Dataset 1: Working with a noisy straight line

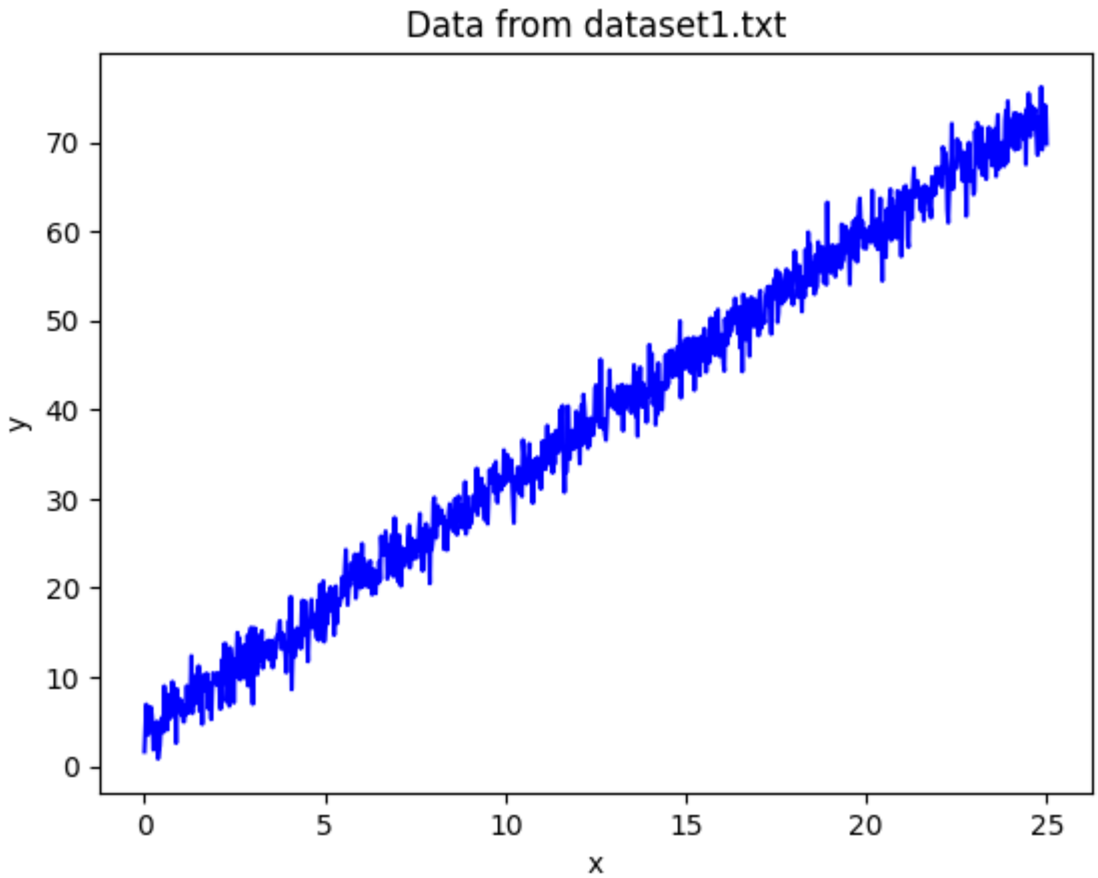
First, we import the required libraries and modules, `numpy` , `matplotlib.pyplot` for plotting the curves and `curve_fit` to fit the data to a required curve. `%matplotlib inline` is a magic command that displays the plots directly below the cell that produced it.

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
%matplotlib inline
```

Loading the data and plotting it

- We use the `loadtxt` function of `numpy` to store the data in `dataset1.txt` in a 2-D array called `data1` .
- We extract its first column and store it to the 1-D array `x_data1` and its second column to the 1-D array `y_data1` .
- Using `plot` function of `matplotlib.pyplot` we plot the data. Using `title` , `xlabel` and `ylabel` from `matplotlib.pyplot` , title and axis labels can be added to the graph.

In [ ]:
```
#loading the data
data1 = np.loadtxt("dataset1.txt")

#extracting and storing into arrays
x_data1=data1[:,0]
y_data1=data1[:,1]

#plotting the data
plt.plot(x_data1,y_data1,"-b")
plt.title("Data from dataset1.txt")
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```


Data from dataset1.txt
y
x
0
10
20
30
40
50
60
70
0
5
10
15
20
25

Fitting the data to a curve

- The data when plotted resembles a straight line with noise added to it. So the curve can be assumed to be in the form of $y = mx + c$.

- We perform linear fitting using `numpy.linalg.lstsq` rather than `scipy.optimize.curve_fit` in this case as it is much faster.

- Let the datapoints be in the form (x_i, y_i) where $i = 1, 2, 3. \ldots n$. A system of n equations can be constructed using these n datapoints:

$$
\begin{gathered}
y_1 = mx_1 + c \\
y_2 = mx_2 + c \\
\ldots \\
y_n = mx_n + c
\end{gathered}
$$

- These set of equations can be restructured into a matrix and the parameters m and c can be approximated by the least squares approximation, using `lstsq` function from `numpy.linalg` module.

$$
\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} \equiv \mathbf{Mp}
$$

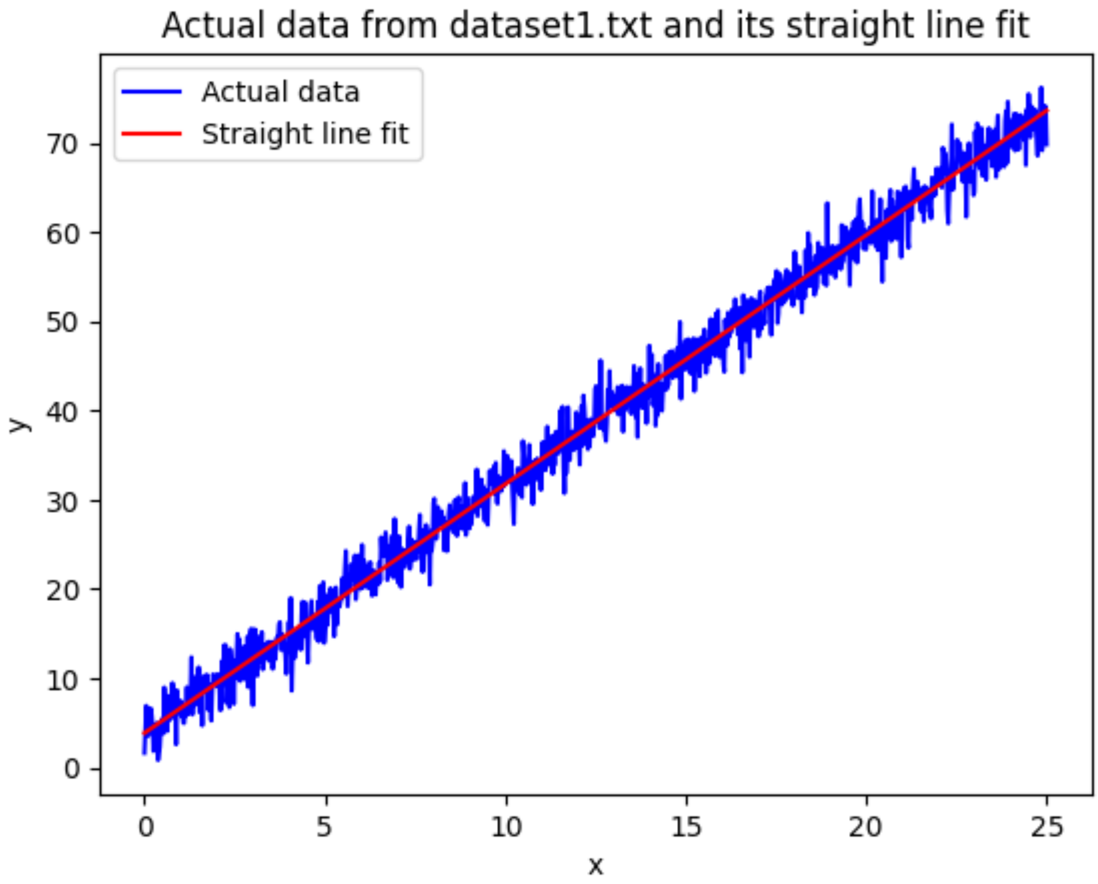
- The required matrix M is created by joining the column vector `x_data1` and a column full of ones of appropriate length, side to side.

- The value of the parameters m and c is estimated by the least squares approximation and their values are printed.

- The actual data is plotted using a blue line and the straight line fit is plotted using a red line in the cell below.

In [ ]:
```
#creating a 2 column matrix M using the x_data1 column vector and a colum
M=np.c_[x_data1, np.ones(len(x_data1))]

#using least square approximation to estimate the parameters m and c
(m, c), _, _, _ = np.linalg.lstsq(M, y_data1, rcond=None)

#printing out the estimatef parameters
print(f"The estimated value of m is {m}")
print(f"The estimated value of c is {c}")

#plotting the actual data using a blue line and its straight line fit usi
plt.plot(x_data1, y_data1,"-b", label="Actual data")
plt.plot(x_data1, m*x_data1+c,"-r",label="Straight line fit")
plt.title("Actual data from dataset1.txt and its straight line fit")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```

```
The estimated value of m is 2.791124245414918
The estimated value of c is 3.848800101430743
```

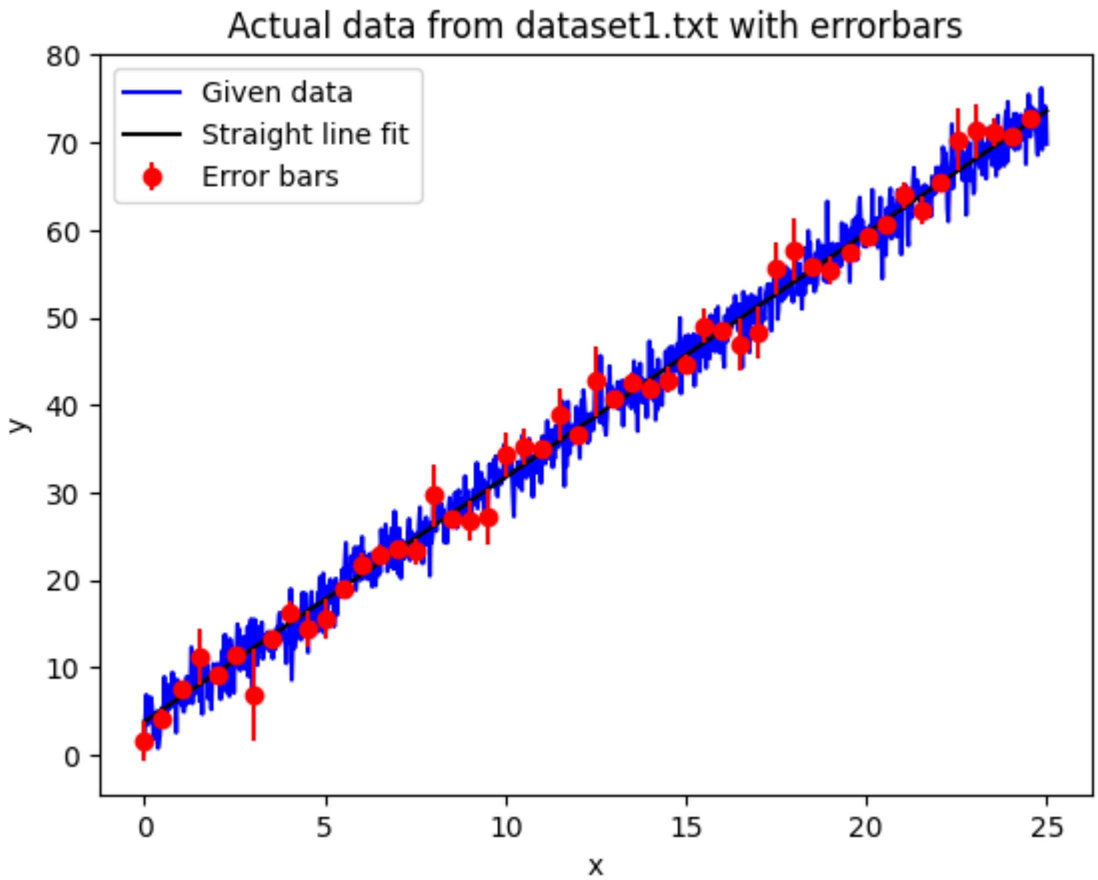


Plotting the errorbars

- The cell below plots the original data in blue and errorbars in red for every 20^{th} datapoint to make it less cluttered.
- On assuming the linearly fit data to be accurate, we calulate the error in y as the absolute value of the difference between the given value and the value predicted by the straight line fit.

In [ ]:

```
#plotting the original data
plt.plot(x_data1,y_data1,"-b",label="Given data")

#plotting the fit curve
plt.plot(x_data1, m*x_data1+c,"-k",label="Straight line fit")

#errorbars for every 20th datapoint
plt.errorbar(x_data1[::20],y_data1[::20], yerr=abs(m*x_data1+c-y_data1)[:

#labelling the axes and title
plt.title("Actual data from dataset1.txt with errorbars")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```



Dataset 2: Fitting a Fourier series to the given data

According to the problem statement, the dataset can be described as a series of sines:

$$y = \sum_{k=1}^{N} a_k \sin(k\omega_0 x)$$

where,

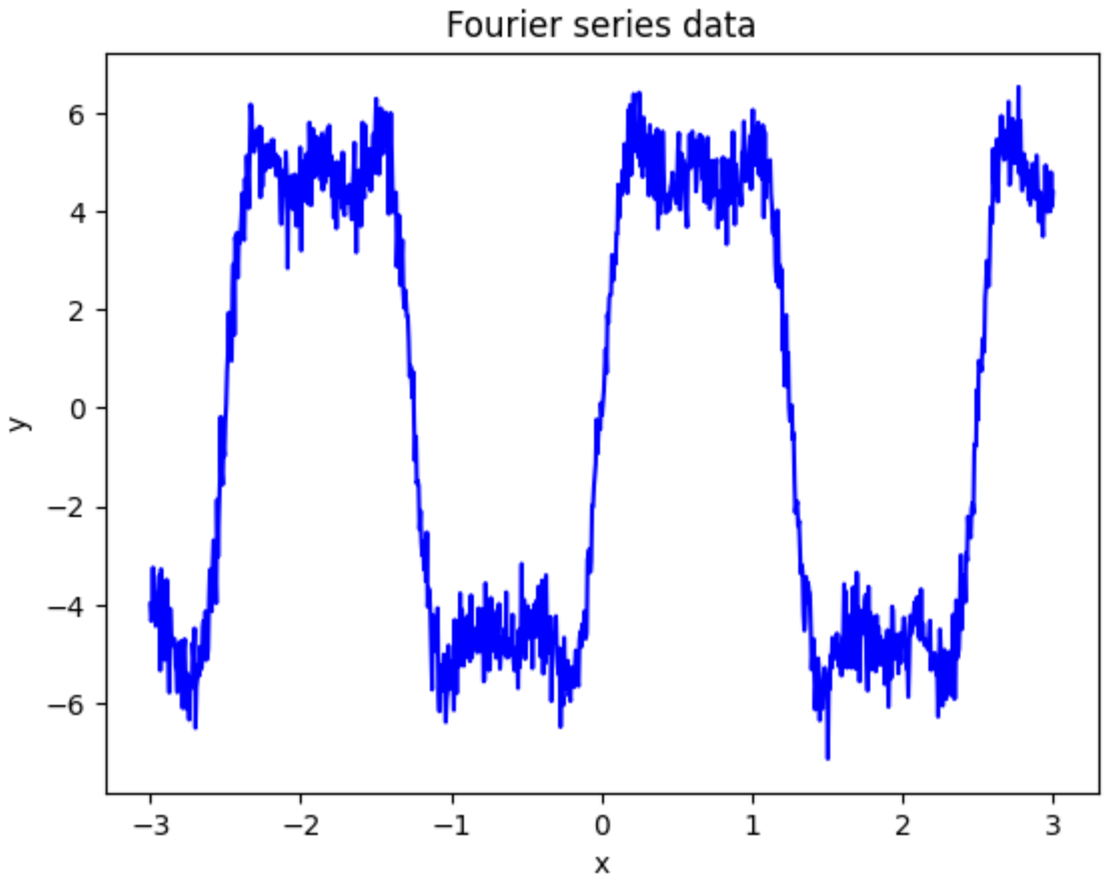
- ω_0 is the fundamental frequency
- a_k refers to the Fourier coefficients
- N is the number of sines in the series

To fit this curve, the fundamental frequency ω_0 should be found first. Then by using the `curve_fit` function from `scipy.optimize`, the Fourier coefficients can be found out.

Loading the data and plotting it

In [ ]:

```
#loading the data
data2 = np.loadtxt("dataset2.txt")

#extracting the data and storing it into arrays
x_data2=data2[:,0]
y_data2=data2[:,1]

#plotting the given data
plt.plot(x_data2,y_data2,"-b")
plt.title("Fourier series data")
plt.xlabel("x")
plt.ylabel("y")
plt.show()
#plt.errorbar(x_data2[::5], y_data2[::5], np.std(), fmt='ro')
```



Estimating the fundamental frequency

- From the graph of the given data, the fundamental time period T_0 can be estimated to be 2.5.
- So, the fundamental angular frequency is given by $\omega_0 = \frac{2\pi}{T_0} = 0.8\pi$.

Fitting a Fourier series on the given data

- In the cell below, we create a function `estimatedfourier` which takes the data set and a variable number of arguments as the input.
- The variable number of arguments are stored is stored in `arg` and they can be accessed by indexing it.
- As we are fitting the data to the curve $y = \sum_{k=1}^{N} a_k \sin(k\omega_0 x)$, the parameters $a_1, a_2, \ldots, a_N$ can be given as arguments to the `estimatedfourier` and it can be accessed inside the function by indexing `arg` , like $a_i =$ `arg[i-1]` .
- The for loop runs for `len(arg)` times to sum over the N terms in the sine series and returns the sum.

In [ ]:

```
#this function can take a variable number of arguments
def estimatedfourier(x,*arg):

    #fundamental angular frequency
    w0=2*np.pi*0.4
    sum=0

    #evaluating the sum of the series for N terms
    for i in range(1,len(arg)+1):
        sum+=arg[i-1]*np.sin(i*w0*x)
    return sum
```

- In the cell below, we define a function `fourierfit` which takes an argument `N`, referring to the number of terms of the sine series to fit the curve to.
- We use the `curve_fit` function from `scipy.optimize` to fit the given data to N terms of the sine series. This can be done by specifying an initial guess list `p0` of length `N`, as an argument of the `curve_fit` function.
- `fourierfit` returns the `parameters` array, containg the estimated values of the fourier coeffiecients a_i. Here $a_i =$ `parameters[i-1]`.

In [ ]:

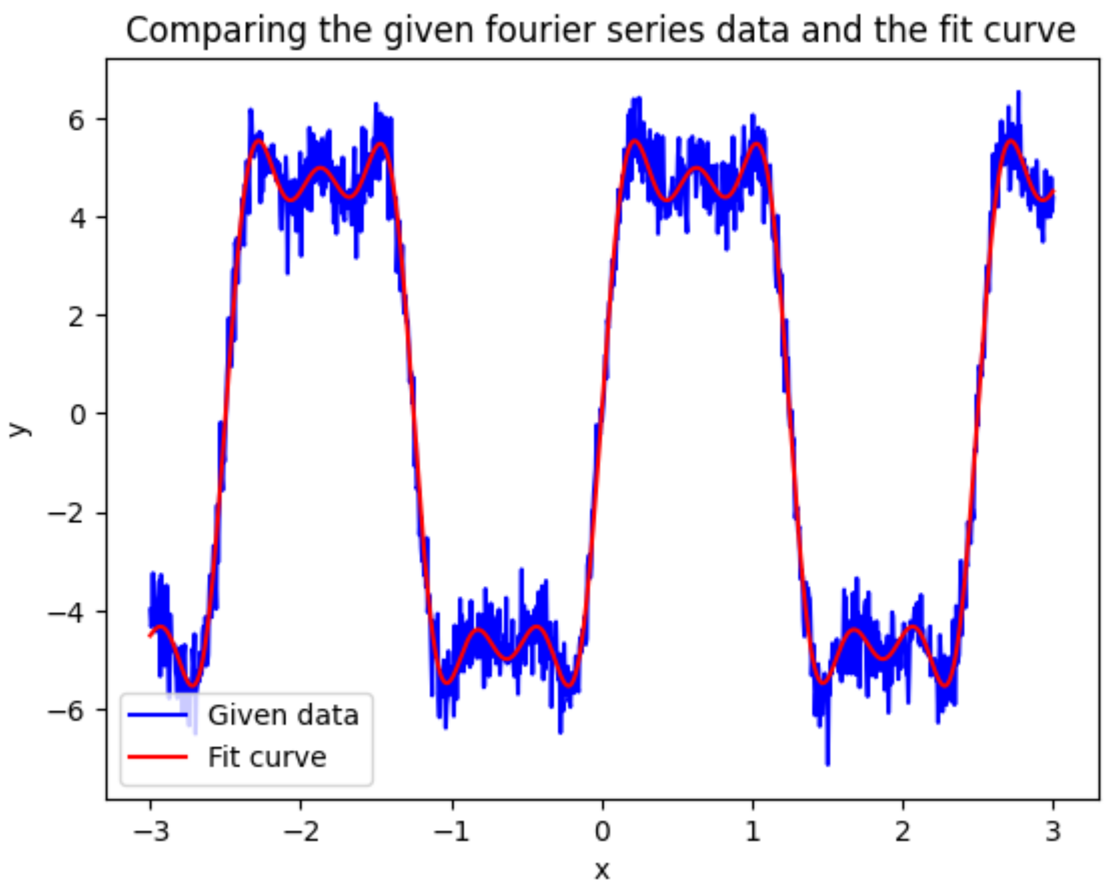
```
#fitting the data to N terms of the sine series
def fourierfit(N):
    parameters,_=curve_fit(estimatedfourier,x_data2,y_data2,p0=[[1.0]*N])
    return parameters

#fitting data to 7 terms of the sine series and printing the coefficients
print(fourierfit(7))
```

```
[ 6.01174927e+00  1.04185231e-03  1.99775867e+00  3.27548924e-02
  9.76536720e-01 -7.10737543e-03  5.59996261e-03]
```

- In the below cell, we plot the given data in blue and the fit curve in red in the same plot, to compare the accuracy of the fit.

In [ ]:

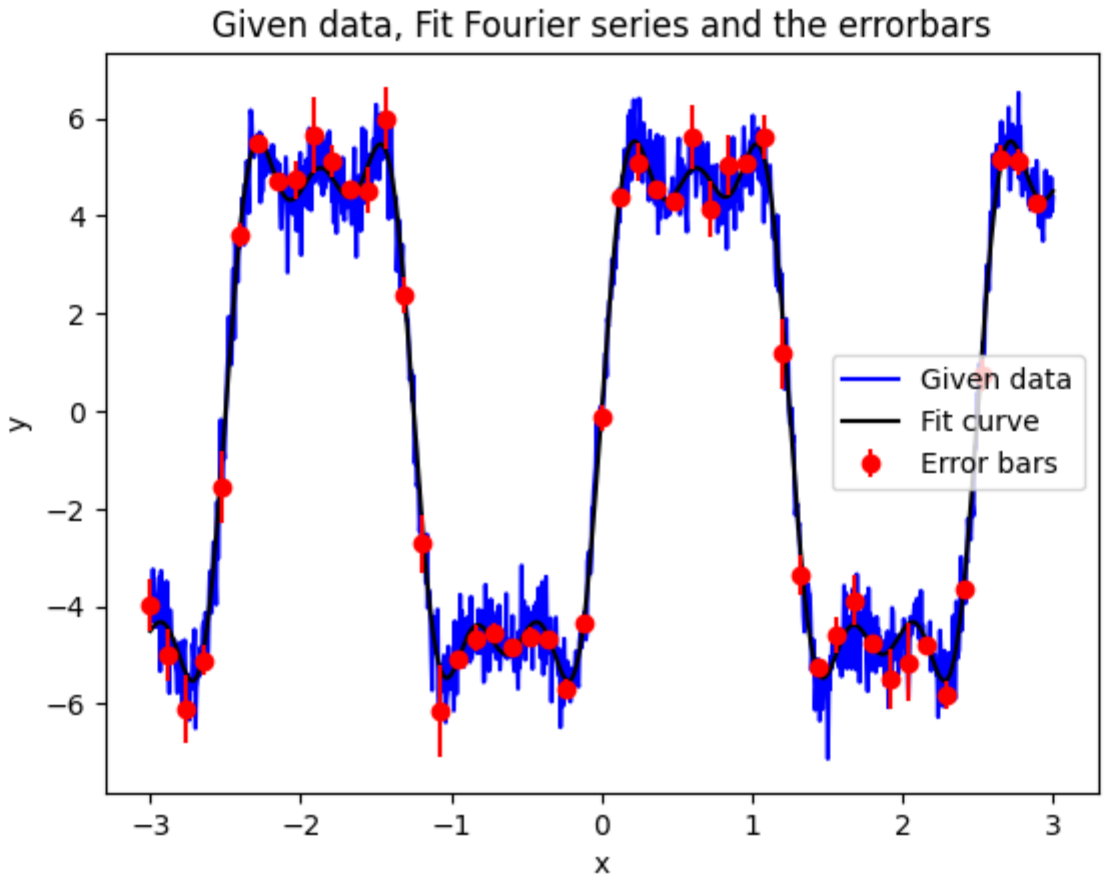
```
plt.plot(x_data2,y_data2,"-b",label="Given data")

#*fourier(7) adds all the elements of the generated paramters (using four
#the function
plt.plot(x_data2,estimatedfourier(x_data2,*fourierfit(7)),"-r", label="Fi

#adding axis labels, legends and title
plt.title("Comparing the given fourier series data and the fit curve")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```



Plotting the errorbars

- In the code below, we plot the curve corresponding to the given data, the fit curve and the errorbars.
- Considering the fit curve as the accurate value, we calculate the parameter `yerr` as the absolute value of the difference between the given `y_data2` and the calculated value using the estimated Fourier coefficients.
- To avoid cluttering, errorbars are plotted for every 20^{th} datapoint.

In [ ]:
```
#original data
plt.plot(x_data2,y_data2,"-b",label="Given data")

#the fit curve (7 term sine series)
plt.plot(x_data2,estimatedfourier(x_data2,*fourierfit(7)),"-k", label="Fi

#plotting errorbars for every 20th datapoint (to avoid cluttering)
plt.errorbar(x_data2[::20], y_data2[::20], yerr=np.abs(y_data2-estimatedf

#adding axis labels, legends and title
plt.title("Given data, Fit Fourier series and the errorbars")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```



Dataset 3: Analysing Spectral radiance and frequency data

The rate of emission of energy per unit area per unit time, per unit frequency interval is called Radiance, which is represented by R. It can be expressed as:

$$R(f,T) = \frac{2hf^3}{c^2(e^{\frac{hf}{kT}} - 1)}$$

where,

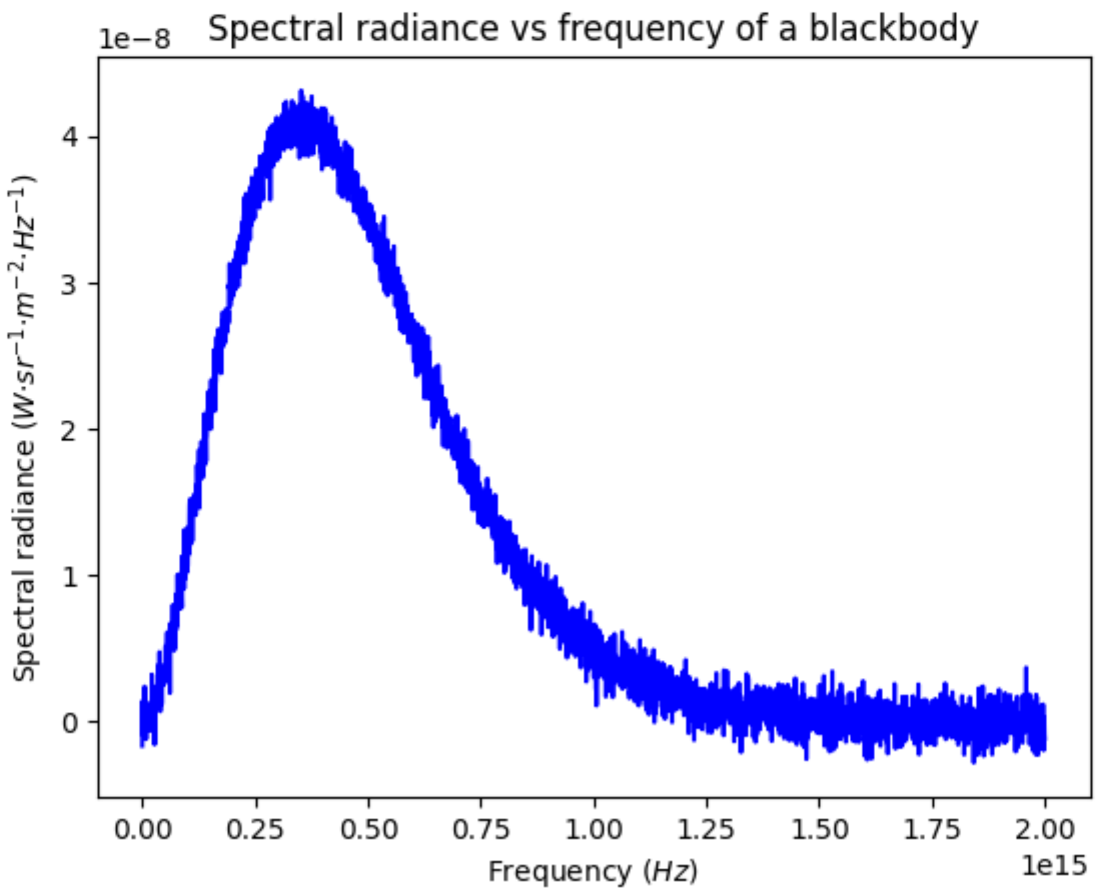
- h is the Planck's constant
- f is the frequency of the radiation
- T refers to the temperature of the blackbody
- $c = 3.0 * 10^8 ms^{-1}$ is the speed of light in vacuum
- $k = 1.38 * 10^{-23} JK^{-1}$ is the Boltzmann constant

Loading the data and plotting it

- We load the data from `dataset3.txt` into a 2-D array called `data3`, and then we seperate that into different columns to store them into the `f` (frequency data) and `R` (spectral radiance data) arrays.

- Using the `plot` command from the `matplotlib.pyplot` module, the graph between the frequency data and the spectral radiance data can be plotted along with the axis labels and the title.

In [ ]:
```
data3 = np.loadtxt("dataset3.txt")

#frequency
f=data3[:,0]

#spectralradiance
R=data3[:,1]

#plotting the given data
plt.plot(f,R,"-b")
plt.title("Spectral radiance vs frequency of a blackbody")
plt.xlabel("Frequency $(Hz)$")
plt.ylabel("Spectral radiance $(W·sr^{-1}·m^{-2}·Hz^{-1})$")
plt.show()
```



Fitting the given data

- We're given the data related to f, R. We also know the values of k (Boltzmann constant) and c (Speed of light in vacuum).
- Using the given data, we need to estimate the parameters h (Planck's constant) and T (absolute temperature).
- We perform non-linear fitting in this case using `scipy.optimize.curve_fit` as the data is to be fit on a non-linear function.

Finding the parameters h and T by curve fitting

In [ ]:

```
#frequency(Hz) vs Spectral radiance of a body  ( W·sr−1·m−2·Hz−1)
def spectralradiance(f,h,T):
    #l for wavelength and T for temperature
    k=1.38e-23
    c=3.0e8
    return 2*h*(np.power(f,3))/((c**2)*(np.exp(h*f/(k*T))-1))
```

In the above cell, we define the `spectralradiance` function which takes the frequency data (f), Planck's constant (h) and absolute temperature (T) as arguments and returns the value of spectral radiance as the output.

In [ ]:

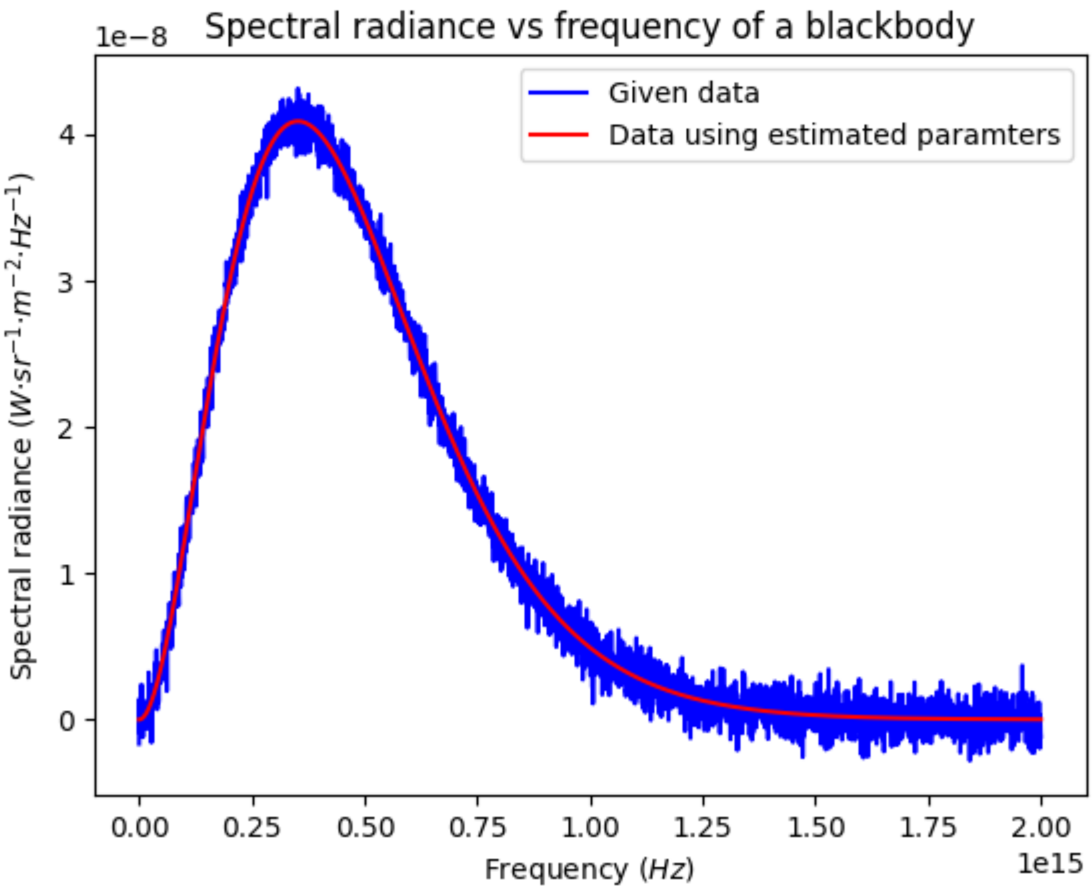
```
#curvefitting using inital guesses for h and T
zp1, zp2= curve_fit(spectralradiance, f,R, p0=[1e-34,5000])

#zp1 contains the value of the parameters while zp2 contains the covarian
h_estimate=zp1[0]
T_estimate=zp1[1]

print(f"Estimated value of h is {h_estimate} Js.")
print(f"Estimated value of T is {T_estimate} K.")
```

```
Estimated value of h is 6.643229747663249e-34 Js.
Estimated value of T is 6011.361514579023 K.
```

- In the above cell, we use the `curve_fit` function of the `scipy` library to find the parameters h and T. By default, the Levenberg–Marquardt algorithm is used.
- As arguments, we give the name of the function `spectralradiance` along with the input and output datasets(`f` contains the frequency data and `R` contains the spectral radiance data).
- `p0` contains the intial guesses of the parameters `h` and `T` respectively in form of a list.
- After some experimentation, the inital guesses `1e-34` for `h` and `5000` for `T` gives a quite an accurate fit.
- The outputs `zp1` contains the calculated value of the parameters `h` and `T` as a list, while `zp2` contains the covariance matrix. We extract the estimated values, store them in `h_estimate` and `T_estimate` and print them.

Calculating the spectral radiance using the estimated parameters and given frequency data

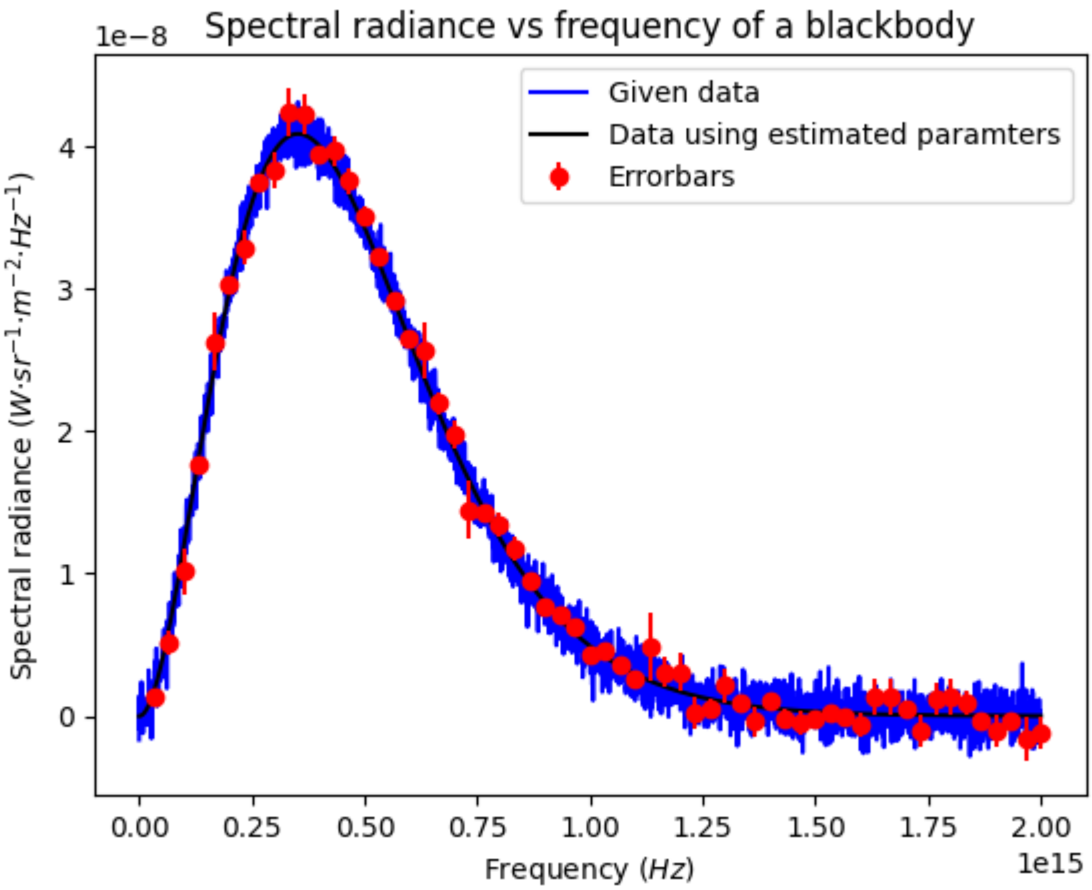
In [ ]:
```
#plotting to check how the curve fit is
plt.plot(f,R,"-b",label='Given data')
plt.plot(f,spectralradiance(f,h_estimate,T_estimate),"-r",label="Data usi

#adding titles, axes labels and legends
plt.title("Spectral radiance vs frequency of a blackbody")
plt.xlabel("Frequency $(Hz)$")
plt.ylabel("Spectral radiance $(W·sr^{-1}·m^{-2}·Hz^{-1})$")
plt.legend()
plt.show()
```



Plotting the errorbars

- We consider the data generated from the fit curve as the actual data and set the `yerr` to the absolute value of the difference between the given data and the output of the `spectralradiance` function with arguments `f`, `h=h_estimate` and `T=T_estimate`.
- To reduce cluttering, errorbars for every 50^{th} point of the dataset is plotted.

In [ ]:
```
#plotting the original data
plt.plot(f,R,"-b",label='Given data')

#plottig the fit curve
plt.plot(f,spectralradiance(f,h_estimate,T_estimate),"-k",label="Data usi

#plotting errorbars for every 50th sample
plt.errorbar(f[::50], R[::50], yerr=np.abs(R-spectralradiance(f,h_estimat

plt.title("Spectral radiance vs frequency of a blackbody")
plt.xlabel("Frequency $(Hz)$")
plt.ylabel("Spectral radiance $(W·sr^{-1}·m^{-2}·Hz^{-1})$")
plt.legend()
plt.show()
```



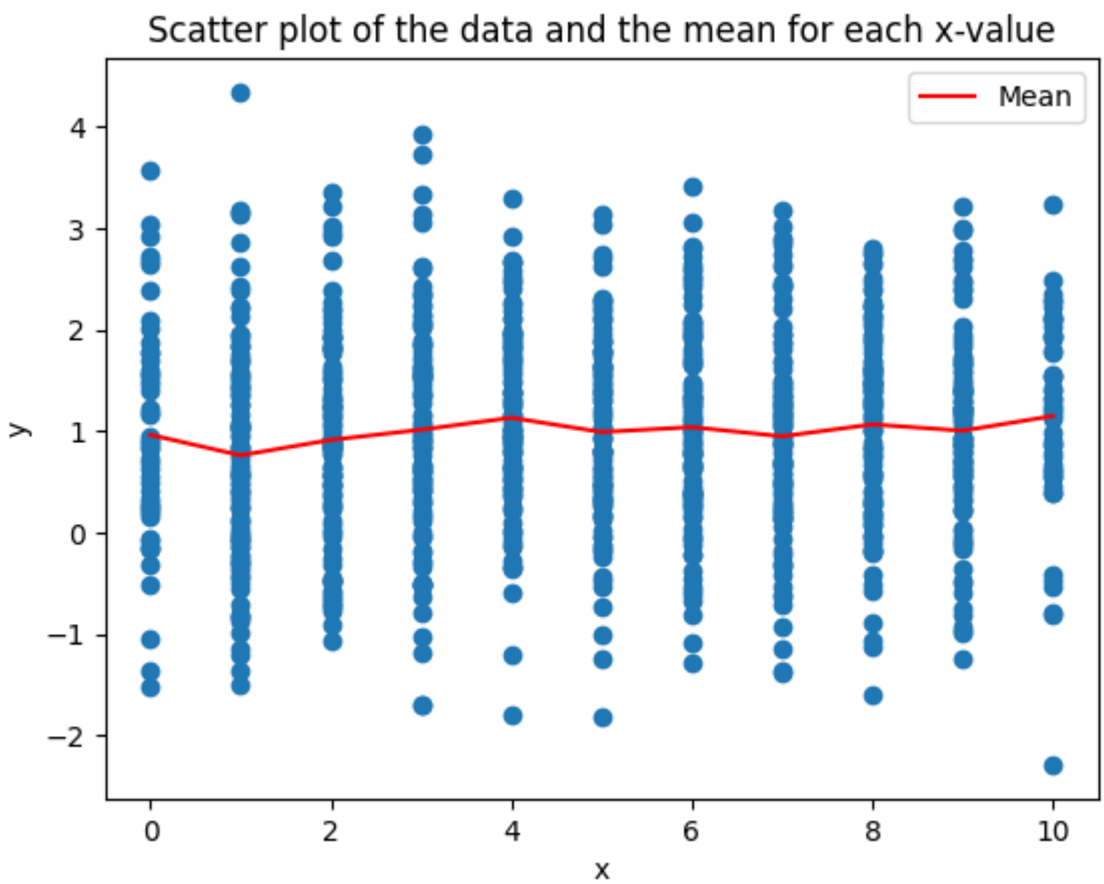
Dataset 4: Examining a scatter plot

Loading the dataset and plotting it

- We load the data in `dataset4.txt` to a 2-D array called `data4` and seperate it into x-values and the y-values.
- As each value in the dataset has a different number of y-values (like 0 and 10, has 50 corresponding y-values, others have 100 correspnding y-values), we use a brute force approach to calculate a list of means `y_list` .
- An array `x_list` is created that has each x value only once from 0 to 10.
- The scatter plot and the mean is plotted in the same graph, using `plt.scatter` and `plt.plot` .

In [ ]:

```
#loading the dataset and extracting the values
data4 = np.loadtxt("dataset4.txt")
x_data4=data4[:,0]
y_data4=data4[:,1]

#this list stores the mean of y-values corresponding to a single x-value
y_list=[0]*11
for i in range(0,11):
    if i==0:
        y_list[0]=np.mean(y_data4[0:50])
    elif i==10:
        y_list[10]=np.mean(y_data4[950:1000])
    else:
        y_list[i]=np.mean(y_data4[100*i-50:100*i+50])


#making the scatter plot and plotting the mean
x_list=np.arange(0,11,1)
plt.scatter(x_data4,y_data4)
plt.plot(x_list,y_list,"-r",label="Mean")

#adding title, labels and legend
plt.title("Scatter plot of the data and the mean for each x-value")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```



Fitting the mean curve

- As the mean curve has some jagged edges, we try to estimate it as a series as a sawtooth signals (as in `scipy.signal.sawtooth`) of different amplitudes and different frequencies, like

$$y = k + \sum_{i=1}^{N} a_i * sawtooth(i\omega_0 x)$$

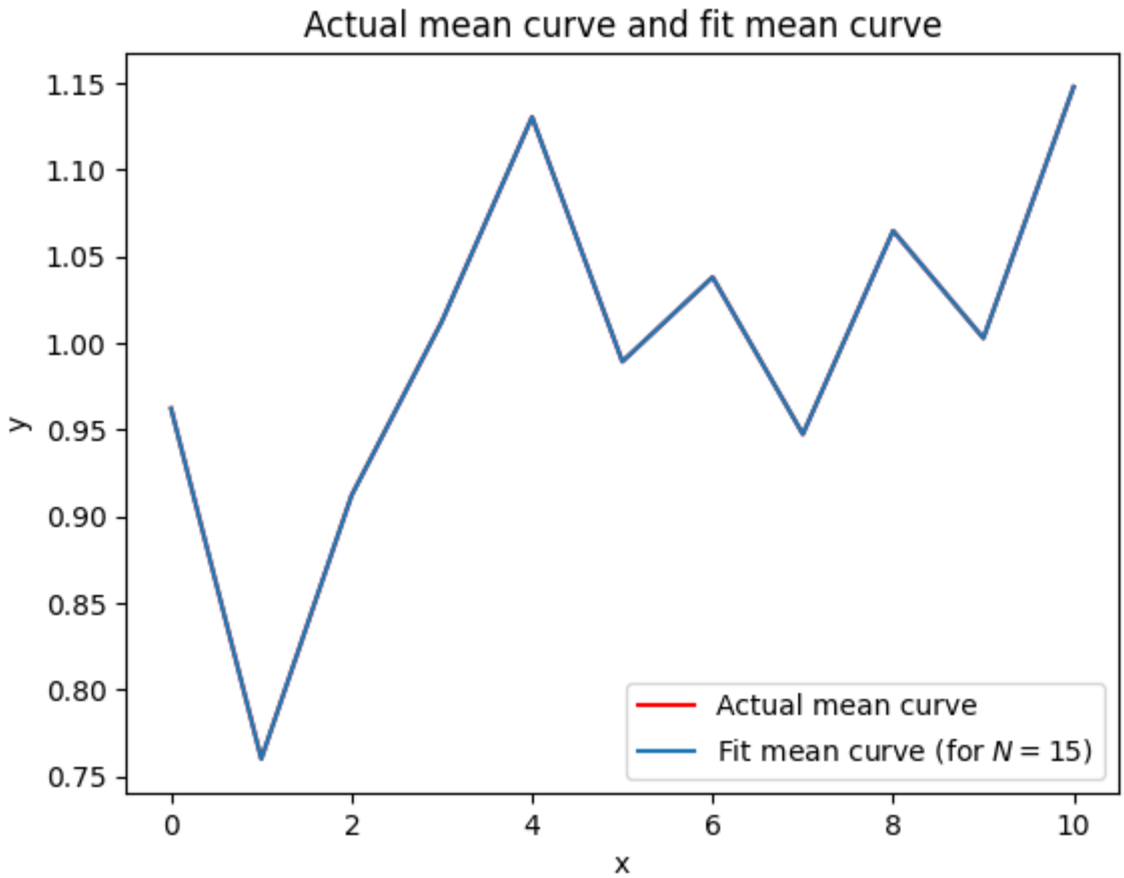
where, $sawtooth(x)$ is a linearly increases from -1 to 1 in a x interval of 0 to 2π.

- After using `curve_fit` and estimating (as a series of 15 signals in this case), we plot the original mean curve and the fit curve.

In [ ]:

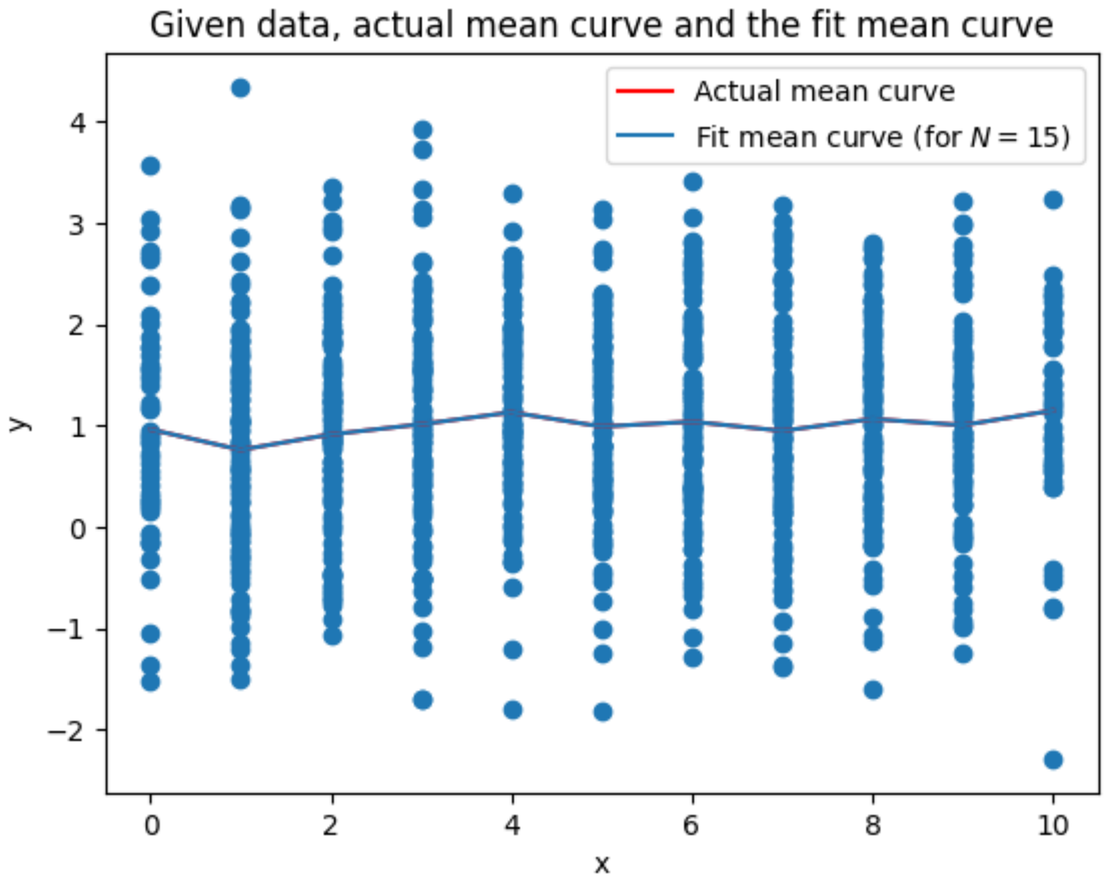
```
from scipy import signal
def estimatedsawtooth(x,*arg):

    #fundamental angular frequency
    w0=arg[0]

    #dc offset
    k=arg[1]

    sum=k

    #evaluating the sum of the series for N terms
    for i in range(1,len(arg)-1):
        sum+=arg[i+1]*signal.sawtooth(i*w0*x)
    return sum

#for now we estimate the data to be a sum of 15 sawtooth signals
values1,_=curve_fit(estimatedsawtooth,x_data4,y_data4,[1.0]*15)
print(values1)

#plotting the original mean curve and the fit curve
plt.plot(x_list,y_list,"-r",label="Actual mean curve")
plt.plot(x_list,estimatedsawtooth(x_list,*values1),label="Fit mean curve

#adding title, labels and legend
plt.title("Actual mean curve and fit mean curve")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```

```
[ 0.99989238  1.09985895  0.14750757  1.30668754  2.43490835 -0.05165847
 -1.15756705 -0.11714291 -1.3944347  -1.28965998 -3.76775717 -1.30699744
  0.98513037  1.47107091  2.87753194]
```



In the below cell, we plot the given data as a scatter plot along with the actual mean curve and the fit mean curve.

In [ ]:
```
#plotting the scatterplot and mean plots in the same graph
plt.scatter(x_data4,y_data4)
plt.plot(x_list,y_list,"-r",label="Actual mean curve")
plt.plot(x_list,estimatedsawtooth(x_list,*values1),label="Fit mean curve

#adding title, labels and legend
plt.title("Given data, actual mean curve and the fit mean curve")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```

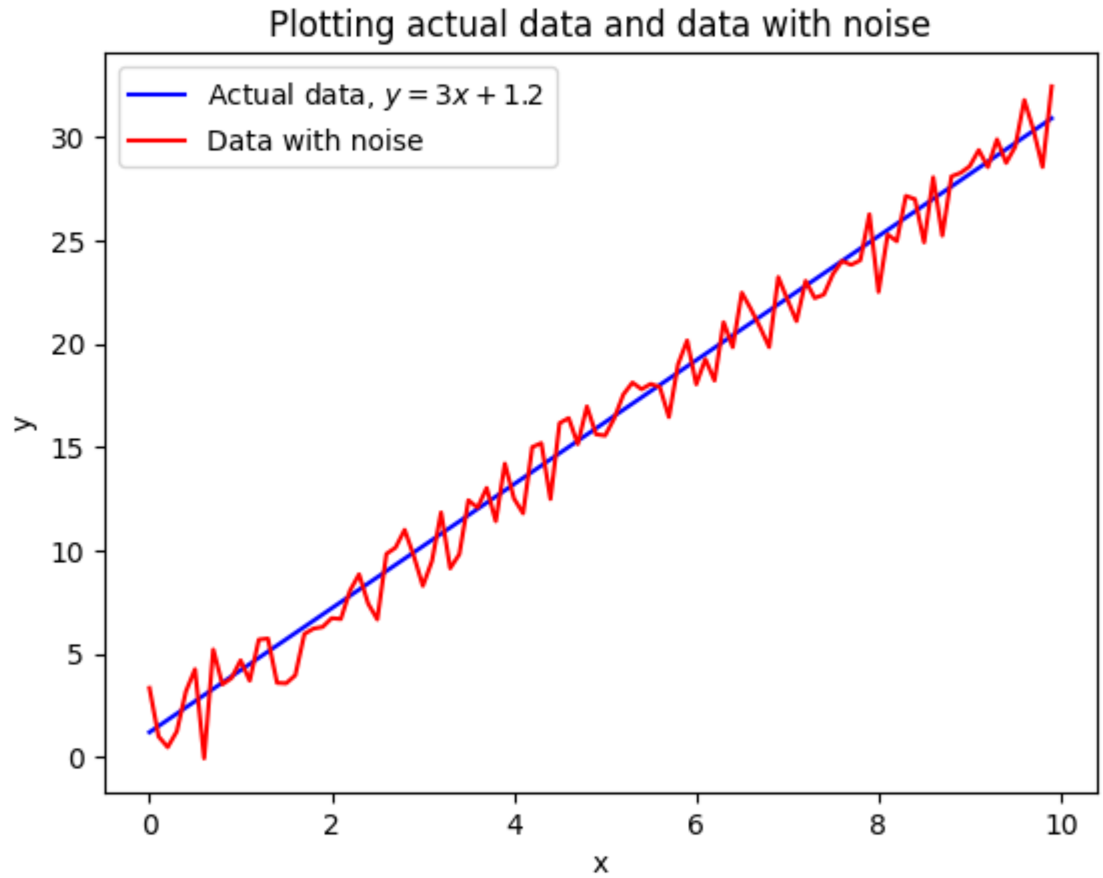


Comparing the speed and accuracy of `lstsq` and `curve_fit` for a straight line fit

Generating the data and plotting it

- As per the problem statement, we use the same dataset used in the class with gaussian noise of same standard deviation.
- Actual data is evaluated by passing the array `t` to the function `stline` (represents the line $y = 3x + 1.2$).
- Noisy data is generated by adding a Gaussian noise of mean 0 and variance 1 to the actual data.

In [ ]:
```
#creating a series of 100 timestamps
t = np.arange(0, 10, 0.1)

#creating a function to represent a straight line
def stline(x, m, c):
    return m * x + c

#the correct data
y = stline(t, 3, 1.2)

#correct data along with Gaussian noise
n = 1 * np.random.randn(len(t))
yn = y + n


#plotting the correct data and the data with noise
plt.plot(t, y,"-b",label="Actual data, $y=3x+1.2$")
plt.plot(t, yn,"-r", label="Data with noise")

#adding labels, legends and titlet, yn,"-r", label="Data with noise"
plt.title("Plotting actual data and data with noise")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```



Comparing the speed of `numpy.linalg.lstsq` and `scipy.optimize.curve_fit`

- In the cell below we define a function `linearfit` that takes the input arrays as the arguments, makes a linear fit to the curve $y = mx + c$ and returns the parameters m and c.

- We call `linearfit` with the arguments `xdata as t and ydata as yn` (noisy y data) and print the estimated m and c, stored in `solution` .

- It takes an average of 26 to $27\mu s$ to make the linear fit.

In [ ]:
```
def linearfit(xdata, ydata):
    M = np.column_stack([xdata, np.ones(len(xdata))])
    # Use the lstsq function to solve for p_1 and p_2
    solution, _, _, _ = np.linalg.lstsq(M, ydata, rcond=None)
    return solution

print(linearfit(xdata=t,ydata=yn))
%timeit linearfit(xdata=t,ydata=yn)
```

```
[3.02089233 0.96521379]
26.6 µs ± 1.01 µs per loop (mean ± std. dev. of 7 runs, 10,000 loops eac
h)
```

- A similar function `curvefit` is defined to take the input arrays as arguments and perform curve fitting using `scipy.optimize.curve_fit and stline` function defined before and return the parameters m and c.

- We call `curvefit` with the arguments `xdata as t and ydata as yn` (noisy y data) and print the estimated m and c, stored in `solution2` .

- It takes an average of around $200\mu s$ to fit the data into the required curve.

In [ ]:
```
def curvefit(xdata, ydata):
    solution2,_=curve_fit(stline, xdata, ydata)
    return solution2

print(curvefit(xdata=t, ydata=yn))
%timeit curvefit(xdata=t, ydata=yn)
```

```
[3.02089233 0.96521382]
214 µs ± 2.72 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

Conclusion

`numpy.linalg.lstsq` is faster than `scipy.optimize.curve_fit` in fitting the above data to a straight line.

Comparing the accuracy of `numpy.linalg.lstsq` and `scipy.optimize.curve_fit`

Both these methods give almost the same error in estimating the parameters m and c of a noisy straight line. An error of 0.696% is obtained on estimating m and an error of 19.56% is obtained on the estimation of c.

In [ ]:

```
actual=np.array([3,1.2])
print(f"Errors by numpy.linalg.lstsq:")
print(f"    Error in estimating m:  {np.abs(actual-linearfit(t,yn))[0]},
print(f"    Error in estimating c:  {np.abs(actual-linearfit(t,yn))[1]},

print(f"Errors by scipy.optimize.curve_fit:")
print(f"    Error in estimating m:  {np.abs(actual-curvefit(t,yn))[0]}, {
print(f"    Error in estimating c:  {np.abs(actual-curvefit(t,yn))[1]}, {
```

```
Errors by numpy.linalg.lstsq:
    Error in estimating m:  0.020892333316681277, 0.6964111105560425 %
    Error in estimating c:  0.23478620662511385, 19.56551721875949 %
Errors by scipy.optimize.curve_fit:
    Error in estimating m:  0.020892332660945367, 0.6964110886981789 %
    Error in estimating c:  0.234786180772244, 19.56551506435367 %
```