

SHA-512 Logic

The algorithm takes as input a message with a maximum length of less than 2^{128} bits and produces as output a 512-bit message digest. The input is processed in 1024-bit blocks. Figure 11.8 depicts the overall processing of a message to produce a digest. This follows the general structure depicted in Figure 11.7. The processing consists of the following steps.

Table 11.3 Comparison of SHA Parameters

	SHA-1	SHA-224	SHA-256	SHA-384	SHA-512
Message Digest Size	160	224	256	384	512
Message Size	$< 2^{64}$	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block Size	512	512	512	1024	1024
Word Size	32	32	32	64	64
Number of Steps	80	64	64	80	80

Note: All sizes are measured in bits.

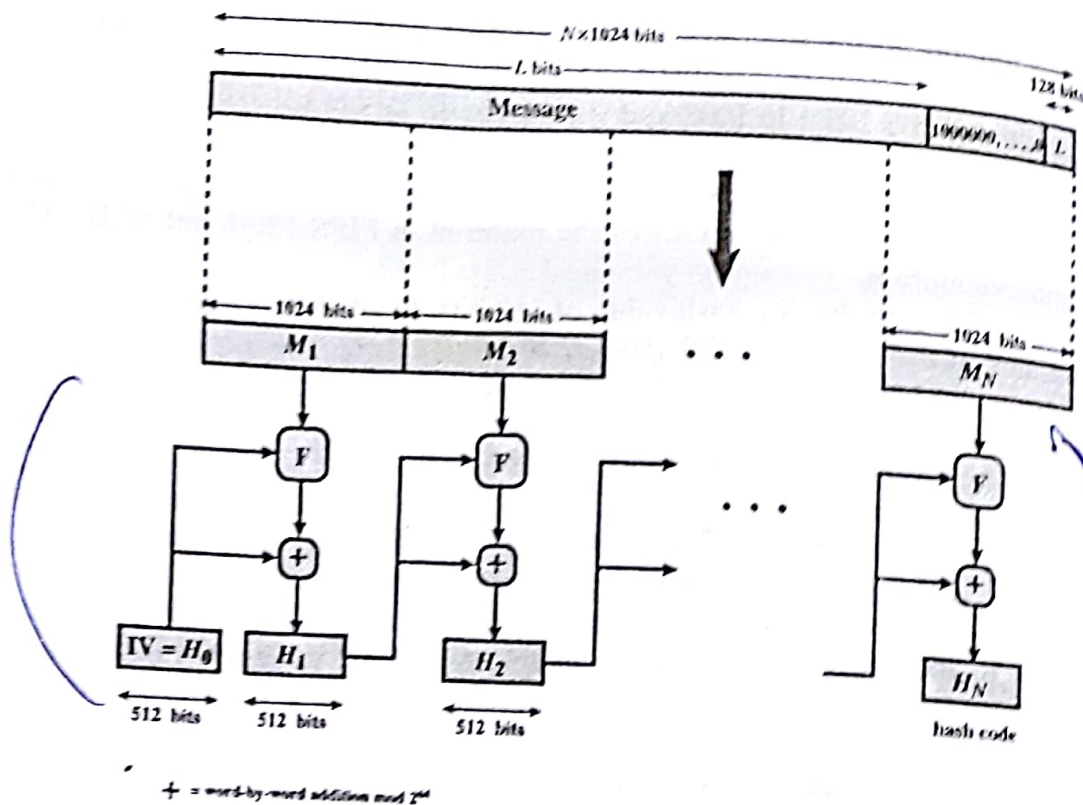


Figure 11.8 Message Digest Generation Using SHA-512

Step 1 Append padding bits. The message is padded so that its length is congruent to 896 modulo 1024 [$\text{length} \equiv 896(\text{mod } 1024)$]. Padding is always added, even if the message is already of the desired length. Thus, the number of padding bits is in the range of 1 to 1024. The padding consists of a single 1 bit followed by the necessary number of 0 bits.

Step 2 Append length. A block of 128 bits is appended to the message. This block is treated as an unsigned 128-bit integer (most significant byte first) and contains the length of the original message (before the padding).

The outcome of the first two steps yields a message that is an integer multiple of 1024 bits in length. In Figure 11.8, the expanded message is represented as the sequence of 1024-bit blocks M_1, M_2, \dots, M_N , so that the total length of the expanded message is $N \times 1024$ bits.

Step 3 Initialize hash buffer. A 512-bit buffer is used to hold intermediate and final results of the hash function. The buffer can be represented as eight 64-bit registers (a, b, c, d, e, f, g, h). These registers are initialized to the following 64-bit integers (hexadecimal values):

a = 6A09E667F3BCC908	e = 510E527FADE682D1
b = BB67AE8584CAA73B	f = 9B05688C2B3E6C1F
c = 3C6EF372FE94F82B	g = 1F83D9ABFB41BD6B
d = A54FF53A5F1D36F1	h = 5BE0CD19137E2179

These values are stored in **big-endian** format, which is the most significant byte of a word in the low-address (leftmost) byte position. These words were obtained by taking the first sixty-four bits of the fractional parts of the square roots of the first eight prime numbers.

Step 4 Process message in 1024-bit (128-word) blocks. The heart of the algorithm is a module that consists of 80 rounds; this module is labeled F in Figure 11.8. The logic is illustrated in Figure 11.9.

Each round takes as input the 512-bit buffer value, $abcdefgh$, and updates the contents of the buffer. At input to the first round, the buffer has the value of the intermediate hash value, (H_{i-1}) . Each round t makes use of a 64-bit value W_t , derived from the current 1024-bit block being processed (M_i). These values are derived using a message schedule described subsequently. Each round also makes use of an additive constant K_t , where $0 \leq t \leq 79$ indicates one of the 80 rounds. These words represent the first 64 bits of the fractional parts of the cube roots of the first 80 prime numbers. The constants provide a "randomized" set of 64-bit patterns, which should eliminate any regularities in the input data. Table 11.4 shows these constants in hexadecimal format (from left to right).

The output of the eightieth round is added to the input to the first round (H_{i-1}) to produce H_i . The addition is done independently for each of the eight

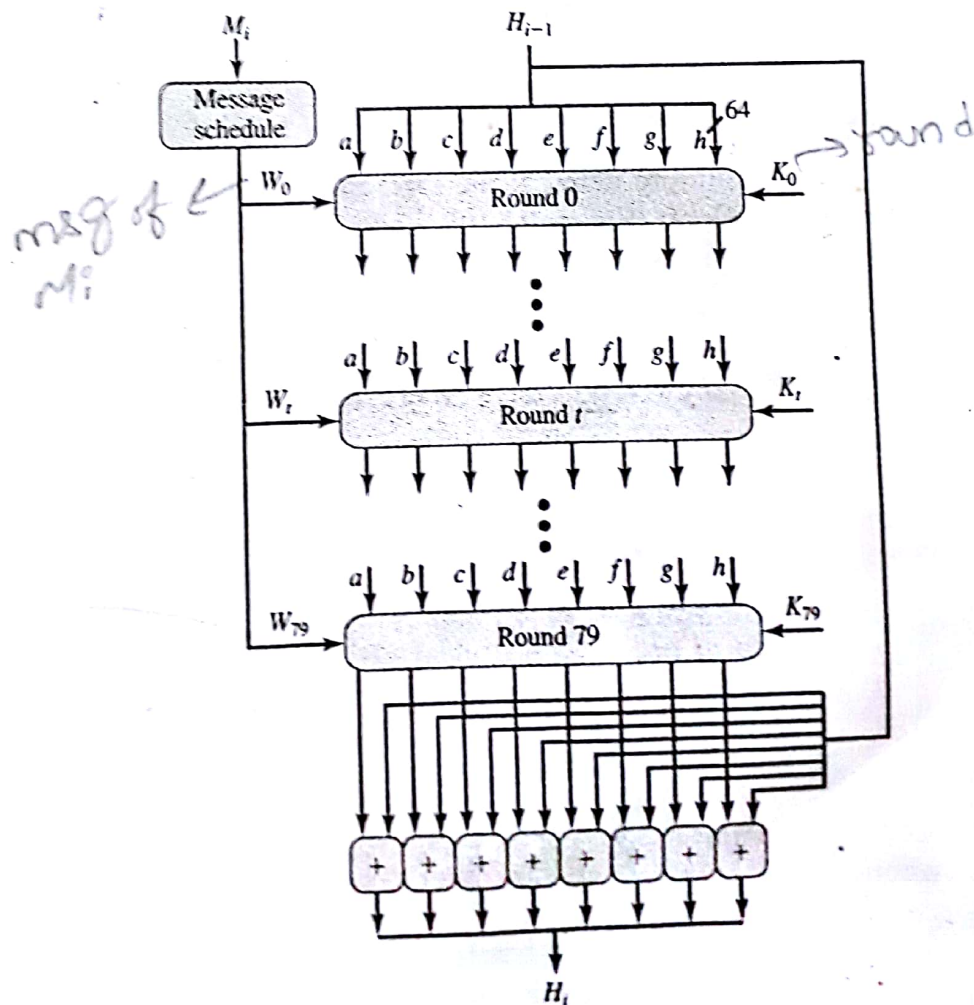


Figure 11.9 SHA-512 Processing of a Single 1024-Bit Block

words in the buffer with each of the corresponding words in H_{i-1} , using addition modulo 2^{64} .

Step 5 Output. After all N 1024-bit blocks have been processed, the output from the N th stage is the 512-bit message digest.

We can summarize the behavior of SHA-512 as follows:

$$H_0 = IV$$

$$H_i = \text{SUM}_{64}(H_{i-1}, \text{abcdefgh}_i)$$

$$MD = H_N$$

where

- IV = initial value of the abcdefgh buffer, defined in step 3
- abcdefgh_i = the output of the last round of processing of the i th message block
- N = the number of blocks in the message (including padding and length fields)
- SUM_{64} = addition modulo 2^{64} performed separately on each word of the pair of inputs
- MD = final message digest value

SHA-512 Round Function

Let us look in more detail at the logic in each of the 80 steps of the processing of one 512-bit block (Figure 11.10). Each round is defined by the following set of equations:

$$T_1 = h + \text{Ch}(e, f, g) + \left(\sum_1^{512} e\right) + W_t + K_t$$

$$T_2 = \left(\sum_0^{512} a\right) + \text{Maj}(a, b, c)$$

$$h = g$$

$$g = f$$

$$f = e$$

$$e = d + T_1$$

$$d = c$$

$$c = b$$

$$b = a$$

$$a = T_1 + T_2$$

where

t = step number; $0 \leq t \leq 79$

$$\text{Ch}(e, f, g) = (e \text{ AND } f) \oplus (\text{NOT } e \text{ AND } g)$$

the conditional function: If e then f else g

$$\text{Maj}(a, b, c) = (a \text{ AND } b) \oplus (a \text{ AND } c) \oplus (b \text{ AND } c)$$

the function is true only if the majority (two or three) of the arguments are true

$$\left(\sum_0^{512} a \right) = \text{ROTR}^{28}(a) \oplus \text{ROTR}^{34}(a) \oplus \text{ROTR}^{39}(a)$$

$$\left(\sum_1^{512} e \right) = \text{ROTR}^{14}(e) \oplus \text{ROTR}^{18}(e) \oplus \text{ROTR}^{41}(e)$$

$\text{ROTR}^n(x)$ = circular right shift (rotation) of the 64-bit argument x by n bits

W_t = a 64-bit word derived from the current 512-bit input block

K_t = a 64-bit additive constant

$+$ = addition modulo 2^{64}

Two observations can be made about the round function.

1. Six of the eight words of the output of the round function involve simply permutation (b, c, d, f, g, h) by means of rotation. This is indicated by shading in Figure 11.10.
2. Only two of the output words (a, e) are generated by substitution. Word e is a function of input variables (d, e, f, g, h), as well as the round word W_t and the constant K_t . Word a is a function of all of the input variables except d , as well as the round word W_t and the constant K_t .

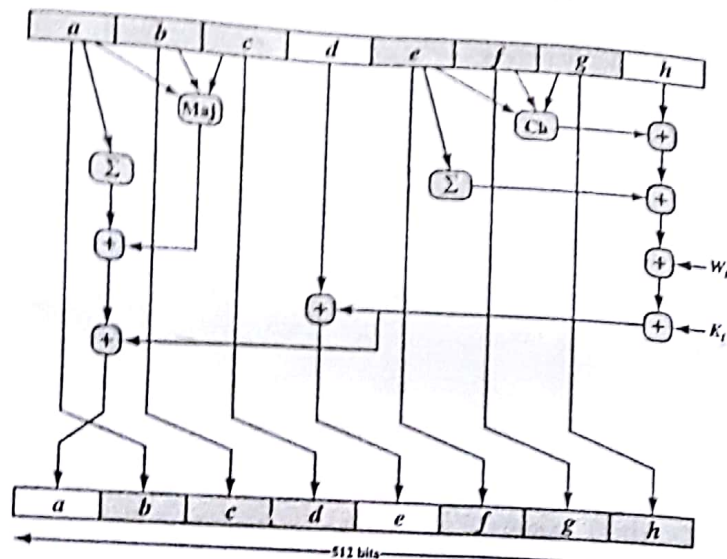


Figure 11.10 Elementary SHA-512 Operation (single round)

It remains to indicate how the 64-bit word values W_t are derived from the 1024-bit message. Figure 11.11 illustrates the mapping. The first 16 values of W_t are taken directly from the 16 words of the current block. The remaining values are defined as

$$W_t = \sigma_1^{512}(W_{t-2}) + W_{t-7} + \sigma_0^{512}(W_{t-15}) + W_{t-16}$$

where

$$\sigma_0^{512}(x) = \text{ROTR}^1(x) \oplus \text{ROTR}^8(x) \oplus \text{SHR}^7(x)$$

$$\sigma_1^{512}(x) = \text{ROTR}^{19}(x) \oplus \text{ROTR}^{61}(x) \oplus \text{SHR}^6(x)$$

$\text{ROTR}^n(x)$ = circular right shift (rotation) of the 64-bit argument x by n bits

$\text{SHR}^n(x)$ = left shift of the 64-bit argument x by n bits with padding by zeros on the right

$+$ = addition modulo 2^{64}

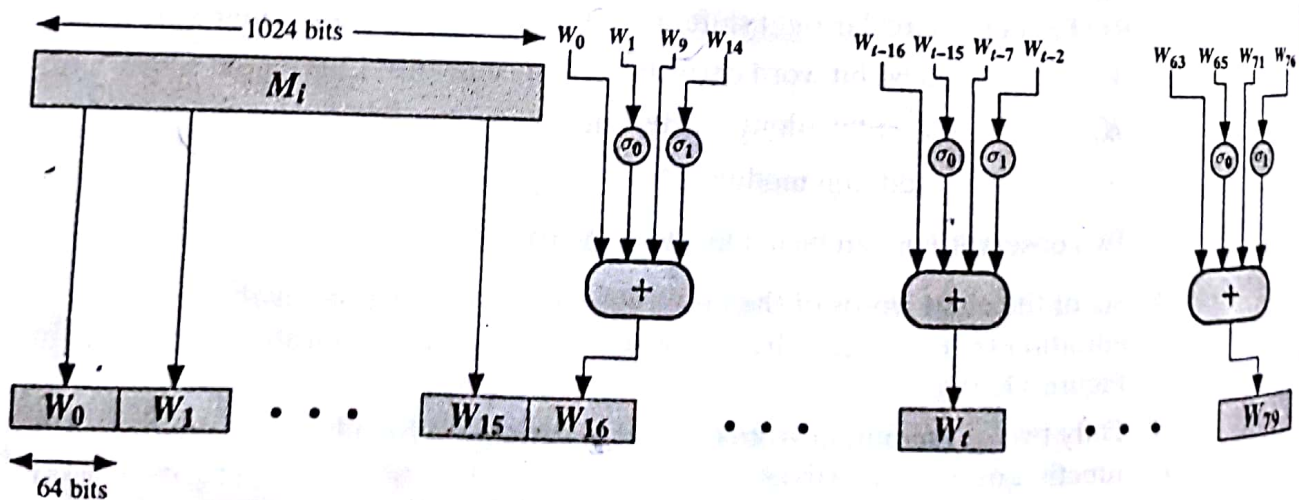


Figure 11.11 Creation of 80-word Input Sequence for SHA-512 Processing of Single Block

Thus, in the first 16 steps of processing, the value of W_t is equal to the corresponding word in the message block. For the remaining 64 steps, the value of W_t consists of the circular left shift by one bit of the XOR of four of the preceding values of W_t , with two of those values subjected to shift and rotate operations. This introduces a great deal of redundancy and interdependence into the message blocks that are compressed, which complicates the task of finding a different message block that maps to the same compression function output.

Figure 11.12 summarizes the SHA-512 logic.

The SHA-512 algorithm has the property that every bit of the hash code is a function of every bit of the input. The complex repetition of the basic function F produces results that are well mixed; that is, it is unlikely that two messages chosen at random, even if they exhibit similar regularities, will have the same hash code. Unless there is some hidden weakness in SHA-512, which has not so far been published, the difficulty of coming up with two messages having the same message digest is on the order of 2^{256} operations, while the difficulty of finding a message with a given digest is on the order of 2^{512} operations.