Sanskar Agrawal
RA1811032010051

Maths Assignment

1) $(A-C) \cap (C-B) = \phi$

$(A-C)\cap(C-B) = \{ x / x \in A \text{ and } x \notin C \text{ and } x \in C \text{ and } x \notin B \}$

$= \{ x / x \in A \text{ and } (x \notin C \text{ and } x \in \bar{C}) \text{ and } x \in \bar{B} \}$

$= \{ x / x \in A \text{ and } x \in \phi \text{ and } x \in \bar{B} \}$

$= \{ x / x \in \phi \text{ and } x \in \bar{B} \}$

$= \{ x / x \in \phi \}$

$= \phi$ = RHS , Hence Proved.

2) $A = \{1, 2, 3\}$
$R = \{ (1,1), (1,3), (2,2), (3,1), (3,3) \}$

$$\therefore M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{R^{-1}} = (M_R)^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{R^2} = M_R * M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

~~5) a^2+b is even~~

3) $f(x) = x+2$

$g(x) = x-2$

$fog = f(g(x)) = f(x-2) = (x-2)+2$
$= x$

$gof = g(f(x)) = g(x+2) = (x+2)-2$
$= x$

$fog = gof$, Hence Proved.

4) $f(x) = ax+b$

$g(x) = 1-x+x^2$

$(g\,of)(x) = 9x^2-9x+3$

$g(f(x)) = g(ax+b) = 1-(ax+b)+(ax+b)^2$

$= 1-ax-b+a^2x^2+b^2+2abx$

$9x^2-9x+3 = (1+b^2-b)+x(2ab-a)+a^2x^2$

$\therefore a^2 = 9$ $\qquad$ $2ab-a = -9$

$\underline{a = \pm 3}$ $\qquad$ Put $a = 3$,

$6b-3 = -9$

$\boxed{b=-1}$

Put $a=-3$,

$-6b+3 = -9$

$\boxed{b=+2}$ $\qquad$ $1+b^2-b = 3$

$\boxed{b=2}$, $\boxed{b=-1}$

$\therefore$ $(a = 3, b = -1)$ and $(a = -3, b = 2)$ both satisfy the eqn.

5) $A = \{a, b, c, d\}$
$R = \{(a,a), (a,c), (a,d), (b,b), (c,a), (c,c), (d,a), (d,d)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

All the diagonal elements of M_R are equal and equal to 1.
M_R is reflexive and thus R is reflexive.

M_R is transitive, because all elements share their corresponding pair i.e. (1, 2) and (2, 1)
$\therefore$ R is transitive.

Also M_R is symmetric along the diagonal, so does the R.

As R is reflexive, transitive & Symmetric in nature, R is an Equivalence Relation.

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5) $A = \{a, b, c, d\}$

$R = \{(a,a), (a,c), (a,d), (b,b), (c,a), (d,a), (d,d)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Since all the elements of the ~~m~~ diagonal of M_R are not equal to 1, it is not reflexive.

Thus it won't be equivalence Relation.

6) $R = \{(1,1), (1,3), (2,2)\}$

$S = \{(1,2), (1,3), (2,1), (2,2), (3,3)\}$

$R \cup S = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,3)\}$

$$M_{R \cup S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R \cap S = \{(1,3), (2,2)\}$

$$M_{R \cap S} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Part - C

1) The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements.

(i) $(A\cup B)' = A' \cap B'$

Let $P = (A\cup B)'$ and $Q = A' \cap B'$

Let x be an arbitrary element of P then, $x \in P = x \in (A\cup B)'$

$\Rightarrow x \notin (A\cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow x \in Q$ $\therefore P \subset Q$ ——— (1)

Let y be an arbitrary element of Q then $y \in Q \Rightarrow y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$y \notin A$ and $y \notin B$

$y \notin (A\cup B)$

$y \in (A\cup B)'$

$y \in P$ $Q \subset P$ ——— (2)

Now combining (1) & (2),

$P = Q$

$(A\cup B)' = A' \cap B'$

(ii) $(A\cap B)' = A' \cup B'$

Let $M = (A\cap B)'$ and $N = A' \cup B'$

Let x be an arbitrary element of M then,

$x \in M \Rightarrow x \in (A\cap B)'$

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$x \notin (A \cap B)$

$x \notin A$ and $x \notin B$

$x \in A'$ and $x \in B'$

$x \in A' \cup B'$

$x \in N \quad \therefore M \subset N$ ——— ①

let y be an arbitrary element of N then, $y \in A' \cup B'$

$y \in A'$ or $y \in B'$

$y \notin A$ or $y \notin B$

$y \notin (A \cap B)$

$y \in (A \cap B)'$

$y \in M \quad \therefore N \subset M$ ——— ②

Combining ① & ②,

$M = N,$

$(A \cap B)' = A' \cup B'$

2) $(A \cap B) - C = (A - C) \cap (B - C)$

Let $x \Rightarrow \{x \mid x \in (A \cap B) - C\}$

$\Rightarrow \{x \mid x \in (A \cap B)$ and $x \notin C\}$

$\Rightarrow \{x \mid ((x \in A)$ and $(x \in B))$ and $x \in C'\}$

$\Rightarrow \{x \mid (x \in A$ and $x \in C')$ and $(x \in B$ and $x \in C')\}$

$\Rightarrow \{x \mid (x \in (A - C))$ and $(x \in (B - C))\}$

$\Rightarrow \{x \mid x \in (A - C)$ and $x \in (B - C)\}$

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$\Rightarrow \{x \mid x \in (A-C) \cap (B-C)\}$

$\Rightarrow$ RHS , Hence Proved.

3) $a^2 + b$

Taking a & b as odd numbers.

$(odd)^2 + odd = even$ (satisfys)

a as odd & b as even (+ve) integers.

$(odd)^2 + even = odd$ (does not satisfy)

a as even & b as odd

$(even)^2 + odd = odd$ (" " " ")

both even (+ve) integers.

$(even)^2 + even = even$ (satisfys)

$\therefore M_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since all the elements in the main diagonal of M_R and equals to 1 each, R is reflexive relation.

Also M_R is symmetric Matrix, R is a symmetric relation.

$M_{R^2} = M_R \cdot M_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore R^2 \subseteq R$ $\therefore$ R is a transitive relation.

Thus R is ~~and~~ equivalence relation.

4) $a^m = a^1 = a$

$\therefore (a,a) \in R \Rightarrow R$ is Reflexive.

Let $(a,b) \in R$ and $(b,a) \in R$

$\Rightarrow b = a^m$ and $a = b^n$ $(m, n \in N)$

$\therefore a = (a^m)^n = a^{mn}$ ——— (1)

$\therefore mn = 1$ at $a = 1$ or $a - 1$

(1) If $mn = 1$

$m = 1, n = 1$

$\Rightarrow a = b$ (from (1))

(2) If $a = 1$, $b = 1^m = 1 = a$

Also, $a = 1^n = 1 = b$

$\Rightarrow a = b$

(3) If $a = -1$, $\Rightarrow b = -1$

$\therefore a = b$

$\therefore$ from (1), (2), (3), R is Antisymmetric.

Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow b = a^m$ and $c = b^n$

$\therefore c = (a^m)^n = a^{mn}$

$\therefore (a,c) \in R$

$\Rightarrow$ R is transitive.

$\Rightarrow$ R is a partial Ordering.

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5) $R = \{(1,1),(1,3),(1,5),(2,3),(2,4)(3,3),(3,5),(4,2),(4,4),(5,4)\}$

$A = \{1, 2, 3, 4, 5\}$

Assuming $W_0 = M_R$,

$$W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$|A| = 5$, so we need to compute W_1, W_2, W_3, W_4, W_5

$\therefore$ W_5 will be outr transitive closure.

Col	Row	(P_i, Q_i)	W_k
1	1, 3, 5	(1,1) (1,3) (1,5)	W_0 (remains same)
1	3, 4	(1,3), (1,4)	$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
1, 2, 3	3, 5	(1,3), (1,5), (2,3), (2,5) (3,5) (3,3)	$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
1, 2, 4, 5	2, 4	(1,2), (1,4) (2,2), (2,4) (4,2), (4,4) (5,2), (5,4)	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

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1, 2, 3   2, 4   (1,2), (1,4)
(2,2) (2,4)
(3,2) (3,4)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$\therefore R^\infty = \{(1,1), (1,2), (1,3), (1,4), (1,5),$
$(2,2), (2,3), (2,4), (2,5),$
$(3,2), (3,3), (3,4), (3,5),$
$(4,2), (4,4), (5,2), (5,4)\}$

6) For composition of functions, it is necessary that the functions are one - one and onto.
Consider f and g to be invertible functions.

Since they are invertible ∴ They are bothe one - one and onto.

∴ gof is also bijective ⇒ gof is invertable.

∴ The composition of invertible functions is also invertible.

7) $A = \{2, 4, 5, 10, 12, 20, 25\}$

(a divides b)

$R = \{(2,2), (2,4), (2,10), (2,12), (2,20), (4,12), (4,20)$
$(4,4), (5,5), (5,10), (5,20), (5,25), (10,10)$
$(10,20), (12,12), (20,20), (25,25)\}$

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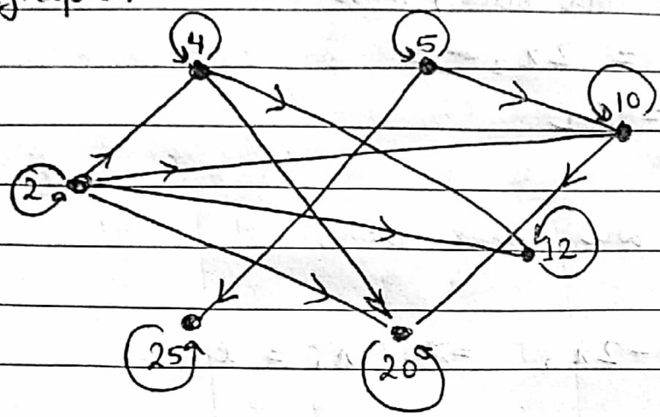
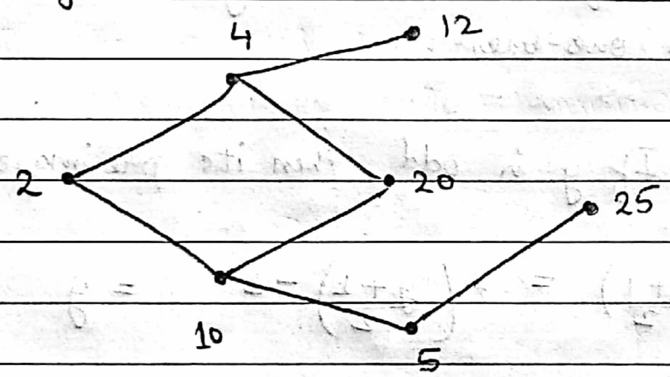
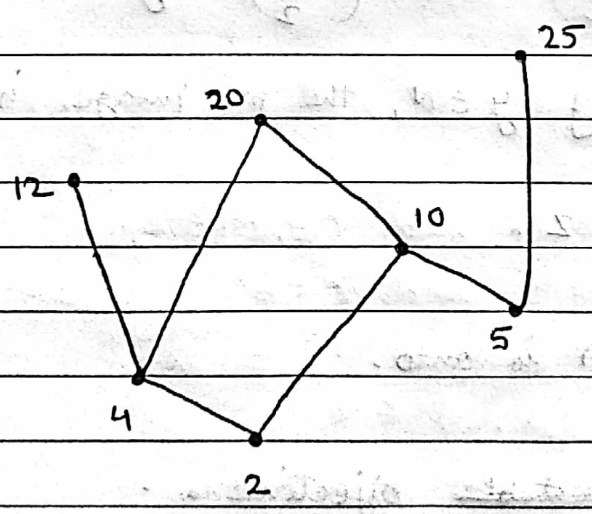
Diagraph :



Hasse Diagram





8) $f(x) = \begin{cases} 2x-1, & x>0 \\ -2x, & x \le 0 \end{cases}$

Let $x_1, x_2 \in Z$ and $f(x_1) = f(x_2)$.
Then either $f(x_1)$ and $f(x_2)$ are both odd or both even.

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If they are both odd, then

$$2x_1 - 1 = 2x_2 - 1$$

$\therefore \ x_1 = x_2$

If they both are even, then,

$$-2x_1 = -2x_2 \Rightarrow x_1 = x_2$$

Thus if $f(x_1) = f(x_2)$ then we get $x_1 = x_2$.

So, f is one-one.

Let $y \in N$. If y is odd, then its pre image is $\frac{y+1}{2}$

Since $f\left(\frac{y+1}{2}\right) = 2\left(\frac{y+1}{2}\right) - 1 = y$

If y is even, then its pre-image is $-\frac{y}{2}$

Since $f\left(\frac{-y}{2}\right) = -2\left(\frac{-y}{2}\right) = y$

Thus for any $y \in N$, the pre image is $\frac{y+1}{2} \in Z$

or $\frac{-y}{2} \in Z$.

Hence, $f(x)$ is onto.

$\therefore$ f is ~~invertible~~ bijective.

10) R {a, b | 3a+b is a multiple of 4}

If $a = b$,

$3a + a = 4a$ (which is divisible by 4)

∴ R is reflexive.

If $3a + b = 4k$, (multiple of 4)

then $3b + a = 3(4k - 3a) + a$

$= 12k - 9a + a$

$= 12k - 8a$

$= 4(3k - 2a)$

(which is divisible by 4)

∴ R is symmetric.

If $3a + b = 4k$ and $3b + c = 4l$ —— (2)

$b = 4k - 3a$ —— (1) put in (2)

$\Rightarrow 3(4k - 3a) + c = 4l$

$12k - 9a + c = 4l$

$\Rightarrow 12k - 12a + 3a + c = 4l$

$3a + c = 4l + 12a - 12k$

$3a + c = 4(l + 3a - 3k)$

(which is divisible by 4)

∴ R is transitive.

As R is Reflexive, Symmetric & Transitive.

∴ R is an Equivalence Relation.