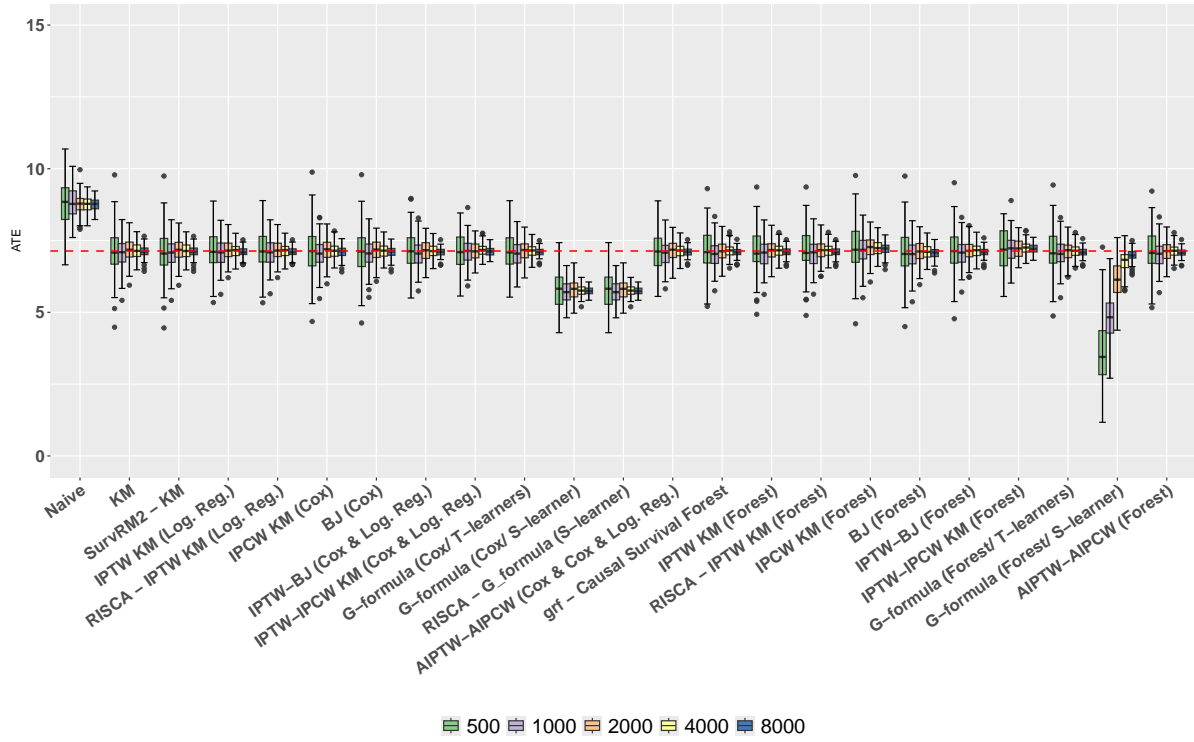


Figure 1: Results of the ATE for the simulation of a RCT with independent censoring.

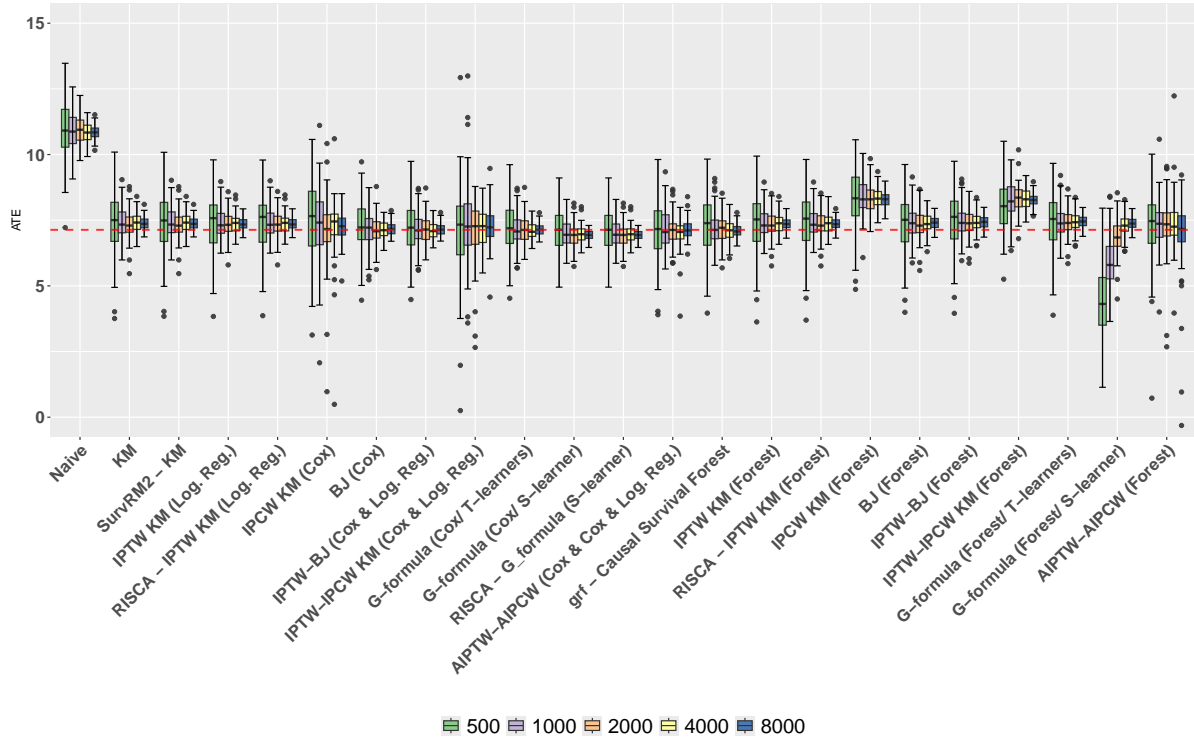


Figure 2: Estimation results of the ATE for the simulation of a RCT with dependent censoring.

1st Qu.: 9.51006	1st Qu.: 12.70	1st Qu.: 2.6983	1st Qu.:0.0000
Median : 22.82636	Median : 18.24	Median : 8.2438	Median :1.0000
Mean : 33.01274	Mean : 31.63	Mean : 21.6324	Mean :0.6894
3rd Qu.: 46.68240	3rd Qu.: 33.10	3rd Qu.: 23.0964	3rd Qu.:1.0000
Max. :208.07788	Max. :727.56	Max. :717.5625	Max. :1.0000

T_tild

Min. : 0.00026
1st Qu.: 1.98842
Median : 5.44379
Mean : 10.94189
3rd Qu.: 14.14035
Max. :131.55968

[1] "Descriptive statistics for group A=1: 886"

X1	X2	X3	X4
Min. :-2.24505	Min. :-2.50433	Min. :-4.6901	Min. :-2.2209
1st Qu.: 0.08697	1st Qu.: 0.07369	1st Qu.: -2.1523	1st Qu.: 0.0607
Median : 0.77618	Median : 0.73579	Median : -1.5859	Median : 0.7213
Mean : 0.75924	Mean : 0.73632	Mean : -1.6223	Mean : 0.7269
3rd Qu.: 1.44977	3rd Qu.: 1.41292	3rd Qu.: -1.1121	3rd Qu.: 1.4250
Max. : 4.14015	Max. : 3.56741	Max. : 0.7896	Max. : 3.8273

C	T1	T0	status
Min. : 0.04381	Min. : 10.00	Min. : 0.0047	Min. :0.0000
1st Qu.: 10.95928	1st Qu.: 12.65	1st Qu.: 2.6482	1st Qu.:0.0000
Median : 25.09587	Median : 18.68	Median : 8.6817	Median :1.0000
Mean : 35.14770	Mean : 35.67	Mean : 25.6705	Mean :0.5102
3rd Qu.: 49.01538	3rd Qu.: 35.17	3rd Qu.: 25.1680	3rd Qu.:1.0000
Max. :269.58579	Max. :952.42	Max. :942.4151	Max. :1.0000

T_tild

Min. : 0.04381
1st Qu.: 10.18937
Median : 13.28362
Mean : 17.12036
3rd Qu.: 20.54031
Max. :144.19037

[1] "Descriptive statistics for group A=0: 1103"

X1	X2	X3	X4
Min. :-2.1918	Min. :-1.7200	Min. :-3.06189	Min. :-1.569

1st Qu.: 0.5403	1st Qu.: 0.5867	1st Qu.: -1.09671	1st Qu.: 0.542
Median : 1.2091	Median : 1.2376	Median : -0.51301	Median : 1.235
Mean : 1.2122	Mean : 1.2426	Mean : -0.51026	Mean : 1.198
3rd Qu.: 1.8535	3rd Qu.: 1.9072	3rd Qu.: 0.03966	3rd Qu.: 1.904
Max. : 5.1993	Max. : 4.0857	Max. : 2.27566	Max. : 3.958
C	T1	T0	status
Min. : 0.00207	Min. : 10.02	Min. : 0.0204	Min. : 0.0000
1st Qu.: 2.22411	1st Qu.: 12.56	1st Qu.: 2.5650	1st Qu.: 0.0000
Median : 6.27230	Median : 17.90	Median : 7.8959	Median : 0.0000
Mean : 14.00106	Mean : 31.58	Mean : 21.5793	Mean : 0.4397
3rd Qu.: 16.01545	3rd Qu.: 31.09	3rd Qu.: 21.0898	3rd Qu.: 1.0000
Max. : 204.96604	Max. : 1067.64	Max. : 1057.6385	Max. : 1.0000
status_tau	T_obs	e	
Min. : 0.0000	Min. : 0.00207	Min. : 0.0000017	
1st Qu.: 0.0000	1st Qu.: 1.19569	1st Qu.: 0.0222539	
Median : 0.0000	Median : 3.30205	Median : 0.0974830	
Mean : 0.4796	Mean : 7.71096	Mean : 0.2001436	
3rd Qu.: 1.0000	3rd Qu.: 8.47812	3rd Qu.: 0.3083314	
Max. : 1.0000	Max. : 148.38035	Max. : 0.9684896	

[1] "Descriptive statistics for group A=1: 897"

X1	X2	X3	X4
Min. : -2.13542	Min. : -2.7342	Min. : -5.3672	Min. : -2.20745
1st Qu.: 0.07218	1st Qu.: 0.1398	1st Qu.: -2.1933	1st Qu.: 0.05196
Median : 0.73042	Median : 0.7572	Median : -1.6242	Median : 0.75344
Mean : 0.71441	Mean : 0.7647	Mean : -1.6495	Mean : 0.72697
3rd Qu.: 1.43056	3rd Qu.: 1.4404	3rd Qu.: -1.0671	3rd Qu.: 1.39642
Max. : 3.68158	Max. : 3.8468	Max. : 0.7538	Max. : 3.43270
C	T1	T0	status
Min. : 0.0123	Min. : 10.01	Min. : 0.0084	Min. : 0.0000
1st Qu.: 2.5781	1st Qu.: 12.79	1st Qu.: 2.7936	1st Qu.: 0.0000
Median : 7.0050	Median : 18.98	Median : 8.9838	Median : 0.0000
Mean : 15.5646	Mean : 34.55	Mean : 24.5518	Mean : 0.1795
3rd Qu.: 17.9349	3rd Qu.: 32.59	3rd Qu.: 22.5949	3rd Qu.: 0.0000
Max. : 408.3617	Max. : 535.60	Max. : 525.6036	Max. : 1.0000
status_tau	T_obs	e	
Min. : 0.0000	Min. : 0.01228	Min. : 0.009642	
1st Qu.: 0.0000	1st Qu.: 2.57807	1st Qu.: 0.605492	
Median : 0.0000	Median : 7.00496	Median : 0.849832	
Mean : 0.2386	Mean : 10.89814	Mean : 0.749911	
3rd Qu.: 0.0000	3rd Qu.: 13.72771	3rd Qu.: 0.967684	

Max. :1.0000 Max. :158.52600 Max. :0.999947

[1] "The ground truth for Obs scenario 1 at time 25 is 7.1"

[1] "The ground truth for Obs scenario 2 at #time 25 is 7.1"

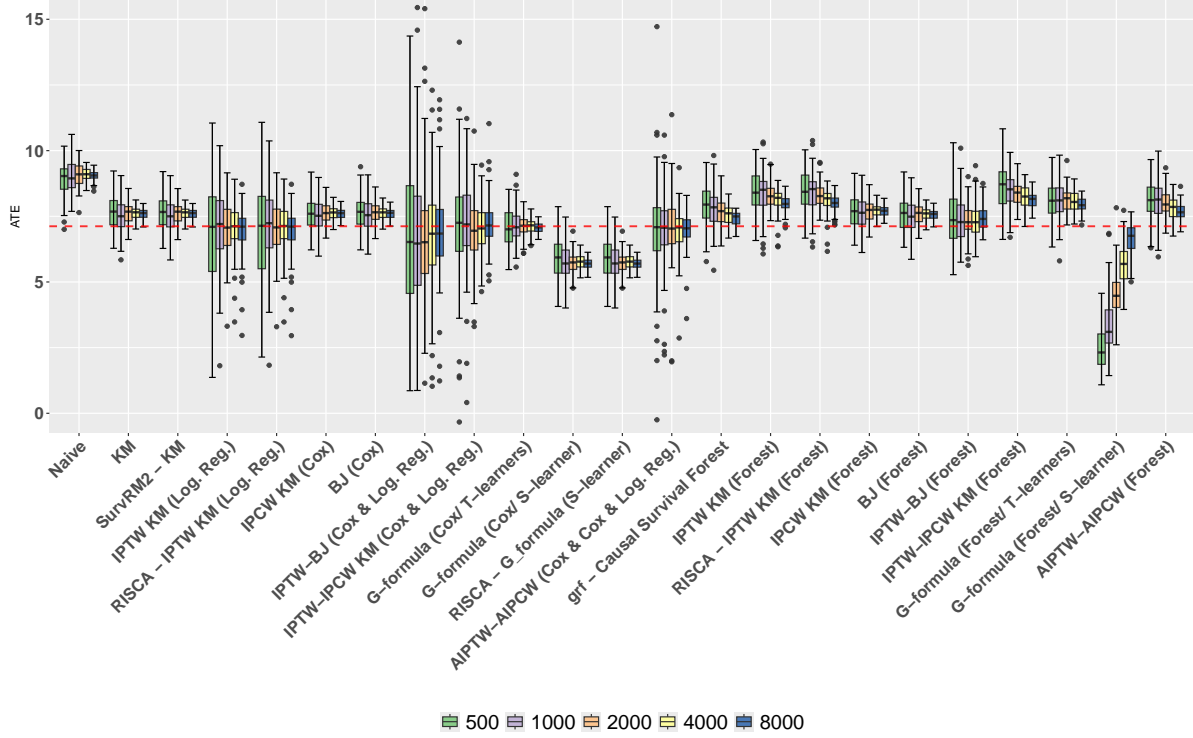


Figure 3: Estimation results of the ATE for the simulation of an observational study with independent censoring.

X1		X2		X3		X4	
Min.	:-2.5835	Min.	:-3.3052	Min.	:-3.27410	Min.	:-2.7741
1st Qu.	:-0.1830	1st Qu.	:-0.2097	1st Qu.	: 0.05041	1st Qu.	:-0.1862
Median	: 0.4562	Median	: 0.4574	Median	: 0.68655	Median	: 0.5089
Mean	: 0.4731	Mean	: 0.4728	Mean	: 0.71901	Mean	: 0.4952
3rd Qu.	: 1.1341	3rd Qu.	: 1.1611	3rd Qu.	: 1.39502	3rd Qu.	: 1.1702
Max.	: 4.0721	Max.	: 4.2091	Max.	: 4.46760	Max.	: 4.8199
tau		A		T0		T1	
Min.	:0.5	Min.	:0.000	Min.	: 0.00000	Min.	: 1.000
1st Qu.	:0.5	1st Qu.	:0.000	1st Qu.	: 0.03414	1st Qu.	: 1.034
Median	:0.5	Median	:1.000	Median	: 0.18426	Median	: 1.184

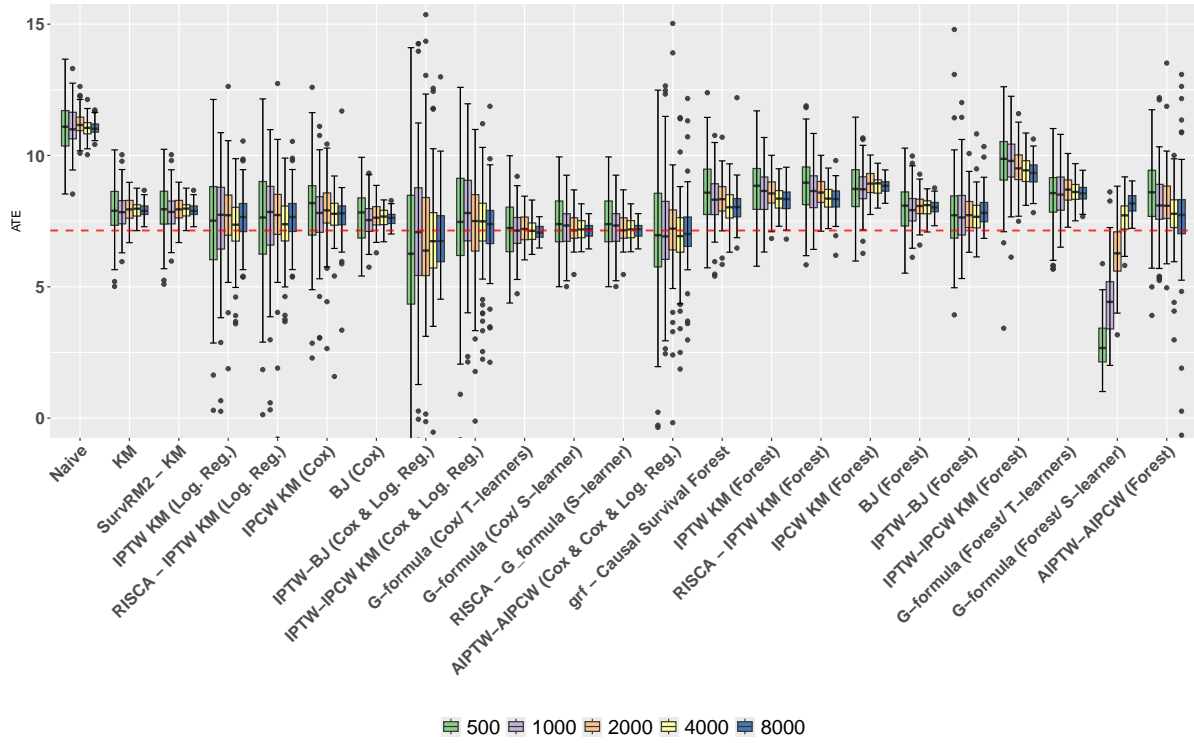


Figure 4: Estimation results of the ATE for the simulation of an observational study with dependent censoring.

Mean	:0.5	Mean	:0.592	Mean	: 0.88261	Mean	: 1.883
3rd Qu.:	0.5	3rd Qu.:	1.000	3rd Qu.:	0.74155	3rd Qu.:	1.742
Max.	:0.5	Max.	:1.000	Max.	:129.39936	Max.	:130.399

C		T_obs		T_obs_tau		status	
Min.	: 0.000	Min.	: 0.00000	Min.	:0.0000002	Min.	:0.0000
1st Qu.:	0.132	1st Qu.:	0.04917	1st Qu.:	0.0491686	1st Qu.:	0.0000
Median	: 0.555	Median	: 0.25320	Median	:0.2532009	Median	:0.0000
Mean	: 6.704	Mean	: 0.55299	Mean	:0.2701239	Mean	:0.4525
3rd Qu.:	1.827	3rd Qu.:	1.00001	3rd Qu.:	0.5000000	3rd Qu.:	1.0000
Max.	:3690.899	Max.	:40.91579	Max.	:0.5000000	Max.	:1.0000

status_tau		censor.status		e	
Min.	:0.000	Min.	:0.0000	Min.	:0.00002
1st Qu.:	0.000	1st Qu.:	0.0000	1st Qu.:	0.42474
Median	:1.000	Median	:1.0000	Median	:0.60172
Mean	:0.614	Mean	:0.5475	Mean	:0.59365
3rd Qu.:	1.000	3rd Qu.:	1.0000	3rd Qu.:	0.82019
Max.	:1.000	Max.	:1.0000	Max.	:0.99992

[1] "Descriptive statistics for group A=0: 816"

X1		X2		X3		C	
Min.	:-2.5376	Min.	:-2.51817	Min.	:-2.53921	Min.	: 0.000
1st Qu.:	-0.1696	1st Qu.:	-0.01548	1st Qu.:	-0.01273	1st Qu.:	0.121
Median	: 0.4797	Median	: 0.67363	Median	: 0.55086	Median	: 0.510
Mean	: 0.4995	Mean	: 0.68994	Mean	: 0.54482	Mean	: 7.752
3rd Qu.:	1.1790	3rd Qu.:	1.40110	3rd Qu.:	1.05513	3rd Qu.:	1.539
Max.	: 3.7169	Max.	: 4.20912	Max.	: 3.04368	Max.	:3690.899

T1		T0		status		T_obs	
Min.	: 1.000	Min.	: 0.00000	Min.	:0.0000	Min.	:0.00000
1st Qu.:	1.024	1st Qu.:	0.02384	1st Qu.:	0.0000	1st Qu.:	0.01611
Median	: 1.130	Median	: 0.13040	Median	:1.0000	Median	:0.07291
Mean	: 1.632	Mean	: 0.63158	Mean	:0.6605	Mean	:0.22610
3rd Qu.:	1.587	3rd Qu.:	0.58689	3rd Qu.:	1.0000	3rd Qu.:	0.27818
Max.	:28.225	Max.	:27.22494	Max.	:1.0000	Max.	:3.85334

status_tau		e	
Min.	:0.0000	Min.	:0.00002
1st Qu.:	0.0000	1st Qu.:	0.23687
Median	:1.0000	Median	:0.44919
Mean	:0.7255	Mean	:0.42923
3rd Qu.:	1.0000	3rd Qu.:	0.59952
Max.	:1.0000	Max.	:0.97938

[1] "Descriptive statistics for group A=1: 1184"

X1	X2	X3	C
Min. : -2.5835	Min. : -3.3052	Min. : -3.27410	Min. : 0.0000
1st Qu.: -0.1901	1st Qu.: -0.3162	1st Qu.: 0.09726	1st Qu.: 0.1409
Median : 0.4490	Median : 0.3208	Median : 0.83010	Median : 0.6042
Mean : 0.4550	Mean : 0.3231	Mean : 0.83905	Mean : 5.9810
3rd Qu.: 1.1113	3rd Qu.: 0.9553	3rd Qu.: 1.66787	3rd Qu.: 2.0396
Max. : 4.0721	Max. : 3.2994	Max. : 4.46760	Max. : 1686.4699

T1	T0	status	T_obs
Min. : 1.000	Min. : 0.00000	Min. : 0.0000	Min. : 0.0000
1st Qu.: 1.044	1st Qu.: 0.04421	1st Qu.: 0.0000	1st Qu.: 0.1409
Median : 1.245	Median : 0.24534	Median : 0.0000	Median : 0.6042
Mean : 2.056	Mean : 1.05562	Mean : 0.3091	Mean : 0.7783
3rd Qu.: 1.879	3rd Qu.: 0.87946	3rd Qu.: 1.0000	3rd Qu.: 1.0831
Max. : 130.399	Max. : 129.39936	Max. : 1.0000	Max. : 40.9158

status_tau	e
Min. : 0.0000	Min. : 0.03365
1st Qu.: 0.0000	1st Qu.: 0.54551
Median : 1.0000	Median : 0.72756
Mean : 0.5372	Mean : 0.70697
3rd Qu.: 1.0000	3rd Qu.: 0.89391
Max. : 1.0000	Max. : 0.99992

[1] "The ground truth for mis scenario at time 0.45 is 0.26"

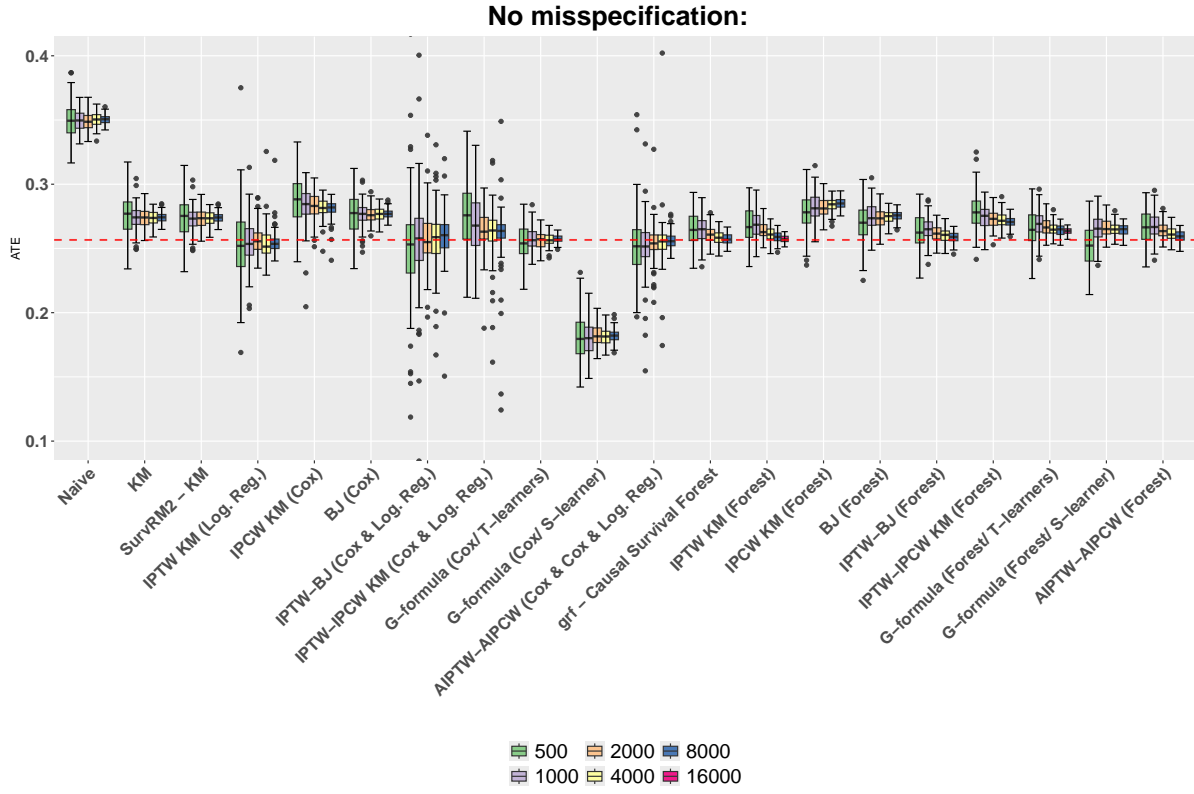


Figure 5: Estimation results of the ATE for the simulation of an observational study with dependent censoring and non linear relationships.

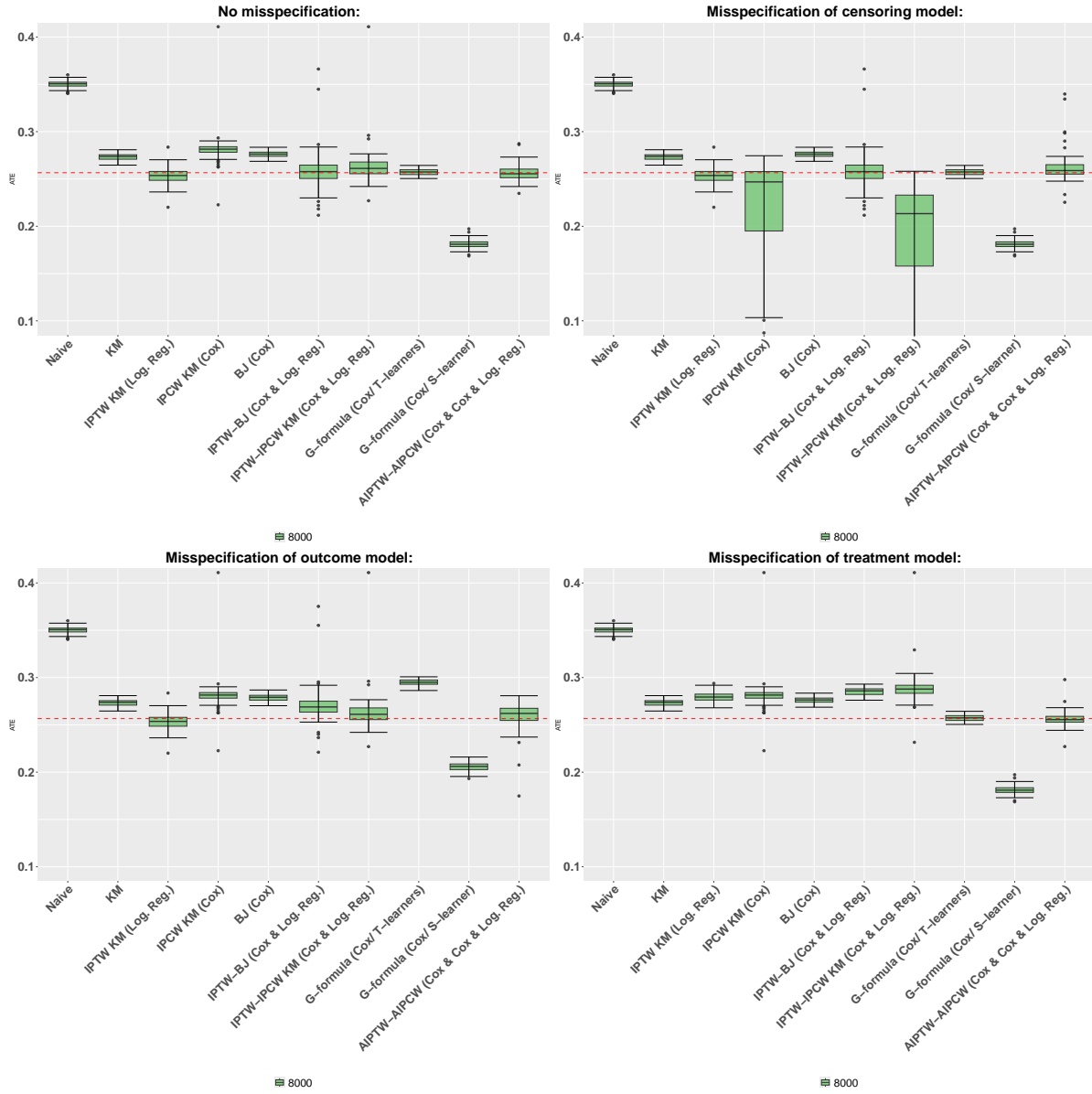


Figure 6: Estimation results of the ATE for an observational study with dependent censoring in case of a single misspecification.

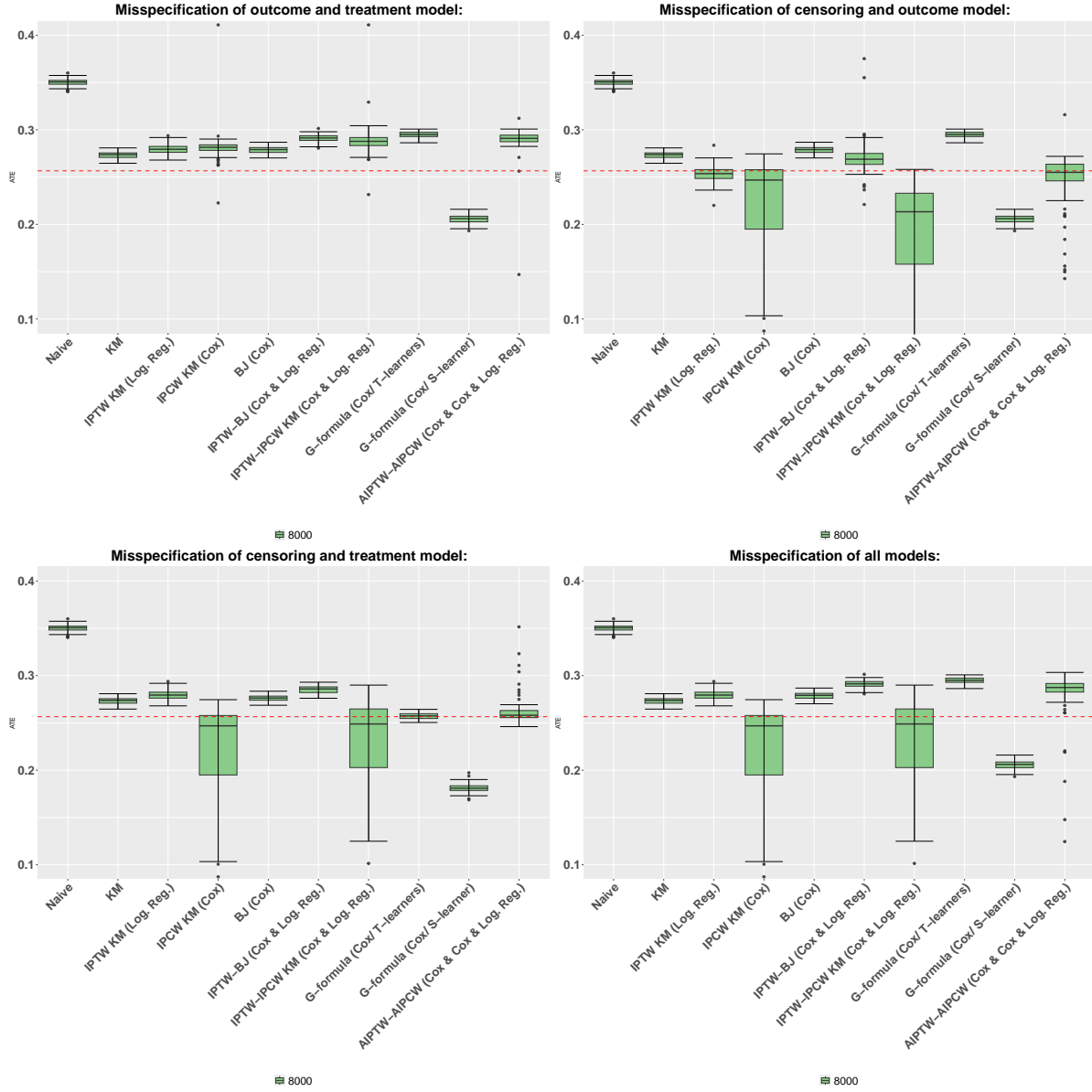


Figure 7: Estimation results of the ATE for an observational study with dependent censoring in case of a two or more misspecifications.