Bayesian Theory and Data Analysis
Assignment #2

Due on February 25, 2016 at 1:00pm

Professor: Daniel Manrique-Vallier STAT S626
INDIANA UNIVERSITY

Santhosh Soundararajan

Problem 1

Hoff 3.4 Mixtures of beta priors: Estimate the probability of teen recidivism based on a study in which there were n = 43 individuals released from incarceration and y = 15 re-offenders within 36 months.
a) Using a beta(2,8) prior for θ, plot p(θ), $p(y|\theta)$ and $p(\theta|y)$ as functions of θ. Find the posterior mean, mode, and standard deviation of θ. Find a 95% quantile-based confidence interval.
[*Solution* : We have the values of a=2, b=8, y=15 and n=43 given from the question.
To compute and plot p(θ), $p(y|\theta)$ and $p(\theta|y)$ as functions of θ, we use the following equations.
The beta distribution

$$p(\theta) = dbeta(\theta, a, b) = dbeta(\theta, 2, 8)$$

The binomial distribution
A random variable Y has a binomial(n, θ) distribution if

$$Pr(Y = y|\theta) = dbinom(y, n, \theta) \Rightarrow p(y|\theta) = dbinom(15, 43, \theta)$$

The posterior distribution

$$p(\theta|y) = dbeta(\theta, a + y, b + ny) = dbeta(\theta, 17, 36)$$

Posterior Mean: It is given by the formula

$$E[\theta|y] = \frac{a + y}{a + b + n} = 0.3207$$

Posterior Mode:

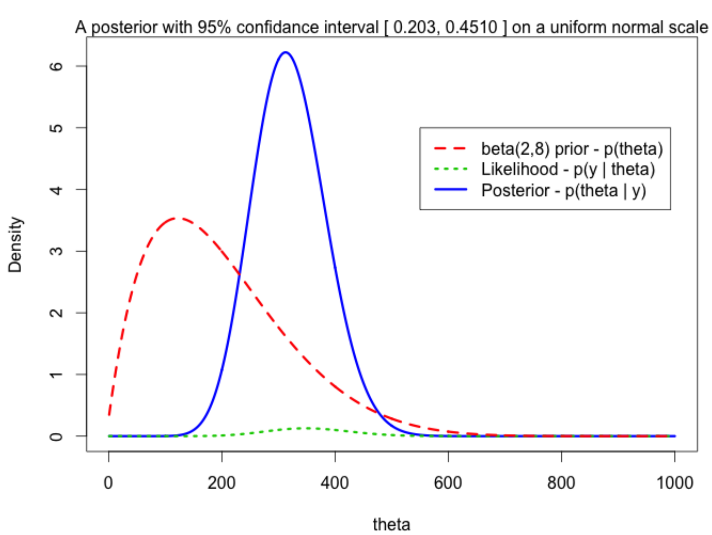
$$mode[\theta|y] = \frac{a + y - 1}{a + b + n - 2} = 0.3137$$

Posterior Standard Deviation:

$$Var[\theta|y] = \frac{E[\theta|y]\ E[1 -\ \theta|y]}{a + b + n + 1} = 0.002577$$

$$StandardDeviation : 0.0507$$

$$\text{"}95\%\ confinterval\text{"} \ = \ [0.203297787819103\ ,\ 0.451023982216632]$$



The R-Code for the same is attached at the end of this doc.

Problem 1 continued on next page. . .

End Solution]

b) Repeat a), but using a beta(8,2) prior for θ

[Solution: So by changing the values to a=8, b=2, we get

$$p(\theta) = dbeta(\theta, a, b) = dbeta(\theta, 8, 2)$$
$$p(y|\theta) = dbinom(n, y, \theta) = dbinom(15, 43, \theta)$$
$$p(\theta|y) = dbeta(\theta, a + y, b + ny) = dbeta(\theta, 17, 36)$$

Posterior Mean: It is given by the formula

$$E[\theta|y] = \frac{a + y}{a + b + n} = 0.4339$$

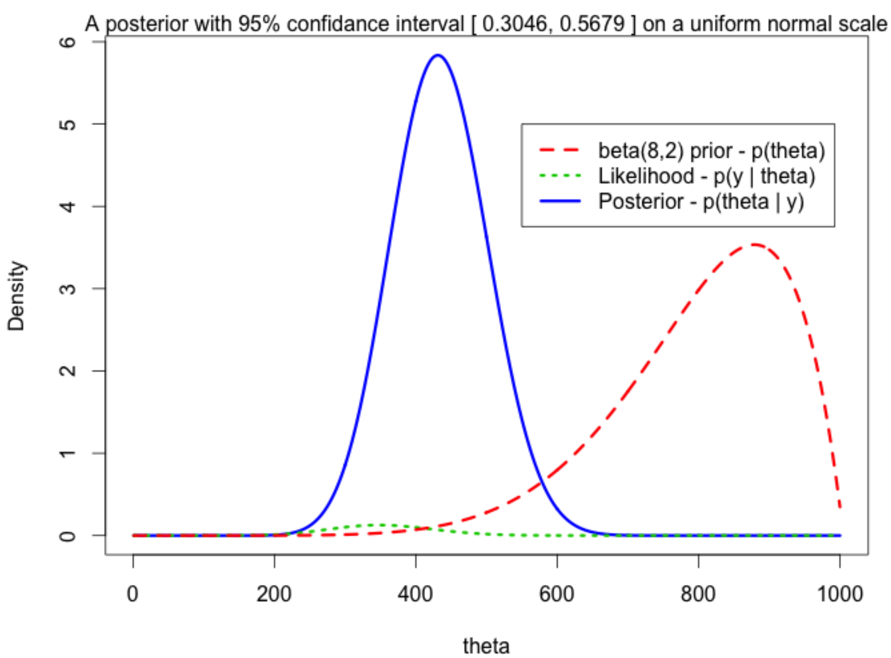
Posterior Mode:

$$mode[\theta|y] = \frac{a + y - 1}{a + b + n - 2} = 0.4313$$

Posterior Standard Deviation:

$$Var[\theta|y] = \frac{E[\theta|y]\ E[1 - \ \theta|y]}{a + b + n + 1} = 0.002577$$
$$StandardDeviation : 0.05077$$

$$\text{"95\%}\ conf interval\text{"} \ = \ [0.304695624711747\ ,\ 0.567952795996458]$$



The R-Code for the same is attached at the end of this doc.

End Solution]

c) Consider the following prior distribution for :

$$p(\theta) = \frac{1}{4}\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[3\theta(1-\theta)^7 + \theta^7(1-\theta)]$$

which is a 75-25% mixture of a beta(2,8) and a beta(8,2) prior distribution. Plot this prior distribution and compare it to the priors in a) and b). Describe what sort of prior opinion this may represent.

[*Solution* : The above equation of mixture prior $p(\theta)$ can be written as:

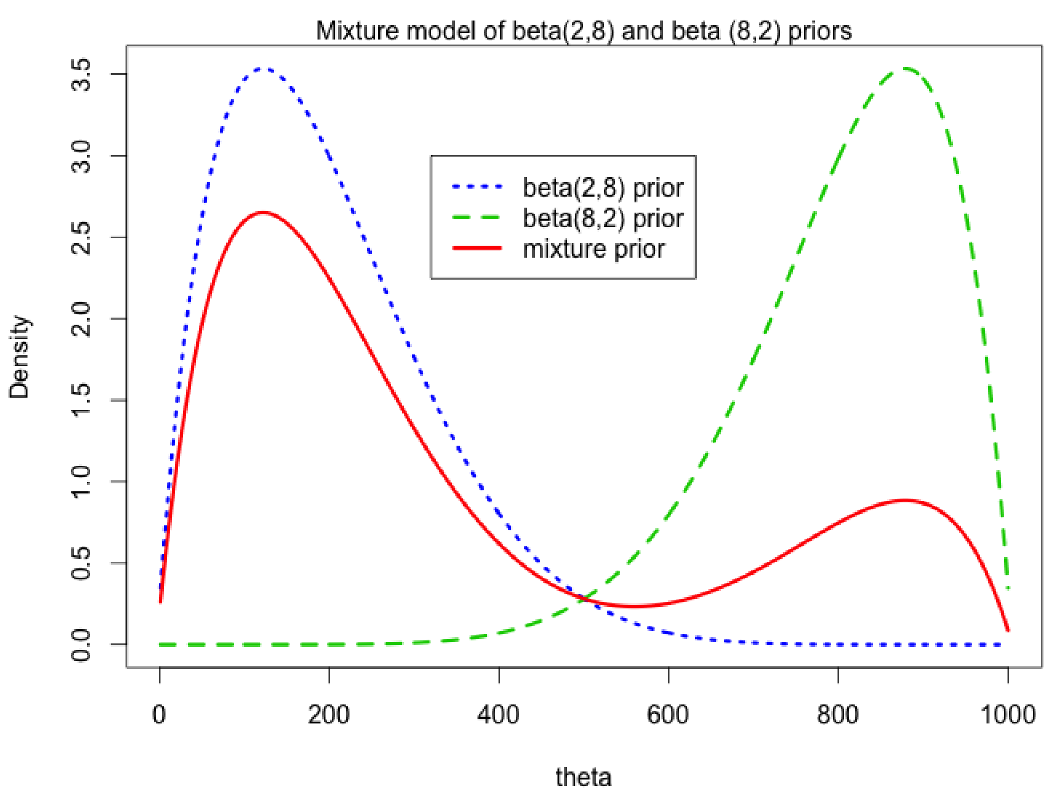
$$p(\theta) = \frac{3}{4}\ [\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta(1-\theta)^7]] \quad + \quad \frac{1}{4}\ [\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta^7(1-\theta)]]$$

$$p(\theta) = \frac{3}{4}\ [\ beta(2,8)\ prior] + \frac{1}{4}\ [\ beta(8,2)\ prior]$$

which is a 75-25% mixture of a beta(2,8) and a beta(8,2) prior distribution as highlighted in the question. We plot this mixture distribution in R:

So the below plotted prior in "red" is the mixture of conjugate prior distributions of beta(2,8) and a beta(8,2). And the process of mixing the distributions look to represent the prior information from **both the priors in a weighted fasion.**

The weights being the percentage of Mixture : 75% of beta(2,8) gives us a curve more inclined towards the beta(2,8) and lesser towards beta(8,2))



The R-Code for the same is attached at the end of this doc.

End Solution]

d) For the prior in c):
i. Write out mathematically $p(\theta) \times p(y|\theta)$ and simplify as much as possible.
[*Solution* : As we know, the expression of $p(\theta)$

$$p(\theta) = \frac{3}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta(1-\theta)^7]) \quad + \quad \frac{1}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta^7(1-\theta)])$$

And

$$p(y|\theta) = \theta^y\ (1-\theta)^{n-y}, \ \ \textit{which is a binomial distribution}$$

in our case

$$p(y|\theta) = \theta^7\ (1-\theta)^{28},$$

$$\Rightarrow p(\theta) \times p(y|\theta) \ = \ (\frac{3}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta(1-\theta)^7]) \ + \ \frac{1}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta^7(1-\theta)]))\ \ (\theta^7\ (1-\theta)^{28})$$

which comes down to this in paper,

$$p(\theta) \times p(y|\theta) \ = \ \frac{1}{4}\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[3\theta^{16}(1-\theta)^{35} + \theta^{22}(1-\theta)^{29}]$$

ii. The posterior distribution is a mixture of two distributions you know. Identify these distributions.

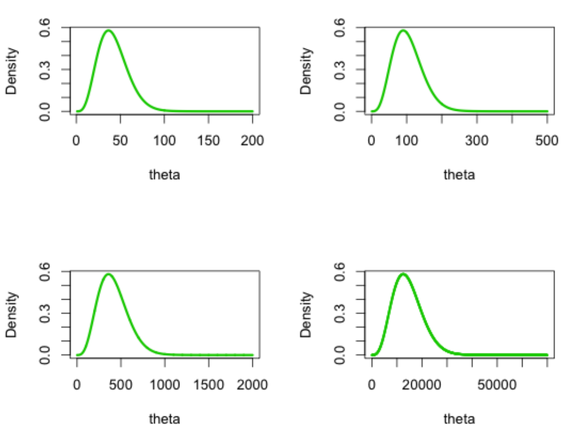
$$We\ know\ that\ Y\ \in\ binomial(n,\theta)$$

$$p(y|\theta) = \theta^y\ (1-\theta)^{n-y}, \ \ \textit{which is a } \textbf{binomial distribution}$$

$$p(\theta)\ =\ dbeta\ (\theta, a, b) \quad \textit{this is the } \textbf{beta prior}$$

These two are the distributions as joint distributions generates a posterior distribution that we are interested in.

iii. On a computer, calculate and plot $p(\theta) \times p(y|\theta)$ for a variety of values. Also find (approximately) the posterior mode, and discuss its relation to the modes in a) and b).



the mode for this distribution is 0.190 and the modes of the distributions from 3.4 a) and b) problems were 0.3137 and 0.4313 respectively.

So we can clearly observe that the mode is **shifting towards the beta(2,8) priors's** distribution because it is the 75% mixture of 75-25% mixture model distribution.
Since our beta(2,8) model had its majority of values near [0.2-0.3], this posterior distribution also tend towards that naturally when we mix the model according to the weights.
End Solution]

e) Find a general formula for the weights of the mixture distribution in d)ii, and provide an interpretation for their values.
[*Solution* :

$$p(\theta) = \frac{3}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta(1-\theta)^7]) \quad + \quad \frac{1}{4}\ (\frac{\Gamma(10)}{\Gamma(2)\Gamma(8)}[\theta^7(1-\theta)])$$

From the above sample we can readily infer that, the $p(\theta)$ is straight forward mixture of the two binomial modesls like this:

$$p(\theta) = \frac{3}{4}beta(2,8) + \frac{1}{4}beta(8,2)$$

and 3/4 and 1/4 being the 75% and 25% split.

So i would generalize this equation as,

$$p_{mixture}(\theta) = \frac{percent_A}{100}beta(A_prior_mixture) + \frac{percent_B}{100}beta(B_prior_mixture)$$

End Solution]

Problem 2

Hoff 3.7 Posterior prediction: Consider a pilot study in which $n_1 = 15$ children enrolled in special education classes were randomly selected and tested for a certain type of learning disability. In the pilot study, $y_1 = 2$ children tested positive for the disability.

a) Using a uniform prior distribution, find the posterior distribution of θ, the fraction of students in special education classes who have the disability.
Find the posterior mean, mode and standard deviation of θ, and plot the posterior density.
[$Solution$: Given: $n_1 = 15$ & $y_1 = 2$
and we know that for $\theta \sim beta(1,1)(uniform)$,

$$(\theta|Y = y) \ \sim beta(1 + y,\ 1 + n - y)$$

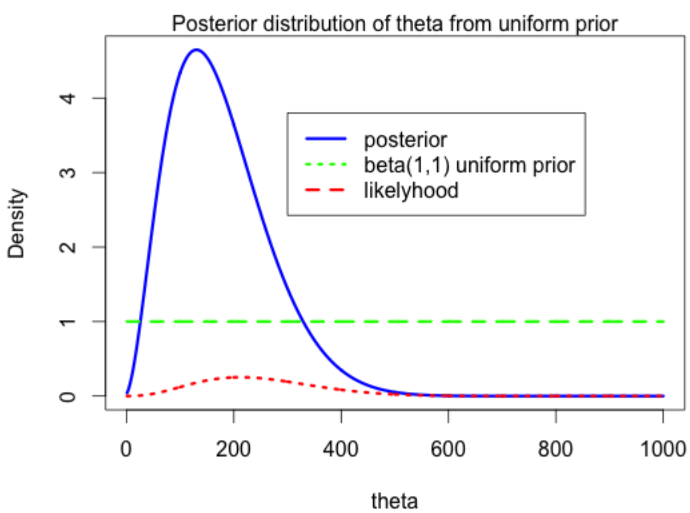
to get the likelihood, assuming this is an i.i.d

$Pr(Y = y|\theta) = \theta^y\,(1-\theta)^{n-y} \ = \ dbinom(y,n,\theta) = dbinom(2,15,\theta) \ which\,is\,a$ **binomial distribution that gives the**

From the prior inference condition of the uniform distribution, we know that

$$p(\theta|Y) = beta(1 + y,\ 1 + n - y) = \ beta(3, 14)$$

The above equations demonstrates that for any i.i.d prior, the $\sum y_i$ is the sufficient data to predict the posterior distribution.



Posterior Mean: It is given by the formula

$$E[\theta|y] = \frac{a + y}{a + b + n} = \frac{3}{17} = 0.1764$$

Posterior Mode:

$$mode[\theta|y] = \frac{a + y - 1}{a + b + n - 2} = \frac{2}{15} = 0.1333$$

Posterior Standard Deviation:

$$Var[\theta|y] = \frac{E[\theta|y]\ E[1 - \ \theta|y]}{a + b + n + 1} = \frac{0.176 * 0.176}{18} = 0.00173$$

$$SD = \sqrt{Var[\theta|y]} = 0.04159$$

End Solution]

Researchers would like to recruit students with the disability to partici- pate in a long-term study, but first they need to make sure they can recruit enough students. Let n2 = 278 be the number of children in special edu- cation classes in this particular school district, and let Y2 be the number of students with the disability.

b) Find $Pr(Y2 = y2|Y1 = 2)$, the posterior predictive distribution of $Y2$, as follows:
i. Discuss what assumptions are needed about the joint distribution of $(Y1, Y2)$ such that the following is true:

[*Solution* :

$$Pr(Y_2 = y_2|Y_1 = 2) = \int_0^1 Pr(Y_2 = y_2|\theta)\ p(\theta|Y_1 = 2)\ \ d\theta$$

For the above joint distribution to be true, **the prior distribution of** Y_1 *should be i.i.d. Poisson with mean* θ [*i.i.d. − conditionally independent and identically distributed*]

And we say that

$$\sum Y_{prior}\ is\ a\ sufficient\ statistic\ to\ come\ to\ a\ Poisson\ Inference$$

And this predictive distribution does not depend on any unknown quantities. All it works with is the Y_1. If it was depended on any other variable containing uncertainty, we would not be able to use it to make predictions.

ii & iii. Now plug in the forms for $Pr(Y_2 = y_2|\theta)$ and $p(|Y_1 = 2)$ in the given integral.

$$Pr(Y_2 = y_2|Y_1 = 2) = \int Pr(Y_2 = y_2|\theta)\ p(\theta|Y_1 = 2)\ \ d\theta$$

Posterior poisson inference is given by,

$$Pr(Y_1...., Y_n = y_n \mid \theta) = \prod_{i=1}^{n} p(y_i|\theta)\ \ = c(y_1, ..., y_n)\ \theta^{\sum y_i} e^{-n\theta}$$

by pluggin in $Pr(Y_1...., Y_n = y_n \mid \theta) \sim Pr(Y_2 = y_2|\theta)$, we get

$$= \int c(y_2)\ \theta^{\sum y_2} e^{-278(\theta)}\ p(\theta|Y_1 = 2)\ \ d\theta,\ since\ we\ know\ n_2\ \ =\ \ 278$$

$$where\ p(\theta) = dgama(\theta,\ a,\ b) = \frac{b^a}{\Gamma(a)}\theta^{a-1}e^{-b\theta}$$

and $p(\theta|Y_1 = 2)$ is a beta distribution of form [Hoff 3.1 Binomial Model(pg-37)]

$$p(\theta|y_1) = \frac{\Gamma(a + 2 + y_1)}{\Gamma(y_1 + 1)\Gamma(a + 2)}\theta^{(y_1+1)-1}(1 - \theta)^{(n_1-y_1+1)-1} = beta(y_1 + 1, n_1 - y_1 + 1)$$

since we know $y_1 = 2$ and $n_1 = 15$

$$p(\theta|y_1 = 2) = beta(3, 14)$$

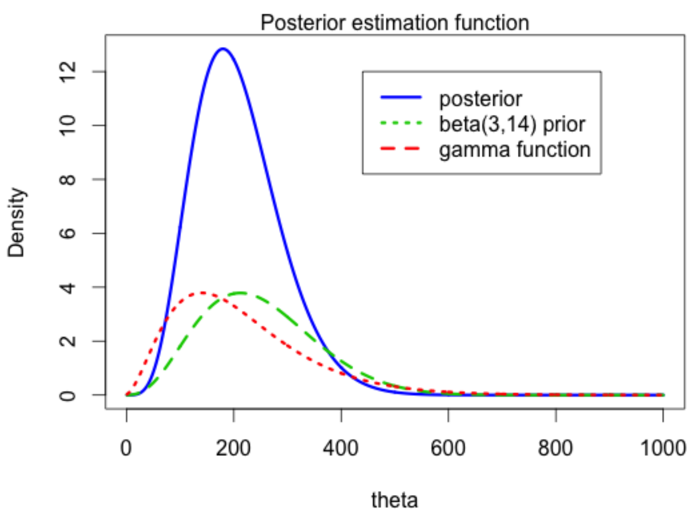
plugging in both the beta form and the posterior poisson form into the integral, we get

$$= \int c(y_2)\ \theta^{\sum y_2} e^{-278(\theta)}\ \frac{\Gamma(a+2+y_1)}{\Gamma(y_1+1)\Gamma(a+2)} \theta^{(y_1+1)-1}(1-\theta)^{(n_1-y_1+1)-1}\ \ d\theta,$$

this looks like a complex integral but can be machine calculated using the dgamma() and beta() functions in R.
End Solution]

End Solution]

c) Plot the function $Pr(Y2 = y2|Y1 = 2)$ as a function of y2. Obtain the mean and standard deviation of Y2, given Y1 = 2.
[*Solution* :



This plot gives us a mean 0.1724 and standard deviation 0.0563 of as shown below in uniform scale:

```
> print(paste0("posterior_mean = ",posterior_mean))
[1] "posterior_mean = 0.172413793103448"
> print(paste0("posterior_mean - 1 = ",posterior_mean_inv))
[1] "posterior_mean - 1 = 0.551724137931034"
> print(paste0("posterior_mode = ",posterior_mode))
[1] "posterior_mode = 0.148148148148148"
> print(paste0("posterior_sd = ",posterior_sd))
[1] "posterior_sd = 0.0563101090294983"
> print(paste0("posterior_var = ",posterior_var))
[1] "posterior_var = 0.00317082837891399"
> print(paste0("95% conf interval = ",CI_95[1], " , " ,CI_95[2]))
[1] "95% conf interval = 0.0606429088192076 , 0.326652669315795"
```

End Solution]

Problem 3

Hoff 3.3 Tumor counts: A cancer laboratory is estimating the rate of tumorigenesis in two strains of mice, A and B. They have tumor count data for 10 mice in strain A and 13 mice in strain B. Type A mice have been well studied, and information from other laboratories suggests that type A mice have tumor counts that are approximately Poisson-distributed with a mean of 12. Tumor count rates for type B mice are unknown, but type B mice are related to type A mice. The observed tumor counts for the two populations are
yA = (12,9,12,14,13,13,15,8,15,6);
yB = (11,11,10,9,9,8,7,10,6,8,8,9,7).

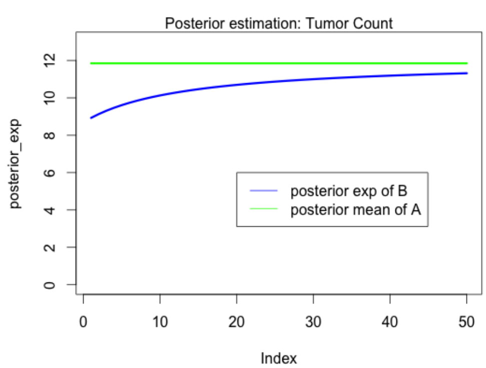
a) Find the posterior distributions, means, variances and 95% quantile- based confidence intervals for θ_A and θ_B , assuming a Poisson sampling distribution for each group and the following prior distribution:

$$\theta_A \sim gamma(120, 10), \theta_B \sim gamma(12, 1), p(A, B) = p(\theta_A) \times p(\theta_B).$$

```
> print(paste0("Population A_mean = ",posterior_mean))
[1] "posterior_mean = 11.65793103448"
> print(paste0("Population_A_var = ",posterior_var))
[1] "posterior_var = 0.60925891399"
> print(paste0("Population A 95% conf interval = ",CI_95[1], " , " ,CI_95[2]))
[1] "Population B 95% conf interval = 10.943206795 , 13.0560308"

> print(paste0("Population A_mean = ",posterior_mean))
[1] "posterior_mean = 8.57903448"
> print(paste0("Population_A_var = ",posterior_var))
[1] "posterior_var = 0.59258391399"
> print(paste0("Population A 95% conf interval = ",CI_95[1], " , " ,CI_95[2]))
[1] "Population B 95% conf interval = 6.94320645795 , 11.0560308"
```

b) Compute and plot the posterior expectation of θ_B under the prior distribution $\theta_B \sim gamma(12 \times n_0, n_0)$ for each value of $n_0 \in (1, 2, ..., 50)$.



c) Should knowledge about population A tell us anything about population B? Discuss whether or not it makes sense to have $p(\theta_A, \theta_B) = p(\theta_A)p(\theta_B)$.
[*Solution* :
Yes it in some way mix the two distributions to form a new shape or a distribution that will give us information on the B given that A is known.
End Solution]

R Code for the above plots:

```
########   3.4 a) Plotting P(theta), P(y | theta) and P(theta | y)   #########
########   & calculating the values of mean, mode, var and SD ##############
a=2
b=8
y=15
n=43
theta = seq(0.005, 0.995, length = 1000)
theta=seq(0,1,by=0.01)
prior = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
posterior = dbeta(theta, a+y, b+n-y)

plot(posterior, col = "blue",typ='l', ylab = "Density", xlab = "theta",lty = 1, lwd = 3)
lines(likelihood, typ='l', col="green3", lty = 3, lwd = 3)
lines(prior, col = "red", lty = 2, lwd = 3)
legend(x=550,y=5.0, c("beta(2,8) prior - p(theta)", "Likelihood - p(y | theta)", "Posterio
       lwd = c(3, 3, 3), col = c("red", "green3", "blue"))

posterior_mean =(a+y)/(a+b+n)
posterior_mean_inv =(b+y)/(a+b+n)
posterior_mode =(a+y-1)/(a+b+n-2)
posterior_var =(posterior_mean * posterior_mean_inv)/(a+b+n+1)
posterior_sd =sqrt(posterior_var)
CI_95=qbeta(c(0.025,0.975),a+y,b+n-y)


print(paste0("posterior_mean = ",posterior_mean))
print(paste0("posterior_mean - 1 = ",posterior_mean_inv))
print(paste0("posterior_mode = ",posterior_mode))
print(paste0("posterior_sd = ",posterior_sd))
print(paste0("posterior_var = ",posterior_var))
print(paste0("95% conf interval = ",CI_95[1], " , " ,CI_95[2]))

mtext("A posterior with 95% confidance interval [ 0.203, 0.4510 ] on a uniform normal scale

###[end 3.4 a)]

########   3.4 b) Plotting P(theta), P(y | theta) and P(theta | y)   #########
########   & calculating the values of mean, mode, var and SD ##############
a=8
b=2
prior = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
posterior = dbeta(theta, a+y, b+n-y)

plot(posterior, col = "blue",typ='l', ylab = "Density", xlab = "theta",lty = 1, lwd = 3)
```

```
lines(likelihood, typ='l', col="green3", lty = 3, lwd = 3)
lines(prior, col = "red", lty = 2, lwd = 3)
legend(x=550,y=5.0, c("beta(8,2) prior - p(theta)", "Likelihood - p(y | theta)", "Posterior
       lwd = c(3, 3, 3), col = c("red", "green3", "blue"))

posterior_mean =(a+y)/(a+b+n)
posterior_mean_inv =(b+y)/(a+b+n)
posterior_mode =(a+y-1)/(a+b+n-2)
posterior_var =(posterior_mean * posterior_mean_inv)/(a+b+n+1)
posterior_sd =sqrt(posterior_var)
CI_95=qbeta(c(0.025,0.975),a+y,b+n-y)

print(paste0("posterior_mean = ",posterior_mean))
print(paste0("posterior_mean - 1 = ",posterior_mean_inv))
print(paste0("posterior_mode = ",posterior_mode))
print(paste0("posterior_sd = ",posterior_sd))
print(paste0("posterior_var = ",posterior_var))
print(paste0("95% conf interval = ",CI_95[1], " , " ,CI_95[2]))

mtext("A posterior with 95% confidance interval [ 0.3046, 0.5679 ] on a uniform normal scal

###[end 3.4 b)]

##########################################################################################
############   3.4 c) Mixture model of beta(2,8) and beta (8,2) priors  ############
theta = seq(0.005, 0.995, length = 1000)
a=2
b=8
prior1 = dbeta(theta, a, b)
a=8
b=2
prior2 = dbeta(theta, a, b)

plot(prior1, typ='l', col="blue", lty = 3, lwd = 3, ylab = "Density", xlab = "theta",ylim =
lines(prior2, col = "green3", lty = 2, lwd = 3)
lines((((3/4)*prior1)+((1/4)*prior2)),typ='l', col="red", lty = 1, lwd = 3)
legend(x=320,y=3.0, c("beta(2,8) prior", "beta(8,2) prior", "mixture prior"), lty = c(3, 2
       lwd = c(3, 3, 3), col = c( "blue","green3","red"))
mtext("Mixture model of beta(2,8) and beta (8,2) priors")

###[end 3.4 c)]

##############################################################################################
####### 3.4 d) iii. Distributions of P(theta) * P(y | theta) for variety of theta lengths
#######

par(mfrow=c(2,2))

a=2
```

```
b=8
theta = seq(0.005, 0.995, length = 200)
prior1 = dbeta(theta, a, b)
a=8
b=2
prior2 = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
plot( ( (likelihood)*(((3/4)*prior1)+((1/4)*prior2)) ),typ='l', col="green3", lty = 1, lwd


a=2
b=8
theta = seq(0.005, 0.995, length = 500)
prior1 = dbeta(theta, a, b)
a=8
b=2
prior2 = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
plot( ( (likelihood)*(((3/4)*prior1)+((1/4)*prior2)) ),typ='l', col="green3", lty = 1, lwd



a=2
b=8
theta = seq(0.005, 0.995, length = 2000)
prior1 = dbeta(theta, a, b)
a=8
b=2
prior2 = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
plot( ( (likelihood)*(((3/4)*prior1)+((1/4)*prior2)) ),typ='l', col="green3", lty = 1, lwd


a=2
b=8
theta = seq(0.005, 0.995, length = 70000)
prior1 = dbeta(theta, a, b)
a=8
b=2
prior2 = dbeta(theta, a, b)
likelihood = dbinom(y,n,theta)
plot( ( (likelihood)*(((3/4)*prior1)+((1/4)*prior2)) ),typ='l', col="green3", lty = 1, lwd

###[end 3.4 d)]
################################################################################

###########    3.7 a) Uniform posterior distribution of theta    ############
a=3
b=14
```

```
theta = seq(0.005, 0.995, length = 1000)
posterior = dbeta(theta, a, b)
prior = dbeta(theta,1,1)
likelihood = dbinom(2,15,theta)
plot(posterior, col = "blue",typ='l', ylab = "Density", xlab = "theta",lty = 1, lwd = 3)
lines(prior, col = "green", lty = 2, lwd = 3)
lines(likelihood, col = "red", lty = 3, lwd = 3)
legend(x=300,y=3.8, c("posterior","beta(1,1) uniform prior","likelyhood"), lty = c(1, 3, 2
       lwd = c(3, 3, 3), col = c( "blue","green","red"))
mtext("Posterior distribution of theta from uniform prior")
###[end 3.7 a)]

########################## 3.7 c) Plotting Pr(Y2 | Y1=2) ####################################

y=3
n=14
a=2
b=13
theta = seq(0.005, 0.995, length = 1000)

prior = dbeta(theta,(y+1), (n-y+1))
likelihood = dgamma(theta,y,n)
posterior = (prior * likelihood)
plot(posterior, col = "blue",typ='l', ylab = "Density", xlab = "theta",lty = 1, lwd = 3)
lines(prior, col = "green3", lty = 2, lwd = 3)
lines(likelihood, col = "red", lty = 3, lwd = 3)
legend(x=440,y=12, c("posterior","beta(3,14) prior","gamma function"), lty = c(1, 3, 2),
       lwd = c(3, 3, 3), col = c( "blue","green3","red"))

mtext("Posterior estimation function")

posterior_mean =(a+y)/(a+b+n)
posterior_mean_inv =(b+y)/(a+b+n)
posterior_mode =(a+y-1)/(a+b+n-2)
posterior_var =(posterior_mean * posterior_mean_inv)/(a+b+n+1)
posterior_sd =sqrt(posterior_var)
CI_95=qbeta(c(0.025,0.975),a+y,b+n-y)

print(paste0("posterior_mean = ",posterior_mean))
print(paste0("posterior_mean - 1 = ",posterior_mean_inv))
print(paste0("posterior_mode = ",posterior_mode))
print(paste0("posterior_sd = ",posterior_sd))
print(paste0("posterior_var = ",posterior_var))
print(paste0("95% conf interval = ",CI_95[1], " , " ,CI_95[2]))


################################## 3.3 #########################

aA=120
```

```
bA=10
nA=10
yA=c(12, 9, 12, 14, 13, 13, 15, 8, 15, 6)
total=sum(yA)
posterior_mean=(aA+total)/(bA+nA)
n0=seq(1,50)
aB=12*n_0
bB=n_0
nB=13
yB=c(11,11,10,9,9,8,7,10,6,8,8,9,7)
totalB=sum(yB)

posterior_exp = (aB+totalB)/(bB+nB)
plot(posterior_exp, type = "l", col="blue" , ylim = c(0,13))
lines(n_0,as.vector(rep(posterior_mean,50)), col="green")
legend(x = 20, y=6, legend = c("posterior exp of B","posterior mean of A"), lty = c(1,1), c
```

```
####################################################################
```

REFERENCES: **Peter D. Hoff -** A First Course in Bayesian Statistical Methods.