Table 1.2 Security Services (X.800)

1(a)

AUTHENTICATION

The assurance that the communicating entity is the one that it claims to be.

Peer Entity Authentication
Used in association with a logical connection to provide confidence in the identity of the entities connected.

Data-Origin Authentication
In a connectionless transfer, provides assurance that the source of received data is as claimed.

ACCESS CONTROL

The prevention of unauthorized use of a resource (i.e., this service controls who can have access to a resource, under what conditions access can occur, and what those accessing the resource are allowed to do).

DATA CONFIDENTIALITY

The protection of data from unauthorized disclosure.

Connection Confidentiality
The protection of all user data on a connection.

Connectionless Confidentiality
The protection of all user data in a single data block

Selective-Field Confidentiality
The confidentiality of selected fields within the user data on a connection or in a single data block.

Traffic-Flow Confidentiality
The protection of the information that might be derived from observation of traffic flows.

DATA INTEGRITY

The assurance that data received are exactly as sent by an authorized entity (i.e., contain no modification, insertion, deletion, or replay).

Connection Integrity with Recovery
Provides for the integrity of all user data on a connection and detects any modification, insertion, deletion, or replay of any data within an entire data sequence, with recovery attempted.

Connection Integrity without Recovery
As above, but provides only detection without recovery.

Selective-Field Connection Integrity
Provides for the integrity of selected fields within the user data of a data block transferred over a connection and takes the form of determination of whether the selected fields have been modified, inserted, deleted, or replayed.

Connectionless Integrity
Provides for the integrity of a single connectionless data block and may take the form of detection of data modification. Additionally, a limited form of replay detection may be provided.

Selective-Field Connectionless Integrity
Provides for the integrity of selected fields within a single connectionless data block; takes the form of determination of whether the selected fields have been modified.

NONREPUDIATION

Provides protection against denial by one of the entities involved in a communication of having participated in all or part of the communication.

Nonrepudiation, Origin
Proof that the message was sent by the specified party.

Nonrepudiation, Destination
Proof that the message was received by the specified party.

1(b)

SPECIFIC SECURITY MECHANISMS

May be incorporated into the appropriate protocol layer in order to provide some of the OSI security services.

Encipherment
The use of mathematical algorithms to transform data into a form that is not readily intelligible. The transformation and subsequent recovery of the data depend on an algorithm and zero or more encryption keys.

Digital Signature
Data appended to, or a cryptographic transformation of, a data unit that allows a recipient of the data unit to prove the source and integrity of the data unit and protect against forgery (e.g., by the recipient).

Access Control
A variety of mechanisms that enforce access rights to resources.

Data Integrity
A variety of mechanisms used to assure the integrity of a data unit or stream of data units.

Authentication Exchange
A mechanism intended to ensure the identity of an entity by means of information exchange.

Traffic Padding
The insertion of bits into gaps in a data stream to frustrate traffic analysis attempts.

Routing Control
Enables selection of particular physically secure routes for certain data and allows routing changes, especially when a breach of security is suspected.

Notarization
The use of a trusted third party to assure certain properties of a data exchange.

PERVASIVE SECURITY MECHANISMS

Mechanisms that are not specific to any particular OSI security service or protocol layer.

Trusted Functionality
That which is perceived to be correct with respect to some criteria (e.g., as established by a security policy).

Security Label
The marking bound to a resource (which may be a data unit) that names or designates the security attributes of that resource.

Event Detection
Detection of security-relevant events.

Security Audit Trail
Data collected and potentially used to facilitate a security audit, which is an independent review and examination of system records and activities.

Security Recovery
Deals with requests from mechanisms, such as event handling and management functions, and takes recovery actions.

2(a)

Playfair Cipher

The best-known multiple-letter encryption cipher is the Playfair, which treats digrams in the plaintext as single units and translates these units into ciphertext digrams.[3]

The Playfair algorithm is based on the use of a 5 × 5 matrix of letters constructed using a keyword. Here is an example, solved by Lord Peter Wimsey in Dorothy Sayers's *Have His Carcase*:[4]

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

In this case, the keyword is *monarchy*. The matrix is constructed by filling in the letters of the keyword (minus duplicates) from left to right and from top to bottom, and then filling in the remainder of the matrix with the remaining letters in alphabetic order. The letters I and J count as one letter. Plaintext is encrypted two letters at a time, according to the following rules:

1. Repeating plaintext letters that are in the same pair are separated with a filler letter, such as x, so that balloon would be treated as ba lx lo on.
2. Two plaintext letters that fall in the same row of the matrix are each replaced by the letter to the right, with the first element of the row circularly following the last. For example, ar is encrypted as RM.
3. Two plaintext letters that fall in the same column are each replaced by the letter beneath, with the top element of the column circularly following the last. For example, mu is encrypted as CM.
4. Otherwise, each plaintext letter in a pair is replaced by the letter that lies in its own row and the column occupied by the other plaintext letter. Thus, hs becomes BP and ea becomes IM (or JM, as the encipherer wishes).

The Playfair cipher is a great advance over simple monoalphabetic ciphers. For one thing, whereas there are only 26 letters, there are 26 × 26 = 676 digrams, so

[3]This cipher was actually invented by British ... of his friend ...

that identification of individual digrams is more difficult. Furthermore, the relative frequencies of individual letters exhibit a much greater range than that of digrams, making frequency analysis much more difficult. For these reasons, the Playfair cipher was for a long time considered unbreakable. It was used as the standard field system by the British Army in World War I and still enjoyed considerable use by the U.S. Army and other Allied forces during World War II.

Despite this level of confidence in its security, the Playfair cipher is relatively easy to break, because it still leaves much of the structure of the plaintext language intact. A few hundred letters of ciphertext are generally sufficient.

One way of revealing the effectiveness of the Playfair and other ciphers is shown in Figure 2.6, based on [SIMM93]. The line labeled *plaintext* plots the frequency distribution of the more than 70,000 alphabetic characters in the *Encyclopaedia Britannica* article on cryptology.[5] This is also the frequency distribution of any monoalphabetic substitution cipher, because the frequency values for individual letters are the same, just with different letters substituted for the original letters. The plot was developed in the following way: The number of occurrences of each letter in the text was counted and divided by the number of occurrences of the letter e (the most frequently used letter). As a result, e has a relative frequency of 1, t of about 0.76, and so on. The points on the horizontal axis correspond to the letters in order of decreasing frequency.

Figure 2.6 also shows the frequency distribution that results when the text is encrypted using the Playfair cipher. To normalize the plot, the number of occurrences of each letter in the ciphertext was again divided by the number of occurrences of e

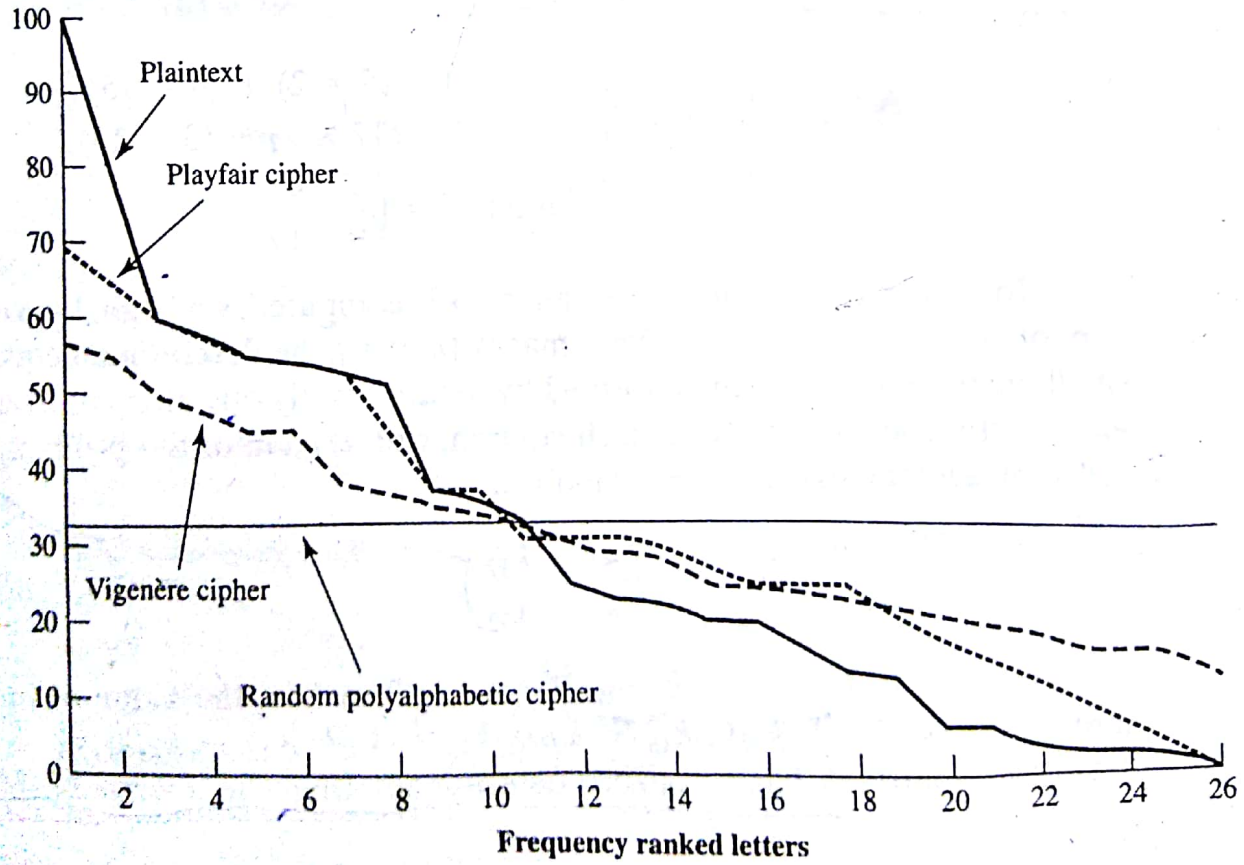


Figure 2.6 Relative Frequency of Occurrence of Letters

Hill Cipher[6]

Another interesting multiletter cipher is the Hill cipher, developed by the mathematician Lester Hill in 1929.

CONCEPTS FROM LINEAR ALGEBRA Before describing the Hill cipher, let us briefly review some terminology from linear algebra. In this discussion, we are concerned with matrix arithmetic modulo 26. For the reader who needs a refresher on matrix multiplication and inversion, see Appendix E.

We define the inverse $\mathbf{M}^{-1}$ of a square matrix $\mathbf{M}$ by the equation $\mathbf{M}(\mathbf{M}^{-1}) = \mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. $\mathbf{I}$ is a square matrix that is all zeros except for ones along the main diagonal from upper left to lower right. The inverse of a matrix does not always exist, but when it does, it satisfies the preceding equation. For example,

$$\mathbf{A} = \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} \qquad \mathbf{A}^{-1} \bmod 26 = \begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{pmatrix} (5 \times 9) + (8 \times 1) & (5 \times 2) + (8 \times 15) \\ (17 \times 9) + (3 \times 1) & (17 \times 2) + (3 \times 15) \end{pmatrix}$$
$$= \begin{pmatrix} 53 & 130 \\ 156 & 79 \end{pmatrix} \bmod 26 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To explain how the inverse of a matrix is computed, we begin by with the concept of determinant. For any square matrix $(m \times m)$, the **determinant** equals the sum of all the products that can be formed by taking exactly one element from each row and exactly one element from each column, with certain of the product terms preceded by a minus sign. For a 2×2 matrix,

$$\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$$

the determinant is $k_{11}k_{22} - k_{12}k_{21}$. For a 3×3 matrix, the value of the determinant is $k_{11}k_{22}k_{33} + k_{21}k_{32}k_{13} + k_{31}k_{12}k_{23} - k_{31}k_{22}k_{13} - k_{21}k_{12}k_{33} - k_{11}k_{32}k_{23}$. If a square matrix $\mathbf{A}$ has a nonzero determinant, then the inverse of the matrix is

computed as $[\mathbf{A}^{-1}]_{ij} = (\det \mathbf{A})^{-1}(-1)^{i+j}(\mathrm{D}_{ji})$, where (D_{ji}) is the subdeterminant formed by deleting the jth row and the ith column of $\mathbf{A}$, $\det(\mathbf{A})$ is the determinant of $\mathbf{A}$, and $(\det \mathbf{A})^{-1}$ is the multiplicative inverse of $(\det \mathbf{A}) \bmod 26$.

Continuing our example,

$$\det \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} = (5 \times 3) - (8 \times 17) = -121 \bmod 26 = 9$$

We can show that $9^{-1} \bmod 26 = 3$, because $9 \times 3 = 27 \bmod 26 = 1$ (see Chapter 4 or Appendix E). Therefore, we compute the inverse of $\mathbf{A}$ as

$$\mathbf{A} = \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix}$$

$$\mathbf{A}^{-1} \bmod 26 = 3\begin{pmatrix} 3 & -8 \\ -17 & 5 \end{pmatrix} = 3\begin{pmatrix} 3 & 18 \\ 9 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 54 \\ 27 & 15 \end{pmatrix} = \begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix}$$

THE HILL ALGORITHM This encryption algorithm takes m successive plaintext letters and substitutes for them m ciphertext letters. The substitution is determined by m linear equations in which each character is assigned a numerical value ($a = 0, b = 1, \ldots, z = 25$). For $m = 3$, the system can be described as

$$c_1 = (k_{11}p_1 + k_{12}p_2 + k_{13}p_3) \bmod 26$$
$$c_2 = (k_{21}p_1 + k_{22}p_2 + k_{23}p_3) \bmod 26$$
$$c_3 = (k_{31}p_1 + k_{32}p_2 + k_{33}p_3) \bmod 26$$

This can be expressed in terms of row vectors and matrices:[7]

$$(c_1\ c_2\ c_3) = (p\ p_2\ p_3)\begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \bmod 26$$

or

$$\mathbf{C} = \mathbf{PK} \bmod 26$$

where $\mathbf{C}$ and $\mathbf{P}$ are row vectors of length 3 representing the plaintext and ciphertext, and $\mathbf{K}$ is a 3×3 matrix representing the encryption key. Operations are performed mod 26.

For example, consider the plaintext "paymoremoney" and use the encryption key

$$\mathbf{K} = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$$

[7] Some cryptography books express the plaintext and ciphertext as column vectors, so that the column vector is placed after the matrix rather than the row vector placed before the matrix. Sage uses row vectors, so we adopt that convention.

2 THE DATA ENCRYPTION STANDARD

The most widely used encryption scheme is based on the Data Encryption Standard (DES) adopted in 1977 by the National Bureau of Standards, now the National Institute of Standards and Technology (NIST), as Federal Information Processing Standard 46 (FIPS PUB 46). The algorithm itself is referred to as the Data Encryption Algorithm (DEA).[7] For DES, data are encrypted in 64-bit blocks using a 56-bit key. The algorithm transforms 64-bit input in a series of steps into a 64-bit output. The same steps, with the same key, are used to reverse the encryption.

The DES enjoys widespread use. It has also been the subject of much controversy concerning how secure the DES is. To appreciate the nature of the controversy, let us quickly review the history of the DES.

In the late 1960s, IBM set up a research project in computer cryptography led by Horst Feistel. The project concluded in 1971 with the development of an algorithm with the designation LUCIFER [FEIS73], which was sold to Lloyd's of London for use in a cash-dispensing system, also developed by IBM. LUCIFER is a Feistel block cipher that operates on blocks of 64 bits, using a key size of 128 bits. Because of the promising results produced by the LUCIFER project, IBM embarked on an effort to develop a marketable commercial encryption product that ideally could be implemented on a single chip. The effort was headed by Walter Tuchman and Carl Meyer, and it involved not only IBM researchers but also outside consultants and technical advice from the National Security Agency (NSA). The outcome of this effort was a refined version of LUCIFER that was more resistant to cryptanalysis but that had a reduced key size of 56 bits, in order to fit on a single chip.

In 1973, the National Bureau of Standards (NBS) issued a request for proposals for a national cipher standard. IBM submitted the results of its Tuchman–Meyer project. This was by far the best algorithm proposed and was adopted in 1977 as the Data Encryption Standard.

Before its adoption as a standard, the proposed DES was subjected to intense criticism, which has not subsided to this day. Two areas drew the critics' fire. First, the key length in IBM's original LUCIFER algorithm was 128 bits, but that of the proposed system was only 56 bits, an enormous reduction in key size of 72 bits. Critics feared that this key length was too short to withstand brute-force attacks. The second area of concern was that the design criteria for the internal structure of DES, the S-boxes, were classified. Thus, users could not be sure that the internal structure of DES was free of any hidden weak points that would enable NSA to decipher messages without benefit of the key. Subsequent events, particularly the recent work on differential cryptanalysis, seem to indicate that DES has a very strong internal structure. Furthermore, according to IBM participants, the only changes that were made to the proposal were changes to the S-boxes, suggested by NSA, that removed vulnerabilities identified in the course of the evaluation process.

Whatever the merits of the case, DES has flourished and is widely used, especially in financial applications. In 1994, NIST reaffirmed DES for federal use for another five years; NIST recommended the use of DES for applications other than the protection of classified information. In 1999, NIST issued a new version of its standard (FIPS PUB 46-3) that indicated that DES should be used only for legacy systems and that triple DES (which in essence involves repeating the DES algorithm three times on the plaintext using two or three different keys to produce the ciphertext) be used. We study triple DES in Chapter 6. Because the underlying encryption and decryption algorithms are the same for DES and triple DES, it remains important to understand the DES cipher.

DES Encryption

The overall scheme for DES encryption is illustrated in Figure 3.5. As with any encryption scheme, there are two inputs to the encryption function: the plaintext to be encrypted and the key. In this case, the plaintext must be 64 bits in length and the key is 56 bits in length.

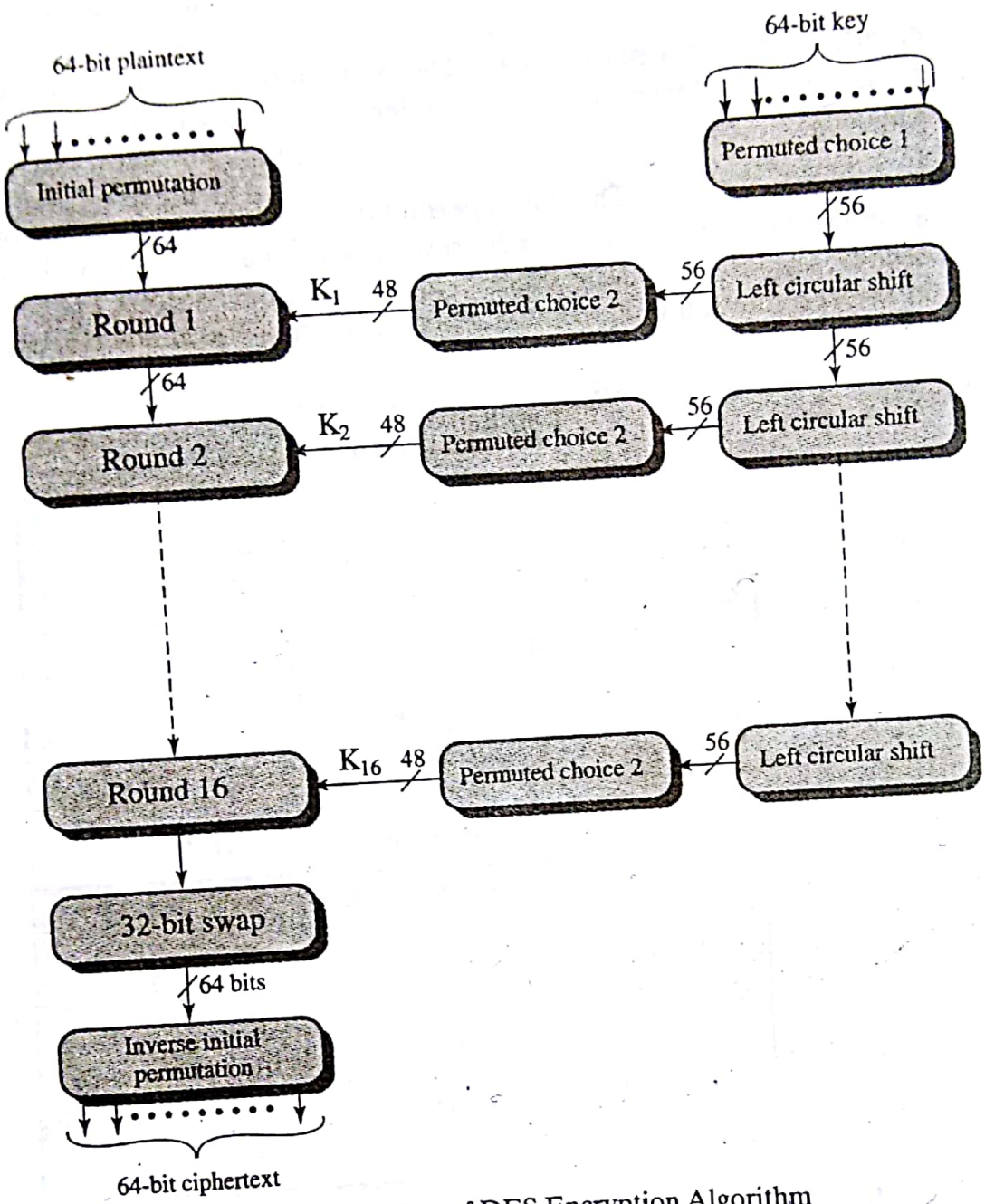


Figure 3.5 General Depiction of DES Encryption Algorithm

Looking at the left-hand side of the figure, we can see that the processing of the plaintext proceeds in three phases. First, the 64-bit plaintext passes through an initial permutation (IP) that rearranges the bits to produce the *permuted input*. This is followed by a phase consisting of sixteen rounds of the same function, which involves both permutation and substitution functions. The output of the last (sixteenth) round consists of 64 bits that are a function of the input plaintext and the key. The left and right halves of the output are swapped to produce the **preoutput**. Finally, the preoutput is passed through a permutation $[IP^{-1}]$ that is the inverse of the initial permutation function, to produce the 64-bit ciphertext. With the exception of the initial and final permutations, DES has the exact structure of a Feistel cipher, as shown in Figure 3.3.

The right-hand portion of Figure 3.5 shows the way in which the 56-bit key is used. Initially, the key is passed through a permutation function. Then, for each of the sixteen rounds, a *subkey* (K_i) is produced by the combination of a left

circular shift and a permutation. The permutation function is the same for each round, but a different subkey is produced because of the repeated shifts of the key bits.

Initial Permutation The initial permutation and its inverse are defined by tables, as shown in Tables 3.2a and 3.2b, respectively. The tables are to be interpreted as follows. The input to a table consists of 64 bits numbered from 1 to 64. The 64 entries in the permutation table contain a permutation of the numbers from 1 to 64. Each

Table 3.2 Permutation Tables for DES

(a) Initial Permutation (IP)

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

(b) Inverse Initial Permutation (IP^{-1})

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

(c) Expansion Permutation (E)

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

(d) Permutation Function (P)

16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

entry in the permutation table indicates the position of a numbered input bit in the output, which also consists of 64 bits.

To see that these two permutation functions are indeed the inverse of each other, consider the following 64-bit input M:

$$\begin{array}{cccccccc}
M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 \\
M_9 & M_{10} & M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\
M_{17} & M_{18} & M_{19} & M_{20} & M_{21} & M_{22} & M_{23} & M_{24} \\
M_{25} & M_{26} & M_{27} & M_{28} & M_{29} & M_{30} & M_{31} & M_{32} \\
M_{33} & M_{34} & M_{35} & M_{36} & M_{37} & M_{38} & M_{39} & M_{40} \\
M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} & M_{47} & M_{48} \\
M_{49} & M_{50} & M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\
M_{57} & M_{58} & M_{59} & M_{60} & M_{61} & M_{62} & M_{63} & M_{64}
\end{array}$$

where M_i is a binary digit. Then the permutation $X = (\text{IP}(M)$ is as follows:

$$\begin{array}{cccccccc}
M_{58} & M_{50} & M_{42} & M_{34} & M_{26} & M_{18} & M_{10} & M_2 \\
M_{60} & M_{52} & M_{44} & M_{36} & M_{28} & M_{20} & M_{12} & M_4 \\
M_{62} & M_{54} & M_{46} & M_{38} & M_{30} & M_{22} & M_{14} & M_6 \\
M_{64} & M_{56} & M_{48} & M_{40} & M_{32} & M_{24} & M_{16} & M_8 \\
M_{57} & M_{49} & M_{41} & M_{33} & M_{25} & M_{17} & M_9 & M_1 \\
M_{59} & M_{51} & M_{43} & M_{35} & M_{27} & M_{19} & M_{11} & M_3 \\
M_{61} & M_{53} & M_{45} & M_{37} & M_{29} & M_{21} & M_{13} & M_5 \\
M_{63} & M_{55} & M_{47} & M_{39} & M_{31} & M_{23} & M_{15} & M_7
\end{array}$$

If we then take the inverse permutation $Y = \text{IP}^{-1}(X) = \text{IP}^{-1}(\text{IP}(M))$, it can be seen that the original ordering of the bits is restored.

Details of Single Round Figure 3.6 shows the internal structure of a single round. Again, begin by focusing on the left-hand side of the diagram. The left and right halves of each 64-bit intermediate value are treated as separate 32-bit quantities, labeled L (left) and R (right). As in any classic Feistel cipher, the overall processing at each round can be summarized in the following formulas:

$$\begin{aligned}
L_i &= R_{i-1} \\
R_i &= L_{i-1} \oplus F(R_{i-1}, K_i)
\end{aligned}$$

The round key K_i is 48 bits. The R input is 32 bits. This R input is first expanded to 48 bits by using a table that defines a permutation plus an expansion that involves duplication of 16 of the R bits (Table 3.2c). The resulting 48 bits are XORed with K_i. This 48-bit result passes through a substitution function that produces a 32-bit output, which is permuted as defined by Table 3.2d.

The role of the S-boxes in the function F is illustrated in Figure 3.7. The substitution consists of a set of eight S-boxes, each of which accepts 6 bits as input and produces 4 bits as output. These transformations are defined in Table 3.3, which is

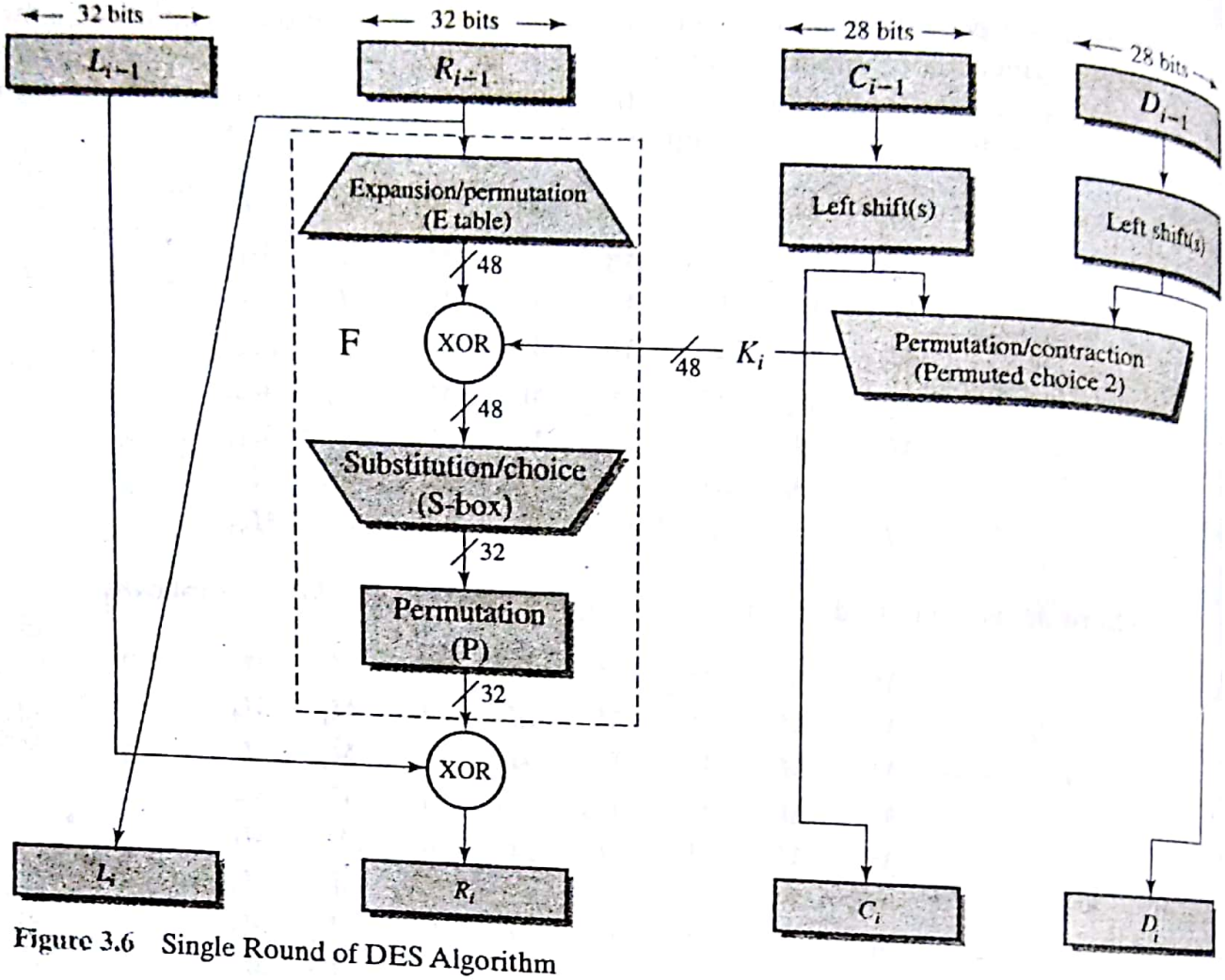


Figure 3.6 Single Round of DES Algorithm

interpreted as follows. The first and last bits of the input to box S_i form a 2-bit binary number to select one of four substitutions defined by the four rows in the table for S_i. The middle four bits select one of the sixteen columns. The decimal value in the cell selected by the row and column is then converted to its 4-bit representation to produce the output. For example, in S_1, for input 011001, the row is 01 (row 1) and the column is 1100 (column 12). The value in row 1, column 12 is 9, so the output is 1001.

Each row of an S-box defines a general reversible substitution. Figure 3.2 may be useful in understanding the mapping. The figure shows the substitution for row 0 of box S_1.

The operation of the S-boxes is worth further comment. Ignore for the moment the contribution of the key (K_i). If you examine the expansion table, you see that the 32 bits of input are split into groups of 4 bits and then become groups of 6 bits by taking the outer bits from the two adjacent groups. For example, if part of the input word is

```
... efgh ijkl mnop ...
```

this becomes

```
... defghi hijklm lmnopq ...
```

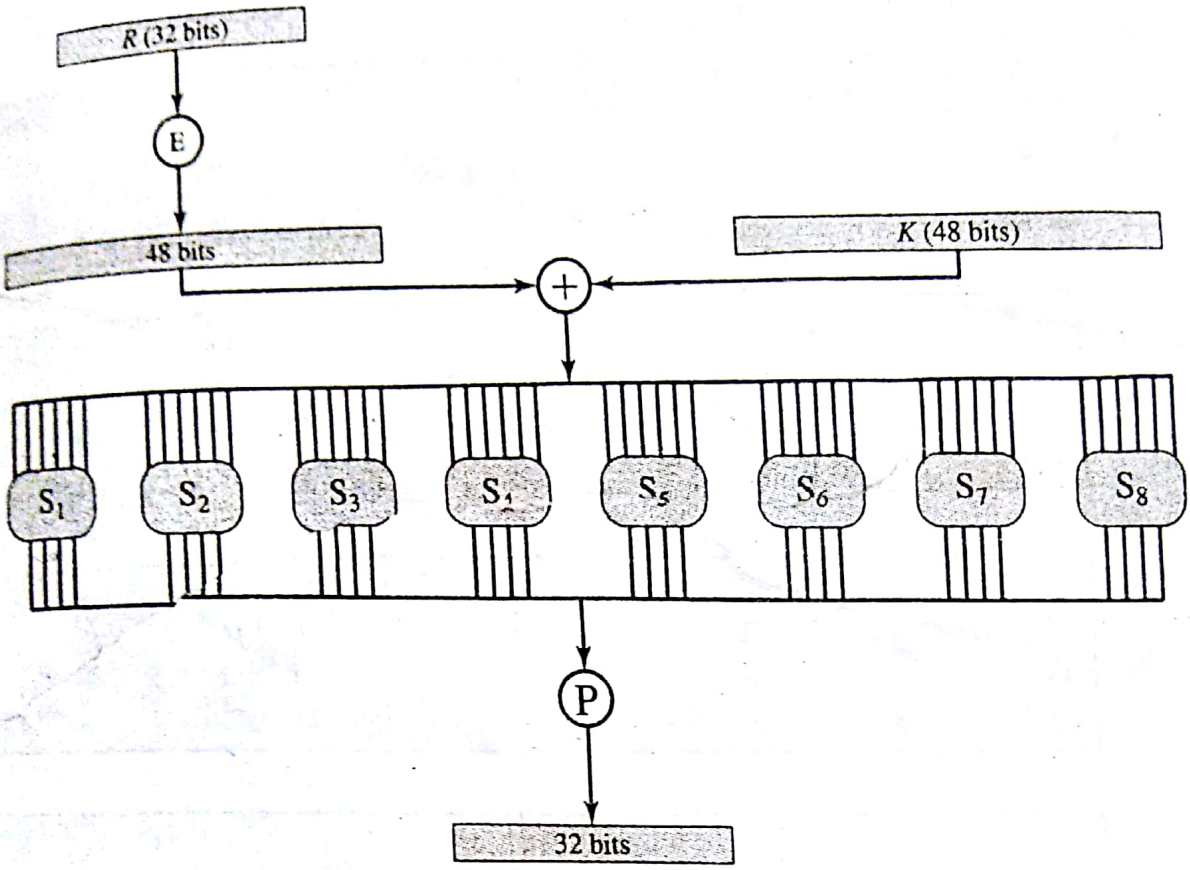


Figure 3.7 Calculation of F(R, K)

The outer two bits of each group select one of four possible substitutions (one row of an S-box). Then a 4-bit output value is substituted for the particular 4-bit input (the middle four input bits). The 32-bit output from the eight S-boxes is then permuted, so that on the next round, the output from each S-box immediately affects as many others as possible.

KEY GENERATION Returning to Figures 3.5 and 3.6, we see that a 64-bit key is used as input to the algorithm. The bits of the key are numbered from 1 through 64; every eighth bit is ignored, as indicated by the lack of shading in Table 3.4a. The key is first subjected to a permutation governed by a table labeled Permuted Choice One (Table 3.4b). The resulting 56-bit key is then treated as two 28-bit quantities, labeled C_0 and D_0. At each round, C_{i-1} and D_{i-1} are separately subjected to a circular left shift or (rotation) of 1 or 2 bits, as governed by Table 3.4d. These shifted values serve as input to the next round. They also serve as input to the part labeled Permuted Choice Two (Table 3.4c), which produces a 48-bit output that serves as input to the function $F(R_{i-1}, K_i)$.

DES Decryption

As with any Feistel cipher, decryption uses the same algorithm as encryption, except that the application of the subkeys is reversed.

3.4 THE STRENGTH OF DES

Since its adoption as a federal standard, there have been lingering concerns about the level of security provided by DES. These concerns, by and large, fall into two areas: key size and the nature of the algorithm.

The Use of 56-Bit Keys

With a key length of 56 bits, there are 2^{56} possible keys, which is approximately 7.2×10^{16} keys. Thus, on the face of it, a brute-force attack appears impractical. Assuming that, on average, half the key space has to be searched, a single machine performing one DES encryption per microsecond would take more than a thousand years (see Table 2.2) to break the cipher.

However, the assumption of one encryption per microsecond is overly conservative. As far back as 1977, Diffie and Hellman postulated that the technology existed to build a parallel machine with 1 million encryption devices, each of which could perform one encryption per microsecond [DIFF77]. This would bring the average search time down to about 10 hours. The authors estimated that the cost would be about $20 million in 1977 dollars.

DES finally and definitively proved insecure in July 1998, when the Electronic Frontier Foundation (EFF) announced that it had broken a DES encryption using a special-purpose "DES cracker" machine that was built for less than $250,000. The attack took less than three days. The EFF has published a detailed description of the machine, enabling others to build their own cracker [EFF98]. And, of course, hardware prices will continue to drop as speeds increase, making DES virtually worthless.

It is important to note that there is more to a key-search attack than simply running through all possible keys. Unless known plaintext is provided, the analyst must be able to recognize plaintext as plaintext. If the message is just plain text in English, then the result pops out easily, although the task of recognizing English would have to be automated. If the text message has been compressed before encryption, then recognition is more difficult. And if the message is some more general type of data, such as a numerical file, and this has been compressed, the problem becomes even more difficult to automate. Thus, to supplement the brute-force approach, some degree of knowledge about the expected plaintext is needed, and some means of automatically distinguishing plaintext from garble is also needed. The EFF approach addresses this issue as well and introduces some automated techniques that would be effective in many contexts.

Fortunately, there are a number of alternatives to DES, the most important of which are AES and triple DES, discussed in Chapters 5 and 6, respectively.

The Nature of the DES Algorithm

Another concern is the possibility that cryptanalysis is possible by exploiting the characteristics of the DES algorithm. The focus of concern has been on the eight substitution tables, or S-boxes, that are used in each iteration. Because the design criteria for these boxes, and indeed for the entire algorithm, were not made public, there is a suspicion that the boxes were constructed in such a way that cryptanalysis

is possible for an opponent who knows the weaknesses in the S-boxes. This assertion is tantalizing, and over the years a number of regularities and unexpected behaviors of the S-boxes have been discovered. Despite this, no one has so far succeeded in discovering the supposed fatal weaknesses in the S-boxes.[9]

Timing Attacks

We discuss timing attacks in more detail in Part Two, as they relate to public-key algorithms. However, the issue may also be relevant for symmetric ciphers. In essence, a timing attack is one in which information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertexts. A timing attack exploits the fact that an encryption or decryption algorithm often takes slightly different amounts of time on different inputs. [HEVI99] reports on an approach that yields the Hamming weight (number of bits equal to one) of the secret key. This is a long way from knowing the actual key, but it is an intriguing first step. The authors conclude that DES appears to be fairly resistant to a successful timing attack but suggest some avenues to explore. Although this is an interesting line of attack, it so far appears unlikely that this technique will ever be successful against DES or more powerful symmetric ciphers such as triple DES and AES.

4(a)

Monoalphabetic Ciphers

With only 25 possible keys, the Caesar cipher is far from secure. A dramatic increase in the key space can be achieved by allowing an arbitrary substitution. Before proceeding, we define the term *permutation*. A **permutation** of a finite set of elements S is an ordered sequence of all the elements of S, with each element appearing exactly once. For example, if $S = \{a, b, c\}$, there are six permutations of S:

abc, acb, bac, bca, cab, cba

In general, there are $n!$ permutations of a set of n elements, because the first element can be chosen in one of n ways, the second in $n - 1$ ways, the third in $n - 2$ ways, and so on.

$n(n-1)(n-2)\ldots\ldots$

Recall the assignment for the Caesar cipher:

```
plain:  a b c d e f g h i j k l m n o p q r s t u v w x y z
cipher: D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
```

If, instead, the "cipher" line can be any permutation of the 26 alphabetic characters, then there are 26! or greater than 4×10^{26} possible keys. This is 10 orders of magnitude greater than the key space for DES and would seem to eliminate brute-force techniques for cryptanalysis. Such an approach is referred to as a **monoalphabetic substitution cipher**, because a single cipher alphabet (mapping from plain alphabet to cipher alphabet) is used per message.

There is, however, another line of attack. If the cryptanalyst knows the nature of the plaintext (e.g., noncompressed English text), then the analyst can exploit the regularities of the language. To see how such a cryptanalysis might

proceed, we give a partial example here that is adapted from one in [SINK66]. The ciphertext to be solved is

```
UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ
```

As a first step, the relative frequency of the letters can be determined and compared to a standard frequency distribution for English, such as is shown in Figure 2.5 (based on [LEWA00]). If the message were long enough, this technique alone might be sufficient, but because this is a relatively short message, we cannot expect an exact match. In any case, the relative frequencies of the letters in the ciphertext (in percentages) are as follows:

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	T 2.50	I 0.83	N 0.00
O 7.50	X 4.17	A 1.67	J 0.83	R 0.00
M 6.67				

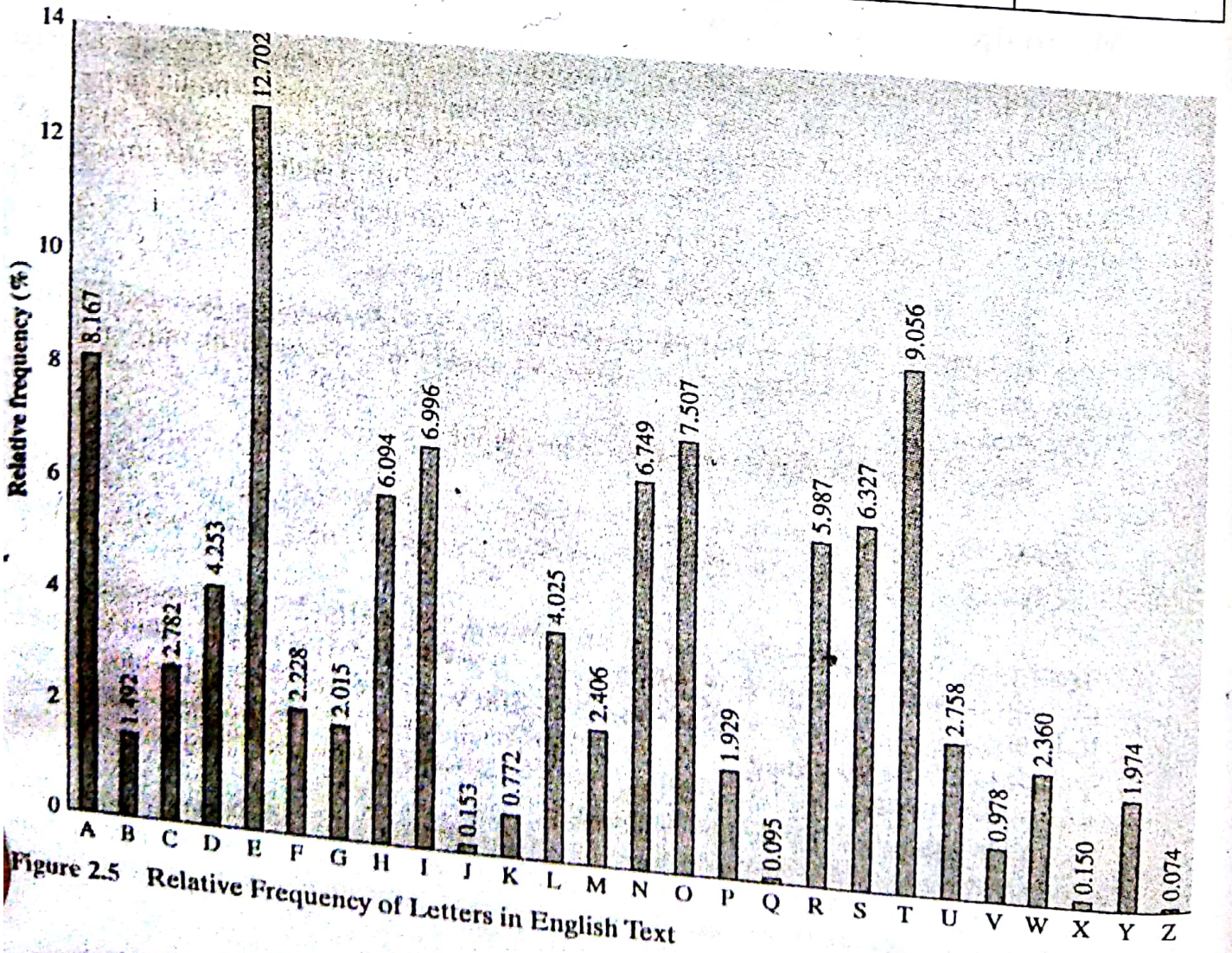


Figure 2.5 Relative Frequency of Letters in English Text

Comparing this breakdown with Figure 2.5, it seems likely that cipher letters P and Z are the equivalents of plain letters e and t, but it is not certain which is which. The letters S, U, O, M, and H are all of relatively high frequency and probably correspond to plain letters from the set {a, h, i, n, o, r, s}. The letters with the lowest frequencies (namely, A, B, G, Y, I, J) are likely included in the set {b, j, k, q, v, x, z}.

There are a number of ways to proceed at this point. We could make some tentative assignments and start to fill in the plaintext to see if it looks like a reasonable "skeleton" of a message. A more systematic approach is to look for other regularities. For example, certain words may be known to be in the text. Or we could look for repeating sequences of cipher letters and try to deduce their plaintext equivalents.

A powerful tool is to look at the frequency of two-letter combinations, known as **digrams**. A table similar to Figure 2.5 could be drawn up showing the relative frequency of digrams. The most common such digram is th. In our ciphertext, the most common digram is ZW, which appears three times. So we make the correspondence of Z with t and W with h. Then, by our earlier hypothesis, we can equate P with e. Now notice that the sequence ZWP appears in the ciphertext, and we can translate that sequence as "the." This is the most frequent trigram (three-letter combination) in English, which seems to indicate that we are on the right track.

Next, notice the sequence ZWSZ in the first line. We do not know that these four letters form a complete word, but if they do, it is of the form th_t. If so, S equates with a.

So far, then, we have

```
UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAIZ
 t a        e  e te  a that e e a       a
VUEPHZHMDZSHZOWSFPAPPDTSVPQUZWYMXUZUHSX
  e t   ta t ha e ee  a e  th    t  a
EPYEPOPDZSZUFPOMBZWPFUPZHMDJUDTMOHMQ
 e  e e tat  e  .the   t
```

Only four letters have been identified, but already we have quite a bit of the message. Continued analysis of frequencies plus trial and error should easily yield a solution from this point. The complete plaintext, with spaces added between words, follows:

```
it was disclosed yesterday that several informal but
direct contacts have been made with political
representatives of the viet cong in moscow
```

Monoalphabetic ciphers are easy to break because they reflect the frequency data of the original alphabet. A countermeasure is to provide multiple substitutes, known as homophones, for a single letter. For example, the letter e could be assigned a number of different cipher symbols, such as 16, 74, 35, and 21, with each homophone assigned to a letter in rotation or randomly. If the number of symbols assigned to each letter is proportional to the relative frequency of that letter, then single-letter frequency information is completely obliterated. The great mathematician Carl

in the plaintext. The resulting plot therefore shows the extent to which the frequency distribution of letters, which makes it trivial to solve substitution ciphers, is masked by encryption. If the frequency distribution information were totally concealed in the encryption process, the ciphertext plot of frequencies would be flat, and cryptanalysis using ciphertext only would be effectively impossible. As the figure shows, the Playfair cipher has a flatter distribution than does plaintext, but nevertheless, it reveals plenty of structure for a cryptanalyst to work with.

Polyalphabetic Ciphers

Another way to improve on the simple monoalphabetic technique is to use different monoalphabetic substitutions as one proceeds through the plaintext message. The general name for this approach is **polyalphabetic substitution cipher**. All these techniques have the following features in common:

1. A set of related monoalphabetic substitution rules is used.
2. A key determines which particular rule is chosen for a given transformation.

Vigenère Cipher The best known, and one of the simplest, polyalphabetic ciphers is the Vigenère cipher. In this scheme, the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers with shifts of 0 through 25. Each cipher is denoted by a key letter, which is the ciphertext letter that substitutes for the plaintext letter a. Thus, a Caesar cipher with a shift of 3 is denoted by the key value *d*.

We can express the Vigenère cipher in the following manner. Assume a sequence of plaintext letters $P = p_0, p_1, p_2, \ldots, p_{n-1}$ and a key consisting of the sequence of letters $K = k_0, k_1, k_2, \ldots, k_{m-1}$, where typically $m < n$. The sequence of ciphertext letters $C = C_0, C_1, C_2, \ldots, C_{n-1}$ is calculated as follows:

$$\begin{aligned} C = C_0, C_1, C_2, \ldots, C_{n-1} &= \mathrm{E}(K, P) = \mathrm{E}[(k_0, k_1, k_2, \ldots, k_{m-1}), (p_0, p_1, p_2, \ldots, p_{n-1})] \\ &= (p_0 + k_0) \bmod 26, (p_1 + k_1) \bmod 26, \ldots, (p_{m-1} + k_{m-1}) \bmod 26, \\ &\quad (p_m + k_0) \bmod 26, (p_{m+1} + k_1) \bmod 26, \ldots, (p_{2m-1} + k_{m-1}) \bmod 26, \ldots \end{aligned}$$

Thus, the first letter of the key is added to the first letter of the plaintext, mod 26, the second letters are added, and so on through the first m letters of the plaintext. For the next m letters of the plaintext, the key letters are repeated. This process continues until all of the plaintext sequence is encrypted. A general equation of the encryption process is

$$C_i = (p_i + k_{i \bmod m}) \bmod 26 \tag{2.3}$$

Compare this with Equation (2.1) for the Caesar cipher. In essence, each plaintext character is encrypted with a different Caesar cipher, depending on the corresponding key character. Similarly, decryption is a generalization of Equation (2.2):

$$p_i = (C_i - k_{i \bmod m}) \bmod 26 \tag{2.4}$$

To encrypt a message, a key is needed that is as long as the message. Usually, the key is a repeating keyword. For example, if the keyword is *deceptive*, the message "we are discovered save yourself" is encrypted as

```
key:          deceptivedeceptivedeceptive
plaintext:    wearediscoveredsaveyourself
ciphertext:   ZICVTWQNGRZGVTWAVZHCQYGLMGJ
```

Expressed numerically, we have the following result.

key	3	4	2	4	15	19	8	21	4	3	4	2	4	15
plaintext	22	4	0	17	4	3	8	18	2	14	21	4	17	4
ciphertext	25	8	2	21	19	22	16	13	6	17	25	6	21	19

key	19	8	21	4	3	4	2	4	15	19	8	21	4
plaintext	3	18	0	21	4	24	14	20	17	18	4	11	5
ciphertext	22	0	21	25	7	2	16	24	6	11	12	6	9

The strength of this cipher is that there are multiple ciphertext letters for each plaintext letter, one for each unique letter of the keyword. Thus, the letter frequency information is obscured. However, not all knowledge of the plaintext structure is lost. For example, Figure 2.6 shows the frequency distribution for a Vigenère cipher with a keyword of length 9. An improvement is achieved over the Playfair cipher, but considerable frequency information remains.

It is instructive to sketch a method of breaking this cipher, because the method reveals some of the mathematical principles that apply in cryptanalysis.

First, suppose that the opponent believes that the ciphertext was encrypted using either monoalphabetic substitution or a Vigenère cipher. A simple test can be made to make a determination. If a monoalphabetic substitution is used, then the statistical properties of the ciphertext should be the same as that of the language of the plaintext. Thus, referring to Figure 2.5, there should be one cipher letter with a relative frequency of occurrence of about 12.7%, one with about 9.06%, and so on. If only a single message is available for analysis, we would not expect an exact match of this small sample with the statistical profile of the plaintext language. Nevertheless, if the correspondence is close, we can assume a monoalphabetic substitution.

If, on the other hand, a Vigenère cipher is suspected, then progress depends on determining the length of the keyword, as will be seen in a moment. For now, let us concentrate on how the keyword length can be determined. The important insight that leads to a solution is the following: If two identical sequences of plaintext letters occur at a distance that is an integer multiple of the keyword length, they will generate identical ciphertext sequences. In the foregoing example, two instances of the sequence "red" are separated by nine character positions. Consequently, in both cases, r is encrypted using key letter *e*, e is encrypted using key letter *p*, and d is encrypted using key letter *t*. Thus, in both cases, the ciphertext sequence is VTW. We indicate this above by underlining the relevant ciphertext letters and shading the relevant ciphertext numbers.

An analyst looking at only the ciphertext would detect the repeated sequences VTW at a displacement of 9 and make the assumption that the keyword is either three or nine letters in length. The appearance of VTW twice could be by chance

and not reflect identical plaintext letters encrypted with identical key letters. However, if the message is long enough, there will be a number of such repeated ciphertext sequences. By looking for common factors in the displacements of the various sequences, the analyst should be able to make a good guess of the keyword length.

Solution of the cipher now depends on an important insight. If the keyword length is m, then the cipher, in effect, consists of m monoalphabetic substitution ciphers. For example, with the keyword DECEPTIVE, the letters in positions 1, 10, 19, and so on are all encrypted with the same monoalphabetic cipher. Thus, we can use the known frequency characteristics of the plaintext language to attack each of the monoalphabetic ciphers separately.

The periodic nature of the keyword can be eliminated by using a nonrepeating keyword that is as long as the message itself. Vigenère proposed what is referred to as an **autokey system**, in which a keyword is concatenated with the plaintext itself to provide a running key. For our example,

```
key:          deceptivewearediscoveredsav
plaintext:    wearediscoveredsaveyourself
ciphertext:   ZICVTWQNGKZEIIGASXSTSLVVWLA
```

Even this scheme is vulnerable to cryptanalysis. Because the key and the plaintext share the same frequency distribution of letters, a statistical technique can be applied. For example, e enciphered by e, by Figure 2.5, can be expected to occur with a frequency of $(0.127)^2 \approx 0.016$, whereas t enciphered by t would occur only about half as often. These regularities can be exploited to achieve successful cryptanalysis.[8]

Vernam Cipher The ultimate defense against such a cryptanalysis is to choose a keyword that is as long as the plaintext and has no statistical relationship to it. Such a system was introduced by an AT&T engineer named Gilbert Vernam in 1918. His system works on binary data (bits) rather than letters. The system can be expressed succinctly as follows (Figure 2.7):

$$c_i = p_i \oplus k_i$$

where

p_i = ith binary digit of plaintext
k_i = ith binary digit of key
c_i = ith binary digit of ciphertext
$\oplus$ = exclusive-or (XOR) operation

Compare this with Equation (2.3) for the Vigenère cipher.

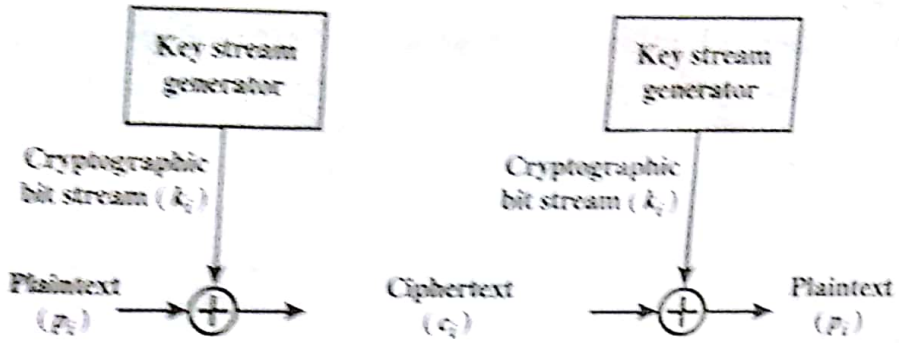


Figure 2.7 Vernam Cipher

Thus, the ciphertext is generated by performing the bitwise XOR of the plaintext and the key. Because of the properties of the XOR, decryption simply involves the same bitwise operation:

$$p_i = c_i \oplus k_i$$

which compares with Equation (2.4).

The essence of this technique is the means of construction of the key. Vernam proposed the use of a running loop of tape that eventually repeated the key, so that in fact the system worked with a very long but repeating keyword. Although such a scheme, with a long key, presents formidable cryptanalytic difficulties, it can be broken with sufficient ciphertext, the use of known or probable plaintext sequences, or both.

4(b)

Table 2.1 Types of Attacks on Encrypted Messages

Type of Attack	Known to Cryptanalyst
Ciphertext Only	• Encryption algorithm • Ciphertext
Known Plaintext	• Encryption algorithm • Ciphertext • One or more plaintext–ciphertext pairs formed with the secret key
Chosen Plaintext	• Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key
Chosen Ciphertext	• Encryption algorithm • Ciphertext • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key
Chosen Text	• Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

6.1 MULTIPLE ENCRYPTION AND TRIPLE DES

6 Given the potential vulnerability of DES to a brute-force attack, there has been considerable interest in finding an alternative. One approach is to design a completely new algorithm, of which AES is a prime example. Another alternative, which would preserve the existing investment in software and equipment, is to use multiple encryption with DES and multiple keys. We begin by examining the

simplest example of this second alternative. We then look at the widely accepted triple DES (3DES) approach.

Double DES

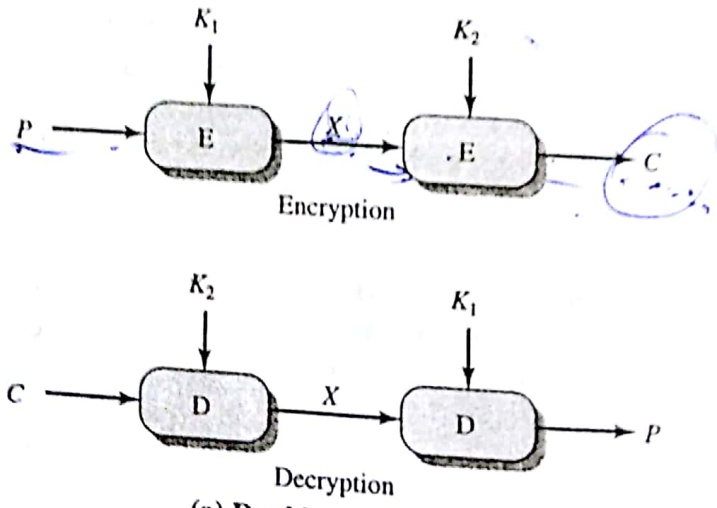
The simplest form of multiple encryption has two encryption stages and two keys (Figure 6.1a). Given a plaintext P and two encryption keys K_1 and K_2, ciphertext is generated as

$$C = E(K_2, E(K_1, P))$$

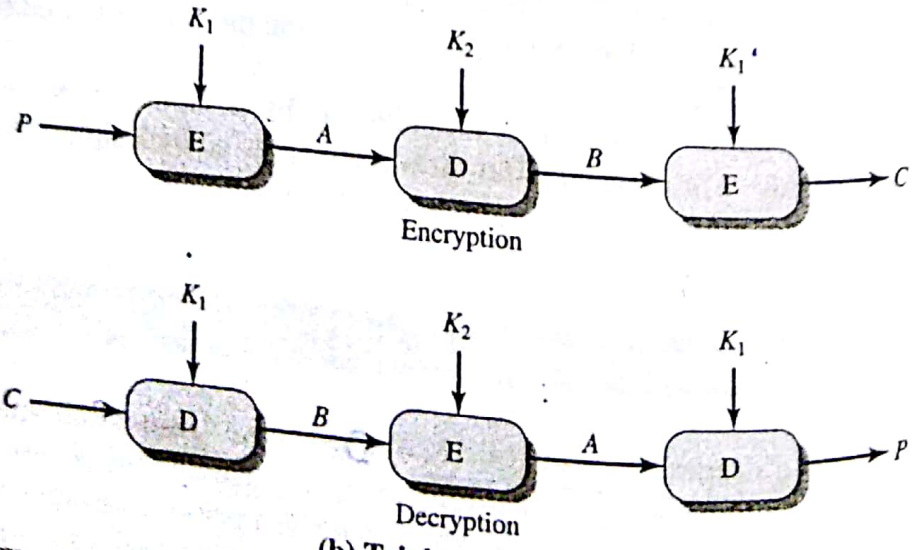
Decryption requires that the keys be applied in reverse order:

$$P = D(K_1, D(K_2, C))$$

For DES, this scheme apparently involves a key length of $56 \times 2 = 112$ bits, resulting in a dramatic increase in cryptographic strength. But we need to examine the algorithm more closely.



(a) Double encryption



(b) Triple encryption

Figure 6.1 Multiple Encryption

Reduction to a Single Stage Suppose it were true for DES, for all 56-bit key values, that given any two keys K_1 and K_2, it would be possible to find a key K_3 such that

$$E(K_2, E(K_1, P)) = E(K_3, P) \tag{6.1}$$

If this were the case, then double encryption, and indeed any number of stages of multiple encryption with DES, would be useless because the result would be equivalent to a single encryption with a single 56-bit key.

On the face of it, it does not appear that Equation (6.1) is likely to hold. Consider that encryption with DES is a mapping of 64-bit blocks to 64-bit blocks. In fact, the mapping can be viewed as a permutation. That is, if we consider all 2^{64} possible input blocks, DES encryption with a specific key will map each block into a unique 64-bit block. Otherwise, if, say, two given input blocks mapped to the same output block, then decryption to recover the original plaintext would be impossible. With 2^{64} possible inputs, how many different mappings are there that generate a permutation of the input blocks? The value is easily seen to be

$$(2^{64})! = 10^{34738000000000000000} > (10^{10^{20}})$$

On the other hand, DES defines one mapping for each different key, for a total number of mappings:

$$2^{56} < 10^{17}$$

Therefore, it is reasonable to assume that if DES is used twice with different keys, it will produce one of the many mappings that are not defined by a single application of DES. Although there was much supporting evidence for this assumption, it was not until 1992 that the assumption was proven [CAMP92].

Meet-in-the-Middle Attack Thus, the use of double DES results in a mapping that is not equivalent to a single DES encryption. But there is a way to attack this scheme, one that does not depend on any particular property of DES but that will work against any block encryption cipher.

The algorithm, known as a **meet-in-the-middle attack**, was first described in [DIFF77]. It is based on the observation that, if we have

$$C = E(K_2, E(K_1, P))$$

then (see Figure 6.1a)

$$X = E(K_1, P) = D(K_2, C)$$

Given a known pair, (P, C), the attack proceeds as follows. First, encrypt P for all 2^{56} possible values of K_1. Store these results in a table and then sort the table by the values of X. Next, decrypt C using all 2^{56} possible values of K_2. As each decryption is produced, check the result against the table for a match. If a match occurs, then test the two resulting keys against a new known plaintext–ciphertext pair. If the two keys produce the correct ciphertext, accept them as the correct keys.

For any given plaintext P, there are 2^{64} possible ciphertext values that could be produced by double DES. Double DES uses, in effect, a 112-bit key, so that

there are 2^{112} possible keys. Therefore, on average, for a given plaintext P, the number of different 112-bit keys that will produce a given ciphertext C is $2^{112}/2^{64} = 2^{48}$. Thus, the foregoing procedure will produce about 2^{48} false alarms on the first (P, C) pair. A similar argument indicates that with an additional 64 bits of known plaintext and ciphertext, the false alarm rate is reduced to $2^{48-64} = 2^{-16}$. Put another way, if the meet-in-the-middle attack is performed on two blocks of known plaintext–ciphertext, the probability that the correct keys are determined is $1 - 2^{-16}$. The result is that a known plaintext attack will succeed against double DES, which has a key size of 112 bits, with an effort on the order of 2^{56}, which is not much more than the 2^{55} required for single DES.

Triple DES with Two Keys

An obvious counter to the meet-in-the-middle attack is to use three stages of encryption with three different keys. This raises the cost of the meet-in-the-middle attack to 2^{112}, which is beyond what is practical now and far into the future. However, it has the drawback of requiring a key length of $56 \times 3 = 168$ bits, which may be somewhat unwieldy.

As an alternative, Tuchman proposed a triple encryption method that uses only two keys [TUCH79]. The function follows an encrypt-decrypt-encrypt (EDE) sequence (Figure 6.1b):

$$C = \mathrm{E}(K_1, \mathrm{D}(K_2, \mathrm{E}(K_1, P)))$$
$$P = \mathrm{D}(K_1, \mathrm{E}(K_2, \mathrm{D}(K_1, C)))$$

There is no cryptographic significance to the use of decryption for the second stage. Its only advantage is that it allows users of 3DES to decrypt data encrypted by users of the older single DES:

$$C = \mathrm{E}(K_1, \mathrm{D}(K_1, \mathrm{E}(K_1, P))) = \mathrm{E}(K_1, P)$$
$$P = \mathrm{D}(K_1, \mathrm{E}(K_1, \mathrm{D}(K_1, C))) = \mathrm{D}(K_1, C)$$

3DES with two keys is a relatively popular alternative to DES and has been adopted for use in the key management standards ANS X9.17 and ISO 8732.[1]

Currently, there are no practical cryptanalytic attacks on 3DES. Coppersmith [COPP94] notes that the cost of a brute-force key search on 3DES is on the order of $2^{112} \approx (5 \times 10^{33})$ and estimates that the cost of differential cryptanalysis suffers an exponential growth, compared to single DES, exceeding 10^{52}.

It is worth looking at several proposed attacks on 3DES that, although not practical, give a flavor for the types of attacks that have been considered and that could form the basis for more successful future attacks.

The first serious proposal came from Merkle and Hellman [MERK81]. Their plan involves finding plaintext values that produce a first intermediate value of $A = 0$

[1] American National Standard (ANS): *Financial Institution Key Management (Wholesale)*. From its title, X9.17 appears to be a somewhat obscure standard [illegible] have been adopted for use in oth[illegible]

(Figure 6.1b) and then using the meet-in-the-middle attack to determine the two keys. The level of effort is 2^{56}, but the technique requires 2^{56} chosen plaintext–ciphertext pairs which is a number unlikely to be provided by the holder of the keys.

A known-plaintext attack is outlined in [VANO90]. This method is an improvement over the chosen-plaintext approach but requires more effort. The attack is based on the observation that if we know A and C (Figure 6.1b), then the problem reduces to that of an attack on double DES. Of course, the attacker does not know A, even if P and C are known, as long as the two keys are unknown. However, the attacker can choose a potential value of A and then try to find a known (P, C) pair that produces A. The attack proceeds as follows.

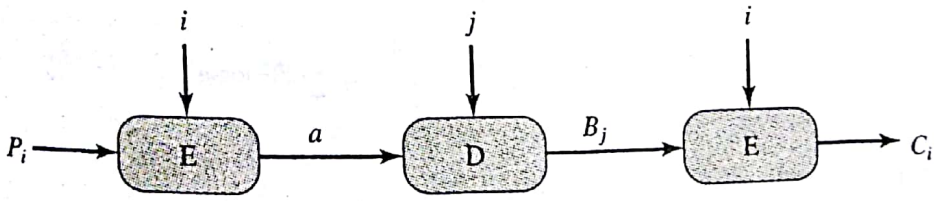
1. Obtain n (P, C) pairs. This is the known plaintext. Place these in a table (Table 1) sorted on the values of P (Figure 6.2b).
2. Pick an arbitrary value a for A, and create a second table (Figure 6.2c) with entries defined in the following fashion. For each of the 2^{56} possible keys $K_1 = i$, calculate the plaintext value P_i that produces a:

$$P_i = \mathrm{D}(i, a)$$

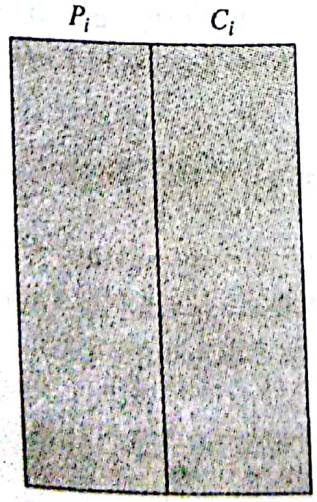
For each P_i that matches an entry in Table 1, create an entry in Table 2 consisting of the K_1 value and the value of B that is produced for the (P, C) pair from Table 1, assuming that value of K_1:

$$B = \mathrm{D}(i, C)$$

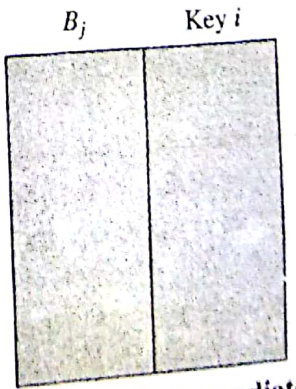
At the end of this step, sort Table 2 on the values of B.



(a) Two-key triple encryption with candidate pair of keys



(b) Table of n known plaintext-ciphertext pairs, sorted on P



(c) Table of intermediate values and candidate keys

Figure 6.2 Known-Plaintext Attack on Triple DES

3. We now have a number of candidate values of K_1 in Table 2 and are in a position to search for a value of K_2. For each of the 2^{56} possible keys $K_2 = j$, calculate the second intermediate value for our chosen value of a:

$$B_j = \mathrm{D}(j, a)$$

At each step, look up B_j in Table 2. If there is a match, then the corresponding key i from Table 2 plus this value of j are candidate values for the unknown keys (K_1, K_2). Why? Because we have found a pair of keys (i, j) that produce a known (P, C) pair (Figure 6.2a).

4. Test each candidate pair of keys (i, j) on a few other plaintext–ciphertext pairs. If a pair of keys produces the desired ciphertext, the task is complete. If no pair succeeds, repeat from step 1 with a new value of a.

For a given known (P, C), the probability of selecting the unique value of a that leads to success is $1/2^{64}$. Thus, given n (P, C) pairs, the probability of success for a single selected value of a is $n/2^{64}$. A basic result from probability theory is that the expected number of draws required to draw one red ball out of a bin containing n red balls and $N - n$ green balls is $(N + 1)/(n + 1)$ if the balls are not replaced. So the expected number of values of a that must be tried is, for large n,

$$\frac{2^{64} + 1}{n + 1} \approx \frac{2^{64}}{n}$$

Thus, the expected running time of the attack is on the order of

$$\left(2^{56}\right)\frac{2^{64}}{n} = 2^{120 - \log_2 n}$$

Triple DES with Three Keys

Although the attacks just described appear impractical, anyone using two-key 3DES may feel some concern. Thus, many researchers now feel that three-key 3DES is the preferred alternative (e.g., [KALI96a]). Three-key 3DES has an effective key length of 168 bits and is defined as

$$C = \mathrm{E}(K_3, \mathrm{D}(K_2, \mathrm{E}(K_1, P)))$$

Backward compatibility with DES is provided by putting $K_3 = K_2$ or $K_1 = K_2$.

A number of Internet-based applications have adopted three-key 3DES, including PGP and S/MIME, both discussed in Chapter 18.

6.2 ELECTRONIC CODE BOOK

A block cipher takes a fixed-length block of text of length b bits and a key as input and produces a b-bit block of ciphertext. If the amount of plaintext to be encrypted is greater than b bits, then the block cipher can still be used by breaking the plaintext

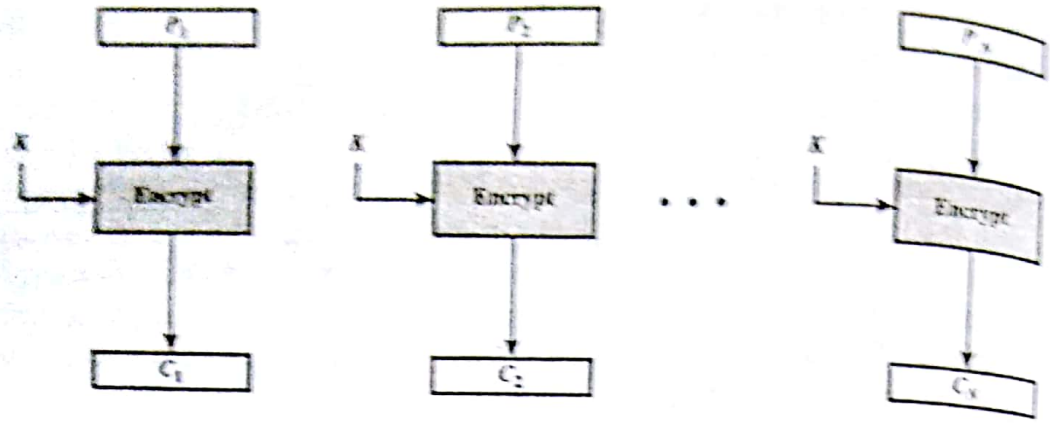
up into b-bit blocks. When multiple blocks of plaintext are encrypted using the same key, a number of security issues arise. To apply a block cipher in a variety of applications, five *modes of operation* have been defined by NIST (SP 800-38A). In essence, a mode of operation is a technique for enhancing the effect of a cryptographic algorithm or adapting the algorithm for an application, such as applying a block cipher to a sequence of data blocks or a data stream. The five modes are intended to cover a wide variety of applications of encryption for which a block cipher could be used. These modes are intended for use with any symmetric block cipher, including triple DES and AES. The modes are summarized in Table 6.1 and described in this and the following sections.

The simplest mode is the **electronic codebook (ECB)** mode, in which plaintext is handled one block at a time and each block of plaintext is encrypted using the same key (Figure 6.3). The term *codebook* is used because, for a given key, there is a unique ciphertext for every b-bit block of plaintext. Therefore, we can imagine a gigantic codebook in which there is an entry for every possible b-bit plaintext pattern showing its corresponding ciphertext.

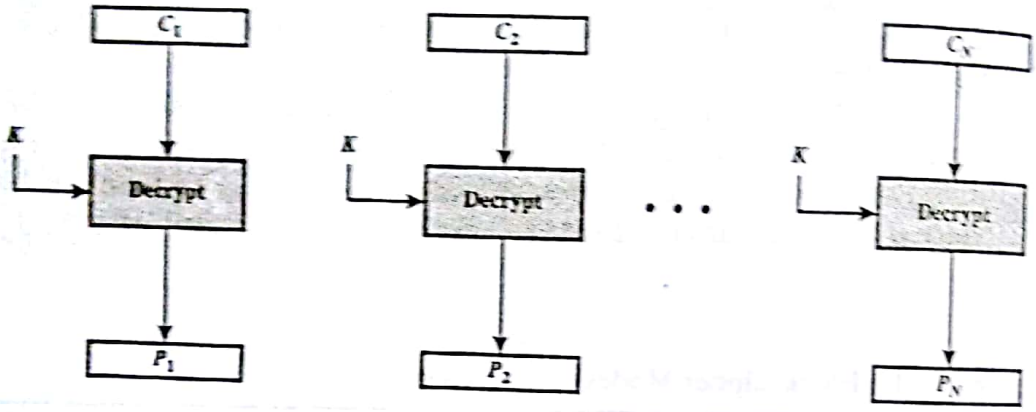
For a message longer than b bits, the procedure is simply to break the message into b-bit blocks, padding the last block if necessary. Decryption is performed one block at a time, always using the same key. In Figure 6.3, the plaintext (padded as necessary) consists of a sequence of b-bit blocks, $P_1, P_2, \ldots, P_N$; the

Table 6.1 Block Cipher Modes of Operation

Mode	Description	Typical Application
Electronic Codebook (ECB)	Each block of 64 plaintext bits is encoded independently using the same key.	• Secure transmission of single values (e.g., an encryption key)
Cipher Block Chaining (CBC)	The input to the encryption algorithm is the XOR of the next 64 bits of plaintext and the preceding 64 bits of ciphertext.	• General-purpose block-oriented transmission • Authentication
Cipher Feedback (CFB)	Input is processed s bits at a time. Preceding ciphertext is used as input to the encryption algorithm to produce pseudorandom output, which is XORed with plaintext to produce next unit of ciphertext.	• General-purpose stream-oriented transmission • Authentication
Output Feedback (OFB)	Similar to CFB, except that the input to the encryption algorithm is the preceding encryption output, and full blocks are used.	• Stream-oriented transmission over noisy channel (e.g., satellite communication)
Counter (CTR)	Each block of plaintext is XORed with an encrypted counter. The counter is incremented for each subsequent block.	• General-purpose block-oriented transmission • Useful for high-speed requirements



(a) Encryption



(b) Decryption

Figure 6.3 Electronic Codebook (ECB) Mode

corresponding sequence of ciphertext blocks is $C_1, C_2, \ldots, C_N$. We can define ECB mode as follows.

ECB	$C_j = E(K, P_j)$ $j = 1, \ldots, N$	$P_j = D(K, C_j)$ $j = 1, \ldots, N$

The ECB method is ideal for a short amount of data, such as an encryption key. Thus, if you want to transmit a DES or AES key securely, ECB is the appropriate mode to use.

The most significant characteristic of ECB is that if the same b-bit block of plaintext appears more than once in the message, it always produces the same ciphertext.

For lengthy messages, the ECB mode may not be secure. If the message is highly structured, it may be possible for a cryptanalyst to exploit these regularities. For example, if it is known that the message always starts out with certain predefined fields, then the cryptanalyst may have a number of known plaintext–ciphertext pairs to work with. If the message has repetitive elements with a period of repetition a multiple of b bits, then these elements can be identified by the analyst. This may help in the analysis or may provide an opportunity for substituting or rearranging blocks.

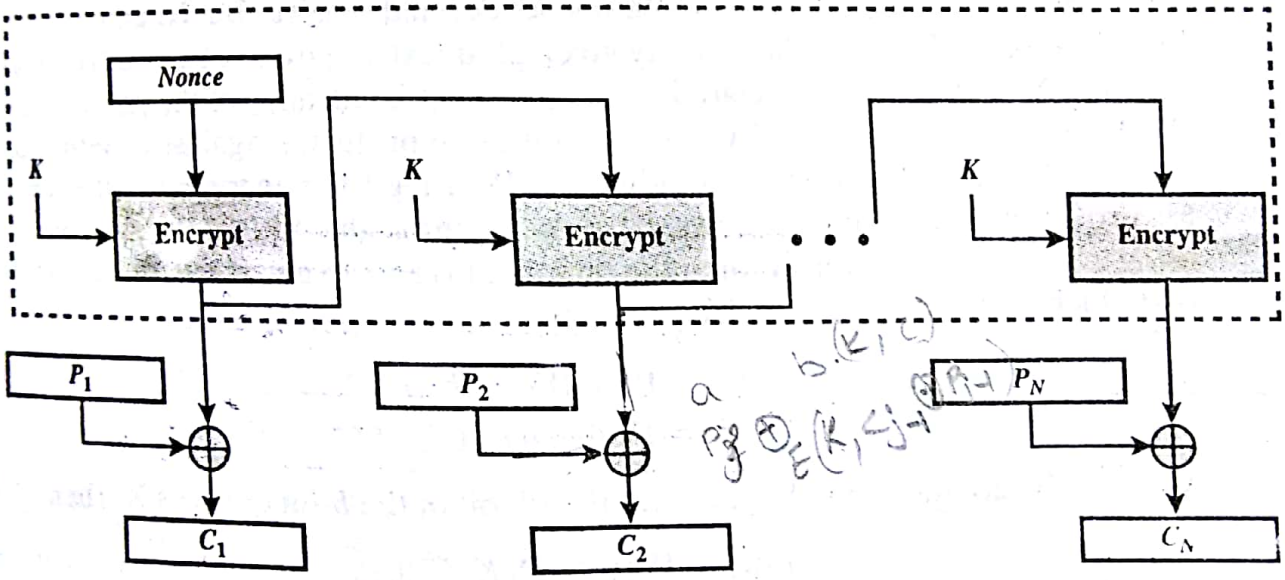
6.3 OUTPUT FEEDBACK MODE

The **output feedback** (OFB) mode is similar in structure to that of CFB. As can be seen in Figure 6.6, it is the output of the encryption function that is fed back to the shift register in OFB, whereas in CFB, the ciphertext unit is fed back to the shift register. The other difference is that the OFB mode operates on full blocks of plaintext and ciphertext, not on an s-bit subset. Encryption can be expressed as

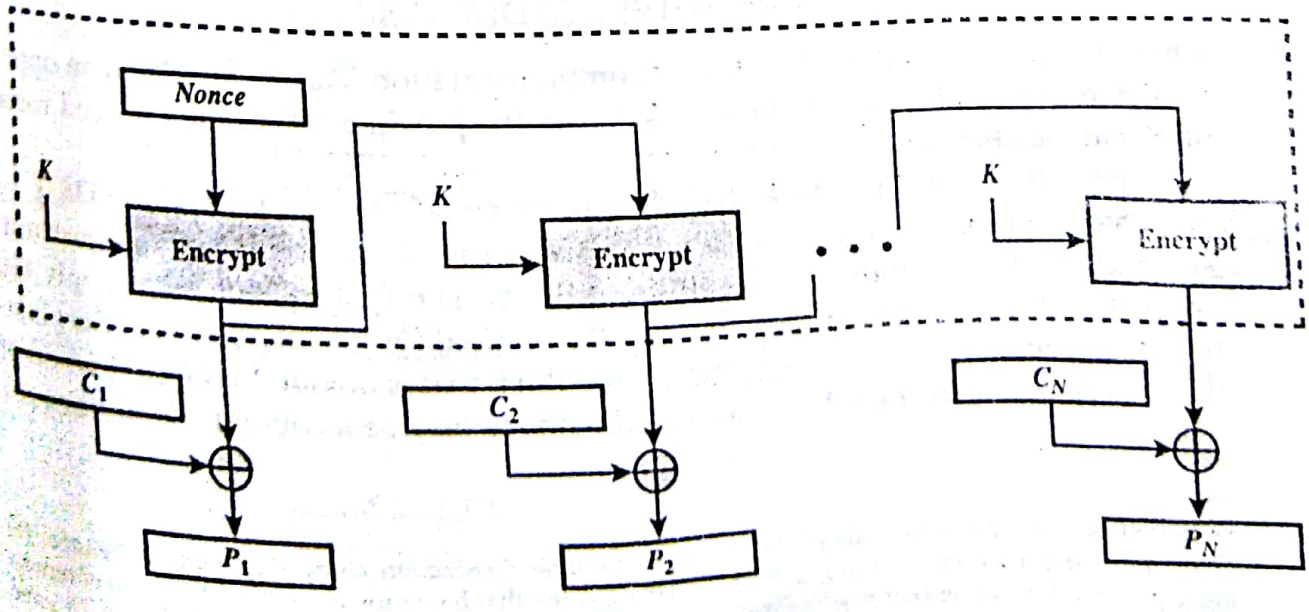
$$C_j = P_j \oplus \mathrm{E}(K, [C_{j-i} \oplus P_{j-1}])$$

By rearranging terms, we can demonstrate that decryption works.

$$P_j = C_j \oplus \mathrm{E}(K, [C_{j-1} \oplus P_{j-1}])$$



(a) Encryption



(b) Decryption

Figure 6.6 Output Feedback (OFB) Mode

Then

$$D(K, C_j) = D(K, E(K, [C_{j-1} \oplus P_j]))$$
$$D(K, C_j) = C_{j-1} \oplus P_j$$
$$C_{j-1} \oplus D(K, C_j) = C_{j-1} \oplus C_{j-1} \oplus P_j = P_j$$

To produce the first block of ciphertext, an initialization vector (IV) is XORed with the first block of plaintext. On decryption, the IV is XORed with the output of the decryption algorithm to recover the first block of plaintext. The IV is a data block that is that same size as the cipher block. We can define CBC mode as

CBC	$C_1 = E(K, [P_1 \oplus IV])$ $C_j = E(K, [P_j \oplus C_{j-1}])$ $j = 2, \ldots, N$	$P_1 = D(K, C_1) \oplus IV$ $P_j = D(K, C_j) \oplus C_{j-1}$ $j = 2, \ldots, N$

The IV must be known to both the sender and receiver but be unpredictable by a third party. In particular, for any given plaintext, it must not be possible to predict the IV that will be associated to the plaintext in advance of the generation of the IV. For maximum security, the IV should be protected against unauthorized changes. This could be done by sending the IV using ECB encryption. One reason for protecting the IV is as follows: If an opponent is able to fool the receiver into using a different value for IV, then the opponent is able to invert selected bits in the first block of plaintext. To see this, consider

$$C_1 = E(K, [IV \oplus P_1])$$
$$P_1 = IV \oplus D(K, C_1)$$

Now use the notation that $X[i]$ denotes the ith bit of the b-bit quantity X. Then

$$P_1[i] = IV[i] \oplus D(K, C_1)[i]$$

Then, using the properties of XOR, we can state

$$P_1[i]' = IV[i]' \oplus D(K, C_1)[i]$$

where the prime notation denotes bit complementation. This means that if an opponent can predictably change bits in IV, the corresponding bits of the received value of P_1 can be changed.

For other possible attacks based on prior knowledge of IV, see [VOYD83].

So long as it is unpredictable, the specific choice of IV is unimportant. Sp800-38a recommends two possible methods: The first method is to apply the encryption function, under the same key that is used for the encryption of the plaintext, to a nonce.[2] The nonce must be a data block that is unique to each execution of the encryption operation. For example, the nonce may be a counter, a timestamp, or

[2] NIST SP-800-90 (*Recommendation for Random Number Generation Using Deterministic Random Bit Generators*) defines nonce as follows: A time-varying value that has at most a negligible chance of repeating, e.g., a random value that is generated anew for each use, a timestamp, a sequence number, or some combination of these.

a message number. The second method is to generate a random data block using a random number generator.

In conclusion, because of the chaining mechanism of CBC, it is an appropriate mode for encrypting messages of length greater than b bits.

In addition to its use to achieve confidentiality, the CBC mode can be used for authentication. This use is described in Chapter 12.

6.4 CIPHER FEEDBACK MODE

For AES, DES, or any block cipher, encryption is performed on a block of b bits. In the case of DES, $b = 64$ and in the case of AES, $b = 128$. However, it is possible to convert a block cipher into a stream cipher, using one of the three modes to be discussed in this and the next two sections: **cipher feedback** (CFB) mode, **output feedback** (OFB) mode, and **counter** (CTR) mode. A stream cipher eliminates the need to pad a message to be an integral number of blocks. It also can operate in real time. Thus, if a character stream is being transmitted, each character can be encrypted and transmitted immediately using a character-oriented stream cipher.

One desirable property of a stream cipher is that the ciphertext be of the same length as the plaintext. Thus, if 8-bit characters are being transmitted, each character should be encrypted to produce a ciphertext output of 8 bits. If more than 8 bits are produced, transmission capacity is wasted.

Figure 6.5 depicts the CFB scheme. In the figure, it is assumed that the unit of transmission is s bits; a common value is $s = 8$. As with CBC, the units of plaintext are chained together, so that the ciphertext of any plaintext unit is a function of all the preceding plaintext. In this case, rather than blocks of b bits, the plaintext is divided into *segments* of s bits.

First, consider encryption. The input to the encryption function is a b-bit shift register that is initially set to some initialization vector (IV). The leftmost (most significant) s bits of the output of the encryption function are XORed with the first segment of plaintext P_1 to produce the first unit of ciphertext C_1, which is then transmitted. In addition, the contents of the shift register are shifted left by s bits, and C_1 is placed in the rightmost (least significant) s bits of the shift register. This process continues until all plaintext units have been encrypted.

For decryption, the same scheme is used, except that the received ciphertext unit is XORed with the output of the encryption function to produce the plaintext unit. Note that it is the *encryption* function that is used, not the decryption function. This is easily explained. Let $\text{MSB}_s(X)$ be defined as the most significant s bits of X. Then

$$C_1 = P_1 \oplus \text{MSB}_s[\text{E}(K, \text{IV})]$$

Therefore, by rearranging terms:

$$P_1 = C_1 \oplus \text{MSB}_s[\text{E}(K, \text{IV})]$$

The same reasoning holds for subsequent steps in the process.

We can define CFB mode as follows.

For the last plaintext block, which may be a partial block of u bits, the most significant u bits of the last output block are used for the XOR operation; the remaining $b-u$ bits are discarded. Unlike the ECB, CBC, and CFB modes, we do not need to use padding because of the structure of the CTR mode.

As with the OFB mode, the initial counter value must be a nonce; that is, T_1 must be different for all of the messages encrypted using the same key. Further, all T_i values across all messages must be unique. If, contrary to this requirement, a counter value is used multiple times, then the confidentiality of all of the plaintext blocks corresponding to that counter value may be compromised. In particular, if any plaintext block that is encrypted using a given counter value is known, then the output of the encryption function can be determined easily from the associated ciphertext block. This output allows any other plaintext blocks that are encrypted using the same counter value to be easily recovered from their associated ciphertext blocks.

One way to ensure the uniqueness of counter values is to continue to increment the counter value by 1 across messages. That is, the first counter value of the each message is one more than the last counter value of the preceding message.

[LIPM00] lists the following advantages of CTR mode.

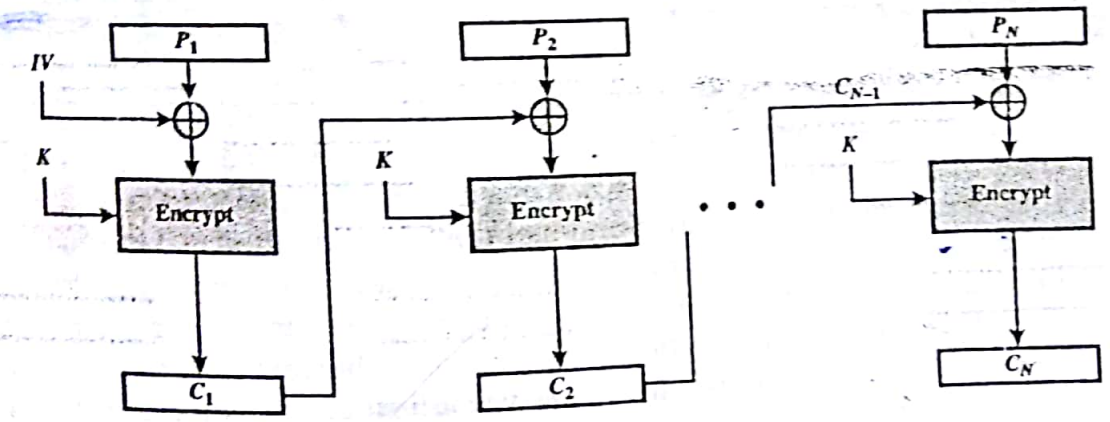
- **Hardware efficiency:** Unlike the three chaining modes, encryption (or decryption) in CTR mode can be done in parallel on multiple blocks of plaintext or ciphertext. For the chaining modes, the algorithm must complete the computation on one block before beginning on the next block. This limits the maximum throughput of the algorithm to the reciprocal of the time for one execution of block encryption or decryption. In CTR mode, the throughput is only limited by the amount of parallelism that is achieved.
- **Software efficiency:** Similarly, because of the opportunities for parallel execution in CTR mode, processors that support parallel features, such as aggressive pipelining, multiple instruction dispatch per clock cycle, a large number of registers, and SIMD instructions, can be effectively utilized.
- **Preprocessing:** The execution of the underlying encryption algorithm does not depend on input of the plaintext or ciphertext. Therefore, if sufficient memory is available and security is maintained, preprocessing can be used to prepare the output of the encryption boxes that feed into the XOR functions, as in Figure 6.7. When the plaintext or ciphertext input is presented, then the only computation is a series of XORs. Such a strategy greatly enhances throughput.
- **Random access:** The ith block of plaintext or ciphertext can be processed in random-access fashion. With the chaining modes, block C_i cannot be computed until the $i-1$ prior block are computed. There may be applications in which a ciphertext is stored and it is desired to decrypt just one block; for such applications, the random access feature is attractive.
- **Provable security:** It can be shown that CTR is at least as secure as the other modes discussed in this section.
- **Simplicity:** Unlike ECB and CBC modes, CTR mode requires only the implementation of the encryption algorithm and not the decryption algorithm. This matters most when the decryption algorithm differs substantially from the encryption algorithm, as it does for AES. In addition, the decryption key scheduling need not be implemented.

6.3 CIPHER BLOCK CHAINING MODE

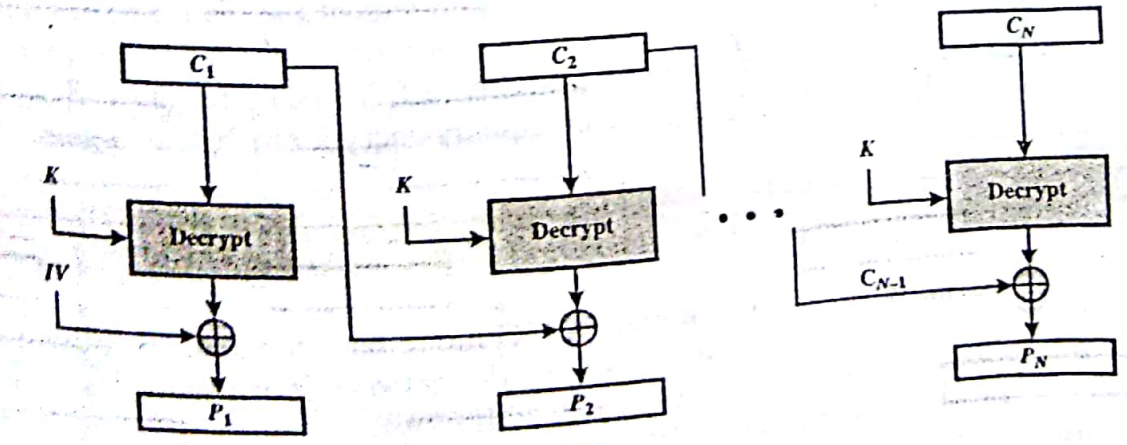
To overcome the security deficiencies of ECB, we would like a technique in which the same plaintext block, if repeated, produces different ciphertext blocks. A simple way to satisfy this requirement is the **cipher block chaining** (CBC) mode (Figure 6.4). In this scheme, the input to the encryption algorithm is the XOR of the current plaintext block and the preceding ciphertext block; the same key is used for each block. In effect, we have chained together the processing of the sequence of plaintext blocks. The input to the encryption function for each plaintext block bears no fixed relationship to the plaintext block. Therefore, repeating patterns of b bits are not exposed. As with the ECB mode, the CBC mode requires that the last block be padded to a full b bits if it is a partial block.

For decryption, each cipher block is passed through the decryption algorithm. The result is XORed with the preceding ciphertext block to produce the plaintext block. To see that this works, we can write

$$C_j = \mathrm{E}(K, [C_{j-1} \oplus P_j])$$



(a) Encryption



(b) Decryption

Figure 6.4 Cipher Block Chaining (CFB) Mode

We can define OFB mode as follows.

OFB	$I_1 = Nonce$	$I_1 = Nonce$
	$I_j = O_{j-1}$ $j = 2, \ldots, N$	$I_j = \text{LSB}_{b-s}(I_{j-1}) \parallel C_{j-1}$ $j = 2, \ldots, N$
	$O_j = \text{E}(K, I_j)$ $j = 1, \ldots, N$	$O_j = \text{E}(K, I_j)$ $j = 1, \ldots, N$
	$C_j = P_j \oplus O_j$ $j = 1, \ldots, N-1$	$P_j = C_j \oplus O_j$ $j = 1, \ldots, N-1$
	$C_N^* = P_N^* \oplus \text{MSB}_u(O_N)$	$P_N^* = C_N^* \oplus \text{MSB}_u(O_N)$

Let the size of a block be b. If the last block of plaintext contains u bits (indicated by *), with $u < b$, the most significant u bits of the last output block O_N are used for the XOR operation; the remaining $b-u$ bits of the last output block are discarded.

As with CBC and CFB, the OFB mode requires an initialization vector. In the case of OFB, the IV must be a nonce; that is, the IV must be unique to each execution of the encryption operation. The reason for this is that the sequence of encryption output blocks, O_i, depends only on the key and the IV and does not depend on the plaintext. Therefore, for a given key and IV, the stream of output bits used to XOR with the stream of plaintext bits is fixed. If two different messages had an identical block of plaintext in the identical position, then an attacker would be able to determine that portion of the O_i stream.

One advantage of the OFB method is that bit errors in transmission do not propagate. For example, if a bit error occurs in C_1, only the recovered value of P_1 is affected; subsequent plaintext units are not corrupted. With CFB, C_1 also serves as input to the shift register and therefore causes additional corruption downstream.

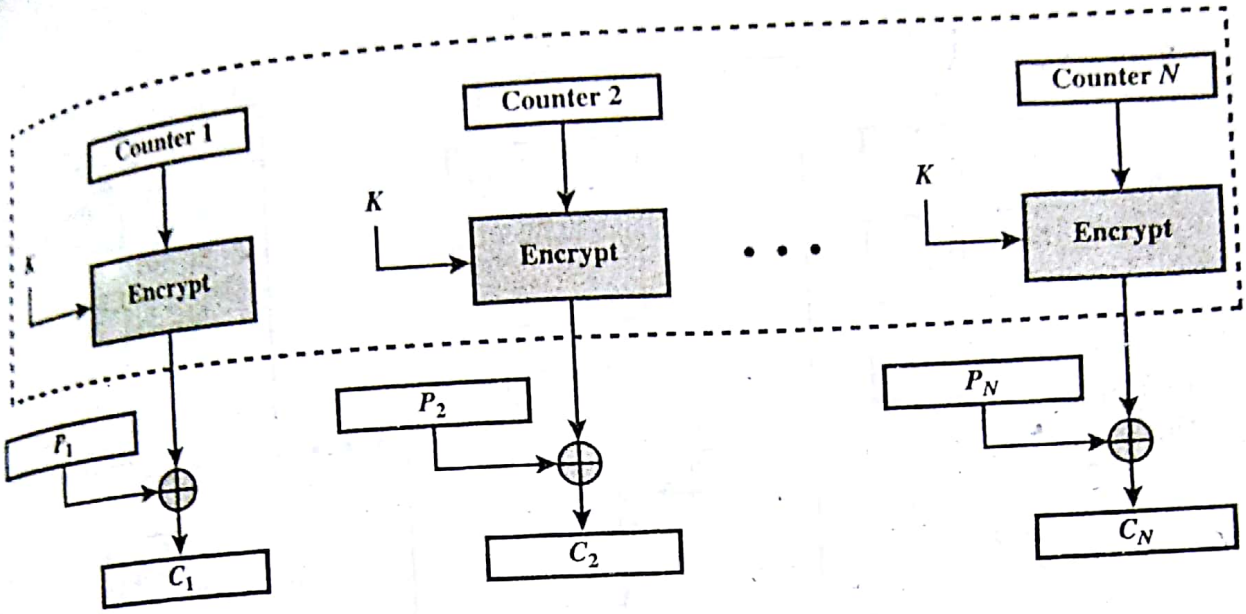
The disadvantage of OFB is that it is more vulnerable to a message stream modification attack than is CFB. Consider that complementing a bit in the ciphertext complements the corresponding bit in the recovered plaintext. Thus, controlled changes to the recovered plaintext can be made. This may make it possible for an opponent, by making the necessary changes to the checksum portion of the message as well as to the data portion, to alter the ciphertext in such a way that it is not detected by an error-correcting code. For a further discussion, see [VOYD83].

OFB has the structure of a typical stream cipher, because the cipher generates a stream of bits as a function of an initial value and a key, and that stream of bits is XORed with the plaintext bits (see Figure 3.1). The generated stream that is XORed with the plaintext is itself independent of the plaintext; this is highlighted by dashed boxes in Figure 6.6. One distinction from the stream ciphers we discuss in Chapter 7 is that OFB encrypts plaintext a full block at a time, where typically a block is 64 or 128 bits. Many stream ciphers encrypt one byte at a time.

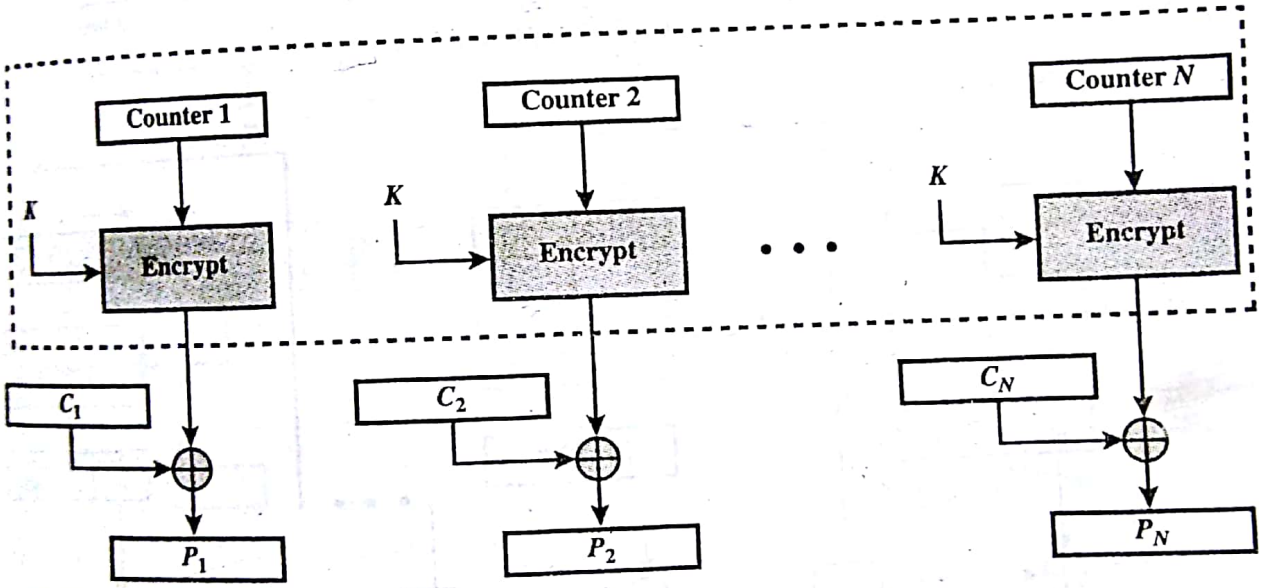
6.6 COUNTER MODE

Although interest in the **counter** (CTR) mode has increased recently with applications to ATM (asynchronous transfer mode) network security and IP sec (IP security), this mode was proposed early on (e.g., [DIFF79]).

Figure 6.7 depicts the CTR mode. A counter equal to the plaintext block size is used. The only requirement stated in SP 800-38A is that the counter value must be



(a) Encryption



(b) Decryption

Figure 6.7 Counter (CTR) Mode

different for each plaintext block that is encrypted. Typically, the counter is initialized to some value and then incremented by 1 for each subsequent block (modulo 2^b, where b is the block size). For encryption, the counter is encrypted and then XORed with the plaintext block to produce the ciphertext block; there is no chaining. For decryption, the same sequence of counter values is used, with each encrypted counter XORed with a ciphertext block to recover the corresponding plaintext block. Thus, the initial counter value must be made available for decryption. Given a sequence of counters $T_1, T_2, \ldots, T_N$, we can define CTR mode as follows.

CTR	$C_j = P_j \oplus \mathrm{E}(K, T_j) \quad j = 1, \ldots, N-1$ $C_N^* = P_N^* \oplus \mathrm{MSB}_u[\mathrm{E}(K, T_N)]$	$P_j = C_j \oplus \mathrm{E}(K, T_j) \quad j = 1, \ldots, N-1$ $P_N^* = C_N^* \oplus \mathrm{MSB}_u[\mathrm{E}(K, T_N)]$

3.6 BLOCK CIPHER DESIGN PRINCIPLES

5

Although much progress has been made in designing block ciphers that are cryptographically strong, the basic principles have not changed all that much since the work of Feistel and the DES design team in the early 1970s. It is useful to begin this discussion by looking at the published design criteria used in the DES effort. Then we look at three critical aspects of block cipher design: the number of rounds, design of the function F, and key scheduling.

DES Design Criteria

The criteria used in the design of DES, as reported in [COPP94], focused on the design of the S-boxes and on the P function that takes the output of the S-boxes (Figure 3.7). The criteria for the S-boxes are as follows.

1. No output bit of any S-box should be too close a linear function of the input bits. Specifically, if we select any output bit and any subset of the six input bits, the fraction of inputs for which this output bit equals the XOR of these input bits should not be close to 0 or 1, but rather should be near 1/2.
2. Each row of an S-box (determined by a fixed value of the leftmost and rightmost input bits) should include all 16 possible output bit combinations.
3. If two inputs to an S-box differ in exactly one bit, the outputs must differ in at least two bits.
4. If two inputs to an S-box differ in the two middle bits exactly, the outputs must differ in at least two bits.
5. If two inputs to an S-box differ in their first two bits and are identical in their last two bits, the two outputs must not be the same.
6. For any nonzero 6-bit difference between inputs, no more than eight of the 32 pairs of inputs exhibiting that difference may result in the same output difference.
7. This is a criterion similar to the previous one, but for the case of three S-boxes.

Coppersmith pointed out that the first criterion in the preceding list was needed because the S-boxes are the only nonlinear part of DES. If the S-boxes were linear (i.e., each output bit is a linear combination of the input bits), the entire algorithm would be linear and easily broken. We have seen this phenomenon with the Hill cipher, which is linear. The remaining criteria were primarily aimed at thwarting differential cryptanalysis and at providing good confusion properties.

The criteria for the permutation P are as follows.

1. The four output bits from each S-box at round i are distributed so that two of them affect (provide input for) "middle bits" of round $(i + 1)$ and the other two affect end bits. The two middle bits of input to an S-box are not shared with adjacent S-boxes. The end bits are the two left-hand bits and the two right-hand bits, which are shared with adjacent S-boxes.
2. The four output bits from each S-box affect six different S-boxes on the next round, and no two affect the same S-box.
3. For two S-boxes j, k, if an output bit from S_j affects a middle bit of S_k on the next round, then an output bit from S_k cannot affect a middle bit of S_j. This implies that, for $j = k$, an output bit from S_j must not affect a middle bit of S_j.

These criteria are intended to increase the diffusion of the algorithm.

Number of Rounds

The cryptographic strength of a Feistel cipher derives from three aspects of the design: the number of rounds, the function F, and the key schedule algorithm. Let us look first at the choice of the number of rounds.

The greater the number of rounds, the more difficult it is to perform cryptanalysis, even for a relatively weak F. In general, the criterion should be that the number of rounds is chosen so that known cryptanalytic efforts require greater effort than a simple brute-force key search attack. This criterion was certainly used in the design of DES. Schneier [SCHN96] observes that for 16-round DES, a differential cryptanalysis attack is slightly less efficient than brute force: The differential cryptanalysis attack requires $2^{55.1}$ operations,[10] whereas brute force requires 2^{55}. If DES had 15 or fewer rounds, differential cryptanalysis would require less effort than a brute-force key search.

This criterion is attractive, because it makes it easy to judge the strength of an algorithm and to compare different algorithms. In the absence of a cryptanalytic breakthrough, the strength of any algorithm that satisfies the criterion can be judged solely on key length.

Design of Function F

The heart of a Feistel block cipher is the function F. As we have seen, in DES, this function relies on the use of S-boxes. This is also the case for many other symmetric block ciphers. However, we can make some general comments about the criteria for designing F. After that, we look specifically at S-box design.

Design Criteria for F The function F provides the element of confusion in a Feistel cipher. Thus, it must be difficult to "unscramble" the substitution performed by F. One obvious criterion is that F be nonlinear, as we discussed previously. The more nonlinear F, the more difficult any type of cryptanalysis will be. There are several measures of nonlinearity, which are beyond the scope of this book. In rough terms, the more difficult it is to approximate F by a set of linear equations, the more nonlinear F is.

Several other criteria should be considered in designing F. We would like the algorithm to have good avalanche properties. Recall that, in general, this means that a change in one bit of the input should produce a change in many bits of the output. A more stringent version of this is the **strict avalanche criterion (SAC)** [WEBS86], which states that any output bit j of an S-box should change with probability 1/2 when any single input bit i is inverted for all i, j. Although SAC is expressed in terms of S-boxes, a similar criterion could be applied to F as a whole. This is important when considering designs that do not include S-boxes.

Another criterion proposed in [WEBS86] is the **bit independence criterion (BIC)**, which states that output bits j and k should change independently when any single input bit i is inverted for all i, j, and k. The SAC and BIC criteria appear to strengthen the effectiveness of the confusion function.

S-Box Design One of the most intense areas of research in the field of symmetric block ciphers is that of S-box design. The papers are almost too numerous to

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[10] Recall that differential cryptanalysis of DES requires 2^{47} chosen plaintext. If all you have to work with is known plaintext, then you must sort through a large quantity of known plaintext–ciphertext pairs looking for the useful ones. This brings the level of effort up to $2^{55.1}$.

count.[11] Here we mention some general principles. In essence, we would like any change to the input vector to an S-box to result in random-looking changes to the output. The relationship should be nonlinear and difficult to approximate with linear functions.

One obvious characteristic of the S-box is its size. An $n \times m$ S-box has n input bits and m output bits. DES has 6×4 S-boxes. The encryption algorithm Blowfish, has 8×32 S-boxes. Larger S-boxes, by and large, are more resistant to differential and linear cryptanalysis [SCHN96]. On the other hand, the larger the dimension n, the (exponentially) larger the lookup table. Thus, for practical reasons, a limit of n equal to about 8 to 10 is usually imposed. Another practical consideration is that the larger the S-box, the more difficult it is to design it properly.

S-boxes are typically organized in a different manner than used in DES. An $n \times m$ S-box typically consists of 2^n rows of m bits each. The n bits of input select one of the rows of the S-box, and the m bits in that row are the output. For example, in an 8×32 S-box, if the input is `00001001`, the output consists of the 32 bits in row 9 (the first row is labeled row 0).

Mister and Adams [MIST96] propose a number of criteria for S-box design. Among these are that the S-box should satisfy both SAC and BIC. They also suggest that all linear combinations of S-box columns should be *bent*. Bent functions are a special class of Boolean functions that are highly nonlinear according to certain mathematical criteria [ADAM90]. There has been increasing interest in designing and analyzing S-boxes using bent functions.

A related criterion for S-boxes is proposed and analyzed in [HEYS95]. The authors define the **guaranteed avalanche (GA)** criterion as follows: An S-box satisfies GA of order γ if, for a 1-bit input change, at least γ output bits change. The authors conclude that a GA in the range of order 2 to order 5 provides strong diffusion characteristics for the overall encryption algorithm.

For larger S-boxes, such as 8×32, the question arises as to the best method of selecting the S-box entries in order to meet the type of criteria we have been discussing. Nyberg, who has written a lot about the theory and practice of S-box design, suggests the following approaches (quoted in [ROBS95b]):

- **Random:** Use some pseudorandom number generation or some table of random digits to generate the entries in the S-boxes. This may lead to boxes with undesirable characteristics for small sizes (e.g., 6×4) but should be acceptable for large S-boxes (e.g., 8×32).
- **Random with testing:** Choose S-box entries randomly, then test the results against various criteria, and throw away those that do not pass.
- **Human-made:** This is a more or less manual approach with only simple mathematics to support it. It is apparently the technique used in the DES design. This approach is difficult to carry through for large S-boxes.
- **Math-made:** Generate S-boxes according to mathematical principles. By using mathematical construction, S-boxes can be constructed that offer proven security against linear and differential cryptanalysis, together with good diffusion.

[11] A good summary of S-box design studies through early 1996 can be found in [SCHN96].

A variation on the first technique is to use S-boxes that are both random and key dependent. An example of this approach is Blowfish, which starts with S-boxes filled with pseudorandom digits and then alters the contents using the key. A tremendous advantage of key-dependent S-boxes is that, because they are not fixed, it is impossible to analyze the S-boxes ahead of time to look for weaknesses.

Key Schedule Algorithm

A final area of block cipher design, and one that has received less attention than S-box design, is the key schedule algorithm. With any Feistel block cipher, the key is used to generate one subkey for each round. In general, we would like to select subkeys to maximize the difficulty of deducing individual subkeys and the difficulty of working back to the main key. No general principles for this have yet been promulgated.

Hall suggests [ADAM94] that, at minimum, the key schedule should guarantee key/ciphertext Strict Avalanche Criterion and Bit Independence Criterion.