Q1.1)

$$P(X_{1.6} = O_{1.6};\ \theta) = \sum_j \alpha_T(j)$$

From the base case, $\alpha_1(j) = P(x_1 | z_1 = s_j) * \pi_j$, we get

'A', t=1

$\alpha_1(1) = 0.28$
$\alpha_1(2) = 0.06$

Now to recursively compute $\alpha_t(j)$ using

$$\alpha_t(j) = P(x_t | z_t = s_j) \sum_i a_{ij}\ \alpha_{t-1}(i)$$

'G', t=2

$\alpha_2(1) = 0.992$
$\alpha_2(2) = 0.0184$

'C', t=3

$\alpha_3(1) = 0.008672$
$\alpha_3(2) = 0.009264$

'G', t=4

$\alpha_4(1) = 0.0042573$
$\alpha_4(2) = 0.0014586$

'T', t=5

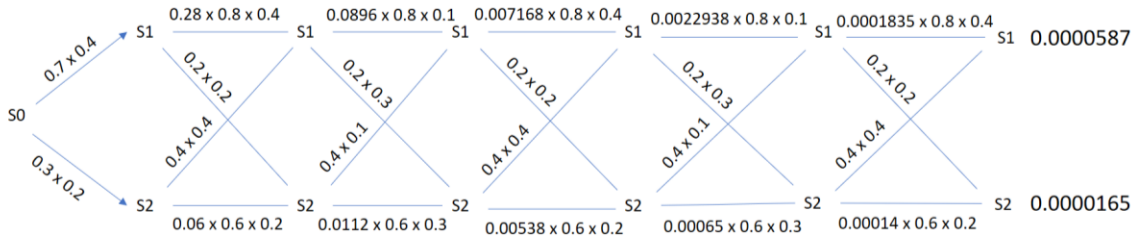
$\alpha_5(1) = 0.0003989$
$\alpha_5(2) = 0.0005180$

'A', t=6

$\alpha_6(1) = 0.0002105$
$\alpha_6(2) = 0.0000781$

$$\boldsymbol{P(X_{1.6} = O_{1.6};\ \theta) = 0.0002105 + 0.0000781 = 0.0002886}$$

Q1.2)



$$P(X_{1.6} = O_{1.6};\ \theta) = 0.0002886$$

$$argmax_{z1:6} P(Z_{1:6} = z_{1:6} |\ X_{1.6} = O_{1.6};\ \theta) = \frac{0.0000587}{0.0002886} = 0.2033957$$

Viterbi best path is 1 1 1 1 1 1

Q1.3)

Compute $x^* = argmax_x P(X_7 = x |\ X_{1.6} = O_{1.6};\ \theta)$

$$= argmax_x \frac{P(X_7 = x, X_{1.6} = O_{1.6};\ \theta)}{P(\ X_{1.6} = O_{1.6})}$$

X='A'

$$\alpha_t(j) = P(x_t | z_t = s_j) \sum_i a_{ij}\ \alpha_{t-1}(i)$$

$\alpha_7(1) = 0.4 * 0.0001996$
$= 0.0000798$
$\alpha_7(2) = 0.2 * 0.000089$
$= 0.0000178$
$\sum_j \alpha_7(j) = 0.0000976$ --------- (1)

X='C'

$\alpha_7(1) = 0.1 * 0.0001996$
$= 0.00001996$
$\alpha_7(2) = 0.3 * 0.000089$
$= 0.0000267$
$\sum_j \alpha_7(j) = 0.0000467$ --------- (2)

X='G'

$\alpha_7(1) = 0.4 * 0.0001996$
$= 0.0000798$
$\alpha_7(2) = 0.2 * 0.000089$
$= 0.0000178$

$\sum_j \alpha_7(j) = 0.0000976$ --------- (3)

X='T'

$\alpha_7(1) = 0.1 * 0.0001996$
$= 0.00001996$
$\alpha_7(2) = 0.3 * 0.000089$
$= 0.0000267$
$\sum_j \alpha_7(j) = 0.0000467$ --------- (4)

From (1), (2), (3), (4)

$argmax_x P(X_7 = x_k, X_{1.6} = O_{1.6};\ \theta) = 0.0000976$ -------- **X is either A or G**

$argmax_x P(X_7 = x |\ X_{1.6} = O_{1.6};\ \theta) = \frac{0.0000976}{0.0002886}$
$= \mathbf{0.3381843}$