ITR Assignment 3

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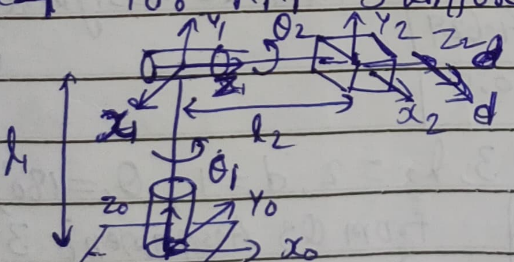
Q1 There are certain configurations for a manipulator at which the velocity components are linearly dependent to each other. In these configurations we cannot solve for joint variables and $\dot{q}$s since we cannot set the velocities independently. ~~Here~~ More formally, the rank of Jacobian matrix $J(q)$ decreases. These configurations are known as singular configurations.

We can check ~~the~~ if the columns of Jacobian are linearly dependent. This means its rank is less than its maximum value. This condition will be a singular configuration.

For a Jacobian, if its columns are close to becoming linearly dependent then it is close to singular configuration. ~~In~~ For a square Jacobian matrix, if its determinant is close to zero, ~~it is~~ the configuration is close to a singular one.

Q4 For RRP Stanford manipulator:




D-H Parameters table:

| | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | $\theta_1 - \frac{\pi}{2}$ | $l_1$ | 0 | $-\pi/2$ |
| 2 | $\theta_2$ | $l_2$ | 0 | $\pi/2$ |
| 3 | 0 | 0 | $d$ | 0 |

Using Q4 from Assignment 2 for the following values: $l_1 = 1$, $l_2 = 2$, $d = 0.5$, $\theta_1 = 60°$, $\theta_2 = 45°$

$$P_0 = \begin{bmatrix} 1.306 \\ 1.555 \\ 0.646 \end{bmatrix}$$

From Q3 of Assignment 3 using the same above values

$$P_0 = \begin{bmatrix} 1.306 \\ 1.555 \\ 0.646 \end{bmatrix}$$

Second example:

Using Q4 from Assignment 2 for:

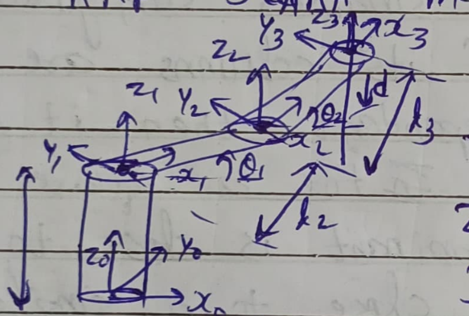
$l_1 = 3$, $l_2 = 2$, $d = 1$, $\theta_1 = 180^\circ$, $\theta_2 = 30^\circ$

$$P_0 = \begin{bmatrix} -2 \\ 0.866 \\ 2.5 \end{bmatrix}$$

From Q3 of Assignment 3 using the same values

$$P_0 = \begin{bmatrix} -2 \\ 0.866 \\ 2.5 \end{bmatrix}$$

For RRP SCARA manipulator




D-H ~~manipul~~ Table

| | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $l_1$ | $l_2$ | 0 |
| 2 | $\theta_2$ | 0 | $l_3$ | 0 |
| 3 | 0 | $-d$ | 0 | 0 |

① For values: $l_1 = 1$, $l_2 = 2$, $l_3 = 3$, $d = 0.5$, $\theta_1 = 60^\circ$, $\theta_2 = 45^\circ$

From Q3 Assignment 2: $P_0 = \begin{bmatrix} 0.224 \\ 4.629 \\ 0.5 \end{bmatrix}$ From Q3 Assignment 3: $P_0 = \begin{bmatrix} 0.224 \\ 4.629 \\ 0.5 \end{bmatrix}$

② For values: $l_1 = 4$, $l_2 = 3$, $l_3 = 2$, $d = 1$, $\theta_1 = 180^\circ$, $\theta_2 = 30^\circ$

From Q3 Assignment 2: $P_0 = \begin{bmatrix} -4.732 \\ -1 \\ 3 \end{bmatrix}$ From Q3 Assignment 3: $P_0 = \begin{bmatrix} -4.732 \\ -1 \\ 3 \end{bmatrix}$

for these values from Q8 assignment 2:

$$J = \begin{bmatrix} 1 & 1 & 0 \\ -4.732 & -1.732 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

from Q3:

$$J = \begin{bmatrix} 1 & 1 & 0 \\ -4.732 & -1.732 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

For values $l_1 = 1,\ l_2 = 2,\ l_3 = 3,\ d = 0.5,\ \theta_1 = 60^\circ,\ \theta_2 = 45^\circ$

From Q8, Assignment 2:

$$J = \begin{bmatrix} -4.629 & -2.898 & 0 \\ 0.224 & -0.776 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

From Q3, Assignment 3:

$$J = \begin{bmatrix} -4.629 & -2.898 & 0 \\ 0.224 & -0.776 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

All the results match from what we calculated before.

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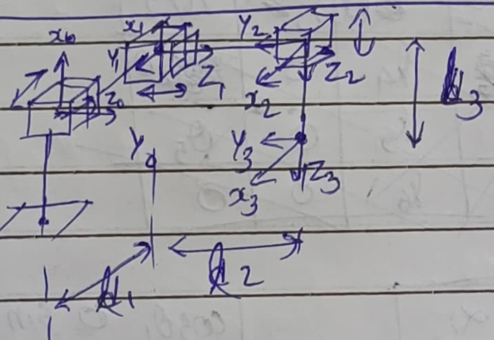
Q5




DH table:

| | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | 0 | $l_1$ | 0 | $-\pi/2$ |
| 2 | $\pi/2$ | $l_2$ | 0 | $-\pi/2$ |
| 3 | 0 | $l_3$ | 0 | 0 |

$$\therefore A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},\quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix},\quad A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~Putting values~~

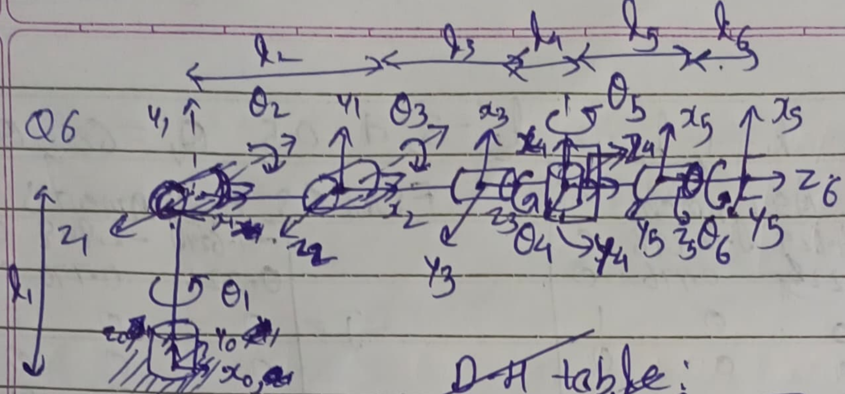
$$\therefore T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & -1 & -l_3 \\ 0 & -1 & 0 & l_2 \\ -1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting values $l_1 = 3,\ l_2 = 2,\ l_3 = 1$ in Q3

we get $T = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

We get the same results.

Q6



~~D-H table:~~

| | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $l_1$ | 0 | $\pi/2$ |
| 2 | $\theta_2$ | 0 | $l_2$ | 0 |
| 3 | $\theta_3+\frac{\pi}{2}$ | 0 | $l_3$ | $\pi/2$ |
| 4 | $\theta_4$ | $l_4$ | 0 | 0 |
| 5 | $\theta$ | $l_5$ | 0 | $\theta_5$ |
| 6 | $\theta_6$ | $l_6$ | 0 | 0 |

D.H. Table:

| | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | 0 | 0 | $\pi/2$ |
| 2 | $\theta_2$ | 0 | $l_2$ | 0 |
| 3 | $\theta_3+\frac{\pi}{2}$ | 0 | 0 | $\pi/2$ |
| 4 | $\theta_4+\frac{\pi}{2}$ | $l_3$ | 0 | $\pi/2$ |
| 5 | $\theta_5+\pi$ | 0 | 0 | $\pi/2$ |
| 6 | $\theta_6$ | $l_6$ | 0 | 0 |

$$\therefore A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $\theta_i = 0$ for all $i$, by multiplication, we get:

$$T_0^6 = \begin{bmatrix} 0 & 0 & 1 & l_2+l_3+l_6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting $l_2 = 1$, $l_3 = 2$ and $l_1 = 6$ in (3)
we get

$$T = \begin{bmatrix} 0 & 0 & 1 & 6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is the expected result.

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Q7

Direct Drive :

The transmittion of torque happens without gears. This gives a quicker response time and no backlash. In this case, the motor is directly attached to the joint. ~~This~~ Hence this has the least cost. But, for this drive we need a high torque, low rpm motor.

Remotely Driven Joint:

In this arrangement, Motor is in the base frame and the joint is driven by using a mechanism to transmit the torque for eg, gears. This is easy to implement. The weight of the motor will be not a problem since motor is at the base.

5-bar Parallelogram :

In this arrangement, two motors are ~~on~~ at the base frame and a 5-bar mechanism is arranged in which 4-bars form parallelogram and the remaining 1 bar and one of the 4-bars act as 2R mechanism. It is smaller in size than the other two. It has ~~not~~ similar advantages as Remotely Driven Joints.

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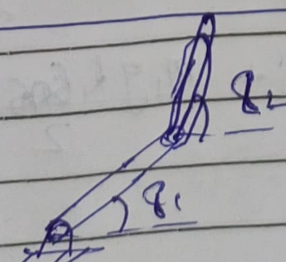
Q8




$$v_{c_1} = \begin{bmatrix} -l_2 \sin(q_1/2) \\ l_2 \cos(q_1/2) \\ 0 \end{bmatrix} \dot{q}_1$$

$$V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin(q_2)/2 \\ l_1 \cos q_1 & l_2 \cos(q_2)/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad \begin{matrix} \omega_1 = \dot{q}_1 \hat{k} \\ \omega_2 = \dot{q}_2 \hat{k} \end{matrix}$$

$$K.E : K = \frac{1}{2} \sum_{i=1}^{2} (m_i v_{c_i}^T v_{c_i} + \omega_i^T I_i \omega_i) = \sum \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

Here,
$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & \frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2} \\ \frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2} & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

$$P.E : V = \frac{m_1 g l_1 \sin q_1}{2} + m_2 g l_1 \sin q_1 + \frac{m_2 g l_2 \sin q_2}{2}$$

We have equations:

$$\sum_j d_{kj}(q) \ddot{q}_j + \sum_{i,j} c_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k$$

$$c_{ijk} = \frac{1}{2} \left[ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial d_k} \right]$$

$$\therefore c_{111} = c_{121} = c_{211} = c_{212} = c_{122} = c_{222} = 0$$

$$c_{112} = \frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}, \quad c_{221} = -\frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = \frac{m_1 g l_1 \cos q_1}{2} + m_2 g l_1 \cos q_1$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = \frac{m_2 g l_2 \cos q_2}{2}$$

Hence the equations are:

$$\tau_1 = \left( \frac{m_1 l_1^2}{4} + m_2 l_2^2 + \frac{m l_1^2}{12} \right) \ddot{q}_1 + \frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2} \ddot{q}_2 - \frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1) \dot{q}_1^2 + \frac{m_1 g l_1 \cos q_1}{2} + m_2 g l_1 \cos q_1$$

$$\tau_2 = \frac{m_2 l_1 l_2 \cos(q_2 - q_1)\ddot{q}_1}{2} + \frac{m_2 l_2^2 \ddot{q}_2}{3} + \frac{m_2 l_1 l_2 \sin(q_2 - q_1)\dot{q}_1^2}{2}$$
$$+ m_2 g l_2 \cos q_2 / 2$$

These are same equations as derived in Miniproject-1

Q 11

We know from Euler Lagrange's Equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k \quad , \quad k = 1, 2 \ldots n \quad \text{--- } (1)$$

Here, $L = K - V$

$$\text{K.E.} : K = \frac{1}{2}\sum_{i=1}^{n} (m_i v_{c_i}^T v_{c_i} + \omega_i^T I_i \omega_i) = \sum_{i=1} \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

$$\text{P.E} \quad V = V(q)$$

$$\therefore L = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i \dot{q}_j - V(q)$$

$$\therefore \frac{\partial L}{\partial \dot{q}_k} = \sum_{i=1}^{n} d_{kj}(q)\dot{q}_j \quad \left(\frac{\partial V(q)}{\partial \dot{q}_k} = 0 \text{ since } V(q) \text{ is not dependent on } \dot{q}\right)$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_{j}^{n} d_{kj}(q)\ddot{q}_j + \sum_{j}^{n} \frac{d}{dt}(d_{kj}(q)\dot{q}_j)$$

$$= \sum_{j}^{n} d_{kj}(q)\ddot{q}_j + \sum_{i,j}^{n} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i \dot{q}_j \quad \text{--- } (2)$$

$$\text{And } \frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j}^{n} \frac{\partial d_{ij}}{\partial q_k}\dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k} \quad \text{--- } (3)$$

Substituting (2) and (3) in (1)

$$\therefore \sum_{j}^{n} d_{kj}\ddot{q}_j + \sum_{i,j}^{n}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \quad \text{--- } (4)$$

Since $D(q)$ is symmetric,

$$\sum_{i,j}^{n}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i \dot{q}_j = \frac{1}{2}\sum_{i,j}^{n}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_i \dot{q}_j$$

Hence equation (4) becomes:

$$\sum_{j}^{n} d_{kj}\ddot{q}_j + \sum_{i,j}^{n} \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

Christoffel symbols:

Putting $c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$

$$\therefore \sum_{j}^{n} d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

Here, $\phi_k(q) = \frac{\partial V}{\partial q_k}$

We write the eq$^n$ in matrix form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$