Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Law of Total probability $P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$

$P[B] = P[B \cap A] + P[B \cap A^c]$

Independence $P[A \mid B] = P[A]$

$P[A \cap B] = P[A]\ P[B]$

Among 100 students, 60 have taken the computer vision (CV) module, 50 have taken natural language processing (NLP). Also, it is seen that 20 have taken both CV and NLP. Given that a person has taken NLP, what is the probability that he has also taken CV?

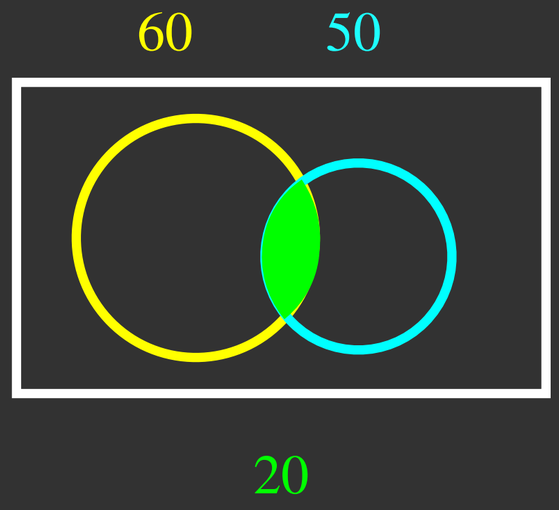
A: Students who have taken CV

What is $P[A]$?

$$P[A] = \frac{60}{100}$$

What is $P[A \cap B]$?

$$P[A \cap B] = \frac{20}{100}$$



B: Students who have taken NLP

What is $P[B]$?

$$P[B] = \frac{50}{100}$$

What is $P[A \mid B]$?

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{20/100}{50/100} = \frac{20}{50}$$

A family has 2 children, at least one of them is a girl. What is the probability that both are girls?

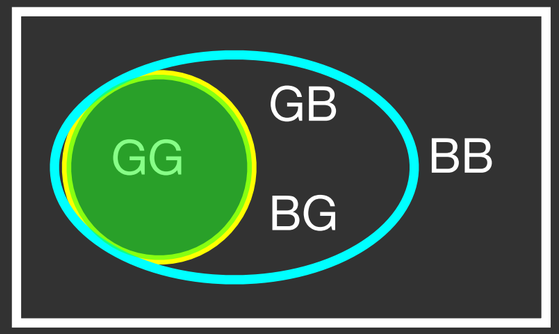
Sample space: {BB, BG, GB, GG}

A: Event that both are girls

As a subset of sample space, what is A?

$$A = \{GG\}$$

$$P[A] = \frac{1}{4}$$



B: Event that there is at least one girl

As a subset of sample space, what is B?

$$B = \{BG, GB, GG\}$$

$$P[B] = \frac{3}{4}$$

What is $A \cap B$? $A \cap B = \{GG\}$

$$P[A \cap B] = \frac{1}{4}$$

What is $P[A \mid B]$?

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} = \frac{1/4}{3/4} = \frac{1}{3}$$

In a university, 30% of faculty members are females. Of the female faculty members, 60% have a PHD. Of the male faculty members, 40% have a PHD.

What is the probability that a randomly chosen faculty member is a female and has PHD?

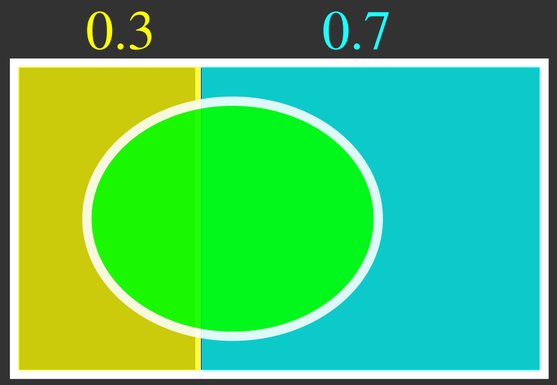
What is the probability that a randomly chosen faculty member is a male and has PHD?

What is the probability that a randomly chosen faculty member has a PHD?

What is the probability that a randomly chosen PHD holder is female?

$P[F] = 0.3 \quad P[\text{ phd } | F] = 0.6 \qquad P[\text{ phd } \cap F] = P[\text{ phd } | F]\ P[F] = 0.6 * 0.3 = 0.18$

$P[M] = 0.7 \quad P[\text{ phd } | M] = 0.4 \qquad P[\text{ phd } \cap M] = P[\text{ phd } | M]\ P[M] = 0.4 * 0.7 = 0.28$



$$P[\text{ phd }] = P[\text{ phd } \cap F\ ] + P[\text{ phd } \cap M\ ] = 0.18 + 0.28 = 0.46$$

$$P[\text{ phd }] = P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M] = 0.6 * 0.3 + 0.4 * 0.7 = 0.46$$

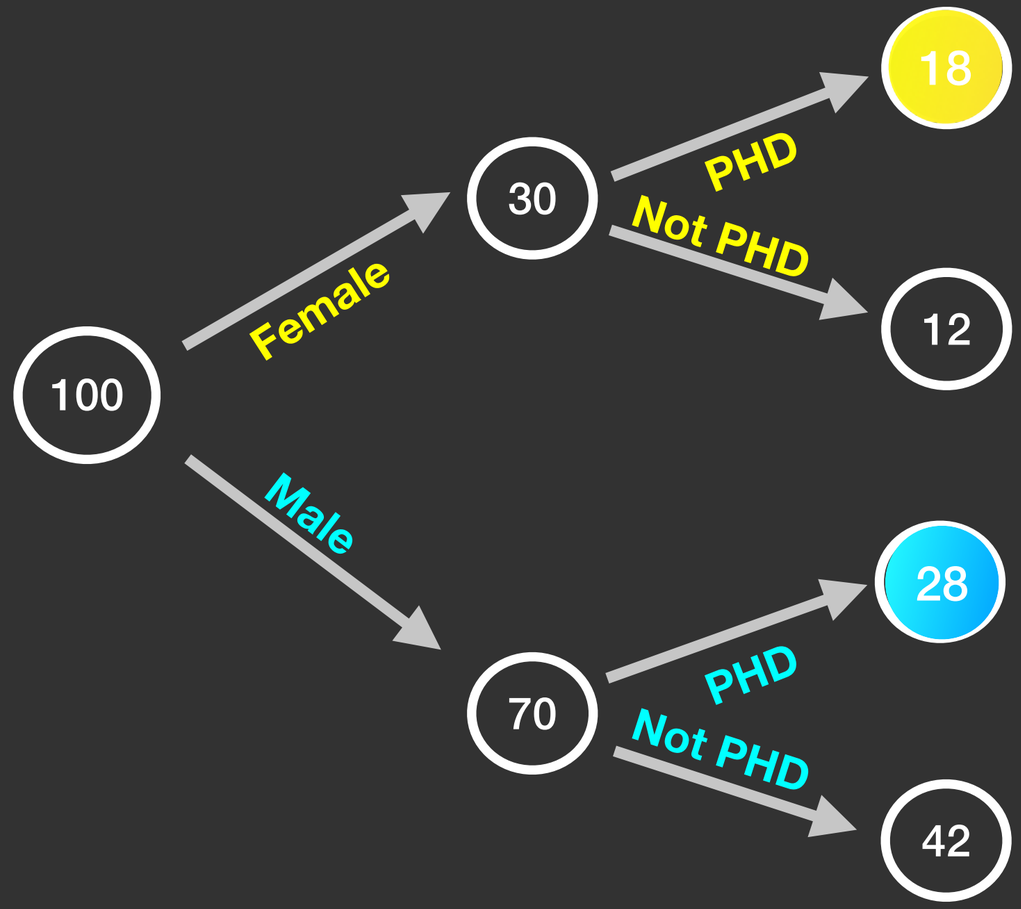
$$P[F | \text{ phd }] = \frac{P[F \cap \text{ phd }]}{P[\text{ phd }]} = \frac{0.18}{0.46} = 0.39$$

$$P[F | \text{ phd }] = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd }]} = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M]} = \frac{0.6 * 0.3}{0.6 * 0.3 + 0.4 * 0.7} = 0.39$$

$P[F] = 0.3 \quad P[\text{ phd } | F] = 0.6$

$P[M] = 0.7 \quad P[\text{ phd } | M] = 0.4$

$$P[F | \text{ phd }] = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M]}$$



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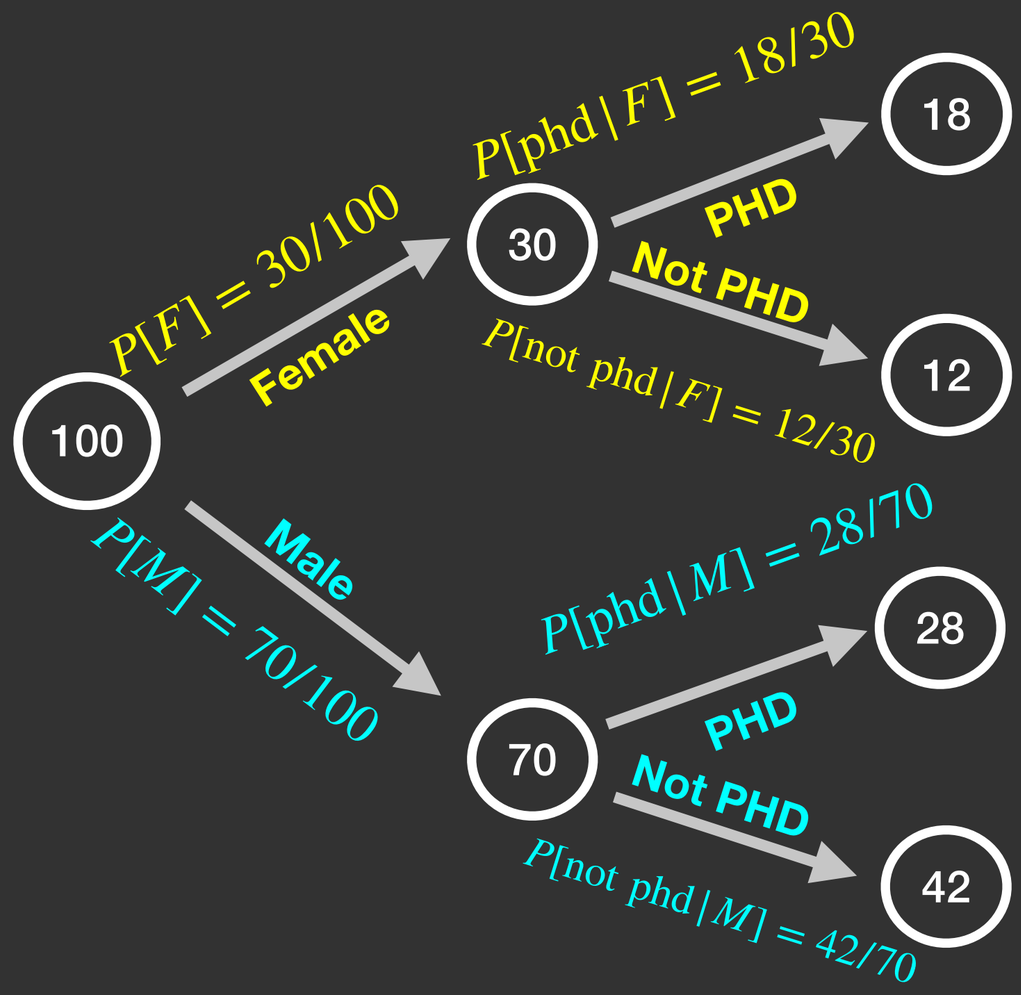
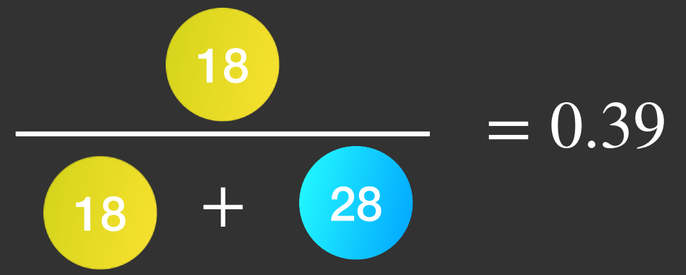
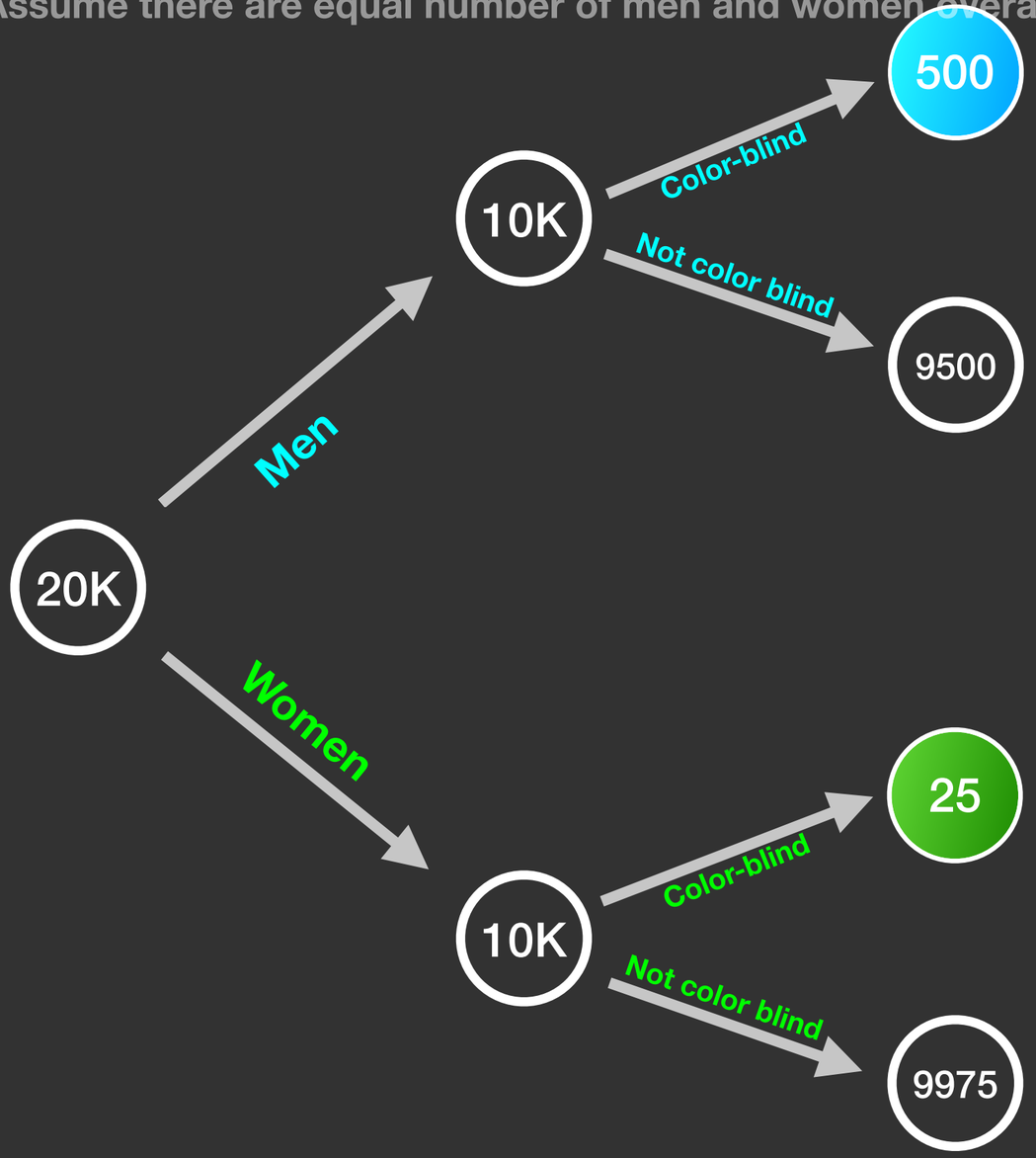
$P[M] = 0.7 \quad P[\text{ phd } | M] = 0.4$

$$P[F | \text{ phd }] = \frac{P[\text{ phd } | F]\ P[F]}{P[\text{ phd } | F]\ P[F] + P[\text{ phd } | M]\ P[M]}$$





Suppose 5 percent of men and 0.25 percent of the women are color-blind. A random color-blind person is chosen. What is the probability of this person being male? Assume there are equal number of men and women overall.



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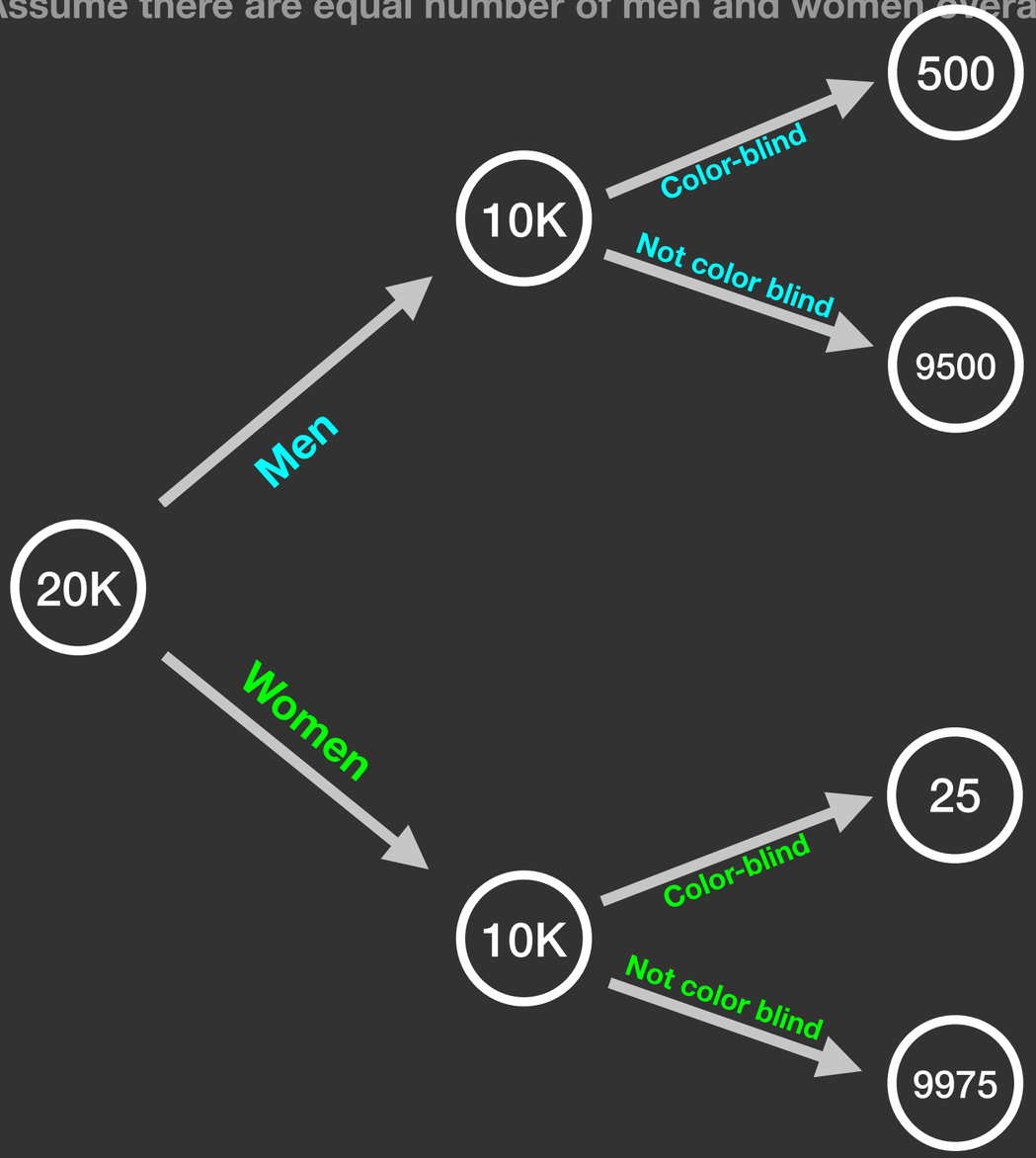

$$\frac{500}{500 + 25} = 0.9523$$

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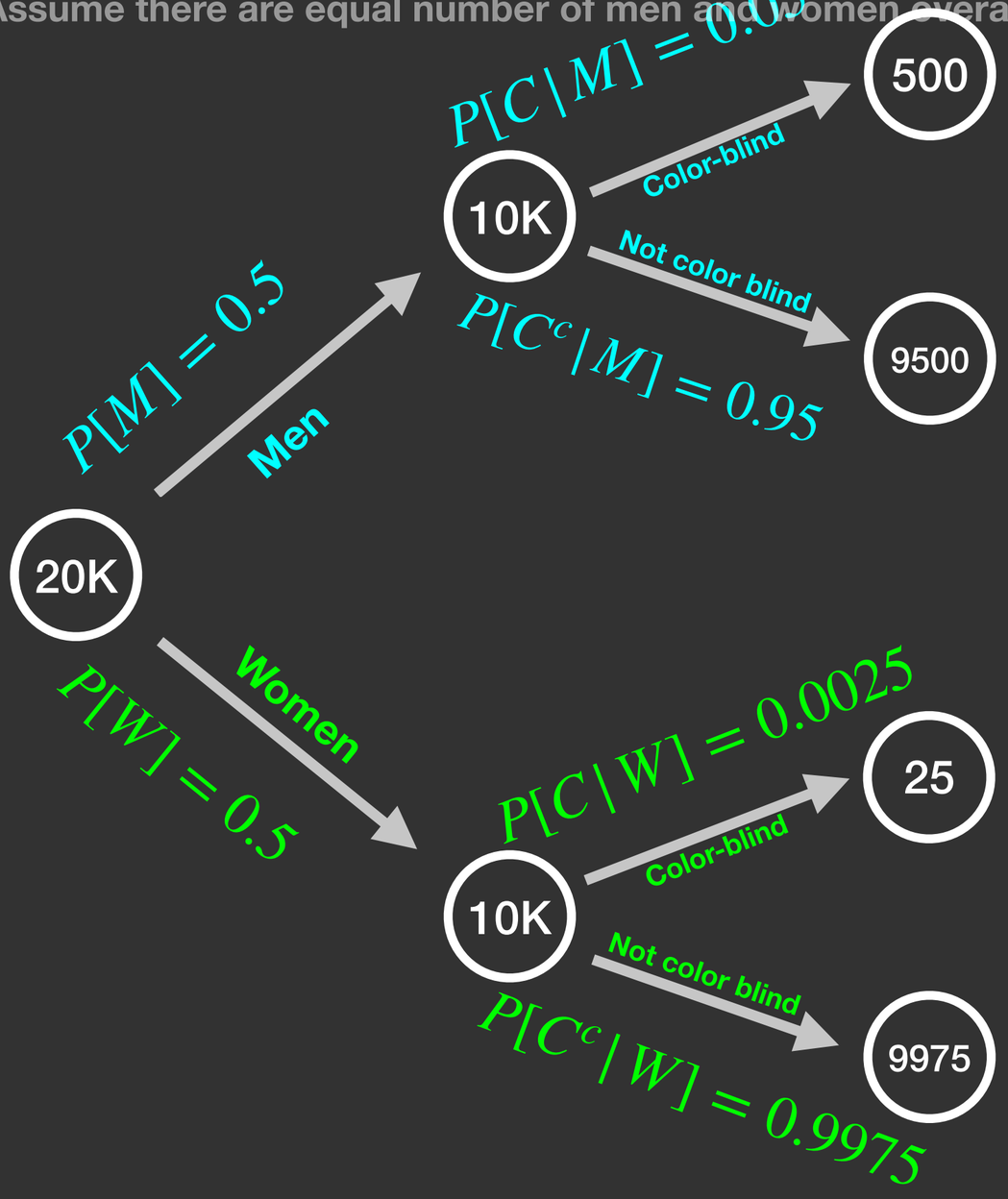

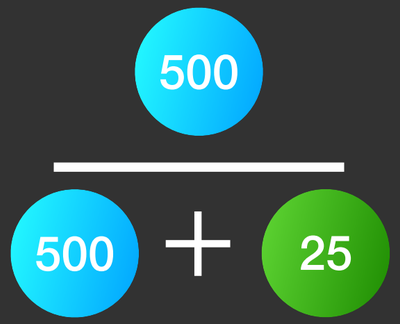

$$= 0.9523$$

$$P[M|C] = \frac{P[C|M]P[M]}{P[C]}$$

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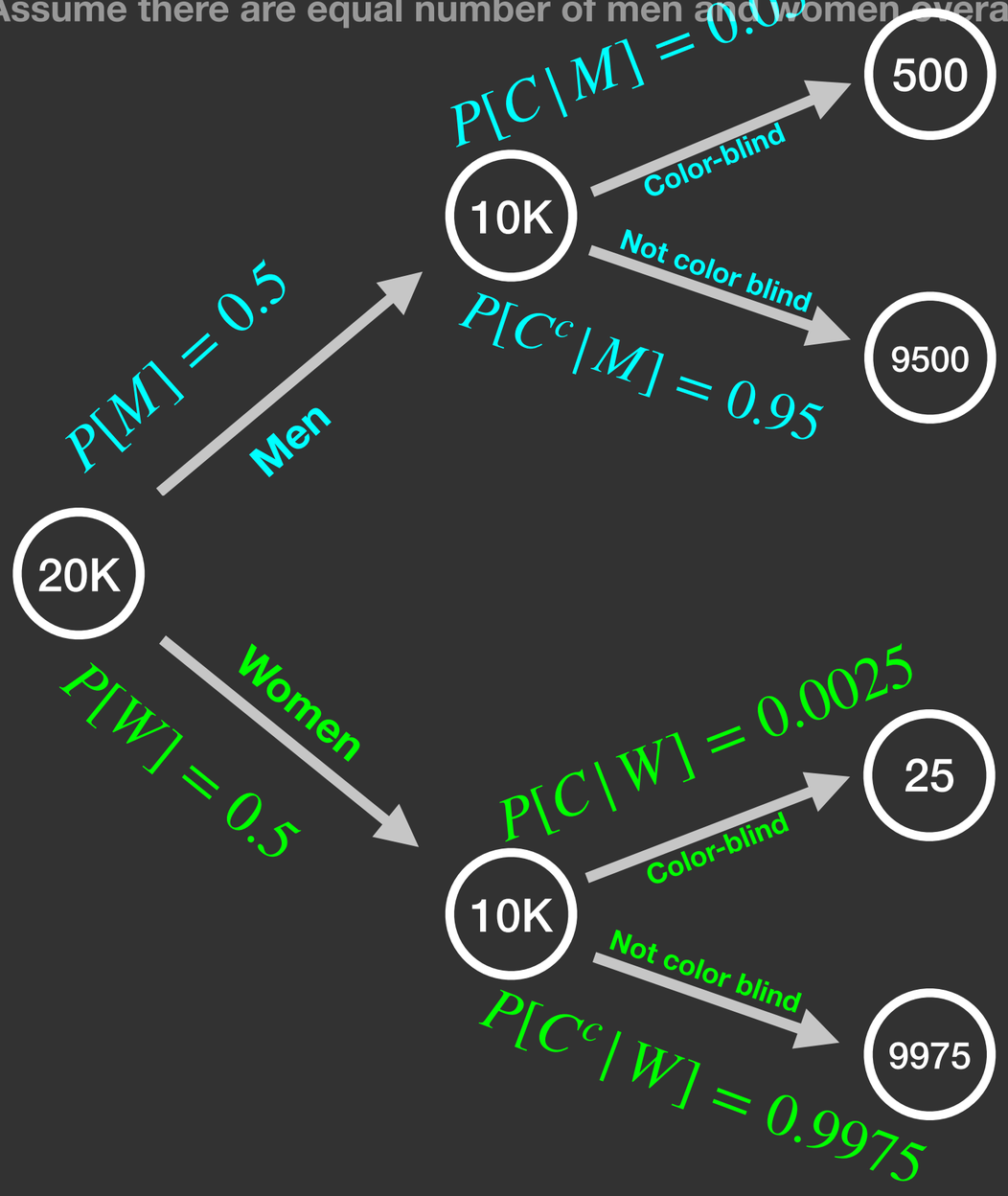

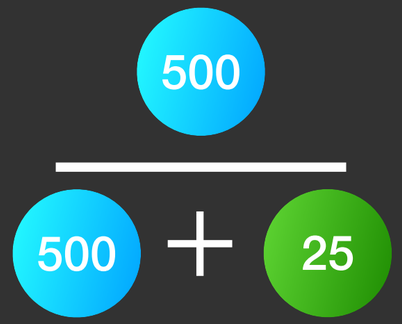

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$$P[M|C] = \frac{P[C|M]P[M]}{P[C|M]P[M] + P[C|M^c]P[M^c]}$$

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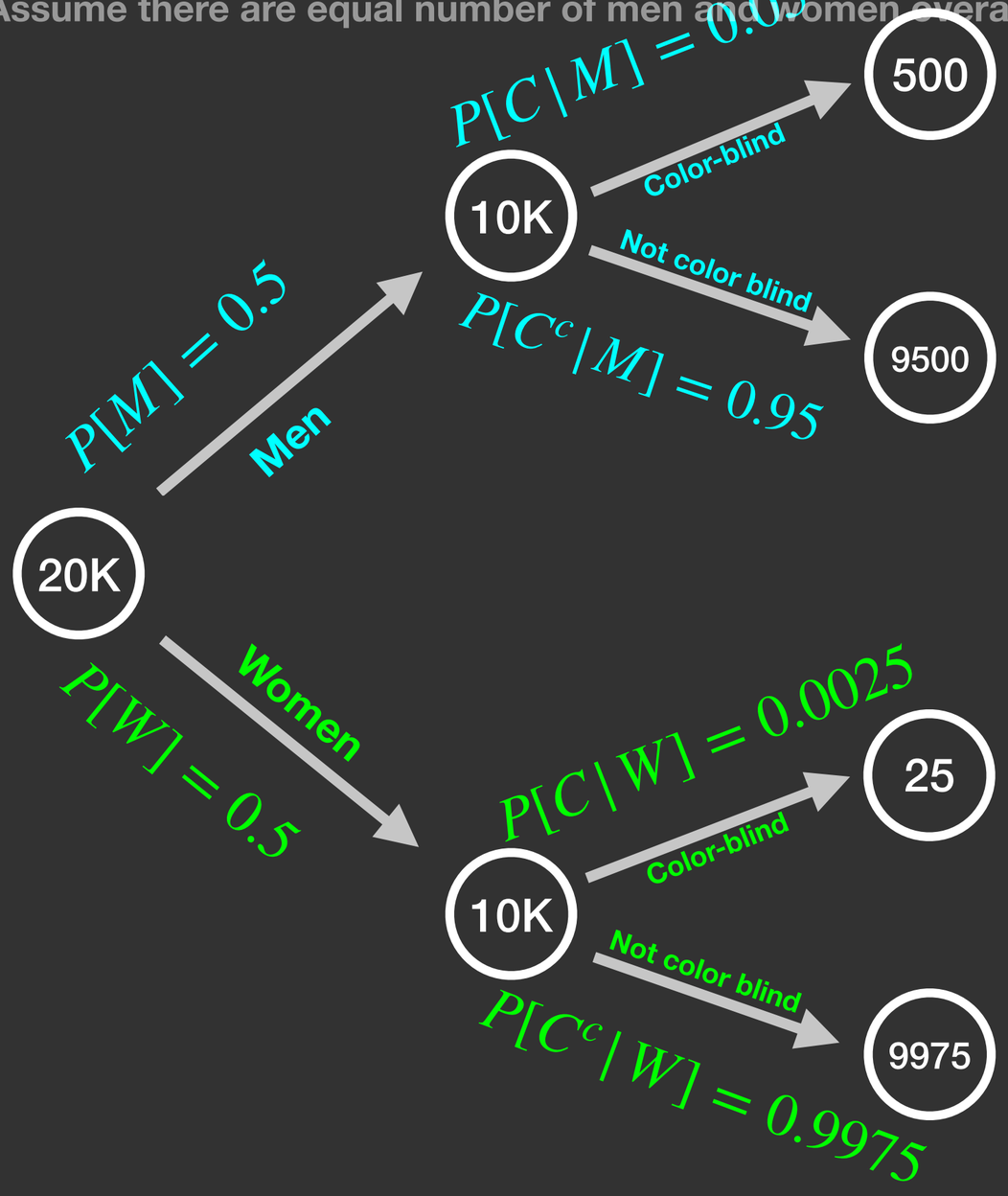

$$\frac{500}{500 + 25} = 0.9523$$

$$P[M|C] = \frac{P[C|M]P[M]}{P[C|M]P[M] + P[C|M^c]P[M^c]}$$

$$= \frac{\frac{500}{10K}\frac{10K}{20K}}{\frac{500}{10K}\frac{10K}{20K} + \frac{25}{10K}\frac{10K}{20K}}$$

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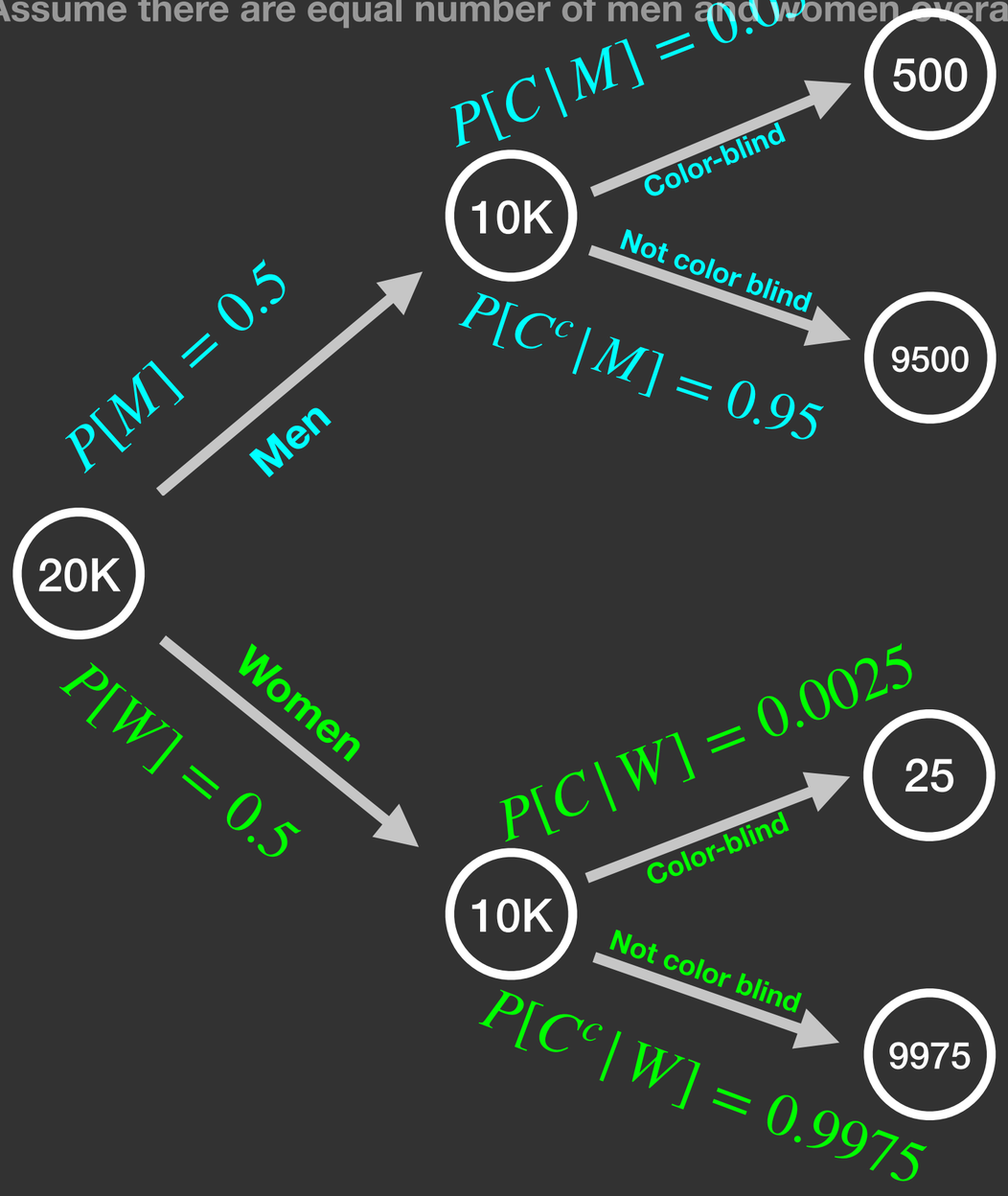


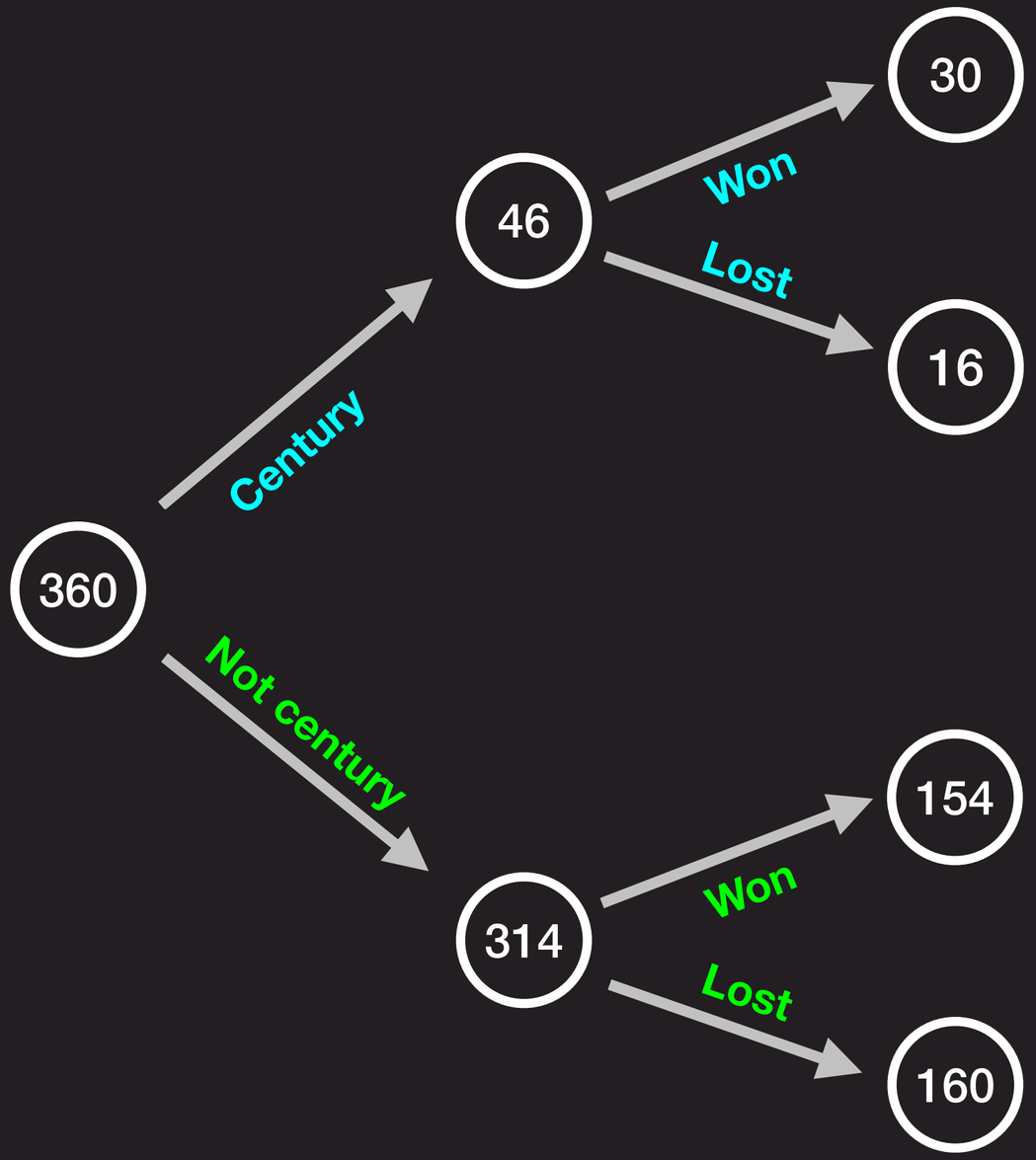
$$\frac{500}{500 + 25} = 0.9523$$

$$P[M|C] = \frac{P[C|M]P[M]}{P[C|M]P[M] + P[C|M^c]P[M^c]}$$

$$= \frac{500}{500 + 25}$$

$$= 0.9523$$

The probability of Sachin scoring a century is 46/360. Probability of winning when he scores a century is 30/46. Probability of winning when he does not score a century is 154/314. Given that a match was won, what is the probability that he scored a century.



$$\frac{30}{30 + 154} = 0.163$$