13th March
Monday

Problem Solving Session

Let's start @ 9:05

Agenda

- DBn
  ≥
- Assignment ✓

2 S_1 S_2 - - - - - - - - S_{2000}

Y N Y N ... Y N

Yes $\rightarrow p$

No $\rightarrow (1-p)$

2000 times a coin toss

$p(H) = 0.3$

$p(T) = 0.7$

$\boxed{\text{Binomial db}^{n}}$

3

The manager of the DC sequentially calls a few store managers to see if they need replenishment. We need to monitor how many calls are needed till the first time a store manager asks for replenishment. Which distribution best charaterizes this?

C_1 C_2 C_n

$P(X=5)$ = Prob. of getting first heads on 5^{th} toss

$T\ T\ T\ T\ H \Rightarrow (1-p)^4 \cdot p$

$(1-p)$ p

$$\Rightarrow \boxed{P(X=k) = (1-p)^{k-1} \cdot p}$$

(p)

$(1-p)$

4

Weekly sales has mean of 1000 and std dev of 200. BOH is 1300. What is the probability of needing replenishment??

Store is going out of stock.

↓

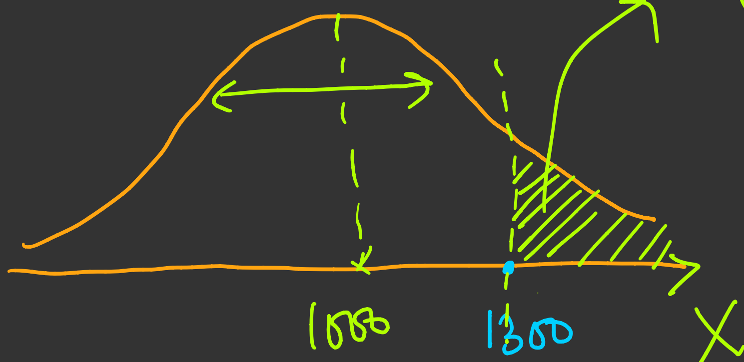
Weekly sale > 1300

given.

$\mu = 1000$

$\sigma = 200$

$P(X > 1300)$



i) Z Score of 1300

$$Z = \left(\frac{X - \mu}{\sigma}\right) = \left(\frac{1300 - 1000}{200}\right)$$

$$Z = \frac{3}{2} = \underline{\underline{1.5}}$$

$$P(Z > 1.5) = 1 - \text{norm.cdf}(1.5)$$

Prob. of store needing replenishment = 0.06

or 6%

(b) =

. **Suppose there are 2000 stores, each of which needs replenishment with probability 0.06. What is the expected number of stores that needs replenishment?**

100 times. ✓

$P(H) = 40\%$ → $p = 0.4$

$P(T) = 60\%$

40 times

$\underbrace{100}_{n} \times \underbrace{0.4}_{p}$

Expected value $= np$.

$$\boxed{E(X) = n \cdot p}$$

2000×0.06

$= \underline{120}$.

The manager of the DC sequentially calls a few store managers to see if they need replenishment. How many calls is he expected to make before the first request for replenishment?

Die. $SS = \{1, 2, 3, 4, 5, 6\}$

$\{6\} \rightarrow$ Success $\qquad P(\{6\}) = 1/6$

. Expected # of rolls before getting 1st 6.

Roll #	Success
1	$(1/6) \approx 0.166$
x	1

$$\frac{1}{x} = \frac{(1/6)}{1}$$

$$\Rightarrow x = 6 = \boxed{x = 1/p}$$

Process : geometric

$$x = \frac{1}{p} = \frac{1}{0.06}$$

$$x = 17 \; *$$

$$\boxed{E(x) = \frac{1}{p}}$$

✓

Expected number of days

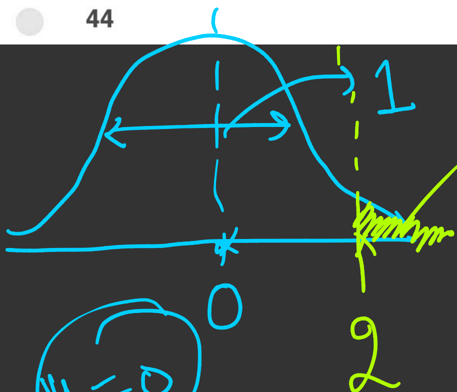
You are drawing from a random variable that is normally distributed with **mean 0** and **standard deviation 1**, X ~ N(0, 1), once a day.

Approximately what is the expected number of days you have to draw until you get a value **greater than** 2?

HINTS ?	Hint 1	Solution Approach	Complete Solution

You will get full points if and only if you give CORRECT ANSWER in first attempt. All later attempts will get you ZERO score.

- 30
- 10
- 50
- 44



$\mu = 0$

$\boxed{\sigma = 1}$

-0.9
0.2
0.7
$\vdots$
2

Prob. of drawing value > 2.

$P(X > 2) = 1 - \text{norm.cdf}(2)$

$= 0.023$ ✓

$p = 0.023$

- drawing process is geometric.

$$\text{days} = \frac{1}{p} = \underline{44}$$

Why Z scores of std. Normal dbⁿ.

Pediatrician

Underweight (thin)

Normal / Chubby

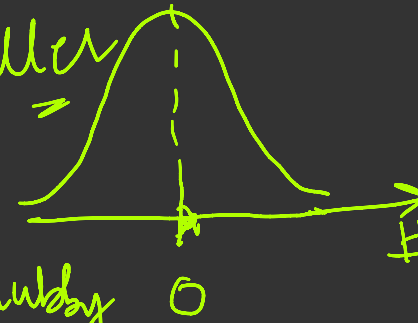
→ weight

→ height

$$w \rightarrow (\mu_w, \sigma_w) \Rightarrow \hat{w} = \left(\frac{w - \mu_w}{\sigma_w}\right)$$

$$h \rightarrow (\mu_h, \sigma_h) \Rightarrow \hat{h} = \left(\frac{h - \mu_h}{\sigma_h}\right)$$

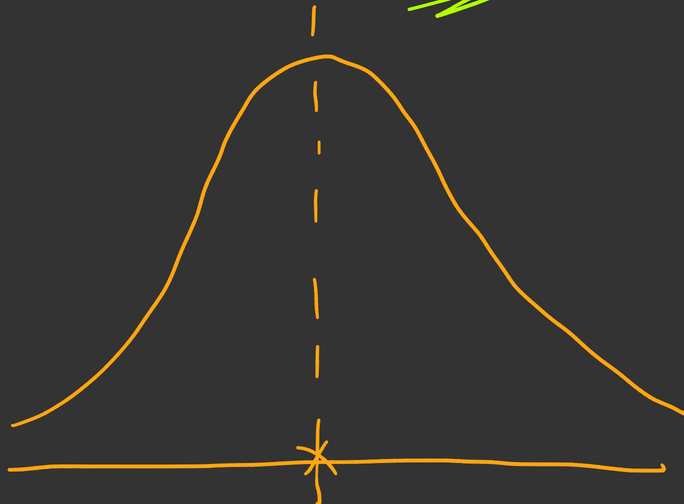
Baby	$\hat{w}$	$\hat{h}$	
1	0.5	1	→ taller
2	1.3	0	→ chubby
3	-0.2	1.1	→ thin & tall
4	0	0	→ normal / avg



# of std. dev^n away

Standardized #'s ✓

Cricket Virat. $\longrightarrow$ 60 Runs/match.



$\mu = 35$

$\sigma = 10$

$$Z\text{score} = \frac{60-35}{10}$$

$$= 2.5$$

$40 \longrightarrow 40 - 35/10 = 0.5$

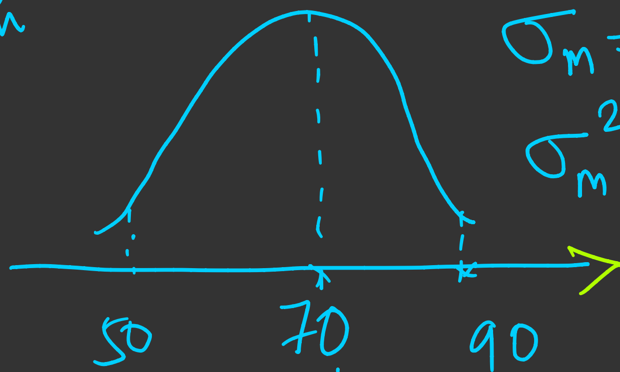
$35 \longrightarrow \quad = 0$

$30 \longrightarrow \quad -5/10 = -0.5$

$= 2 \ldots$

Central Limit Theorem:

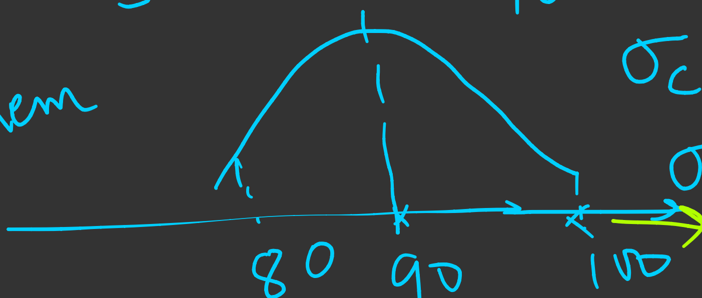
math



$\sigma_m = 10$

$\sigma_m^2 = 100$

chem



$\sigma_c = 5$

$\sigma_c^2 = 25$

Total marks

good	bad
M+C	M+C
M	C
C	M

Total = maths + chem.

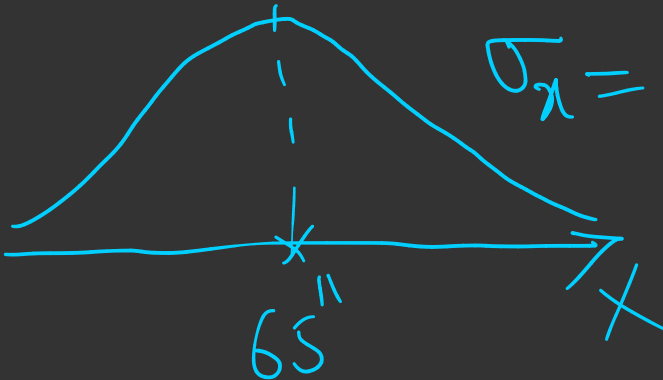
new var

$$\sigma_T^2 = \sigma_m^2 + \sigma_c^2 = 100 + 25$$
$$= 125$$

$$\text{Variance}(\text{math} + \text{chem}) = \text{Var}(\text{math}) + \text{Var}(\text{chem})$$

$$\Rightarrow \boxed{\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2)}$$

only cond^n → X_1 & X_2 are independent.



$\sigma_x = 2.5$

1 inch = 2.54 cm

$$\boxed{Y = 2.54(X)}$$

$$\boxed{\sigma_y = 2.54\,\sigma_x}$$

$$\sigma_y^2 = (2.54)^2\,\sigma_x^2$$

$$\mathrm{Var}(2.54X) = (2.54)^2 \cdot \mathrm{Var}(X)$$

$$\boxed{\mathrm{Var}(aX) = a^2\,\mathrm{Var}(X)}$$

$X_1, X_2, \text{------}, X_n$ Same $\rightarrow \mu$

Same $\rightarrow \sigma$

$Var(aX) = a^2 Var X$

$Var\left(\frac{X}{2}\right) = \frac{1}{4} \cdot Var X$

$$\cdot\, Var\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{4} Var(X_1 + X_2)$$

$$= \frac{1}{4}\left[Var(X_1) + Var X_2\right]$$

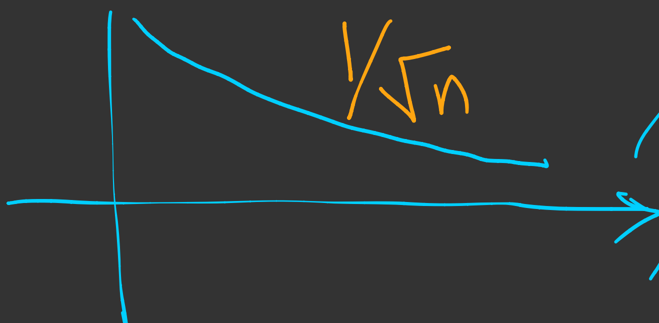
$$= \frac{1}{4} \cdot \left[\sigma^2 + \sigma^2\right] = \frac{\sigma^2}{2}.$$

$$\cdot\, Var\left(\frac{X_1 + X_2 + X_3}{3}\right) = \frac{1}{9}\left[\sigma^2 + \sigma^2 + \sigma^2\right) = \frac{\sigma^2}{3}$$

$$\operatorname{Var}\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right) = \frac{\sigma^2}{n}$$

$$\boxed{\text{Standard deviation Sample means} = \frac{\sigma}{\sqrt{n}}}$$

Standard Error.



Sample size

$$\left(\frac{X_1 + \cdots + X_n}{n}\right) \text{ follows Normal dbn}$$

with mean $= \mu$

std. devn $= \dfrac{\sigma}{\sqrt{n}}$ $\qquad \mathcal{N}\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$

$$\left(\frac{X_1 + \cdots + X_n}{n}\right) \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$\left(X_1 + \cdots + X_n\right) \sim N\left(n\mu, \sqrt{n} \cdot \sigma\right)$$

$$\operatorname{Var}(X_1 + \cdots + X_n) = n \cdot \sigma^2$$

$$\text{std. dev} = \sqrt{n}\,\sigma$$

Q.) Batteries mean life $= 5$ weeks $= \mu$.

std. dev. $= 1.5$ weeks $= \sigma$.

What is the prob. of needing 13 or more Batteries in 1 year?

Sol^n. When do you need 13 or more batteries?

$\hookrightarrow$ when first 12 are dead.

$\Rightarrow$ 5 weeks for 12 Batteries

$X_i \longrightarrow i^{th}$ Battery.

$\downarrow$

$E(X) = 5$ weeks

$X_1 + X_2 + \cdots + X_{12} = Y$

$\Rightarrow E(Y) = 5 \times 12 = 60$ weeks

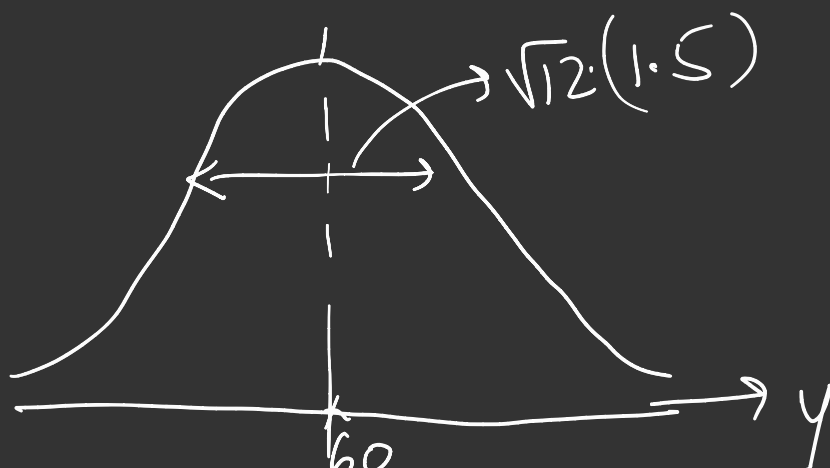
$\Rightarrow Var(X_i) = \sigma^2 = (1.5)^2.$

$$Var(Y) = Var(X_1 + \cdots + X_{12})$$
$$= 12 \times (1.5)^2$$

$$(X_1 + X_2 + \cdots + X_{12}) = Y \sim N(60, \sqrt{12}(1.5))$$

$$\sim N(n\mu, \sqrt{n} \cdot \sigma)$$

*



$Y \rightarrow$ Combined life of 12 Batteries.

$\Rightarrow P(Y < 52)$

when the lifetime of first 12 Batteries together is < 52 weeks

$\Rightarrow$ Z score of 52

$$= \frac{52 - 60}{\sqrt{12}\,(1.5)}$$

$$= \text{norm.cdf}(\quad)$$

$$= 0.0618 \checkmark$$