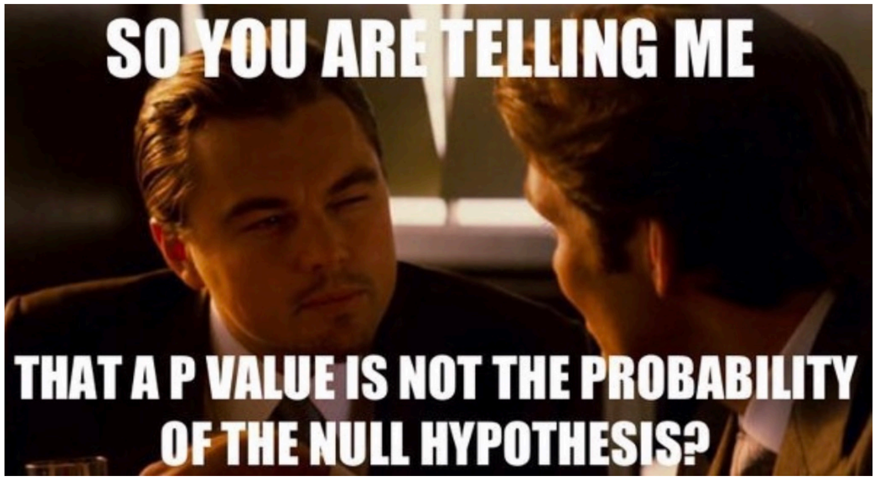
20th March'23

Hypothesis Testing 3



Let's start @ 9:05

Agenda

- More varieties of hypothesis testing
- Practical Code Implementation
- Real world version of A/B testing

Drug recovery :

Mean recovery days .

drug 1 → 110 (n_1)

[8, 9, 11, 7 _ _ _ _, 5]

$m_1 \simeq 7.1$ days.

drug 2 . → 120 (n_2)

[12, 4, 7, 8, _ _ _]

$m_2 \simeq 8.07$ days

two sample z test

One Sample test

fixed #?

① **Supply chain**

$H_0: \mu = 1800$, $H_A: \mu > 1800$

② **IQ of children**

$H_0: \mu = 100$, $H_A: \mu \neq 100$

③ **Drug Vs Nodrug**

$H_0: \mu = 15 \text{ days}$

$H_A: \mu < 15 \text{ days}$

Two Sample Test -

① **Supply chain**

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$

② **Two drug**

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 \neq \mu_2$.

drug eg

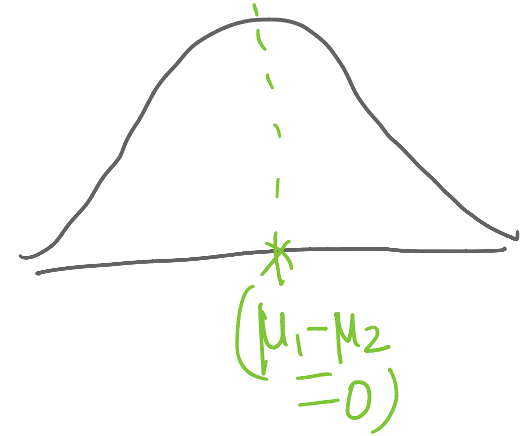
$H_0: \mu_1 - \mu_2 = 0 \quad (\mu_1 = \mu_2)$ → d_1 & d_2 not diff.

$H_A: \mu_1 - \mu_2 \neq 0 \quad (\mu_1 \neq \mu_2)$ d_1 & d_2 are diff.

- If low p value → reject H_0
- If high p value → do not reject H_0

$D_1 \longrightarrow m_1 \approx 7$

$D_2 \longrightarrow m_2 \approx 8$



$H_0 : \mu_1 = \mu_2$

$(\mu_1 - \mu_2) = 0$

- $H_A : \mu_1 \neq \mu_2$

two tailed test

pvalue ≈ 0

left tail

$H_A : \mu_1 < \mu_2$

low p value

① Two tailed Test

$m_1 = 7$, $m_2 = 8$

$H_0: \mu_1 = \mu_2$

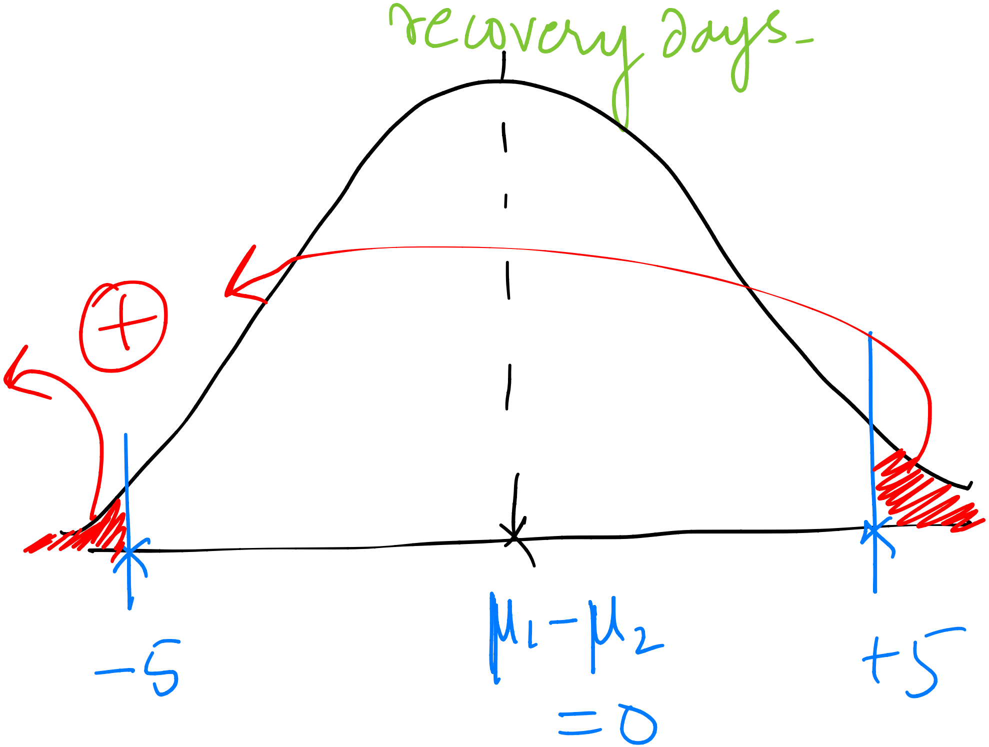
$H_A: \mu_1 \neq \mu_2$

↓

p_value ≈ 0

↓

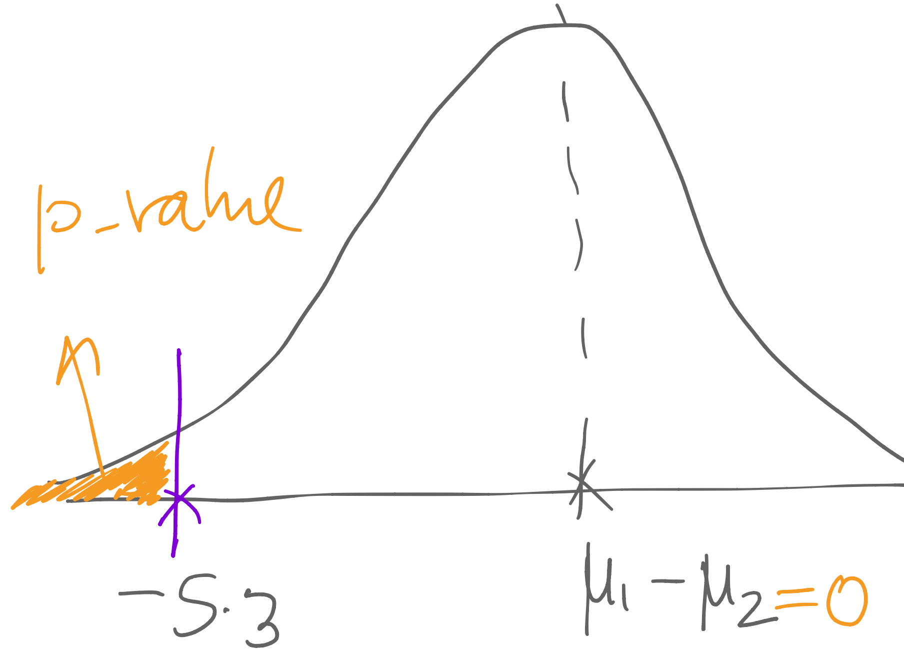
There is statistically sigf^t diff b/w μ_1 & μ_2.



② Left tailed

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 < \mu_2$



Reject H_0

Conclude $\mu_1 < \mu_2$

i.e. recovery days of $D_1 < D_2 \rightarrow$ SS.

Youtube Ads

Currently → 1 Ad → 2 Ads

avg. watch time may ↓

how to quantify this

A/B testing

⇒ Control group (A)

historical version (1 Ad)

Treatment Group (B)

new version (2 Ads)

n_1

[3.4, 2.4, 1.2, 7, - - - -]

Mean $m_1 = 3.6$ hrs

n_2

[7.1, 7.2, 1, 0.5, - - -]

$\underbrace{m_2 = 3.05 \text{ hrs}}_{\text{Avg watchtime}}$

1. $H_0 : \mu_1 = \mu_2$

   $H_A : \mu_1 \neq \mu_2$

2. $$\left(\frac{m_1 - m_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right) \rightarrow \text{Test statistic } Z$$

3 Two

4 p-value

5 $\alpha \longrightarrow$ business

* $Var[X] = \sigma^2$

$Var[2X] = 4\sigma^2$

$Var[\frac{1}{2}X] = \frac{1}{4}\sigma^2$

$Var[-X] = \underline{\sigma^2}$

* $Var[X+Y] = Var[X] + Var[Y]$

$\Rightarrow$ M_1 is sample mean of size n_1 $M_1 = \frac{(X_1 + --- + X_{n_1})}{n_1}$

if each $X_i \rightarrow Var\ \sigma_1^2$

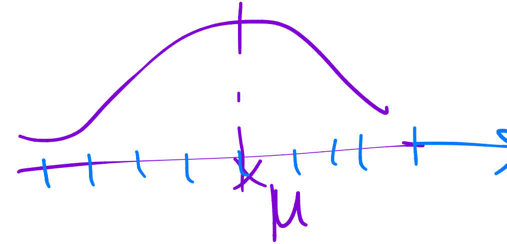
$\Rightarrow Var[M_1] = \frac{\sigma_1^2}{n_1}$; $Var[M_2] = \frac{\sigma_2^2}{n_2}$

$$\Rightarrow \mathrm{Var}[m_1 - m_2] = \mathrm{Var}[m_1] + \mathrm{Var}[m_2]$$

$$\Rightarrow \mathrm{Var}[m_1 - m_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\Rightarrow \mathrm{std.dev}[m_1 - m_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

If $m \to$ sample mean
$\mu \to$ pop. mean



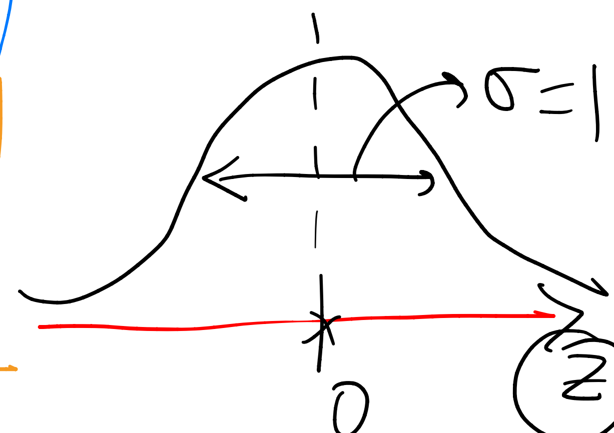
$$m \sim N\left(\mu, \sigma/\sqrt{n}\right)$$

$m \to$ R.V

$$m - \mu \sim N\left(0, \sigma/\sqrt{n}\right)$$

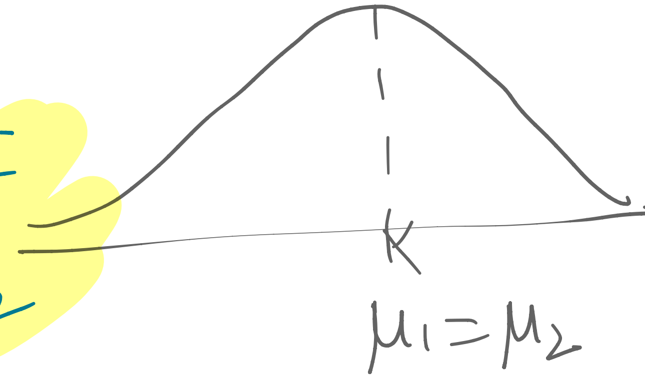
$$\boxed{\frac{m-\mu}{(\sigma/\sqrt{n})} \sim N(0, 1)}$$

$\to$ z score



$H_0: \mu_1 = \mu_2$

$$\text{std. dev}^n\,[m_1 - m_2] = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$



$$E[m_1 - m_2] = 0$$

$\Rightarrow \left[\begin{array}{l} m_1 \rightarrow \text{sample mean} \sim \text{Normal} \\ m_2 \rightarrow \text{Sample mean} \sim \text{Normal}. \end{array}\right.$

- $(m_1 - m_2) \sim$ Normal ?

$$(m_1 - m_2) \sim N\left(0, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$\left(\frac{m_1 - m_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) \sim N(0, 1)$$

↳ test statistic for 2 sample test

$$\left(\frac{M_1 - M_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) \sim N(0,1)$$

σ : popl^n std dev^n

Z test

not known

's' : Sample std. dev^n

n

Inreal

infer, $\sigma \rightarrow$ fm sample

std. devn is not good estimate of popln std. devn.

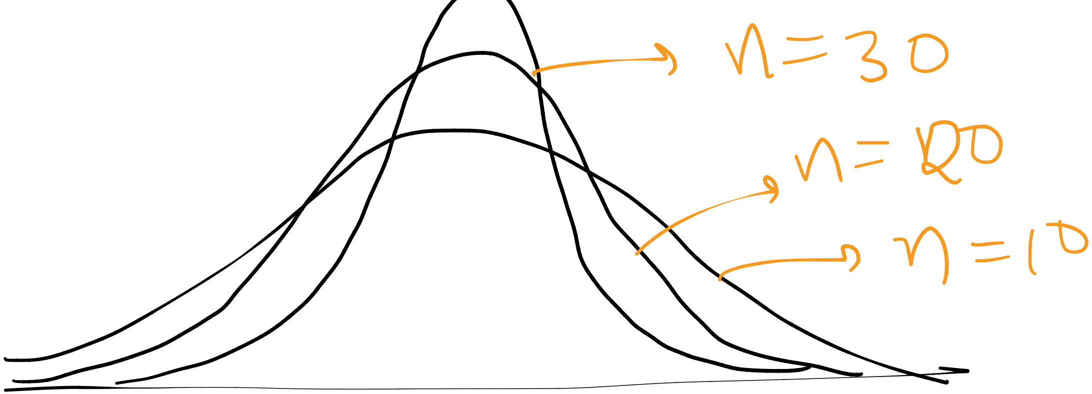
* Sample size < 30

① $\left(\dfrac{m-\mu}{s/\sqrt{n}}\right) \longrightarrow$ One tail test.

$\sim$ T distributⁿ if $n<30$

② $\left(\dfrac{m_1 - m_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}\right) \longrightarrow$ Two tail test

$\sim$ T distribution



* Normal $\rightarrow$ μ, σ
* T-dist $\rightarrow$ 1 paramer $\rightarrow$ degree of freedom
'n'

- $\left(\dfrac{m-\mu}{s/\sqrt{n}} \right) \sim T(dof = n-1)$

50 samp $\rightarrow$ 49

$$\left(\frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}\right) \sim T(dof = n_1 + n_2 - 1)$$

$n_1 = 100$

$n_2 = 200$

299