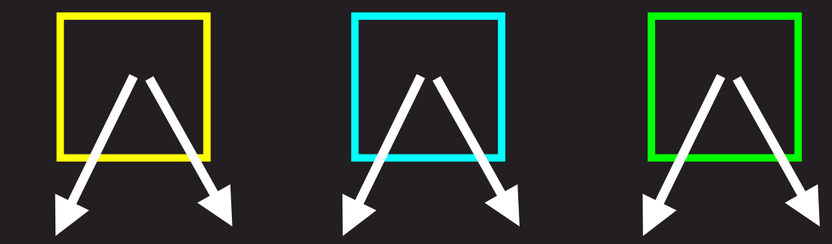
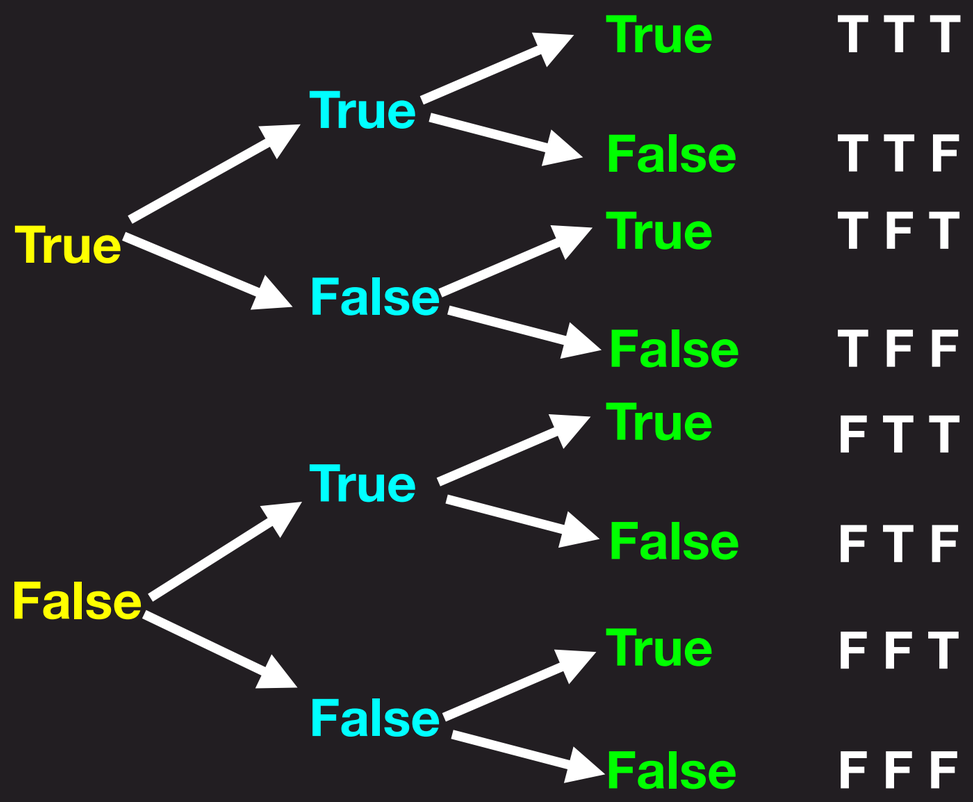
Given 3 True/False questions, in how many ways can we answer them?



2 ways 2 ways 2 ways



Total number of ways = 2 * 2 * 2 = 8

There are 10 girls and 7 boys in a hall.
In how many ways can we pick one boy-girl pair?

Total number of ways = 10 * 7 = 70

(G1, B1), (G1, B2), (G1, B3), (G1, B4), (G1, B5), (G1, B6), (G1, B7),

(G2, B1), (G2, B2), (G2, B3), (G2, B4), (G2, B5), (G2, B6), (G2, B7),

(G3, B1), (G3, B2), (G3, B3), (G3, B4), (G3, B5), (G3, B6), (G3, B7),

(G4, B1), (G4, B2), (G4, B3), (G4, B4), (G4, B5), (G4, B6), (G4, B7),

(G5, B1), (G5, B2), (G5, B3), (G5, B4), (G5, B5), (G5, B6), (G5, B7),

(G6, B1), (G6, B2), (G6, B3), (G6, B4), (G6, B5), (G6, B6), (G6, B7),

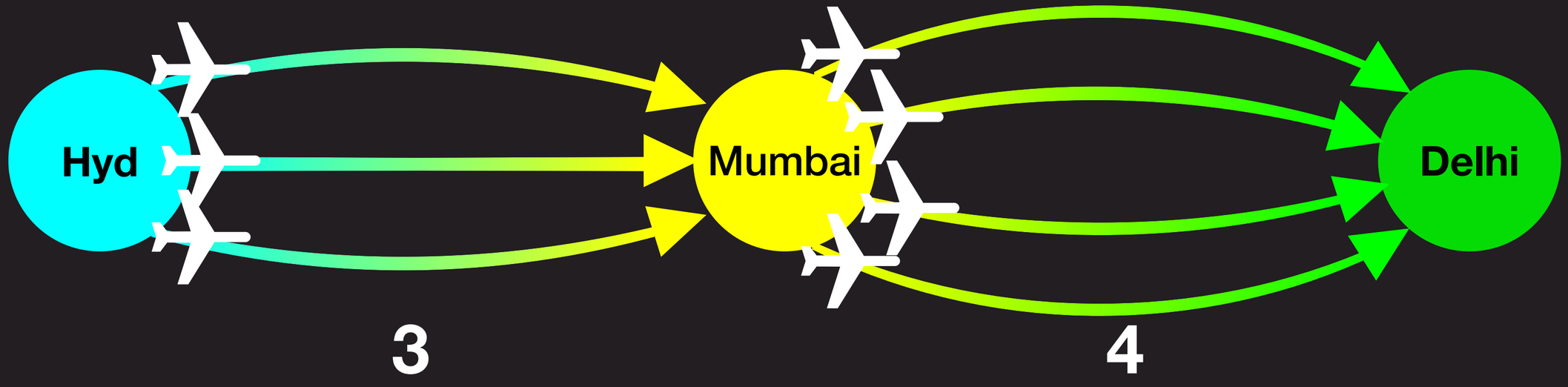
(G7, B1), (G7, B2), (G7, B3), (G7, B4), (G7, B5), (G7, B6), (G7, B7),

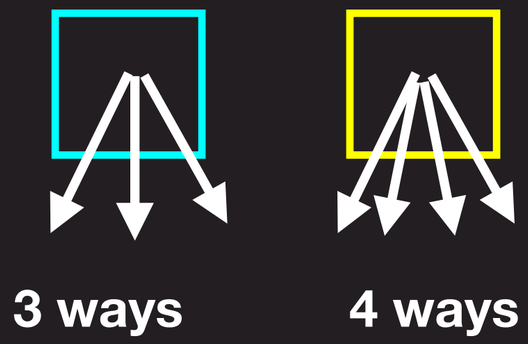
(G8, B1), (G8, B2), (G8, B3), (G8, B4), (G8, B5), (G8, B6), (G8, B7),

(G9, B1), (G9, B2), (G9, B3), (G9, B4), (G9, B5), (G9, B6), (G9, B7),

(G10, B1), (G10, B2), (G10, B3), (G10, B4), (G10, B5), (G10, B6), (G10, B7),

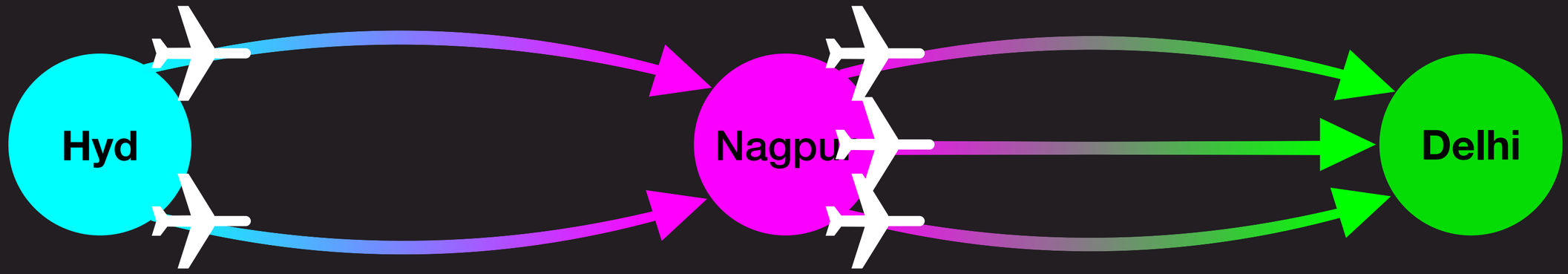
There are 3 ways to move from Hyderabad to Mumbai.
There are 4 ways to move from Mumbai to Delhi.
What are the total ways of moving from Hyderabad to Delhi?





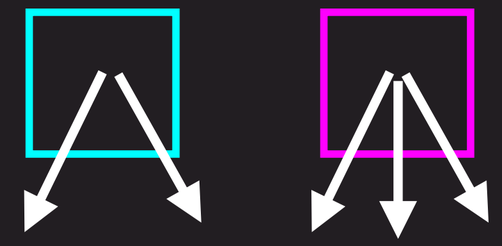
Total number of ways = 3 * 4 = 12

There are 2 ways to move from Hyderabad to Nagpur.
There are 3 ways to move from Nagpur to Delhi.
What are the total ways of moving from Hyderabad to Delhi?



2

3



2 ways

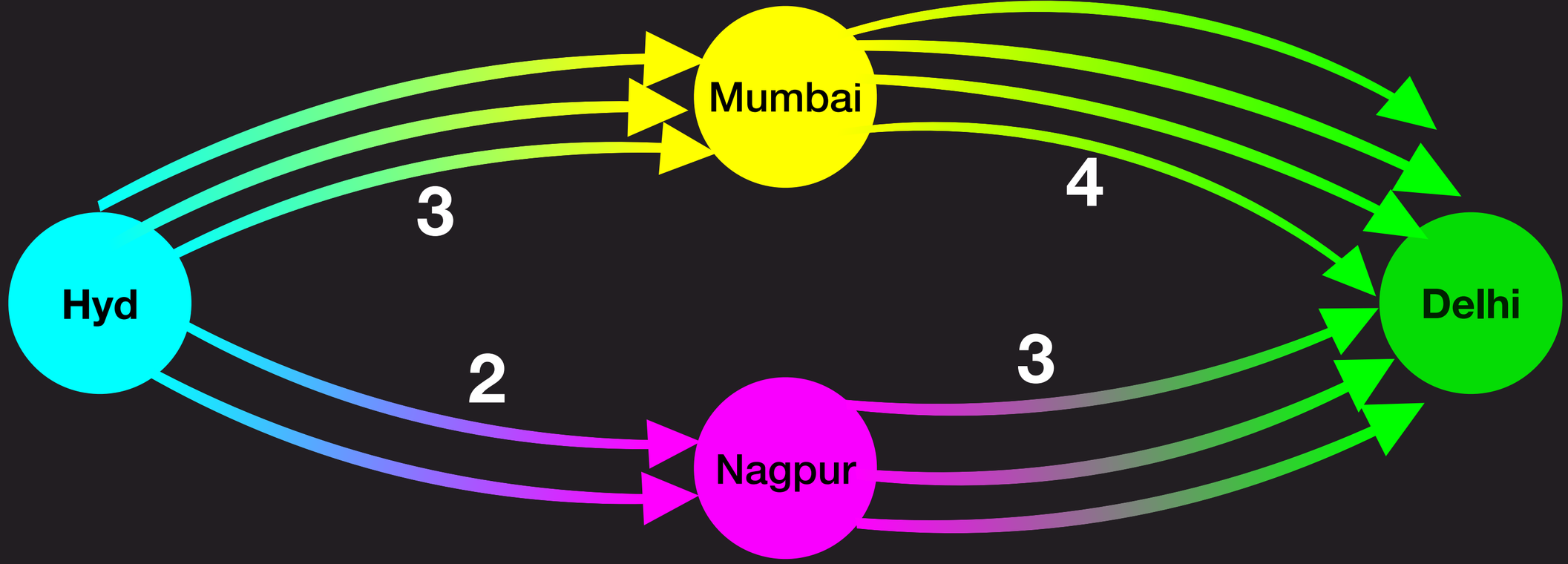
3 ways

Total number of ways = 2 * 3 = 6

There are 3 ways to move from Hyderabad to Mumbai, and 4 ways to move from Mumbai to Delhi.

There are 2 ways to move from Hyderabad to Nagpur, and 3 ways to move from Nagpur to Delhi.

In how many ways can we move from Hyderabad to Delhi?



Via Mumbai 3 * 4 = 12

Via Nagpur 2 * 3 = 6

Total = 12 + 6 = 18

Gift shop

There are 3 pens, 3 rings, 3 chocolates, 5 books, 7 flowers

3 pens

3 rings

3 chocolates

5 books

7 flowers

You can gift one of the following combos:

1 Pen and 1 Book

1 Flower and 1 Chocolate

1 Ring

How many such combinations can you make?

1 Pen and 1 Book	3 * 5 = 15
1 Flower and 1 Chocolates	7 * 3 = 21
1 Ring	3

Total = 15 + 21 + 3 = 39

Permutations

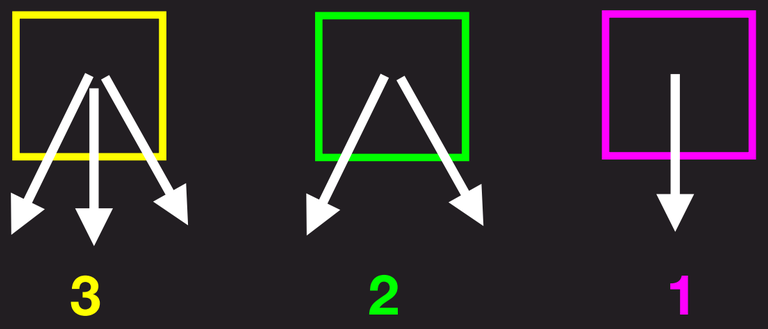
Arrangement of objects

Order matters!

$$(i, j) \neq (j, i)$$

$$a\ b \neq b\ a$$

What is the number of ways of arranging 3 characters: A, B, C?



A → B → C : ABC

A → C → B : ACB

B → A → C : BAC

B → C → A : BCA

C → A → B : CAB

C → B → A : CBA

Total number of ways = 3 * 2 * 1 = 6

This number is called 3 factorial

3! = 3 * 2 * 1

What is the number of ways of arranging 4 characters: A, B, C, D?

4 3 2 1

Total number of ways $= 4! = 4 * 3 * 2 * 1 = 24$

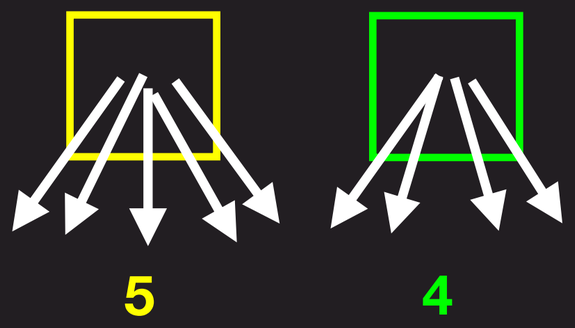
4 factorial

What is the number of ways of arranging N distinct objects?

$$N * (N - 1) * (N - 2) * \cdots * 3 * 2 * 1 = N!$$

N factorial

Given 5 different characters, in how many ways can we arrange them in 2 places?



Total number of ways = $5 * 4 = 20$

$$^{5}P_{2} = 5 * 4 \quad = \frac{5 * 4 * 3 * 2 * 1}{3 * 2 * 1} \quad = \frac{5!}{3!}$$

Given "N" distinct objects, count the number of ways in which can we arrange them in 3 places.

$$^{N}P_3 = N * (N-1) * (N-2)$$

Given "N" distinct objects, count the number of ways in which can we arrange them in 4 places.

$$^{N}P_4 = N * (N-1) * (N-2) * (N-3)$$

Given "N" distinct objects, count the number of ways in which can we arrange them in "k" places.

$$^{N}P_k = N * (N-1) * (N-2) * (N-3) * \cdots * (N-k+1)$$

$$^{N}P_k = \frac{N * (N-1) * (N-2) * (N-3) * \cdots * (N-k+1) * (N-k) * (N-k-1) * \cdots 3 * 2 * 1}{(N-k) * (N-k-1) * \cdots 3 * 2 * 1}$$

$$^{N}P_k = \frac{N!}{(N-k)!}$$

Let us see the same for 5 objects in 2 places

$$^{5}P_2 = 5 * 4 = \frac{5 * 4 * 3 * 2 * 1}{3 * 2 * 1} = \frac{5!}{3!}$$

$$N = 5 \quad k = 2 \quad N - k = 3$$

$$\frac{N!}{(N-k)!} = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

There are 4 players P1, P2, P3, and P4 who can play in the top-order positions of 1, 2, and 3.

How many arrangements of top-order can we make from 3 of these 4 players?

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2

Sachin, Sehwag, Kohli, Rohit

Examples of top-order

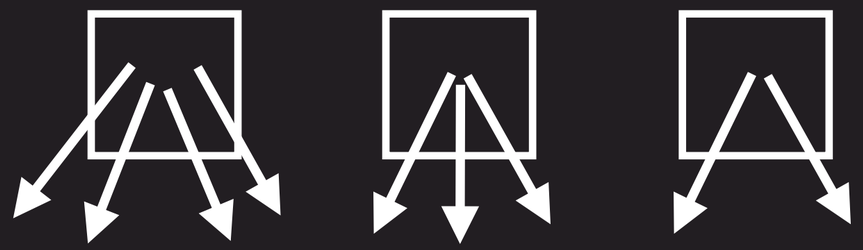
Sachin, Sehwag, Kohli

Sehwag, Sachin, Kohli

Sachin, Rohit, Kohli

Sachin, Kohli, Sehwag

How many more like this?



4 3 2

$$24 = 4 * 3 * 2$$

$${}^{4}P_{3}$$

Sachin, Sehwag, Kohli, Rohit

Suppose we have to select 3 players out of 4 players in our team.

In how many ways can we do this?

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2
P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4

$$^4C_3 = \frac{^4P_3}{3!}$$

$$24 = 4 * 3 * 2$$

$$^4P_3$$

$$\frac{24}{6} = 4$$

$6 \rightarrow 3!$

Combinations

$(i, j) = (j, i)$

$$ {}^{n}C_{k} = \frac{{}^{n}P_{k}}{k!} $$

Suppose we have to select 2 players out of 5 players in our team.

In how many ways can we do this?

$$^{5}C_{2} = \frac{5 * 4}{2 * 1} = 10$$