Lab 4

SVM and performance metrics

Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Introduction

- Objectives
  - SPAM dataset: a binary classification problem
  - Linear and non-linear Support Vector Machine classifiers
  - Parameter validation
  - Performance metrics

SPAM dataset

Each vector in the dataset corresponds to a recevied email

Dataset:

- Classes: c=2 (spam, mail)
- Samples: N=4601 (1813 spam and 2788 mail)
- Features: d=57 frequency of a particular word in the email. The last features correspond to run-length attributes that measure the length of sequences of consecutive capital letters.

Goal: build a personalized spam filter

Dataset preprocessing (to fasten convergence and avoid overfitting):

- Feature binarization (for features 1 to 54)
- Removal of features 55 to 57

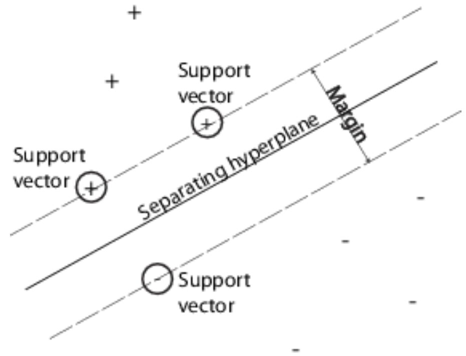
Content of a feature vector

Number	Feature	Number	Feature
1	word_freq_make: continuous.	30	word_freq_labs: continuous.
2	word_freq_address: continuous.	31	word_freq_telnet: continuous.
3	word_freq_all: continuous.	32	word_freq_857: continuous.
4	word_freq_3d: continuous.	33	word_freq_data: continuous.
5	word_freq_our: continuous.	34	word_freq_415: continuous.
6	word_freq_over: continuous.	35	word_freq_85: continuous.
7	word_freq_remove: continuous.	36	word_freq_technology: continuous.
8	word_freq_internet: continuous.	37	word_freq_1999: continuous.
9	word_freq_order: continuous.	38	word_freq_parts: continuous.
10	word_freq_mail: continuous.	39	word_freq_pm: continuous.
11	word_freq_receive: continuous.	40	word_freq_direct: continuous.
12	word_freq_will: continuous.	41	word_freq_cs: continuous.
13	word_freq_people: continuous.	42	word_freq_meeting: continuous.
14	word_freq_report: continuous.	43	word_freq_original: continuous.
15	word_freq_addresses: continuous.	44	word_freq_project: continuous.
16	word_freq_free: continuous.	45	word_freq_re: continuous.
17	word_freq_business: continuous.	46	word_freq_edu: continuous.
18	word_freq_email: continuous.	47	word_freq_table: continuous.
19	word_freq_you: continuous.	48	word_freq_conference: continuous
20	word_freq_credit: continuous.	49	char_freq_;: continuous.
21	word_freq_your: continuous.	50	char_freq_(: continuous.
22	word_freq_font: continuous.	51	char_freq_[: continuous.
23	word_freq_000: continuous.	52	char_freq_!: continuous.
24	word_freq_money: continuous.	53	char_freq_$: continuous.
25	word_freq_hp: continuous.	54	char_freq_#: continuous.
26	word_freq_hpl: continuous.	55	capital_run_length_average: continu
27	word_freq_george: continuous.	56	capital_run_length_longest: continu
28	word_freq_650: continuous.	57	capital_run_length_total: continuou
29	word_freq_lab: continuous.		

SVM

Linear SVM classifier:

Separable classes: An SVM classifier seeks the hyperplane that best separates samples from the two classes



Function to minimize:

$$L = \frac{1}{2}\left\|\mathbf{w}\right\|^2 - \sum_{i=1}^{N} \alpha_i \left(y_i \left(\mathbf{w}^T \mathbf{x}_i + w_0 \right) - 1 \right)$$

We obtain a convex problem depending on α_i ; it can be solved using standard optimization software

$$L = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \alpha_i \alpha_k y_i y_k \mathbf{x}_i^T \mathbf{x}_k \qquad \text{subject to} \quad \begin{cases} \alpha_i \geq 0 \\ \sum_{i=1}^{N} \alpha_i y_i = 0 \end{cases}$$

Classification of a vector x

$$\hat{y} = sign\left(g\left(\mathbf{x}\right)\right) = sign\left(\mathbf{w}^T \mathbf{x} + w_0\right) = sign\left(\sum_{i=1}^{N_{SV}} \alpha_i y_i \mathbf{x}_i^T \mathbf{x} + w_0 \right)$$

SVM

Linear SVM classifier:

Non-separable classes: no hyperplane can separate the classes without error. We permit some training vectors wrongly classified

$$y_i\left(\mathbf{w}^T\mathbf{x}_i + w_o\right) \geq 1 - \xi_i \qquad i = 1,\ldots,N$$

and introduce a penalization for non-null values of ξ_i:

$$L = \tfrac{1}{2}\left\|\mathbf{w}\right\|^2 + P\sum_{i=1}^{N}\xi_i - \sum_{i=1}^{N}\alpha_i\left(y_i\left(\mathbf{w}^T\mathbf{x}_i + w_0\right) - \left(1 - \xi_i\right)\right) - \sum_{i=1}^{N}\beta_i\xi_i$$

The penalization parameter P must be validated

Large P produces overfitting

SVM

Non-linear SVM classifier:

Uses kernel functions

$$L = \sum_{i=1}^{N} \alpha_i - \tfrac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \alpha_i \alpha_k y_i y_k K(\mathbf{x}_i, \mathbf{x}_k) \qquad \text{subject to} \qquad \begin{cases} 0 \le \alpha_i \le P \\ \sum_{i=1}^{N} \alpha_i y_i = 0 \end{cases}$$

Example: a Gaussian Kernel

$$K(\mathbf{x}_i, \mathbf{x}_k) = \exp\left(-\tfrac{1}{\sigma^2} \left\| \mathbf{x}_i - \mathbf{x}_k \right\|^2 \right)$$

σ^2 is a parameter that must be validated

Classification of a vector **x**:

$$\hat{y} = sign\left(\sum_{i=1}^{N_{SV}} \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + w_0 \right)$$

Validation of parameters

Dataset split into 3 subsets:

- Train: X_train, Labels_train, 60%, 70%, etc.
- Validation: X_val, Labels_val, 20%, 15%
- Test: X_test, Labels_test, 20%, 15%

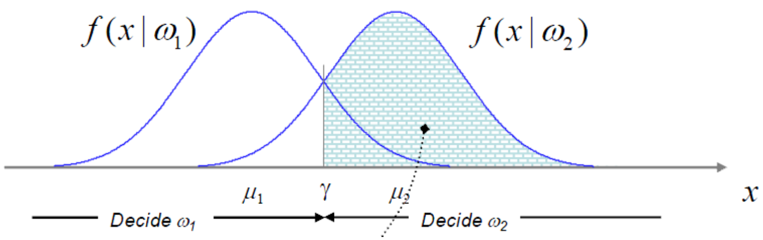
Parameter validation (brute force):

- For each pair of values to test (P, σ_2)
  - train classifier using training set
  - compute validation error
- Select the classifier (parameters) with lowest validation error
- Compute the error on the test set

Performance metrics

w_1 : negative

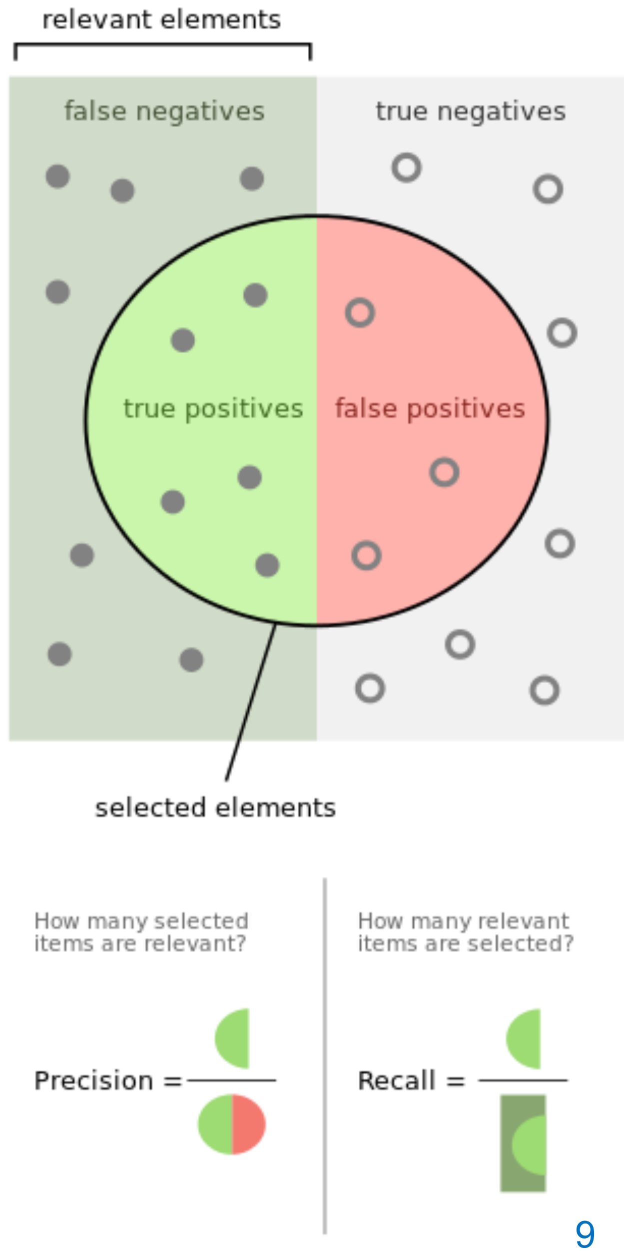
w_2 : positive



$$\text{Precision} = \frac{\text{TP}}{\text{TP}+\text{FP}}, \quad \text{Recall} = \frac{\text{TP}}{\text{TP}+\text{FN}}$$

$$\text{Precision} = \frac{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}}{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}+\Pr\{x>\gamma \mid w_1\}\Pr\{w_1\}} = \\ = \frac{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}}{\Pr\{x>\gamma\}} = \Pr\{w_2 \mid x>\gamma\},$$

$$\text{Recall} = \frac{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}}{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}+\Pr\{x<\gamma \mid w_2\}\Pr\{w_2\}} = \\ = \frac{\Pr\{x>\gamma \mid w_2\}\Pr\{w_2\}}{\Pr\{w_2\}} = \Pr\{x>\gamma \mid w_2\}$$



Performance metrics

Precision, Recall (=Sensitivity), Specificity:

$$P = \frac{TP}{TP + FP} = \frac{\text{\#correctly classified as SPAM}}{\text{\#classified as SPAM}}$$

$$R = Sens = \frac{TP}{TP + FN} = \frac{\text{\#correctly classified as SPAM}}{\text{\#total of SPAM}}$$

$$Sp = \frac{TN}{TN + FP} = \frac{\text{\#correctly classified as MAIL}}{\text{\#total of MAIL}}$$

F score: A measure that combines precision and recall is the harmonic mean of precision and recall, the traditional F-measure or balanced F-score:

$$F_{score} = 2\frac{\mathrm{P} \times \mathrm{R}}{\mathrm{P} + \mathrm{R}}$$

Prior measures:

$$P(\text{class=SPAM}) = \frac{\text{\#SPAM samples in the test set}}{\text{\#samples in the test set}}$$

$$P(\text{class=MAIL}) = \frac{\text{\#MAIL samples in the test set}}{\text{\#samples in the test set}}$$