Q1: The distance of the point $(7,-3,-4)$ from the plane passing through the points $(2,-3,1)$, $(-1,1,-2)$ and $(3,-4,2)$ is :

(A) 4

(B) 5

(C) $5\sqrt{2}$

(D) $4\sqrt{2}$

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(A) 4

(B) 5

(C) $5\sqrt{2}$

(D) $4\sqrt{2}$

Solution:

Equation of Plane is

$$=\begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix}=0$$

$x-z=0$

Distance of $P(7,-3,-4)$ from plane is

$d=\left|\frac{7+4-1}{\sqrt{2}}\right|=5\sqrt{2}$

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Q2: $\lim_{t \to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+....+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(A) n^2+n

(B) n

(C) $\frac{n(n+1)}{2}$

(D) n^2

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Q2: $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+\ldots.+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(A) n^2+n

(B) n

(C) $\frac{n(n+1)}{2}$

(D) n^2

Solution:

$$\lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+\ldots.+n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$$
$$=\lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t}+\ldots..+1\right)^{\sin^2 t}$$
$$=n$$

Q3: Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k},\ \vec{v} = 2\hat{i} + \hat{j} - \hat{k},\ \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(A) 1

(B) $\frac{3}{2}$

(C) 2

(D) $-\frac{2}{3}$

Q3: Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k},\ \vec{v} = 2\hat{i} + \hat{j} - \hat{k},\ \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(A) 1

(B) $\frac{3}{2}$

(C) 2

(D) $-\frac{2}{3}$

Solution:

$\vec{u} = (1, -1, -2)\,,\ \vec{v} = (2, 1, -1)\,,\ \vec{v}.\vec{w} = 2$

$\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v} \qquad \ldots.(1)$

Taking dot with $\vec{w}$ in (1)

$\vec{w}.(\vec{v} \times \vec{w}) = \vec{u}.\vec{w} + \lambda\vec{v}.\vec{w}$

$\Rightarrow 0 = \vec{u}.\vec{w} + 2\lambda$

Taking dot with $\vec{v}$ in (1)

$\vec{v}.(\vec{v} \times \vec{w}) = \vec{u}.\vec{v} + \lambda\vec{v}.\vec{v}$

$\Rightarrow 0 = (2 - 1 + 2) + \lambda(6)$

$\lambda = -\frac{1}{2}$

$\Rightarrow \vec{u}.\vec{w} = -2\lambda = 1$

Q4: The value $\sum_{r=0}^{22} {}^{22}C_r \, {}^{23}C_r$ is

(A) ${}^{45}C_{23}$

(B) ${}^{44}C_{23}$

(C) ${}^{45}C_{24}$

(D) ${}^{44}C_{22}$

Q4: The value $\sum_{r=0}^{22} {}^{22}C_r\ {}^{23}C_r$ is

(A) ${}^{45}C_{23}$

(B) ${}^{44}C_{23}$

(C) ${}^{45}C_{24}$

(D) ${}^{44}C_{22}$

Solution:

$$\sum_{r=0}^{22} {}^{22}C_r .\ {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r .\ {}^{23}C_{23-r}$$
$$= {}^{45}C_{23}$$

Q5: Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

(A) directrix $4x = 3$

(B) directrix $4x = -3$

(C) Length of latus rectum $\frac{3}{2}$

(D) Length of latus rectum 2

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(A) directrix $4x = 3$

(B) directrix $4x = -3$

(C) Length of latus rectum $\frac{3}{2}$

(D) Length of latus rectum 2

Solution:

$y^2 = 24x$
$a = 6$
$xy = 2$
$AB \equiv ty = x + 6t^2 \qquad \ldots.(1)$
$AB \equiv T = S_1$
$kx + hy = 2hk \qquad \ldots..(2)$
From (1) and (2)
$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$
$\Rightarrow$ Then locus is $y^2 = -3x$
Therefore directrix is $4x = 3$

Q6: Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$$x + y + z = 1$$
$$2x + Ny + 2z = 2$$
$$3x + 3y + Nz = 3$$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(A) 18

(B) 19

(C) 20

(D) 21

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Q6: Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

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has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(A) 18

(B) 19

(C) 20

(D) 21

Solution:

$$x+y+z=1$$
$$2x+Ny+2z=2$$
$$3x+3y+Nz=3$$
$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix}=(N-2)(N-3)$$

For unique solution $\Delta \neq 0$

So $N \neq 2,3$

$\Rightarrow P$(system has unique solution) $=\frac{4}{6}$

So $k=4$

Therefore sum $=4+1+4+5+6=20$

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Q7: $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Q7: $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Solution:

$$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$$
$$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{3}$$

Q8: Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(A) 4

(B) 3

(C) 2

(D) $\frac{5}{2}$

Q8: Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(A) 4

(B) 3

(C) 2

(D) $\frac{5}{2}$

Solution:

Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of ΔPQR is $= \frac{1}{2}|\vec{q} \times \vec{r}|$

Area of ΔABC is $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \ \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$

Area of $\Delta ABC = \frac{1}{6}|\vec{q} \times \vec{r}|$

$\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} = 3$

Q9: If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2 B$, then

(A) $AB = I$

(B) $A^2 B = I$

(C) $A^2 = I$ or $B = I$

(D) $A^2 B = BA^2$

Q9: If A and B are two non-zero $n \times n$ matrices such that $A^2 + B = A^2 B$, then

(A) $AB = I$

(B) $A^2 B = I$

(C) $A^2 = I$ or $B = I$

(D) $A^2 B = BA^2$

Solution:

$$A^2 + B = A^2 B$$
$$\left(A^2 - I\right)(B - I) = I \qquad \ldots.(1)$$
$$A^2 + B = A^2 B$$
$$A^2 (B - I) = B$$
$$A^2 = B(B - I)^{-1}$$
$$A^2 = B\left(A^2 - I\right)$$
$$A^2 = BA^2 - B$$
$$A^2 + B = BA^2$$
$$A^2 B = BA^2$$

Q10: Let $y = y(x)$ be the solution of the differential equation $x^3 dy + (xy - 1)\,dx = 0,\ x > 0,\ y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to

(A) 1

(B) e

(C) $2 - e$

(D) 3

Q10: Let $y=y(x)$ be the solution of the differential equation $x^3dy+(xy-1)\,dx=0,\ x>0,\ y\left(\frac{1}{2}\right)=3-e$. Then $y(1)$ is equal to

(A) 1

(B) e

(C) $2-e$

(D) 3

Solution:

$\frac{dy}{dx}=\frac{1-xy}{x^3}=\frac{1}{x^3}-\frac{y}{x^2}$

$\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$

IF $=e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

$y.\ e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}.\frac{1}{x^3}dx$ (put $-\frac{1}{x}=t$)

$y=\frac{1}{x}+1+Ce^{\frac{1}{x}}$

Where C is constant

Put $x=\frac{1}{2}$

$3-e=2+1+Ce^2$

$C=-\frac{1}{e}$

$y(1)=1$

Q11: The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is:

(A) $\frac{25}{3}$

(B) $\frac{22}{3}$

(C) 9

(D) $\frac{23}{3}$

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(A) $\frac{25}{3}$

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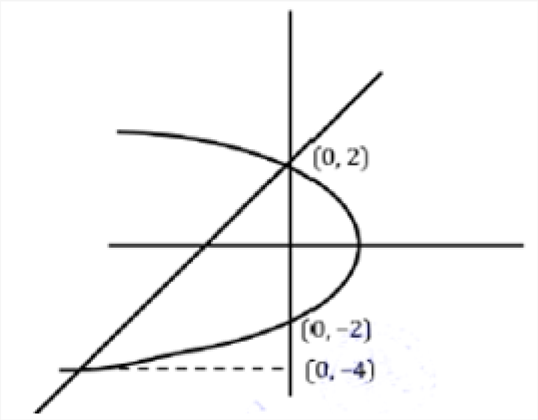
Solution:



$y^2+4x=4$

$y^2=-4(x-1)$

$A=\int_{-4}^{2}\left(\frac{4-y^2}{4}-\frac{y-2}{2}\right)dy=9$

Q12: Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a,b,c are distinct real numbers such that the matrix $\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$ is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is

(A) 6

(B) 3

(C) 9

(D) 12

Q12: Let α be a root of the equation $(a-c)x^2+(b-a)x+(c-b)=0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is

(A) 6

(B) 3

(C) 9

(D) 12

Solution:

$$\Delta = 0 = \begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

$$\Rightarrow \alpha^2 (c-b) - \alpha (c-a) + (b-a) = 0$$

It is singular when $\alpha = 1$

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$$

$$\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= 3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

Q13: The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(A) $13\sqrt{2}$

(B) 31

(C) 26

(D) $20\sqrt{2}$

Q13: The distance of the point $(-1, 9, -16)$ from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(A) $13\sqrt{2}$

(B) 31

(C) 26

(D) $20\sqrt{2}$

Solution:

Equation of line
$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$
G.P on line $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$
Point of intersection of line & plane
$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$
$\lambda = 2$
Point $(5, 1, 8)$
Distance $= \sqrt{36 + 64 + 576} = 26$

Q14: For three positive integers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2 r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

(A) 2

(B) 6

(C) 12

(D) -6

Q14: For three positive integers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

(A) 2

(B) 6

(C) 12

(D) -6

Solution:

$pq^2 = \log_x \lambda$

$qr = \log_y \lambda$

$p^2 r = \log_z \lambda$

$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \quad \ldots.(1)$

$\log_x z = \frac{pq^2}{p^2 r} = \frac{q^2}{pr} \quad \ldots.(2)$

$\log_z y = \frac{p^2 r}{qr} = \frac{p^2}{q} \quad \ldots.(3)$

$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr}$ in A.P.

$\frac{3r}{pq} - 3 = \frac{1}{2}$

$r = \frac{7}{6} pq \quad \ldots.(4)$

$r = pq + 1$

$pq = 6 \quad \ldots.(5)$

$r = 7 \quad \ldots.(6)$

$\frac{3p^2}{q} = 4$

After solving $p = 2$ and $q = 3$

Q15: Let $p, q \in \mathbb{R}$ and $\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right), i=\sqrt{-1}$ then $p+q+q^2$ and $p-q+q^2$ are roots of the equation.

(A) $x^2+4x-1=0$

(B) $x^2-4x+1=0$

(C) $x^2+4x+1=0$

(D) $x^2-4x-1=0$

Q15: Let $p, q \in \mathbb{R}$ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq), i = \sqrt{-1}$ then $p+q+q^2$ and $p-q+q^2$ are roots of the equation.

(A) $x^2+4x-1=0$

(B) $x^2-4x+1=0$

(C) $x^2+4x+1=0$

(D) $x^2-4x-1=0$

Solution:

$(1-\sqrt{3}i)^{200} = 2^{199}(p+iq)$

$2^{200}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^{200} = 2^{199}(p+iq)$

$2\left(-\frac{1}{2}, -i\frac{\sqrt{3}}{2}\right) = p+iq$

$p=-1, q=-\sqrt{3}$

$\alpha = p+q+q^2 = 2-\sqrt{3}$

$\beta = p-q+q^2 = 2+\sqrt{3}$

$\alpha+\beta = 4$

$\alpha.\beta = 1$

Equation $x^2-4x+1=0$

Q16: The relation $R=\{(a,b):\gcd(a,b)=1, 2a\neq b,\ a,b\in\mathbb{Z}\}$ is: ___

(A) transitive but not reflexive

(B) symmetric but not transitive

(C) reflexive but not symmetric

(D) neither symmetric nor transitive

Q16: The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b,\ a, b \in \mathbb{Z}\}$ is: ___

(A) transitive but not reflexive

(B) symmetric but not transitive

(C) reflexive but not symmetric

(D) neither symmetric nor transitive

Solution:

Reflexive : $(a, a) \Rightarrow \gcd\ of\ (a, a) = 1$ As $\gcd(3, 3) = 3$
Which is not true for every $a \in Z$
Symmetric:
Take $a = 2, b = 1 \Rightarrow \gcd(2, 1) = 1$
Also $2a = 4 \neq b$
Now when $a = 1, b = 2 \Rightarrow \gcd(1, 2) = 1$
Also now $2a = 2 = b$
Hence $a = 2b$
$\Rightarrow R$ is not symmetric
Transitive:
Let $a = 14, b = 19, c = 21$
$\gcd(a, b) = 1$
$\gcd(b, c) = 1$
$\gcd(a, c) = 7$
Hence not transitive
$\Rightarrow R$ is neither symmetric nor transitive.

Q17: The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(A) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(B) $(\sim Q) \vee P$

(C) $((\sim P) \vee Q) \wedge (\sim Q)$

(D) $(\sim P) \vee Q$

Q17: The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(A) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(B) $(\sim Q) \vee P$

(C) $((\sim P) \vee Q) \wedge (\sim Q)$

(D) $(\sim P) \vee Q$

Solution:

Let $r = (\sim (P \wedge Q)) \vee ((\sim P) \wedge Q)\,;\ s = ((\sim P) \wedge (\sim Q))$

P	Q	$\sim (P \wedge Q)$	$(\sim P) \wedge Q$	r	s	$r \to s$
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

Option (a): $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
is equivalent to (not of only P) $\wedge$ (not of only Q)
= (Both P, Q) and (neither P nor Q)

Q18: Let $f\left(x\right)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$; Then at $x=0$

(A) f is continuous but f' not differentiable

(B) f is continuous but f' is not continuous

(C) f and f' both are continuous

(D) f' is continuous but not differentiable

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(A) f is continuous but f' not differentiable

(B) f is continuous but f' is not continuous

(C) f and f' both are continuous

(D) f' is continuous but not differentiable

Solution:

Continuity of $f\left(x\right):f\left(0^{+}\right)=h^{2}.\sin\frac{1}{h}=0$

$f\left(0^{-}\right)=\left(-h\right)^{2}.\sin\left(\frac{-1}{h}\right)=0$

$f\left(0\right)=0$

$f\left(x\right)$ is continuous

$$f'\left(0^{+}\right)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\frac{h^{2}.\sin\left(\frac{1}{h}\right)-0}{h}=0$$

$$f'\left(0^{-}\right)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\frac{h^{2}.\sin\left(\frac{1}{-h}\right)-0}{-h}=0$$

$f\left(x\right)$ is differentiable

$$f'\left(x\right)=2x.\sin\left(\frac{1}{x}\right)+x^{2}.\cos\left(\frac{1}{x}\right).\frac{-1}{x^{2}}$$

$$f'\left(x\right)=\begin{cases}2x.\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$$

$\Rightarrow f'\left(x\right)$ is not continuous (as $\cos\left(\frac{1}{x}\right)$ is highly oscillating at $x=0$)

Q19: The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$denotes the greatest integer function, has:

(A) exactly two solutions in $(-\infty, \infty)$

(B) no solution

(C) a unique solution in $(-\infty, 1)$

(D) a unique solution in $(-\infty, \infty)$

Q19: The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$denotes the greatest integer function, has:

(A) exactly two solutions in $(-\infty, \infty)$

(B) no solution

(C) a unique solution in $(-\infty, 1)$

(D) a unique solution in $(-\infty, \infty)$

Solution:

$x^2 - 4x + [x] + 3 = x[x]$
$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$
$\Rightarrow (x-1)(x-3) = [x].(x-1)$
$\Rightarrow x = 1$ or $x - 3 = [x]$
$\Rightarrow x - [x] = 3$
$\Rightarrow \{x\} = 3$ (Not possible)
Only one solution $x = 1$ in $(-\infty, \infty)$

Q20: Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements:

(S1): If $P(A) = 0$, then $A = \phi$

(S2): If $P(A) = 1$, then $A = \Omega$

Then

(A) only (S1) is true

(B) only (S2) is true

(C) both (S1) and (S2) are true

(D) both (S1) and (S2) are false

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Q20: Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements:
(S1): If $P(A) = 0$, then $A = \phi$
(S2): If $P(A) = 1$, then $A = \Omega$
Then

(A) only (S1) is true

(B) only (S2) is true

(C) both (S1) and (S2) are true

(D) both (S1) and (S2) are false

Solution:

$\Omega =$ sample space
$A =$ be an event
$A = \left\{\frac{1}{2}\right\}, \Omega = [0, 1]$
If $P(A) = 0 \Rightarrow A = \phi$
If $P(\bar{A}) = 1 \Rightarrow \bar{A} \neq \Omega$
Then both statement are false

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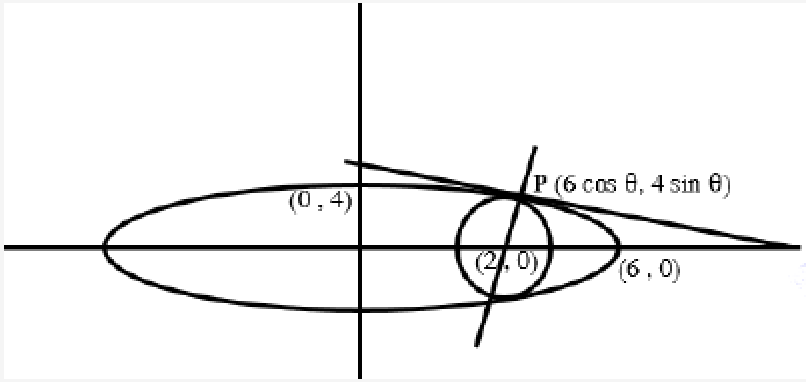
Q21: Let C be the largest circle centred at $(2,0)$ and inscribed in the ellipse $=\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1,\alpha)$ lies on C, then $10\alpha^2$ is equal to ____.

Q21: Let C be the largest circle centred at $(2, 0)$ and inscribed in the ellipse $= \frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to _____.

118.00

Solution:



Equation of normal of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ at any point $P\left(6\cos\theta, 4\sin\theta\right)$ is $3\sec\theta x-2\operatorname{cosec}\theta y=10$ this normal is also the normal of the circle passing through the point $(2,0)$ So,

$6\sec\theta=10$ or $\sin\theta=0$ (Not possible)

$\cos\theta=\frac{3}{5}$ and $\sin\theta=\frac{4}{5}$

So point $P=\left(\frac{18}{5},\frac{16}{5}\right)$

So the largest radius of circle

$r=\frac{\sqrt{320}}{5}$

So the equation of circle $(x-2)^2+y^2=\frac{64}{5}$

Passing it through $(1,\alpha)$

Then $\alpha^2=\frac{59}{5}$

$10\alpha^2=118$

Q22: Suppose $\sum_{r=0}^{2023} r^2 \, {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is ____.

Q22: Suppose $\sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is ____.

1012.00

Solution:

Using result

$\sum_{r=0}^{n} r^2\,{}^{n}C_r = n\left(n+1\right).2^{n-2}$

Then $\sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$

$= 2023 \times \alpha \times 2^{2022}$

So,

$\Rightarrow \alpha = 1012$

Q23: The value of $12\int_{0}^{3}\left|x^{2}-3x+2\right|dx$ is ____

Q23: The value of $12 \int\limits_{0}^{3} \left|x^2 - 3x + 2\right| dx$ is ____

22.00

Solution:

$$12 \int\limits_{0}^{3} \left|x^2 - 3x + 2\right| dx$$

$$= 12 \int\limits_{0}^{3} \left|\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}\right| dx$$

If $x - \frac{3}{2} = t$

$dx = dt$

$$= 24 \int\limits_{0}^{\frac{3}{2}} \left|t^2 - \frac{1}{4}\right| dt$$

$$= 24 \left[-\int\limits_{0}^{\frac{1}{2}} \left(t^2 - \frac{1}{4}\right) dt + \int\limits_{\frac{1}{2}}^{\frac{3}{2}} \left(t^2 - \frac{1}{4}\right) dt\right] = 22$$

Q24: The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is___

Q24: The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is___

60.00

Solution:

Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

Q25: Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of $E\}$ is ____.

Q25: Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of $E\}$ is ____.

5.00

Solution:

$|x|^2 - 2|x| + |\lambda - 3| = 0$
$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$
$(|x| - 1)^2 + |\lambda - 3| = 1$
At $\lambda = 3,\ x = 0$ and 2
At $\lambda = 4 \text{ or } 2$, then $x = 1 \text{ or } -1$
So maximum value of $x + \lambda = 5$

Q26: A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Q26: A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

546.00

Solution:

For at most two language courses

$$= {}^{5}C_{2} \times {}^{7}C_{3} + {}^{5}C_{1} \times {}^{7}C_{4} + {}^{7}C_{5} = \mathbf{546}$$

Q27: Let a tangent to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is ________.

Q27: Let a tangent to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is ________.

7.00

Solution:

Equation of tangent at parametric coordinates point $P(4\cos\theta, 3\sin\theta)$ is $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$

So A is $(4\sec\theta, 0)$ and point B is $(0, 3\operatorname{cosec}\theta)$

Length $AB = \sqrt{16\sec^2\theta + 9\operatorname{cosec}^2\theta}$

$= \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta} \geqslant 7$

Q28: The value of $\frac{8}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}} dx$ is ______.

Q28: The value of $\frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}} dx$ is ______.

2.00

Solution:

$$I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}} dx \qquad \ldots.(1)$$

Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}} dx \qquad \ldots.(2)$$

Adding (1) & (2)

$$2I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} 1\,dx$$

$$I = 2$$

Q29: The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to ______.

Q29: The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to ______.

14.00

Solution:

Shortest distance between the lines

$$=\frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}\right|}$$

$$=\frac{16+12+168}{\left|-4\hat{i}+6\hat{j}-12\hat{k}\right|}=\frac{196}{14}=14$$

Q30: The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}, m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ______.

Q30: The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ______.

12.00

Solution:

$T_4 = 500$ where $a =$ first term

$r =$ common ratio $= \frac{1}{m}, m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$

$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$S_6 - S_5 > 1$ $\frac{a}{m^6} < \frac{1}{2}$

$ar^5 > 1$ $m^3 > 10^3$

$\frac{500}{m^2} > 1$ $m > 10 \quad \ldots.(2)$

$m^2 < 500 \quad \ldots.(1)$

From (1) and (2)

$m = 11, 12, 13........, 22$

So number of possible values of m is 12