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Automatic Fault Detection and Diagnosis for Sensor Based on KPCA

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***Abstract*—Automatic Fault detection and diagnosis for sensor is necessary, which affects the performance of the control system seriously. The KPCA effectively captures the nonlinear relationship of the process variables, which computes principal component in high-dimensional feature space by means of integral operators and nonlinear kernel functions. The KPCA method was used in diagnosing for four familiar sensor faults. At first it detected fault by *Q* statistics, at second it identified fault by T^2 contribution percent variation. The experiment showed the KPCA method had good performance in fault detection and diagnosis.**

Keywords- Sensor; Kernel principal component analysis; Fault detection and diagnosis

I. Introduction

With the development of manufacturing technology, more and more sensors are integrated to the control system. Nowadays the sensors play an important role in the control systems. On the one hand they control instantaneously the system by the parameters they measured; on the other hand they provide information of the system working state to the operators. However, there are usually some faults in the sensor according to their bad work environment and other reasons. Once the sensor faults occurred, they would reduce the reliability of the control system. For example, the hard fault would make the system invalidation or damage the system seriously; the soft fault would make the system working with low efficiency or consuming energy seriously. It is necessary to develop technology of fault detection and diagnosis (FDD) for sensor. The FDD methods for sensor can be divided two types, which are based on model method and based on knowledge method. Recently the PCA as a qualitative model is applied to FDD for sensor. Though it works effectively for FDD to some degree, the PCA is a liner method [1]. In fact, the real control systems are always nonlinear. The nonlinear method would like to be fit the real systems well. Aiming to real control system that is nonlinear, the Kernel Principle Component Analysis (KPCA) is proposed for sensor FDD.

II. Sensor Fault

The sensor faults can be divided four types, which are deviation fault, drift fault, precision grade reduction fault and fail fault. The first three belong to soft faults, and the last one belongs to hard fault. The soft fault is a typical fault, it is difficult to be detected and wouldn't make the system invalidation, but it is very harmful to the system. For instance, the soft fault can lead to outburst fault, reduce the work efficiency, and consume more energy and so on.

The measured value contains three sections

$$x_t = x_r + f_t + u \tag{1}$$

In above equation, x_t is the measured value, x_r is the real value, f_t is the system error, and u is the random measured error. The random error submits zero mean normal distribution $u \sim N(0,\sigma^2)$. σ^2 is the error variance, which is attributing to the measured system or the randomization of measured process, and it is unpredictable. The system error f_t is mainly attributing to the fault. f_t has varied function expression according to the varied faults.

A. Deviation fault

In the deviation fault, the difference between the fault measured value and the normal measured value is a constant. The deviation fault model can be presented

$$f_t = b \tag{2}$$

In above equation, b is a constant.

B. Drift fault

The drift fault is the fault whose value is varying according to time. The drift fault model can be presented

$$f_t = d(t - t_s) \tag{3}$$

In above equation, d is a constant, t_s is the fault appearance moment, and t is the current moment.

C. Precision grade reduction fault

Both the deviation fault and the drift fault appear biases in the measured means. The measured mean of the precision grade reduction fault hasn't changed, but the measured variance has changed. The precision grade reduction fault model can be presented

$$f_t \sim N(0,\sigma_1^2) \tag{4}$$

In above equation, σ_1^2 denotes measured variance.

Combing the equation (4) with the equation (1), we can get

$$x_t = x_r + N(0,\sigma_1^2) + u \tag{5}$$

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Because u submits zero mean normal distribution, the sums of two normal distributions are still submitting the normal distribution according to the probability theory. Then we can get

$$x_t = x_{rt} + N(0, \sigma^2 + \sigma_1^2) \quad (6)$$

From equation (6) we can conclude this fault is similar to the increment of random error variance. So this fault is difficult to be identified.

D. Fail fault

The measured value of fail fault hasn't changed along with the real value, but it holds the invariant reading. The invariable value is usually zero or the max reading.

The fail fault model can be presented

$$f_t = c - x_r - u \quad (7)$$

In above equation, d is a constant.

III. KPCA

KPCA is a nonlinear PCA method that develops in recent years [2-3]. It has the same characteristic of mathematics and statistics to PCA in the feature space, but it can extract more sample information than PCA. To the same performance requirement, KPCA needs less principle components than PCA. Compared with the other nonlinear methods, KPCA is only involved with the calculation of eigenvalue decomposition and needn't solve the nonlinear optimization problem. KPCA maps the input space to the feature, in which the principle component is calculated. KPCA finds a controllable solution by the kernel function, whose essence is to construct the mapping from the input space to the feature space. So KPCA is a PCA method in the input feature. If the application purpose of PCA is to reduce the nonlinear relationship of the given data set $x_k \in R^m (k = 1,2,\cdots,m)$ by diagonal covariance matrix, the covariance matrix can be represented in the feature space

$$C^F = \frac{1}{N}\sum_{i=1}^{N} \varphi(x_i)\varphi(x_i)^T \quad (8)$$

Here, we makes the assumption that $\sum_{i=1}^{N}\varphi(x_i) = 0$, and $\varphi(\bullet)$ is the nonlinear mapping function from the input variable to the F feature space. In order to get the diagonal covariance matrix, it needs to solve the eigenvalue problem

$$\lambda v = C^F v \quad (9)$$

The eigenvalue $\lambda \geq 0$, and $v \in F \setminus \{0\}$. The $C^F v$ can be presented as following

$$C^F v = \left(\frac{1}{N}\sum_{i=1}^{N}\varphi(x_i)\varphi(x_i)^T\right)v = \frac{1}{N}\sum_{i=1}^{N}\langle\varphi(x_i), v\rangle\varphi(x_i) \quad (10)$$

In the above equation, $\langle x, y\rangle$ presents the internal product. The expression $\lambda v = C^F v$ is equivalent to

$$\lambda\langle\varphi(x_k), v\rangle = \langle\varphi(x_k), C^F v\rangle \quad k = 1,2,\cdots N \quad (11)$$

There is coefficient $a_j (j = 1,2,\cdots N)$ that satisfied to

$$v = \sum_{j=1}^{N} a_j \varphi(x_j) \quad (12)$$

Composing the equation (8), (11) and (12), we can get

$$\lambda\sum_{j=1}^{N} a_j\langle\varphi(x_k), \varphi(x_j)\rangle = \frac{1}{N}\sum_{j=1}^{N} a_j\left\langle\varphi(x_k), \sum_{i=1}^{N}\varphi(x_i)\right\rangle\langle\varphi(x_i), \varphi(x_j)\rangle \quad (13)$$

The equation (13) only introduces the internal product of mapping vector in the feature space.

Defining the kernel matrix $K_{N\times N}$, and its element

$$K_{ij} = \langle\varphi(x_i), \varphi(x_j)\rangle, \ 1 \leq i \leq N, 1 \leq j \leq N \quad (14)$$

Because of K is a symmetry matrix, the equation (13) can be simplified by

$$N\lambda\alpha = K\alpha \quad (15)$$

We can calculate the feature vector v of C^F by the feature vector α of K matrix, and this is also the principle component direction.

In this way, the principle component t of the test vector can be extracted by mapping $\varphi(\bullet)$ to eigenvector in the F space as following

$$t_k = \langle v_k, \varphi(x)\rangle = \sum_{j=1}^{N} a_j^k\langle\varphi(x_j), \varphi(x)\rangle \quad (16)$$

In above equation, $k = 1,2,\cdots p$ and p is the number of principle component.

The kernel function $k(x, y) = \langle\varphi(x), \varphi(y)\rangle$ that expressed in the format of internal product introduced in the feature space can avoid calculating the nonlinear mapping. The selection of function determines the mapping φ and the feature space F.

There are three type kernel functions used in KPCA usually, they are radial based function, q order polynomial, and two-layer perceptron [4].

The radial based function is expressed as

$$k(x, x_i) = \exp\left(-\frac{\|x - x_i\|^2}{\sigma^2}\right) \quad (17)$$

In above equation, σ^2 is the specified width of the kernel function.

The q order polynomial is expressed as

$$K(x, x_i) = (x^T x_i + 1)^q \quad (18)$$

The two-layer perceptron is expressed as

$$K(x, x_i) = \tanh(\beta_0 x^T x_i + \beta_1) \quad (19)$$

In the application of KPCA, the data are usually normalized by the mean and variance in the normal state, and the feature data need to be centralizing according to derivation that the mean of data is zero. Here, we can replace the kernel matrix K by the matrix $\hat{K}$ as following.

$$\hat{K} = K - l_N K - K l_N + l_N K l_N \quad (20)$$

In above equation, $l_N \in R^{N\times N}$ and all the elements of it are equal to $\frac{1}{N}$.

IV. FDD FOR SENSOR

A. Fault detection

As the same as PCA, KPCA utilizes the T^2 statistics of Hotelling and Q statistics [5,6]. The Q statistics is also named Square Prognosis Error (SPE). T^2 is the standard square sum of the principle vector, which represents the degree of every sample's deviation from the model in the changing trend and amplitude value, and it represents the measure of inner model change. SPE is another important statistic index of performance detection, it represents the error between the changing trend and the statistical model at each sampling time, and it is the measure of outside data change.

T^2 is defined as following

$$T^2 = [t_1, t_2, \cdots t_p]\Delta^{-1}[t_1, t_2, \cdots t_p]^T \tag{21}$$

In above equation, Δ^{-1} is the inverse of diagonal matrix composed with the principle component value, and t_k is calculated by equation (16).

SPE is defined as following

$$SPE = \left\|\varphi(x) - \varphi_p(x)\right\|^2 \tag{22}$$

In above equation, $\varphi(x)$ is the sum of the product of the score values and eigenvalues, which are corresponding nonzero eigenvalues. The above expression can be reduced as following

$$SPE = \sum_{j=1}^{n} t_j^2 - \sum_{j=1}^{p} t_j^2 \tag{23}$$

The fiducial limit of SPE can be calculated as following

$$SPE_\alpha \sim g\chi_h^2 \tag{24}$$

In above equation, α is the conspicuous level, g denotes the weight parameter of SPE, and h denotes the freedom of SPE. If a and b are respectively the mean and variance of the real SPE value, we can calculate g and h as following

$$g = b/2a\,,\ h = 2a^2/b \tag{25}$$

Here, the SPE statistics is applied to sensor fault detection.

B. Fault diagnosis

After the fault has been detected, we need to find the fault variable and analysis the reason of the fault so as to finish fault diagnosis. SPE index can detect fault but not identify the fault. In the PCA method, there is liner relationship between fault variable and monitoring variable, so the contribution is easily calculated, which is applied to draw contribution scheme. In the KPCA method, there isn't explicit expression in nonlinear transform, and the kernel function can't supply the relationship between the original variable and the monitoring variable, so the method which is similar to contribution scheme in PCA can't applied to fault diagnosis.

In this study, a fault isolation method is proposed, which named the percent variation of contribution variable. This method employs a new contribution scheme solution process, which is applied to resolve the fault variable by original measured variable, and then it isolates the fault by comparing the percent variation of contribution variable fore and after fault occurs so as to we can analysis the fault season further. In this method of fault isolated, we make assumption that the fault occurs to one of the sensor variables at one moment, whose variation of contribution variable will be the biggest fore and after this moment. By this way, we can find the fault sensor according to this biggest variation of contribution variable.

Defining the j original measured variation contributes to the T^2 statistics

$$cntr_{j,i} = \sum_{i=1}^{p}\left|t_i^T x_j / \lambda_i\right| \tag{26}$$

The percent variation of contribution variable is

$$\Delta cper_j = cper_{tf,j} - cper_{tn,j}\,,\ cper_{tn/tf,j} = \frac{cntr_{j,i}}{\sum_{j=1}^{n} cntr_{j,i}} \tag{27}$$

In above equation, p is the number of principle component, t_i is the i nonlinear principle component, x_j is the j sensor measured variable, tn and tf are the moment of normal and fault state, and n is the sensors number of the system.

V. SIMULATION

In this study, the KPCA model for FDD is composed of eight sensor variables, and in the model the width of radial based function $\sigma = 0.7746$, the number of principle component is calculated as following

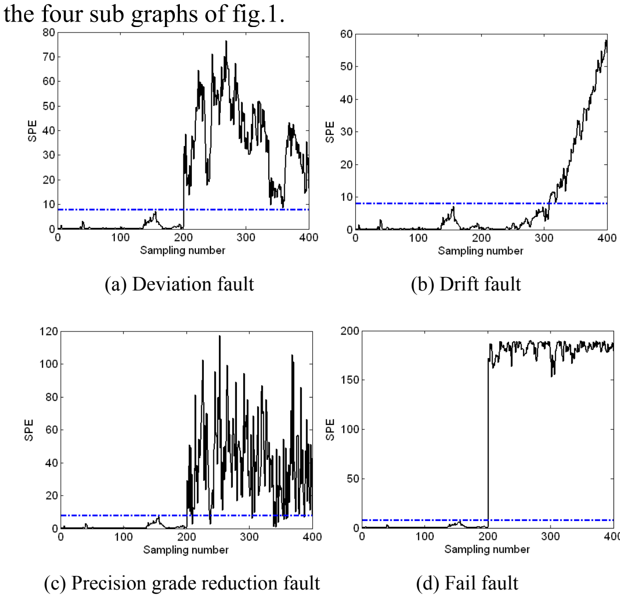
$$\frac{\sum_{k=1}^{p}\lambda_k}{\sum_{k=1}^{m}\lambda_k} \geq c_p \tag{28}$$

In above equation, λ_k is the eigenvalue, p is the number of principle component, m is the number of all the eigenvalues, and c_p is the specified constant. Here, $c_p = 0.85$.

In this model the number of principle component $p = 5$ according to equation (28).

In order to validate the FDD method proposed in this research, the four faults described in the Part two of this paper are introduced to first sensor.

The fault detection result can be respectively illustrated by the four sub graphs of fig.1.



(a) Deviation fault (b) Drift fault

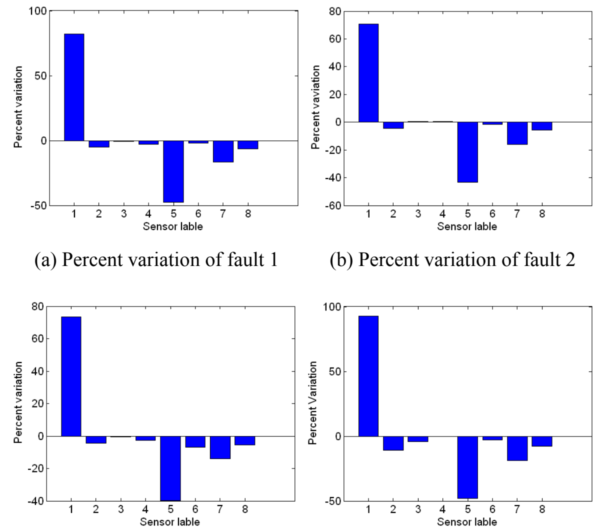
(c) Precision grade reduction fault (d) Fail fault

Fig. 1. Fault detection by *SPE*.

In fig.1 the dash dot line represents the *SPE* detection limit, and the solid line represents the real *SPE* detection value. The first 200 sampling are in normal state, the last 200 sampling are in fault state. In the normal state the real *SPE* detection value is less than the detection limit, which demonstrates there is no fault in the sensor system. In the fault state the real *SPE* detection value is obviously bigger than the detection limit, which indicates there is some fault in the sensor system.

From the demonstrations in fig.1, we can distinctly make out that the four faults introduced are detected exactly because all the results in every sub graphs of fig.1 are coincident to the fact.

After the fault has been detected, we need to isolates the fault so as to make out the fault occurs to which sensor. By the method of percent variation of contribution variable, the fault isolation result can be illustrated respectively by the four sub graphs of fig.2.



(a) Percent variation of fault 1 (b) Percent variation of fault 2

(c) Percent variation of fault 3 (d) Percent variation of fault 4

Fig. 2. Percent variation of contribution variable.

In every sub graphs of fig.2, it demonstrates the first percent variation of contribution variable is the biggest, so we can conclude the fault is located in the first sensor, and this conclusion is coincident to the fact.

On the base of the fault isolated result, we can further analysis which fault type of the fault sensor belongs to according to the four faults model specified in the Part two of this paper.

By now the whole FDD process is finished, the fault sensor and whose fault type are identified.

VI. CONCLUSIONS

Aiming to the nonlinear system and shortage of PCA, the KPCA method is introduced to FDD for sensor of control system. KPCA finds a controllable solution by the kernel function, which constructs the mapping from the input space to the feature space, and in the feature space the principle component is calculated. In this study, the Q statistics is applied to fault detection, and a proposed method named percent variation of contribution variable that based on T^2 statistics is applied to fault diagnosis. The simulation demonstrates that the whole method can detect the fault rapidly and isolate the fault variable correctly. In general, this method can excellently diagnoses the four type faults of sensor that is familiar to us, which are deviation fault, drift fault, precision grade reduction fault and fail fault.

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