

**Q1.1)**

$$K = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} = I$$

$I^T = I$  Thus  $K$  is Symmetric

$$\langle x, Kx \rangle = \langle x, K^T Kx \rangle = \langle Kx, Kx \rangle = \sum x^2 \geq 0$$

Hence,  $K$  is PSD (Positive Semi-Definite)

By Mercer Theorem  $K$  is a valid kernel

**Q1.2)**

Substituting  $\lambda = 0$  and  $K=1$  in training objective we get,

$$J(\alpha) = \frac{1}{2} \alpha^T \alpha - y^T \alpha - \frac{1}{2} y^T y$$

$$\frac{\partial J(\alpha)}{\partial \alpha} = 0 \rightarrow \alpha^* - y^T = 0$$

$$\alpha^* = y^T$$

Substituting back in in training objective,

$$J(\alpha) = \frac{1}{2} y^T y - y^T y - \frac{1}{2} y^T y = 0$$

**Q1.3)**

$$f(x) = [k(x, x_1), k(x, x_2), \dots, k(x, x_n)] \alpha^*$$

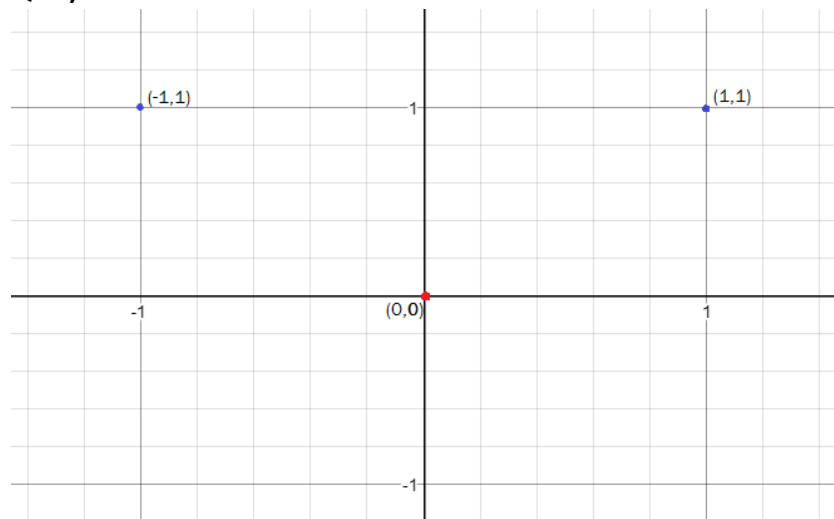
$\forall x, x \neq x_n$ . We know,  $k(x, x_n) = 0$  for the given kernel.

Therefore,  $f(x) = 0$

**Q2.1)**

Not linearly separable because it is evident visually that there cannot exist a line that can separate the points with 100% accuracy.

**Q2.2)**



Yes, the data is now linearly separable in the new feature space

**Q2.3)**

$$K(x_m, x_n) = x_m x_n + x_m^2 x_n^2$$

$$K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Let } \alpha = [\alpha_1, \alpha_2, \alpha_3]$$

$$\alpha^T K \alpha = [\alpha_1, \alpha_2, \alpha_3] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

$$\alpha^T K \alpha = 2\alpha_1^2 + 2\alpha_2^2 \geq 0$$

Thus, **K** is PSD (Positive semi-definite)

**Q2.4)**

$$\varphi(x) = [x, x^2]^T \text{ Let } w = [w_1, w_2]^T$$

$$\min_{w, b, \xi} \sum_n \xi_n + \frac{1}{2} \|w\|^2 \text{ such that, } \forall n (1 - y_n [w_1 x + w_2 x^2 + b]) \leq \xi_n \text{ and } \xi_n \geq 0$$

Dual formulation can be obtained by substituting **K** from **Q2.3)**

$$\max_{\alpha} (\sum_n \alpha_n - \frac{1}{2} \sum_{m,n} (y_m y_n \alpha_m \alpha_n x_m x_n + y_m y_n \alpha_m \alpha_n x_m^2 x_n^2))$$

$$\text{such that, } \forall n \ 0 \leq \alpha_n \leq c \text{ and } \sum_n \alpha_n y_n = 0$$

**Q2.5)**

The dual formulation can be re-written as,

$$\min_{\alpha} (\frac{1}{2} (\sum_{m,n} (y_m y_n \alpha_m \alpha_n x_m x_n + y_m y_n \alpha_m \alpha_n x_m^2 x_n^2) - \sum_n \alpha_n))$$

$$\text{such that, } \forall n \ 0 \leq \alpha_n \leq c \text{ and } \sum_n \alpha_n y_n = 0$$

Expanding the inverse with  $K$  and  $1 \leq m, n \leq 3$  we get,

$$y_1^2 \alpha_1^2 (2) + y_2^2 \alpha_2^2 (2) - (\alpha_1 + \alpha_2 + \alpha_3) = 0$$

$$y_1 \alpha_1 + y_2 \alpha_2 + y_3 \alpha_3 = 0$$

$$\text{we know, } y_1 = -1, y_2 = -1, y_3 = 1$$

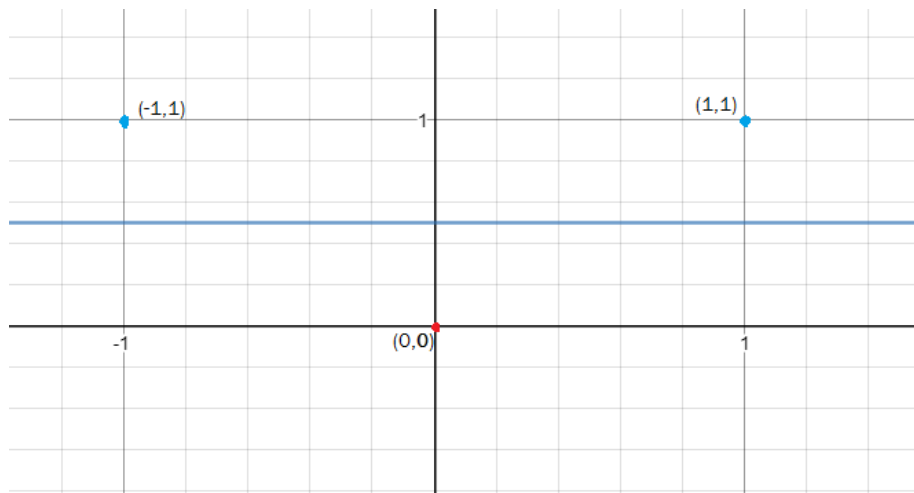
$$\alpha_1 = \alpha_2 \text{ since the equations are symmetrical}$$

$$\text{we then get, } \alpha_3 = 2\alpha_1$$

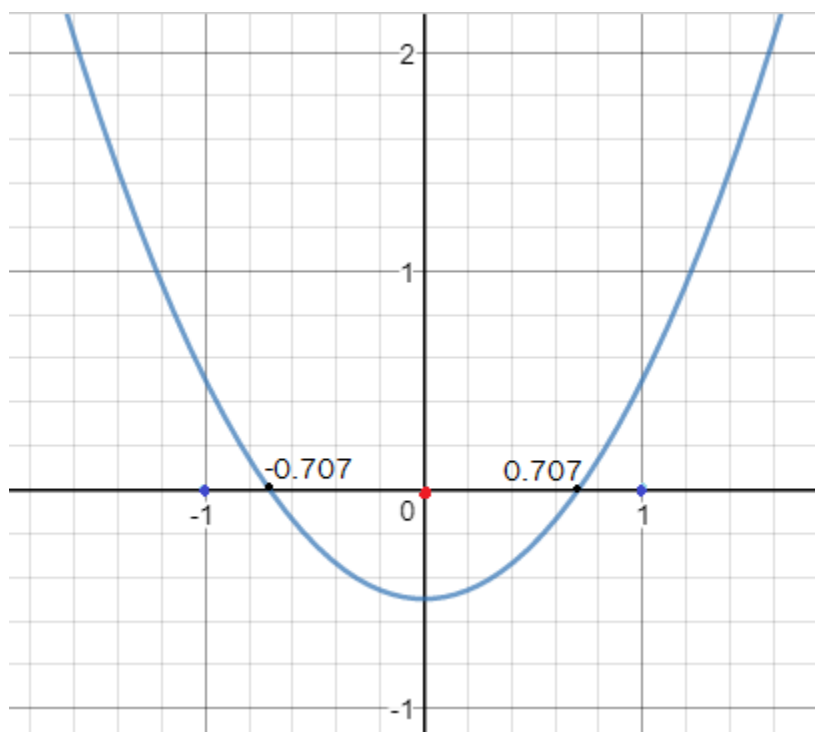
$$\text{Solution: } < \alpha_1, \alpha_1, \alpha_1 > = \{0, 0, 0\} \text{ and } \{1, 1, 2\}$$

$$W = [0, -2] \text{ and } b = 1$$

**Q2.6)**



Plot in 2-Dimensional Feature Space  $\hat{y} = x^2 - 1/2$



Plot in 1-Dimensional Feature Space

**Q3.1)**

$(s, b, d) = (1, 0.5, 1);$

$\epsilon_1 = 0.5$

$\beta_1 = 0$

**Q3.2)**

$w_2(1) = 0.25, w_2(2) = 0.25, w_2(3) = 0.25, w_2(4) = 0.25$

Hence, the algorithm is stuck at  $t = 1$

**Q3.3)**

$$(s, b, d) = (1, -0.5, 1);$$

$$\epsilon_1 = 0.25$$

$$\beta_1 = 0.55$$

**Q3.4)**

$$w_2(1) = 0.17, w_2(2) = 0.17, w_2(3) = 0.17, w_2(4) = 0.51$$

$$(s, b, d) = (1, 0.5, 1);$$

$$\epsilon_2 = 0.17$$

$$\beta_2 = 0.79$$

**Q3.5)**

$$w_2(1) = 0.1, w_2(2) = 0.51, w_2(3) = 0.1, w_2(4) = 0.27$$

$$(s, b, d) = (1, 0.5, 2);$$

$$\epsilon_3 = 0.1$$

$$\beta_3 = 1.1$$

**Q3.6)**

$$F(x) = \text{sign}[0.55h_{(1, -0.5, 1)} + 0.79h_{(1, 0.5, 1)} + 1.1h_{(1, 0.5, 2)}]$$

$$F(x_1) = \text{sign}[0.5 + 0.79 + 1.1] = \text{sign}[2.44] = +1(\text{correctly classified})$$

$$F(x_2) = \text{sign}[-0.55 + 0.79 - 1.1] = \text{sign}[-0.86] = -1(\text{correctly classified})$$

$$F(x_3) = \text{sign}[0.55 + 0.79 - 1.1] = \text{sign}[0.24] = +1(\text{correctly classified})$$

$$F(x_4) = \text{sign}[-0.55 - 0.79 - 1.1] = \text{sign}[-2.44] = -1(\text{correctly classified})$$