8)

[illegible] 1/1

a) $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{1}{(n+2)(n+3)}}_{a_n} =$

$a_n = \dfrac{a}{n+2} - \dfrac{b}{n+3}$, $\dfrac{(n+3)\,a - b(n+2)}{(n+2)(n+3)} = \dfrac{1}{(n+2)(n+3)}$

$(n+3)\,a - b(n+2) = 1$

$\Leftrightarrow an + 3a - bn - 2b = 1 \quad \Leftrightarrow an - bn = 0$ y $3a - 2b = 1$

$\Leftrightarrow \begin{cases} an - bn = 0 \\ 3a - 2b = 1 \end{cases} \Leftrightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 3 & -2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 1 & 1 \end{array}\right]$

$\therefore\ b = 1$ y $a = 1$

así, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \sum_{n=1}^{\infty} \underbrace{\frac{1}{n+2}}_{b_n} - \underbrace{\frac{1}{n+3}}_{b_{n+1}}$, es decir es una telescópica

así, $\boxed{S_n = \lim_{n\to\infty} (b_1 - b_{n+1}) = \frac{1}{3} - 0 = \frac{1}{3}}$