TRANSFORMATIONS OF OBJECTS

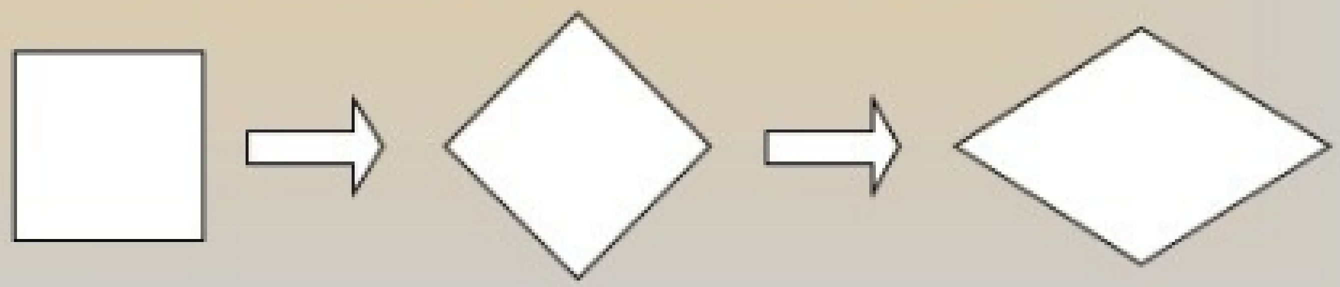
Transformation

- Transformation is the application of some mathematical rules to an object or a pixel to bring about some change to it.
- This change can be a displacement of an object from one point to another or a change in its size or shape.
- Two types of transformations are

  Solid body transformations

  Affine transformations

1. Rigid-body Transformation
   - ✓ Preserves parallelism of lines
   - ✓ Preserves angle and length
   - ✓ e.g. any sequence of $R(\theta)$ and $T(dx,dy)$
2. Affine Transformation
   - ✓ Preserves parallelism of lines
   - ✓ Doesn't preserve angle and length
   - ✓ e.g. any sequence of $R(\theta)$, $S(sx,sy)$ and $T(dx,dy)$

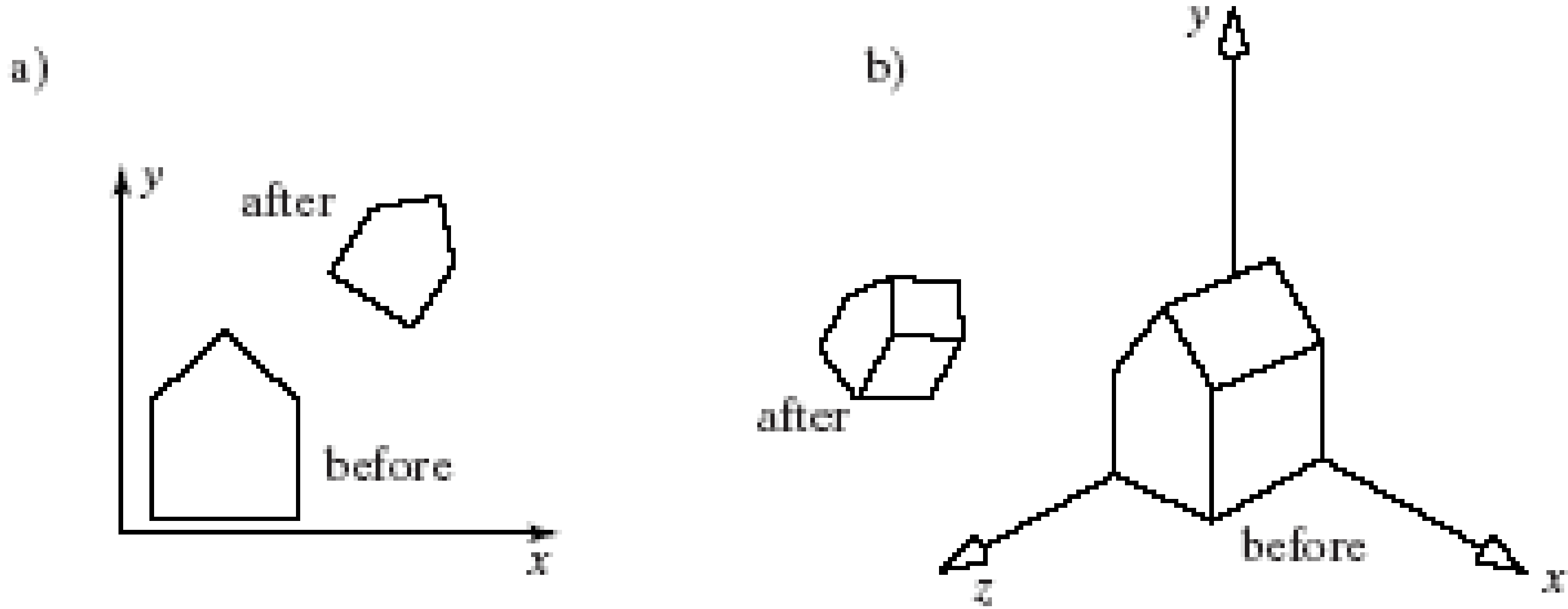


Solid body transformations

- In Solid body transformations shape of the object after the transformation is applied to it is not distorted or changed.
- The shape remains intact after the object is transformed.
- In ‘solid body’ transformations the following properties of the objects are preserved.
  - The distances between the points of the object are preserved.
  - The ratios of the distance between the points of the object are preserved.
  - The angles after and before the transformations remain the same.
  - Parallel lines remain parallel after the transformation.

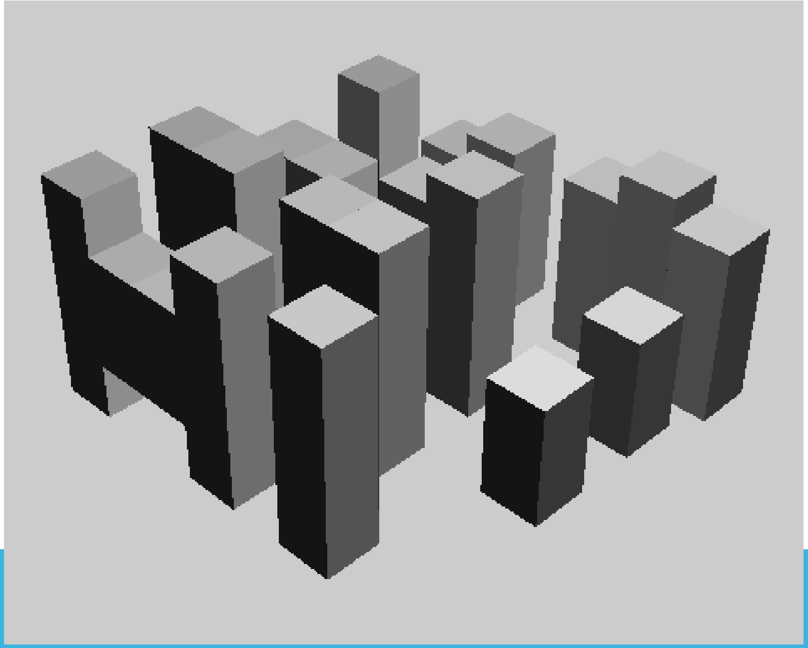
EXAMPLE OF TRANSFORMATIONS

The house has been scaled, rotated and translated, in both 2D and 3D.



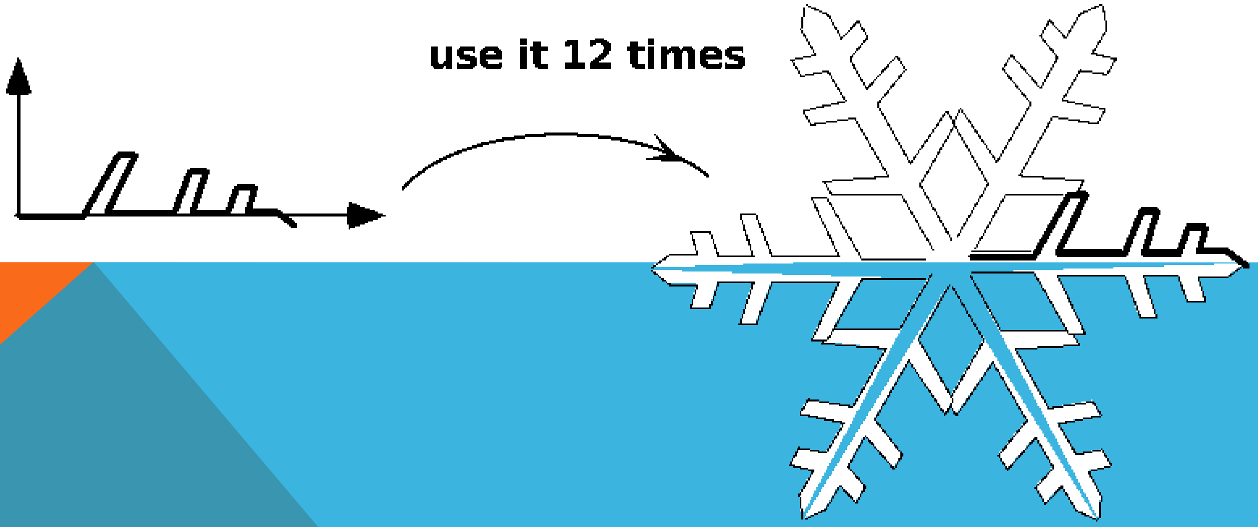
USING TRANSFORMATIONS (2)

In 3D, many cubes make a city.



USING TRANSFORMATIONS (3)

- **The snowflake exhibits symmetries.**
- **We design a single motif and draw the whole shape using appropriate reflections, rotations, and translations of the motif.**

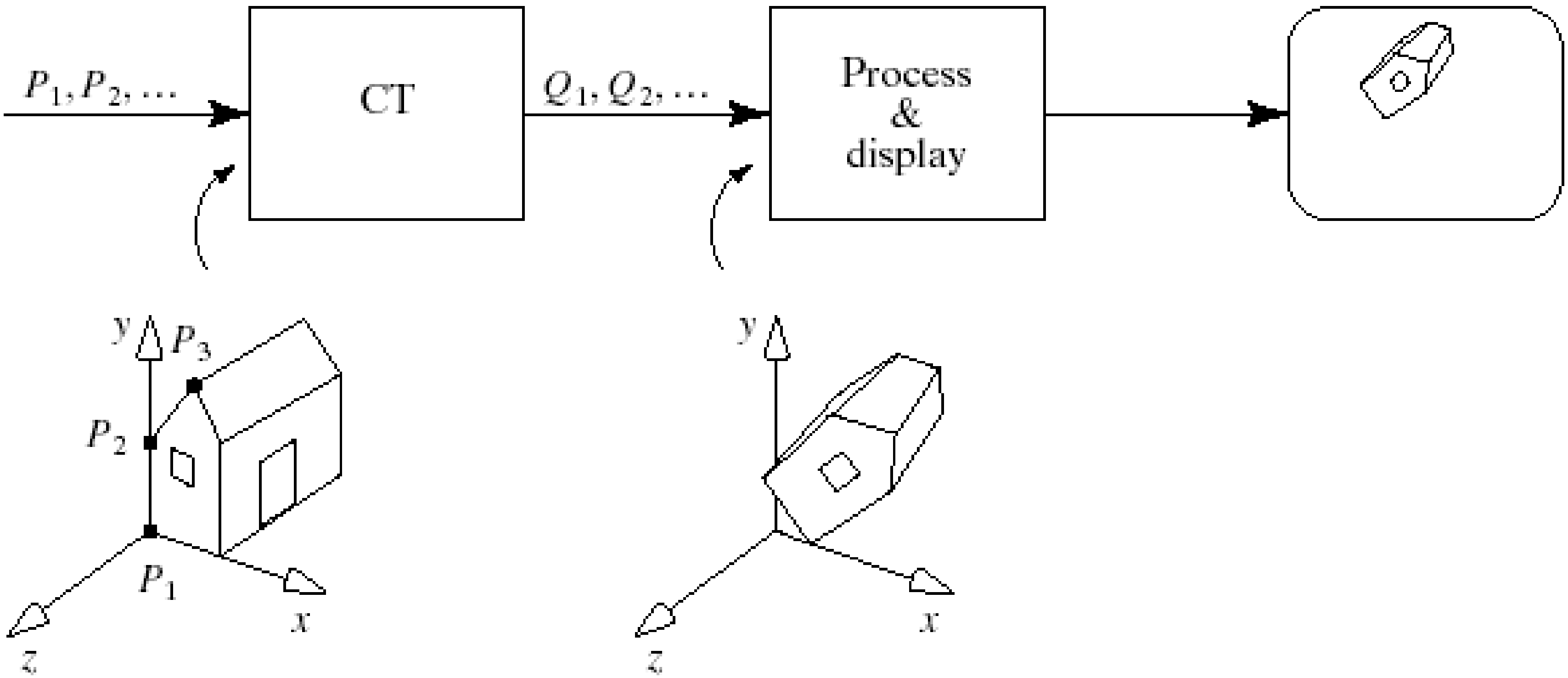


USING TRANSFORMATIONS (5)

- In a computer animation, objects move.
- We make them move by translating and rotating their local coordinate systems as the animation proceeds.
- A number of graphics platforms, including OpenGL, provide a graphics pipeline: a sequence of operations which are applied to all points that are sent through it.
- A drawing is produced by processing each point.

THE OPENGL GRAPHICS PIPELINE

This version is simplified.



Affine transformations

- Affine transformations are the transformations that change the shape of the object they are applied on but they do not distort the object.
- Affine transformations have the following properties.
  - The distances between the points are not preserved.
  - The angles are not preserved.
  - The ratios of the distances between the points are preserved.
  - Parallel lines remain parallel after the transformations are applied.
  - Affine transformations include i) Scaling ii) Shear

AFFINE TRANSFORMATIONS

Matrix form of the affine transformation in 2D:

For a 2D affine transformation the third row of the matrix is always (0, 0, 1).

$$\boxed{\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix}} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$

AFFINE TRANSFORMATIONS (5)

- **Some people prefer to use row matrices to represent points and vectors rather than column matrices: e.g., $P = (P_x, P_y, 1)$**

- **In this case, the *P* vector must *pre-multiply* the matrix, and the transpose of the matrix must be used: $Q = P\,M^T$.**

$$M^T = \begin{pmatrix} m_{11} & m_{21} & 0 \\ m_{12} & m_{22} & 0 \\ m_{13} & m_{23} & 1 \end{pmatrix}$$

AFFINE TRANSFORMATIONS

When vector V is transformed by the same affine transformation as point P, the result is

$$\begin{pmatrix} W_x \\ W_y \\ 0 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ 0 \end{pmatrix}$$

Important: to transform a point *P* into a point *Q*, *post-multiply M* by *P*: Q = M P.

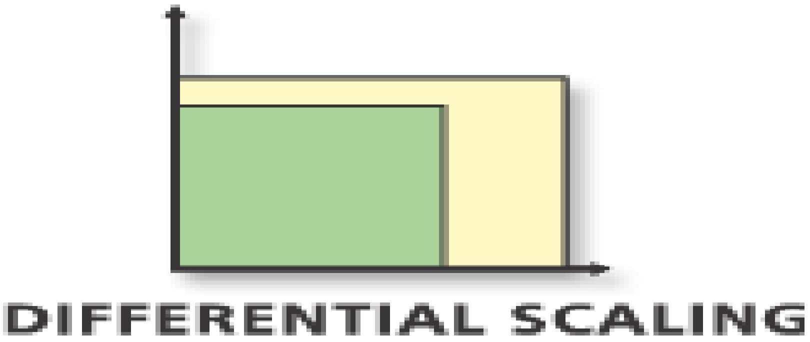
AFFINE TRANSFORMATIONS

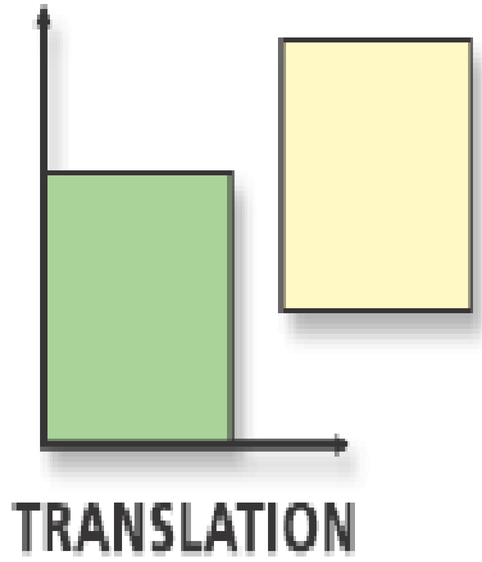
- **Example: find the image Q of point P = (1, 2, 1) using the affine transformation**

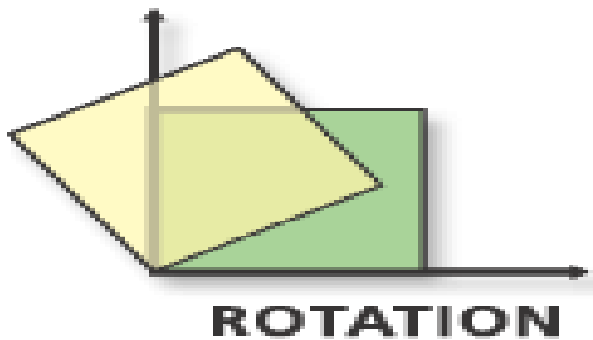
$$M = \begin{pmatrix} 3 & 0 & 5 \\ -2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}; Q = \begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 5 \\ -2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

GEOMETRIC EFFECTS OF AFFINE TRANSFORMATIONS

Combinations of four elementary transformations: (a) a translation, (b) a scaling, (c) a rotation, and (d) a shear (all







TRANSLATIONS

- **The amount *P* is translated does not depend on P's position.**
- **It is meaningless to translate vectors.**
- **To translate a point P by a in the x direction and b in the y direction use the matrix:**

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix} = \begin{pmatrix} Q_x + a \\ Q_y + b \\ 1 \end{pmatrix}$$

- **Only using homogeneous coordinates allow us to include translation as an affine transformation.**

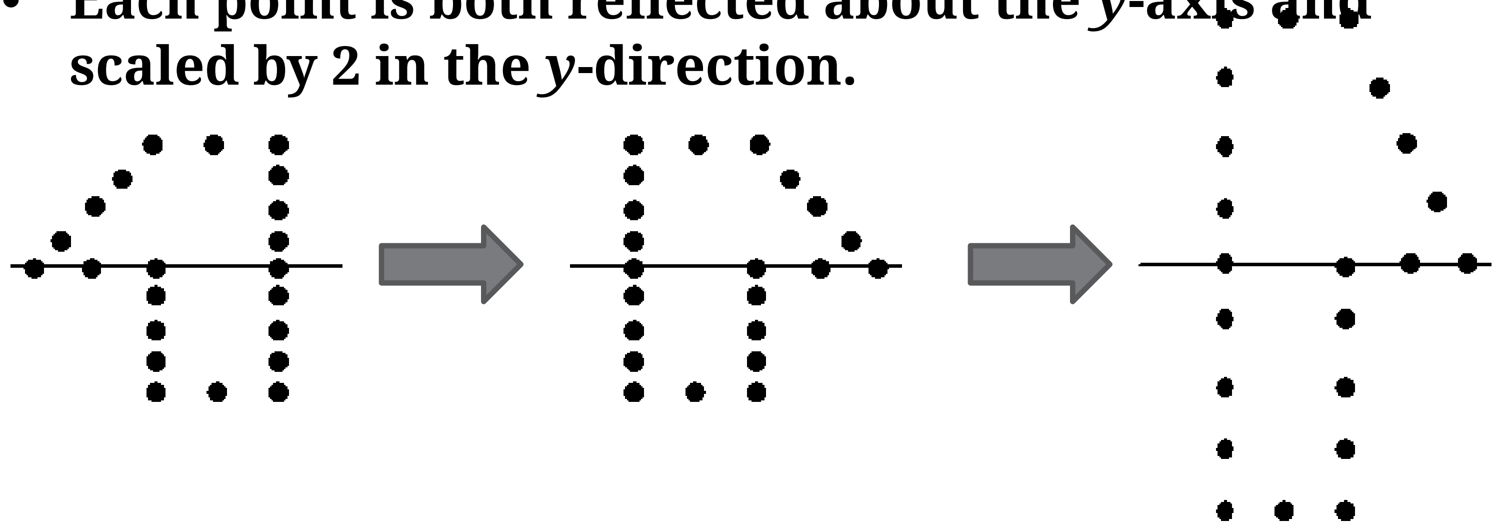
SCALING

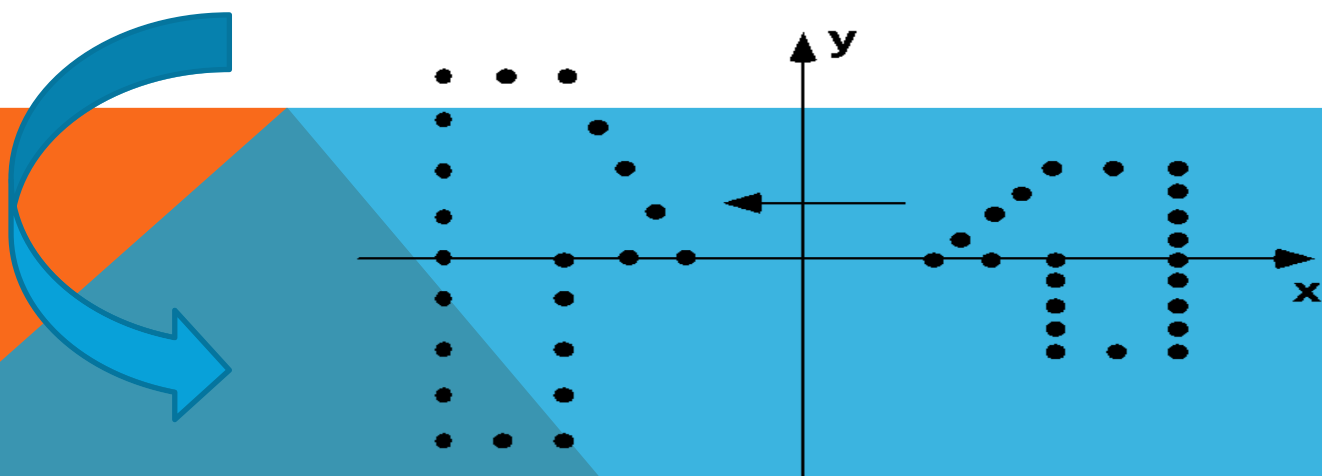
- Scaling is about the origin.
- If $S_x = S_y$ the scaling is uniform; otherwise it distorts the image.
- If S_x or $S_y < 0$, the image is reflected across the x or y axis.
- The matrix form is

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$

EXAMPLE OF SCALING

- **The scaling (*S*x, *S*y) = (-1, 2) is applied to a collection of points.**
- **Each point is both reflected about the *y*-axis and scaled by 2 in the *y*-direction.**





TYPES OF SCALING

- **Pure reflections, for which each of the scale factors is +1 or -1.**
- **A uniform scaling, or a magnification about the origin: $S_x = S_y$, magnification $|S|$.**

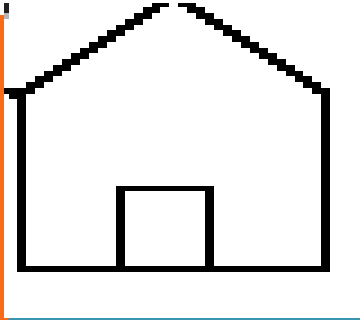
- Reflection also occurs if S_x or S_y is negative.
- If $|S| < 1$, the points will be moved closer to the origin, producing a reduced image.

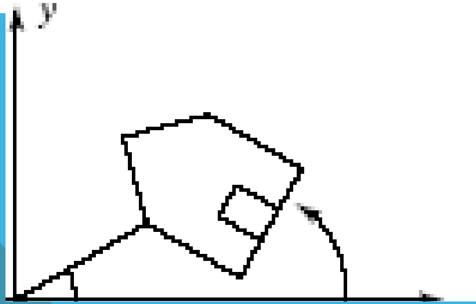
- **If the scale factors are not the same, the scaling is called a differential scaling.**

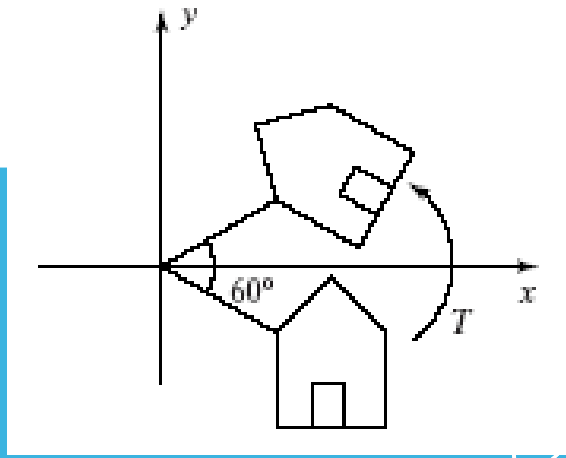
ROTATION

- **Counterclockwise around origin by angle θ:**

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$





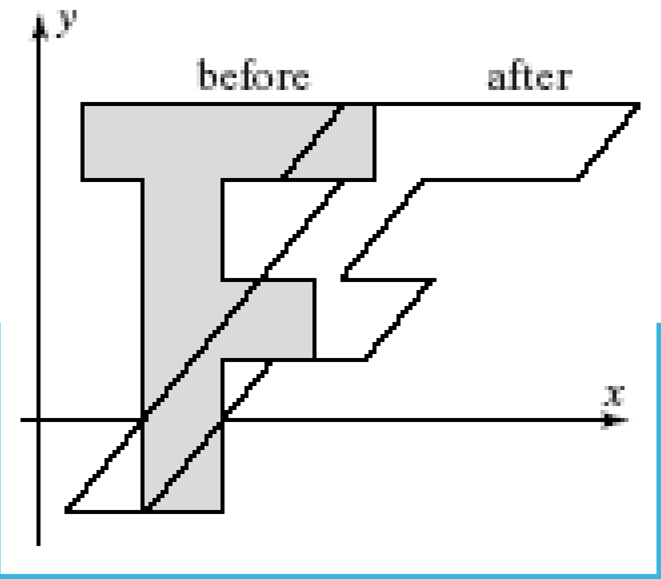


SHEAR

Shear H about origin: x depends linearly on y in the figure.

for example, *italic* letters).

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & h & 0 \\ g & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$



INVERSE TRANSLATION AND SCALING

Inverse of translation T^{-1}:

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$

Inverse of scaling S^{-1}:

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$

INVERSE ROTATION AND SHEAR

- **Inverse of rotation $R^{-1} = R(-\theta)$:**

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix}$$

- **Inverse of shear H^{-1}: generally h=0 or g=0.**

$$\begin{pmatrix} Q_x \\ Q_y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -h & 0 \\ -g & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ 1 \end{pmatrix} \frac{1}{1-gh}$$

COMPOSING AFFINE TRANSFORMATIONS

- **Usually, we want to apply several affine transformations in a particular order to the figures in a scene: for example,**
  - translate by (3, - 4)
  - then rotate by 30^o
  - then scale by (2, - 1) and so on.
- **Applying successive affine transformations is called composing affine transformations.**