Agenda

Case study

Casino case study

Key terms we will see today

Random variable

From sample space to numbers

Empirical Vs Theoretical probability

Empirical means estimating from data. With experiment

Theoretical means computing from rules. Without experiment

Distribution

Rules guiding the process

Expectation

Extension of mean, using distribution

Binomial Distribution

Casino case study

A bag has 3 red and 2 blue balls.

You pick a ball, write its colour, and put it back in the bag. This is done 4 times in total.

If all 4 times, the red ball was drawn, you win Rs 150. In any other case, you lose Rs 10.

Would you play this game?

What are all the outcomes?

0 red   1 red   2 red   3 red   4 red

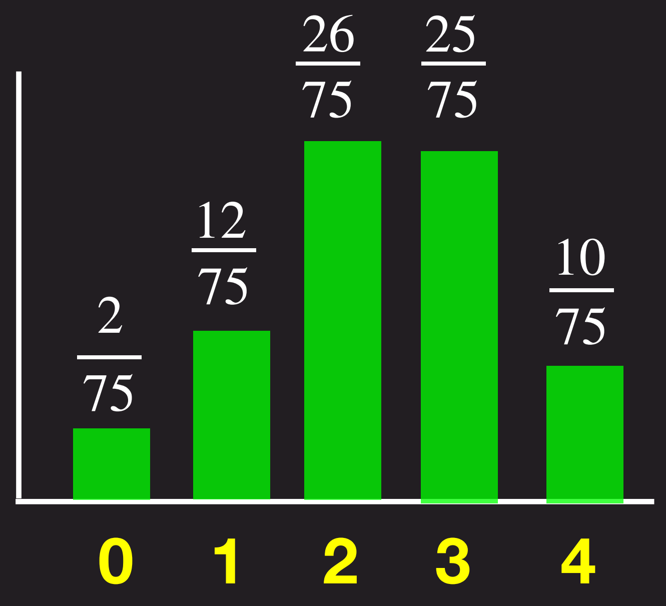
Let "X" denote the number of red balls when you draw 4 balls with replacement

Here, X is an example of what is called a "Random Variable"

Empirical approach: Estimate probability using data

Data from 75 people

$X = 0$ 2 people
$X = 1$ 12 people
$X = 2$ 26 people
$X = 3$ 25 people
$X = 4$ 10 people



X	$P[X]$	$E[X]$
0	$\frac{2}{75}$	$(0)\left(\frac{2}{75}\right)$ +
1	$\frac{12}{75}$	$(1)\left(\frac{12}{75}\right)$ +
2	$\frac{26}{75}$	$(2)\left(\frac{26}{75}\right)$ +
3	$\frac{25}{75}$	$(3)\left(\frac{25}{75}\right)$ +
4	$\frac{10}{75}$	$(4)\left(\frac{10}{75}\right)$

Expectation of X is the weighted average of the values that X takes, with the weights being the probabilities

$$E[X] = (0)\left(\frac{2}{75}\right) + (1)\left(\frac{12}{75}\right) + (2)\left(\frac{26}{75}\right) + (3)\left(\frac{25}{75}\right) + (4)\left(\frac{10}{75}\right) = 2.38$$

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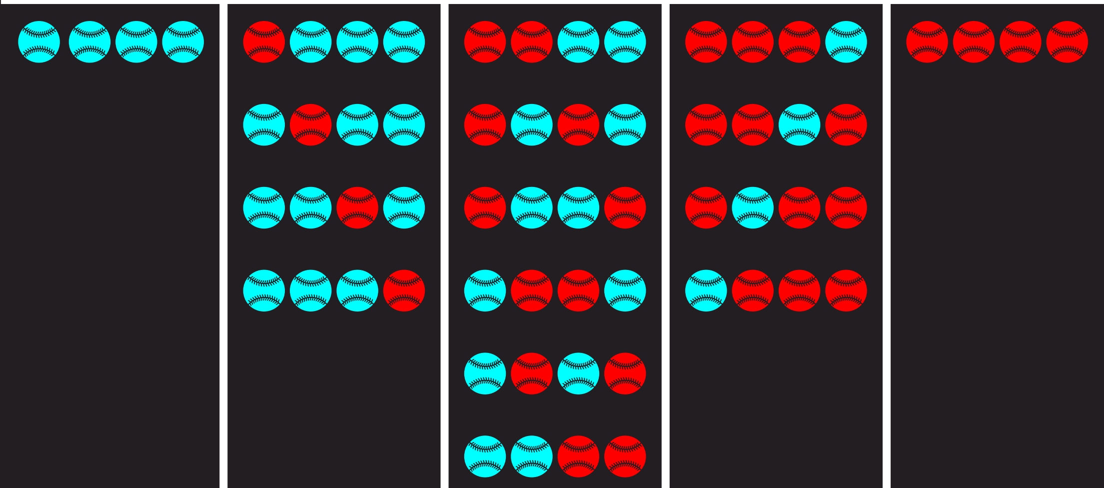


$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{2}{5}$	$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{3}{5}$	$\frac{2}{5}\frac{2}{5}\frac{3}{5}\frac{3}{5}$	$\frac{2}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$	$\frac{3}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

Theoretical approach: Compute probability using rules

What is the probability of 1 red ball in 1 pick?

P[] = 3/5

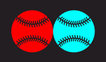
What is the probability of 1 blue ball in 1 pick?

P[] = 2/5

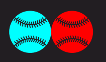
What is the probability of 2 red balls in 2 picks?

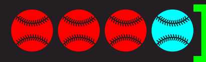
P[] = (3/5)(3/5)

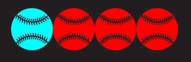
What is the probability of 1 red ball in first pick and 1 blue ball in second?

P[] = (3/5)(2/5)

What is the probability of 1 blue ball in first pick and 1 red ball in second?

P[] = (2/5)(3/5)

P[] = (3/5)(3/5)(3/5)(2/5)

P[] = (3/5)(3/5)(3/5)(2/5)

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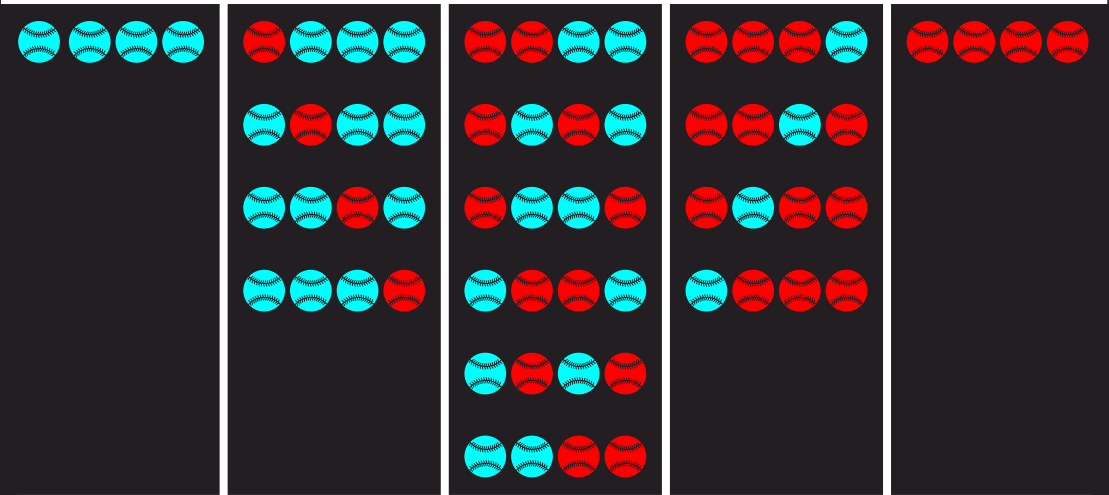

$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{2}{5}$	$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{3}{5}$	$\frac{2}{5}\frac{2}{5}\frac{3}{5}\frac{3}{5}$	$\frac{2}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$	$\frac{3}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

Theoretical approach: Compute probability using rules

X	Number of outcomes	Probability per outcome	$P[X]$	Code
0	${}^4C_0$	$\left(\frac{2}{5}\right)^4$	${}^4C_0\left(\frac{2}{5}\right)^4$	`binom.pmf(k=0, n=4, p=3/5)`
1	${}^4C_1$	$\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$	${}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$	`binom.pmf(k=1, n=4, p=3/5)`
2	${}^4C_2$	$\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2$	${}^4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2$	`binom.pmf(k=2, n=4, p=3/5)`
3	${}^4C_3$	$\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	${}^4C_3\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	`binom.pmf(k=3, n=4, p=3/5)`
4	${}^4C_4$	$\left(\frac{3}{5}\right)^4$	${}^4C_4\left(\frac{3}{5}\right)^4$	`binom.pmf(k=4, n=4, p=3/5)`

$E[X] = 2.4$

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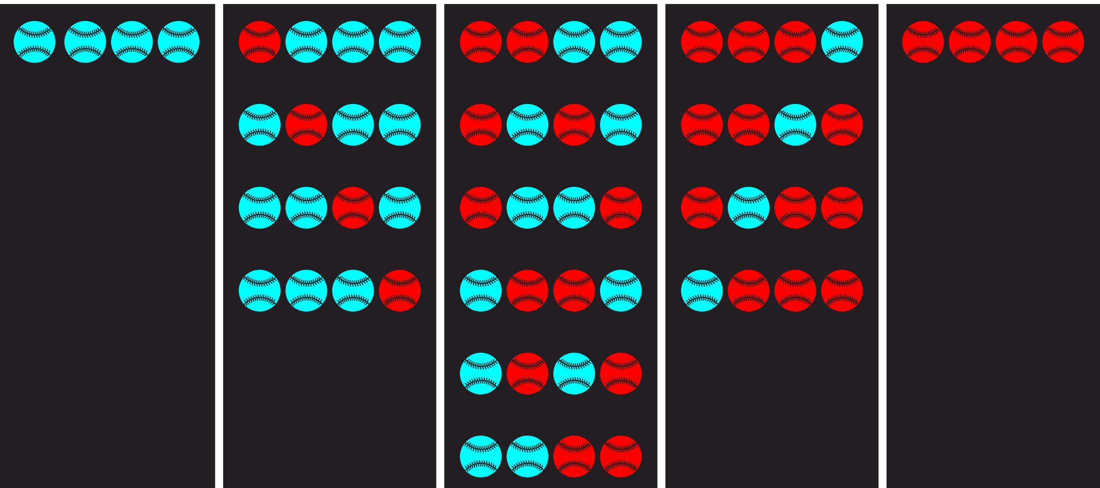


$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{2}{5}$	$\frac{2}{5}\frac{2}{5}\frac{2}{5}\frac{3}{5}$	$\frac{2}{5}\frac{2}{5}\frac{3}{5}\frac{3}{5}$	$\frac{2}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$	$\frac{3}{5}\frac{3}{5}\frac{3}{5}\frac{3}{5}$
${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

Let "X" denote the number of red balls when you draw 4 balls with replacement

Here, X is an example of what is called a "Random Variable"

Let "Y" be the amount won. This is also another example of a random variable

What are all the outcomes for "Y"?

"$Y = 150$" If we get 4 red balls

"$Y = -10$" Otherwise

Y	$P[Y]$	
150	${}^4C_4\left(\frac{3}{5}\right)^4$	**0.1296**
-10	${}^4C_0\left(\frac{2}{5}\right)^4 + {}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1 + {}^4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2 + {}^4C_3\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	**0.8704**

$$E[Y] = (150)(0.1296) + (-10) * (0.8704) \quad = 10.736$$

Quiz Time! **Just for now, please don't answer in chat**

You toss a coin two times. **Suppose you get 1 Rs for every Heads**

Q1) What are the possible amounts that you will receive out of this?

Q2) What is the probability of getting 2 Rs?

Q3) What is the probability of getting 1 Rs?

Q4) What is the probability of getting 0 Rs?

Q5) What is the expected amount you will get?

Sample space

$$S = \{\ HH, HT, TH, TT\ \}$$

Let "X" denote the number of heads

$$X = 0 \longrightarrow \{TT\}$$

$$X = 1 \longrightarrow \{HT, TH\}$$

$$X = 2 \longrightarrow \{HH\}$$

$$E[X] = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{1}{2}\right) + (2)\left(\frac{1}{4}\right) = 1$$

X	$P[X]$	**Binomial**
0	$\frac{1}{4}$	${}^2C_0\left(\frac{1}{2}\right)^2$
1	$\frac{1}{2}$	${}^2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
2	$\frac{1}{4}$	${}^2C_2\left(\frac{1}{2}\right)^2$

Quiz Time! Just for now, please don't answer in chat

You toss two dice.

If both dice are 6, you get 2 Rs

Else if one dice is 6, and another is not 6, then you get 1 Rs

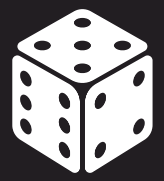
Else, you get 0 Rs

Q 1) What is the probability of getting 0 Rs?

Q 2) What is the probability of getting 1 Rs?

Q 3) What is the probability of getting 2 Rs?

Q 4) What is the expected amount?

D_2 

D_1 

# of 6	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	1
3	0	0	0	0	0	1
4	0	0	0	0	0	1
5	0	0	0	0	0	1
6	1	1	1	1	1	2

X	$P(X)$
0	${}^2C_0\left(\frac{5}{6}\right)^2$
1	${}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$
2	${}^2C_2\left(\frac{1}{6}\right)^2$

$$\frac{5*5}{36}$$

$$\frac{5*1+1*5}{36}$$

$$\frac{1*1}{36}$$

Binomial Distribution

If X is random variable that follows the Binomial distributions with parameters "n" and "p", then

$$P[X = k] = {}^{n}C_{k}\ p^{k}\ (1-p)^{(n-k)}$$