15th Feb. 2023

PROBABILITY CLASS #2

BAYE'S THEOREM

let's start @ 9:05

IPL

CSK

→ (6 ov. , 90 runs , 1 wicket) ☺ → chances of winning is high.

→ (6 Ov. , 20 runs , 4 wicket) ☹ → chances of winning is low.

⇒ P(Win given [6 Ov, 90R , 1W]) → high ~ (0.86)

P(W | (6 Ov. , 90 R , 1W)) → high.

P (W | (6 Ov. 20 R , 4R)) → low.

Conditional Probability.

Whatsapp / Telegram / Google Search

Langugе Models.

w_1 w_2

how are

w_3: you we they

$P(w_3 = \text{you} \mid (w_1 = \text{how}, w_2 = \text{are})) \rightarrow 0.90$

$P(w_3 = \text{we} \mid (\text{"} \quad \text{"})) \rightarrow 0.45$

$P(w_3 = \text{they} \mid (\text{"} \quad \text{"})) \rightarrow 0.20$

Email

Spam

Not spam → ham

- H H H S H → $P(S) = \frac{1}{5}$

$P(S \mid \text{"lottery"})$ → high

Manohar

D_1

Vaibhav

D_2

$$(D_1 + D_2)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ Conditional Probability

$$\boxed{P(A \cap B) = P(B) \cdot P(A|B)}$$

multiplication Rule.

Among 30 faculty members in a department, 5 are females and 25 are males. 3 females and 12 males have a PhD

F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M
F	M	M	M	M	M

$P(F) = \frac{5}{30}$ $\quad P(M) = \frac{25}{30}$

$P(F \cap PhD) = \frac{3}{30}$

$P(M \cap PhD) = \frac{12}{30}$

- $P(PhD) = \frac{15}{30}$

Among those who have done PhD, what fraction are females?

$$P(F \mid Phd) = \frac{3}{15} = \frac{3}{3 + 12}$$

$$\boxed{P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}}$$

$\downarrow$

Bayes theorem

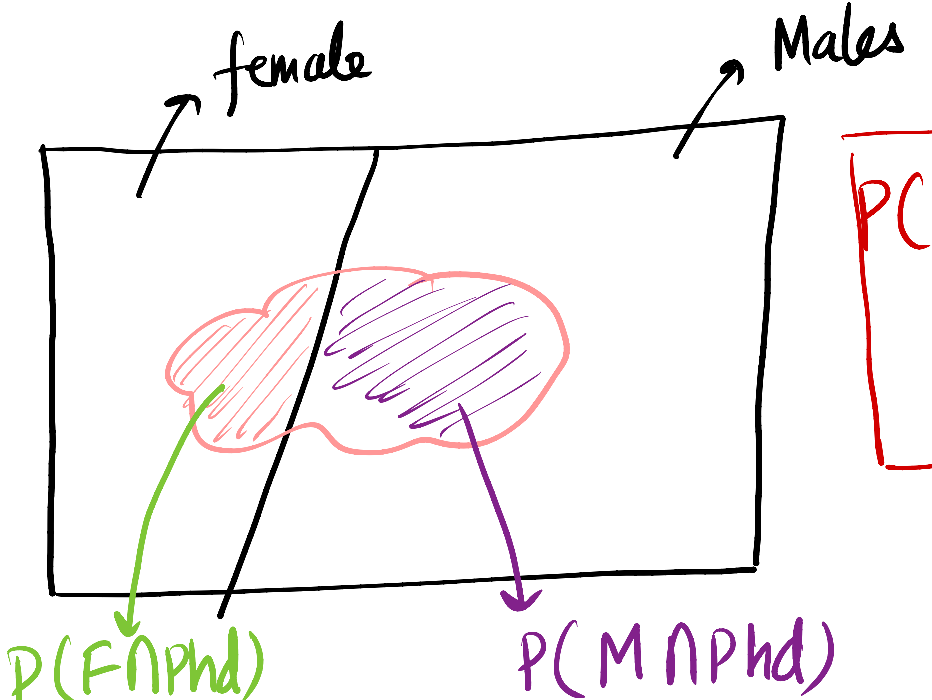
$$P(F \mid Phd) = \frac{P(Phd \mid F) \cdot P(F)}{P(Phd)}$$

$$= \frac{\bullet}{\bullet + \bullet}$$

$\bullet \rightarrow P[Phd \mid F] \cdot P(F) = P(F \cap Phd)$

→ $P(Phd|M) \cdot P(M) = P(M \cap Phd)$

$$P(Phd) = P(F \cap Phd) + P(M \cap Phd)$$



$$\boxed{P(Phd) = \underbrace{P(F \cap Phd)}_{P(Phd|F)\cdot P(F)} + P(M \cap Phd)}$$

* $$\boxed{P(Phd) = P(Phd \mid F) \cdot P(F) + P(Phd \mid M) \cdot P(M)}$$

Total Probability.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \checkmark$$

1.) $P(F) = 0.4 \rightarrow P(Phd \mid F) = 0.3$

$P(M) = 0.6 \rightarrow P(Phd \mid M) = 0.4$

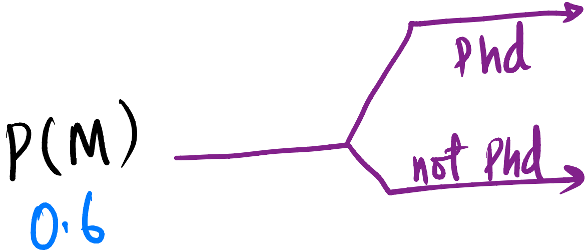
A random person is choosen → Question.

$P(F \mid Phd) \rightarrow ?$

$P(M \mid Phd) \rightarrow ?$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A \mid B) \cdot P(B)$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(B \mid A) \cdot P(B)$$

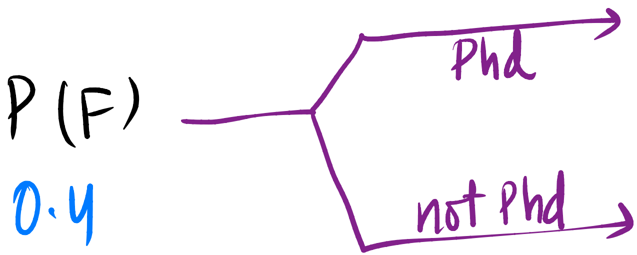


$0.40 = P(Phd \mid M) \rightarrow P(M \cap Phd)$

$= P(Phd \mid M) \cdot P(M)$

$= 0.40 \times 0.6$

$= \underline{0.24}$

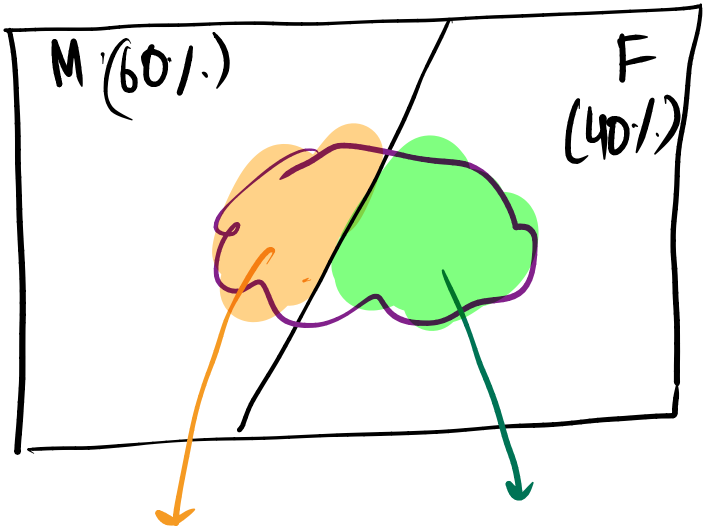


$0.30 = P(Phd \mid F) \rightarrow P(F \cap Phd)$

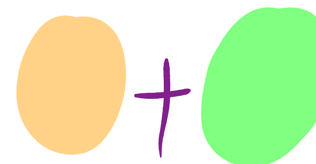
$= P(Phd \mid F) \cdot P(F)$

$= 0.3 \times 0.4$

$= \underline{0.12}$



$$P(Phd) = P(M \cap Phd) + P(F \cap Phd)$$



$P(M \cap Phd)$ $\qquad$ $P(F \cap Phd)$

$$\boxed{P(Phd) = 0.24 + 0.12 = 0.36}$$

Q) A random person is found to be a Phd holder.

(i) what is the prob. that this person is Male?

(ii) ______ Female?

(i) $P(M \mid Phd) = ?$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(M \mid Phd) = \frac{P(M \cap Phd)}{P(Phd)}} = \frac{0.24}{0.36}$$ ✓

(ii) $P(F \mid Phd)$

$$= \frac{P(F \cap Phd)}{P(Phd)} = \frac{0.12}{0.36} \checkmark$$

$$P(F \mid Phd) = \frac{0.12}{0.36}$$

10 : 35

Break

Conditional Probability $\rightarrow$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Multiplication Rule $\rightarrow$ $P(A \cap B) = P(A|B)\,P(B)$

Bayes theorem $\rightarrow$ $P(B|A) = \dfrac{P(A|B) \cdot P(B)}{P(A)}$

Law of total Probability $\rightarrow$ $P(A) = P(A|B) \cdot P(B) + P(A|\bar{B}) \cdot P(\bar{B})$

$P(Phd) = P(Phd \cap F) + P(Phd \cap M)$

$P(Phd) = P(Phd|F) \cdot P(F) + P(Phd|M) \cdot P(M)$

Independent Events :

A and B are said to be independent when

$$\boxed{P(A|B) = P(A)}$$

· throwing a die & throwing a coin independent ?

$$S = \{(H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6)$$
$$(T,1)\ (T,2)\ (T,3)\ (T,4)\ (T,5)\ (T,6)\}$$

A → getting heads in coin toss =
$\{(H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6)\}$

$$P(A) = 6/12 = 1/2$$

B → getting # 3 on a die

$$= \{(H,3)\ (T,3)\} \quad P(B) = \frac{2}{12} = \frac{1}{6}$$

$$P(A \cap B) = P(\{(H,3)\}) = \frac{1}{12}$$

$$\underset{\substack{\downarrow \\ \text{heads}}}{P(A} \mid \underset{\substack{\downarrow \\ \#3}}{B)} = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/6} = \frac{1}{2}$$

$$= P(A)$$

$$\Rightarrow \boxed{P(A \mid B) = P(A)}\ *$$

Conditional Prob.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \boxed{P(A \cap B) = \underbrace{P(A|B)}_{P(A)} \cdot P(B)}$$

A & B are independent

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

$\{\ \} \longrightarrow$ null set

A & B are exclusive events

$A \cap B \longrightarrow \{\ \}$

Conditional Probability $\rightarrow$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Multiplication Rule $\rightarrow$ $P(A \cap B) = P(A|B)\,P(B)$

Bayes theorem $\rightarrow$ $P(B|A) = \dfrac{P(A|B) \cdot P(B)}{P(A)}$

Law of total Probability $\rightarrow$ $P(A) = P(A|B) \cdot P(B) + P(A|\bar{B}) \cdot P(\bar{B})$

$P(Phd) = P(Phd \cap F) + P(Phd \cap M)$

$P(Phd) = P(Phd|F) \cdot P(F) + P(Phd|M) \cdot P(M)$

Independent Events $\rightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$