Last Class

1) Matrix Factorization
2) PCA
3) Singular Value Decom

Today's class

1) MF for clustering
2) Non negative MF
3) Building a Rec Sys with multiple data matrices
4) Netflix Prize Solution

$x_i = (x_{i1}, x_{i2}, x_{i3})$ 3D space

$c_j = (c_{j1}, c_{j2}, c_{j3})$ ✓ Centroid values of j-th cluster

$$||x_i - c_j||^1 = |x_{i1} - c_{j1}| + |x_{i2} - c_{j2}| + |x_{i3} - c_{j3}|$$

$$||x_i - c_j||^2 = |x_{i1} - c_{j1}|^2 + |x_{i2} - c_{j2}|^2 + |x_{i3} - c_{j3}|^2$$

$n = 3, \quad k = 2$

sum = 1

$$Z = \begin{array}{c|cc} & \text{cluster 1} & \text{cluster 2} \\ x_1 & 1 & 0 \\ x_2 & 0 & 1 \\ x_3 & 1 & 0 \end{array}$$

$Z_{0,1}$

0/1

$x_1 \to$ cluster 1

$x_3 \to$ cluster 1

$x_2 \to$ cluster 2

$$\begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.33 & 0.33 & 0.34 \end{bmatrix} \times$$

Z

$$\begin{array}{c|ccc} & C1 & C2 & C3 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 1 & 0 \\ x_4 & 0 & 0 & 1 \end{array}$$

$\to 1$

$\to 1$ (sum)

3×2

row   cols

$$\sum_{j=1}^{k} Z_{ij} = 1$$

$x_3 \to C2$

0/1 4×3

obj fn: $\min_{c_j} \sum_{j=1}^{k} \sum_{i=1 \, (x_i \in S_j)}^{n} \| x_i - c_j \|^2$

$$\boxed{\min_{c_j, Z_{ij}} \sum_{j=1}^{k} \sum_{i=1}^{n} Z_{ij} \| x_i - c_j \|^2}$$

1) $\boxed{\sum_{j=1}^{k} Z_{ij} = 1 \quad \text{sum along rows}}$ GMM

2) $\boxed{Z_{ij} = 0 \text{ or } 1}$

fractional values

$c_j \to (\;,\; , \ldots)$

$c_{ji} \in R$ where R is any real value

no constraint   +ve, −ve, 0

$$\min \; \| \underset{n\times d}{X} - \underset{(n\times k)}{Z} \cdot \underset{(k\times d)}{C^T} \|_F^2$$

K = no. of clusters

$(n\times k) \times (k\times d)$

$n\times d$

$$X = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} & \cdots & x_{i,d} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2,d} \\ \vdots & & & & \\ x_{n,1} & x_{n,2} & \cdots & \cdots & x_{n,d} \end{bmatrix}_{n\times d}$$

$$C = \begin{bmatrix} \uparrow & \uparrow & \uparrow & & \uparrow \\ c_1 & c_2 & c_3 & \cdots & c_k \\ \downarrow & \downarrow & \downarrow & & \downarrow \end{bmatrix}_{d\times k}$$

$$C^T = \begin{bmatrix} \longleftarrow & c_1 & \longrightarrow \\ \longleftarrow & c_2 & \longrightarrow \\ & \vdots & \\ \longleftarrow & c_k & \longrightarrow \end{bmatrix}_{k\times d}$$

$$\begin{bmatrix} a_{11} & a_{12} & \\ & & \end{bmatrix}_{n\times d} - \begin{bmatrix} b_{11} & b_{12} & \\ & & \end{bmatrix}_{n\times d}$$

$(a_{11} - b_{11})^2 + (a_{12} - b_{12})^2 + \vdots$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\|A\|_F^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$$

$$X = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 1 & 7 \\ 7 & 2 & 3 \\ 5 & 0 & 8 \end{bmatrix}$$

4×3

$n \quad d$

$k = 2$

$d \times k$

$$C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

3×2

$Z \cdot C^T$

$$C^T = \begin{array}{cl} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} & \begin{array}{l} c_1 \\ c_2 \end{array} \end{array}$$

2×3

4×3

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \quad Z \cdot C^T$$

4×3

c_1 has 3 points

c_2 has 1 point

$$\min_{Z, C} \| X - Z \cdot C^T \|_F^2$$

data

given

learning

$$\boxed{X \simeq Z \cdot C^T}$$

clustering

MF

$$\| X - Z \cdot C^T \|_F^2 \simeq 0$$

$2 \simeq \rightarrow \boxed{k = d}$

Line Reg $\quad y = WX \quad \sum_{i=1}^{n} (y_i - w \cdot x_i)^2$

$$A = \begin{array}{c|cccc} & i_1 & i_2 & \cdots & i_m \\ \hline u_1 & r_{11} & r_{12} & \cdots & r_{1,m} \\ u_2 & r_{21} & r_{22} & \cdots & r_{2,m} \\ \vdots & \vdots & & & \\ u_n & r_{n,1} & r_{n,2} & \cdots & r_{n,m} \end{array}$$

all ratings from user 1

all ratings for item 1

$A \simeq B \cdot C$

B → user vector matrix

C → item vector matrix

$A_{n \times m} \simeq B \, C^T$ $\quad C = (m \times d)$

B: $(n \times d)$ $\quad C^T$: $(d \times m)$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1,d} \\ b_{21} & b_{22} & \cdots & b_{2,d} \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

User 1 $= [b_{11} \; b_{12} \; \cdots \; b_{1,d}]$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1,d} \\ c_{21} & & & \\ c_{31} & & & \\ \vdots & & & \\ c_{m,1} & c_{m,2} & \cdots & c_{m,d} \end{bmatrix} \quad (m \times d)$$

item 1 $= [c_{11} \; c_{12} \; \cdots \; c_{1,d}]$

$$C^T = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{m,1} \\ c_{12} & c_{22} & & c_{m,2} \\ \vdots & \vdots & & \vdots \\ c_{1,d} & c_{2,d} & & c_{m,d} \end{bmatrix}$$

$$\begin{tikzcd} u_{10} \arrow[r, "L"] & v_2 / i_2 \text{ (video)} \arrow[dl, bend left] \\ u_{100} \arrow[u, bend left] \arrow[u, phantom, "\text{(similar)}"] & \end{tikzcd}$$

user-user similarity

$$\begin{tikzcd} v_2 \arrow[r, "L"] & u_{10} \\ v_5 \arrow[u, phantom, "\text{(similar)}"] \arrow[ur, bend right] & \end{tikzcd}$$

item-item similarity

$$\begin{tikzcd} u_{10} \arrow[r, "D"] & v_{50} \arrow[dl, bend left, "\times"] \\ u_{100} \arrow[u, phantom, "\text{(similar)}"] & \end{tikzcd}$$

user-user similarity from Dislike matrix

$$\begin{tikzcd} V_2 \arrow[r, "D"] & V_{15} \\ v_5 \arrow[ur, leftrightarrow, "\times"] & \end{tikzcd}$$

(similar)

Not recommende