Q 1.1 Condition for Non-Invertibility:

$(D \geq N)$ & $\boldsymbol{Z} = [0,0,0 \ldots..0]^T$ is the only solution for the equation, $\boldsymbol{XZ} = \boldsymbol{0}_N$, where $\boldsymbol{0}_N = [0,0,0 \ldots 0]^T$ is a N-dimensional all zero column vector and $\boldsymbol{X}$ is the design matrix.

Q 1.2

$b^* = argmin_b \sum_n [y_n - (\sum_d w_d x_{nd} + b)]^2$

$$\sum_n 2[y_n - (\sum_d w_d x_{nd} + b)](-1) = 0$$

$$\sum_n y_n - \sum_n \sum_d w_d \, x_{nd} - \sum_n b^* = 0$$

$$\sum_n y_n - \sum_d w_d \sum_n x_{nd} - Nb^* = 0$$

Since: $\frac{1}{N} \sum_n x_{nd} = 0$

$$\sum_n y_n = Nb^* \implies b^* = \frac{\sum_n y_n}{N}$$

Q 2.1

$b^* = min_b - \sum_n \{y_n \log(\sigma(b)) + (1 - y_n) \log(1 - \sigma(b))\}$

$\Rightarrow - \sum_n \{y_n \frac{1}{\sigma(b^*)} \Big(\sigma(b^*)\big(1 - \sigma(b^*)\big)\Big) + (1 - y_n) \frac{1}{(1 - \sigma(b^*))} \Big(-\Big(\sigma(b^*)\big(1 - \sigma(b^*)\big)\Big)\Big) \} = 0$

$\Rightarrow - \sum_n \{y_n \big(1 - \sigma(b^*)\big) + (1 - y_n)(-\big(\sigma(b^*)\big))\} = 0$

$\Rightarrow - \sum_n \{y_n - \big(\sigma(b^*)\big)\} = 0$

$\sum_n \big(\sigma(b^*)\big) = \sum_n y_n$

$b^* = \log(\frac{\sum_n y_n}{N - \sum_n y_n})$