- E and F $\rightarrow$ mutually exclusive events.

$P(E) = 0.5$

$P(E|F) = ?$

$\Rightarrow P(E|F) = \dfrac{P(E\cap F)}{P(F)} = \dfrac{0}{P(F)} = 0$

$P(E\cap F) \rightarrow 0$

* Akshay & Hamza [Tossing a coin] H or T

Whoever gets head first wins?

Akshay starts tossing first.

Akshay → 1st
Hamza → 2nd *

Prob. of Akshay winning the game?

⇒ Solution

$\boxed{P(\text{Akshay Winning})}$

$P(A) = $ 'p'

Sample Space $= \{H, TH, TTH, TTTH, TTTTH, \ldots\}$

Event space for Akshay.

$A = \{H, TTH, TTTTH, \ldots\ldots\}$

head occuring only on odd # of tosses.

Prob. of head → 'p'
Prob. of tail → '1 − p'

$$S = p + (1-p)^2 p + (1-p)^4 \cdot p + (1-p)^6 \cdot p + \ldots$$

↓

total Prob. of Akshay winning $= S$

⇒ let's assume $(1-p)^2 = r$

$$S = p + \cancel{rp} + \cancel{r^2 p} + \cancel{r^3 p} + \cdots$$

$$Sr = \cancel{rp} + \cancel{r^2 p} + \cancel{r^3 p} + \cancel{r^4 p} + \cdots$$

$$\boxed{S - Sr = p}$$

$$r = (1-p)^2$$

$$\Rightarrow S(1-r) = p$$

$$\Rightarrow S = \frac{p}{(1-r)} = \frac{p}{[1-(1-p)^2]}$$

$$\boxed{S = \frac{p}{[1-(1-p)^2]}}$$

final prob. of akshay winning:

$\Rightarrow$ $p = 0.5$ $\quad p = 1/2 = P(H) \Rightarrow P(A) = \dfrac{2}{3}$

$\sim 67\%$.

$$S = \left[\frac{p}{1-(1-p)^2}\right] \qquad p = \frac{1}{2}$$

$$= \frac{\frac{1}{2}}{1-\left[\underbrace{1-\frac{1}{2}}\right]^2} = \frac{1/2}{3/4} = \frac{4}{2\times 3}$$

$$= \frac{2}{3} \checkmark$$

Tennis Game

I^st^ II^nd^ III^rd^

15 30 40

A
B

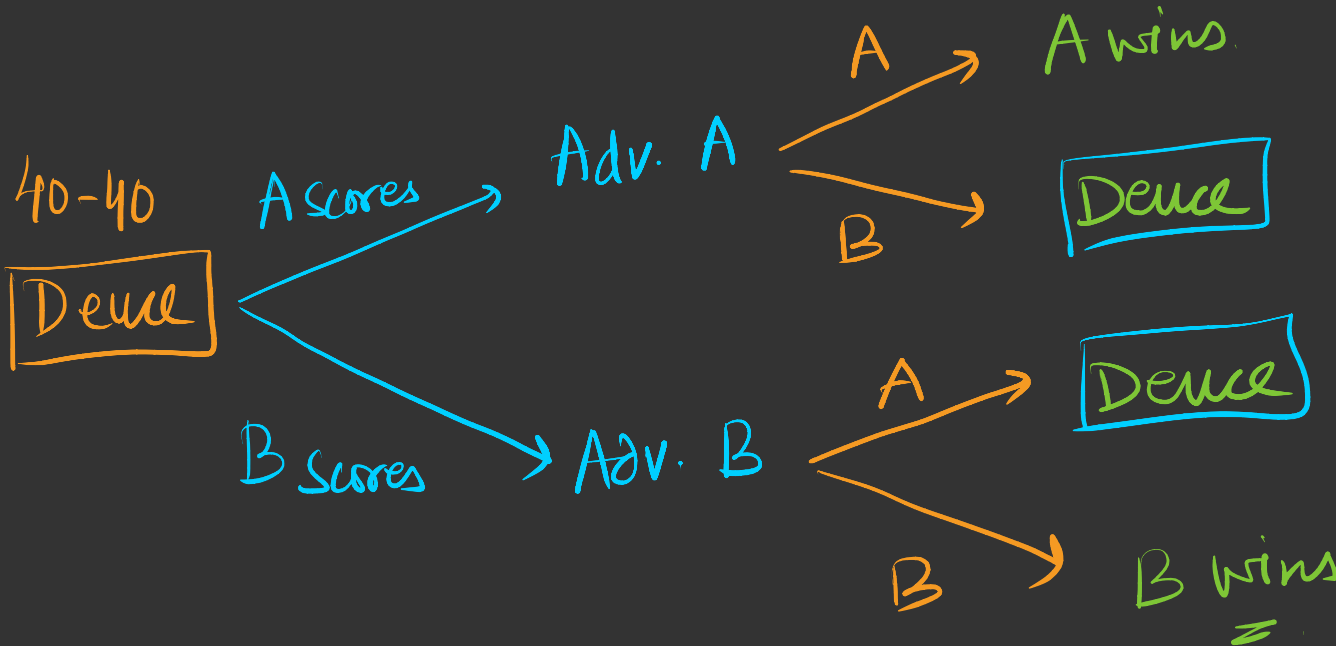
40 - 40 DEUCE

A → PP → A wins

B → PP → B wins

$A \rightarrow P \Rightarrow B \rightarrow P$ } Deuce $\underline{40-40}$

$B \rightarrow P \Rightarrow A \rightarrow P$ } deuce $\underline{40-40}$



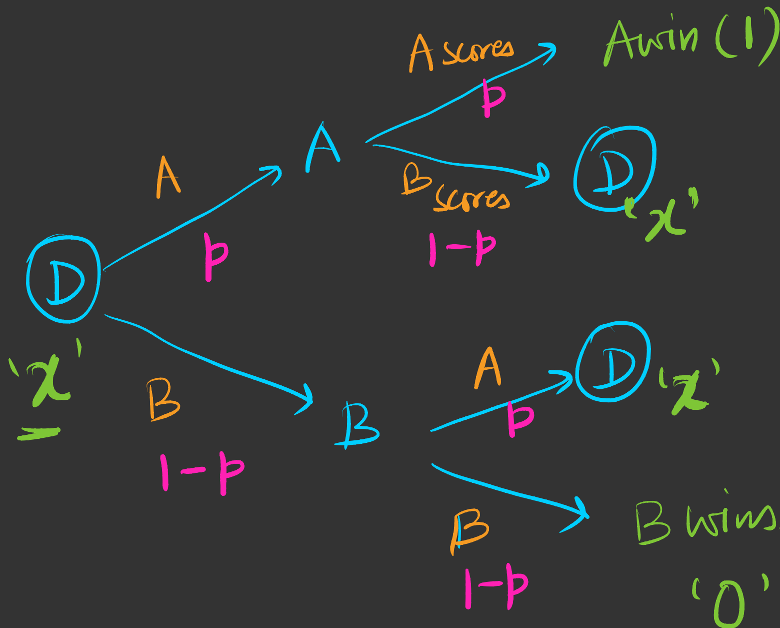
Scoring Prob

A → 'p'

B → '1-p'

What is the Probability of A winning.

P(A win) → ?



lets assume prob. of A winning is 'x'

$P(A) = x$

for D point $P(A \text{ win}) = x =$ Total prob. of A winning.

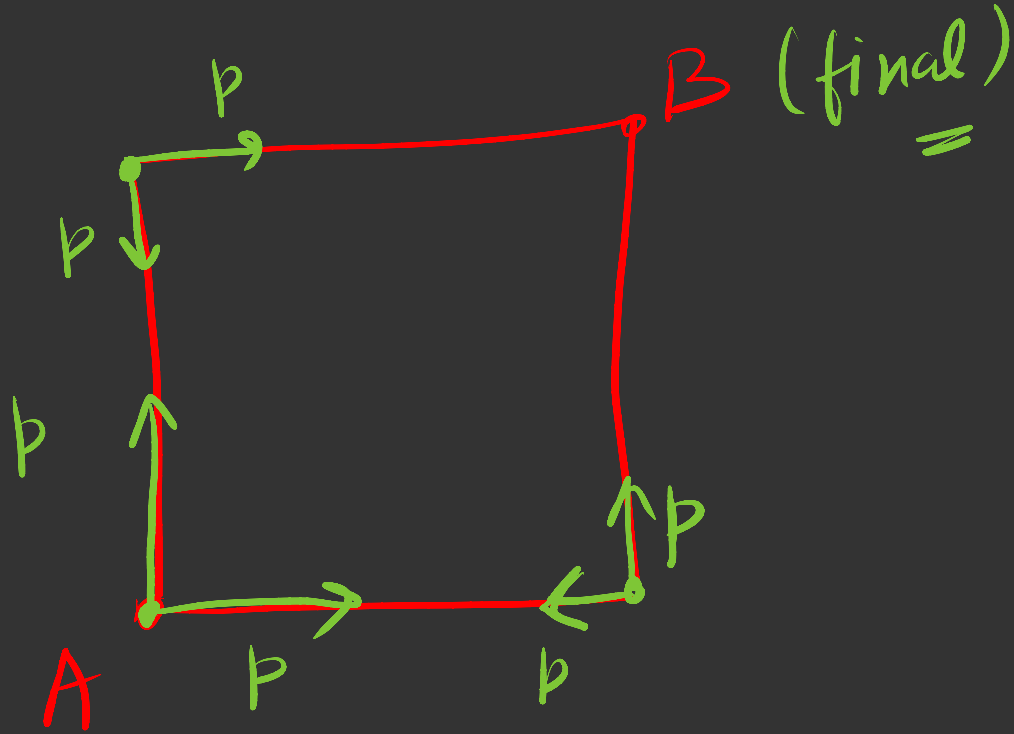
$$x = (p \cdot p \cdot 1) + p \cdot (1-p) \cdot x + (1-p)(p) \cdot x + (1-p)(1-p) \cdot 0$$

$$x = p^2 + 2px(1-p)$$

$$x[1 - 2p(1-p)] = p^2$$

$$x = \frac{p^2}{[1 - 2p(1-p)]}$$

$$\boxed{x = \frac{p^2}{[1 - 2p + 2p^2]}}$$



A
B
(final)
p
p
p
p
p
p