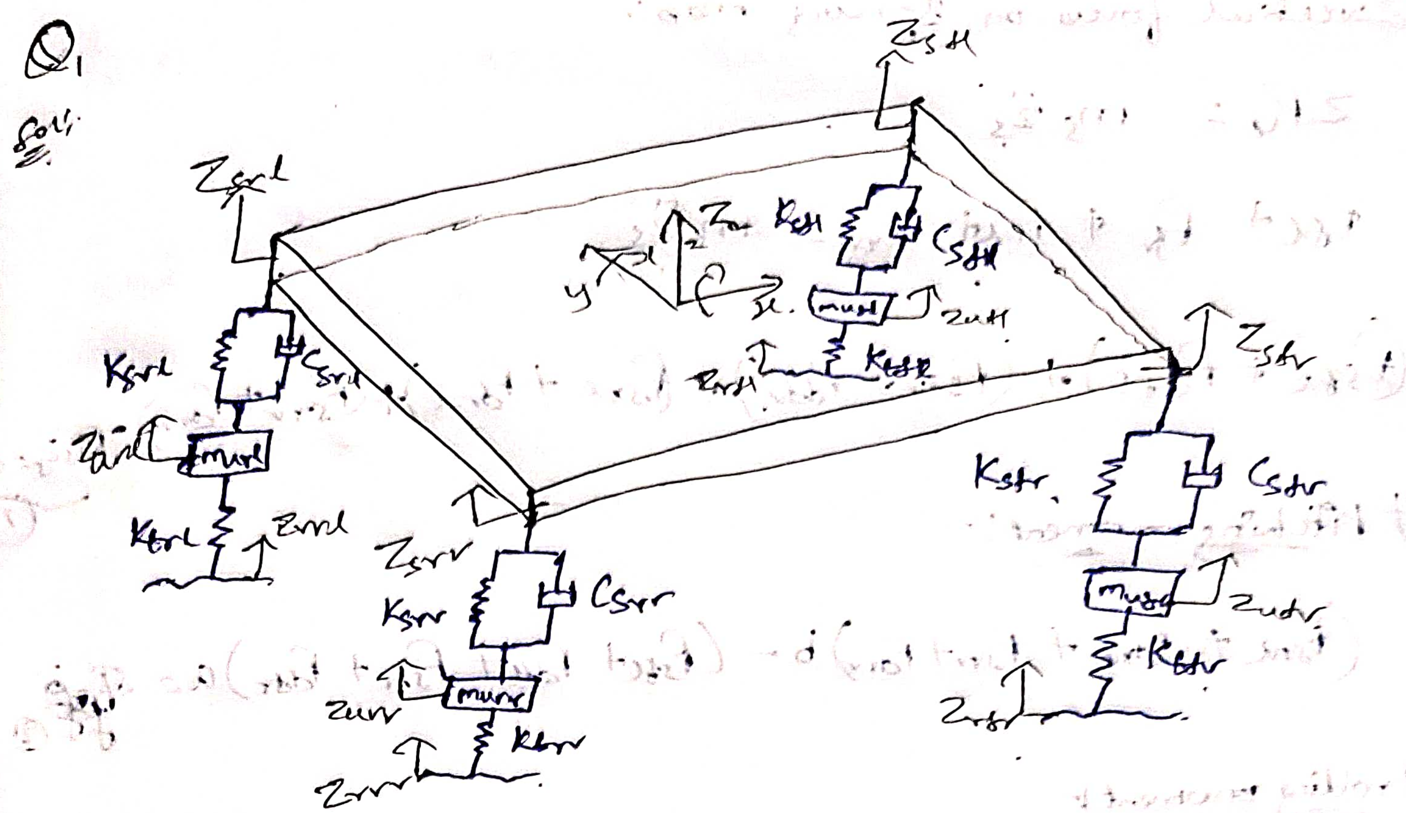
End term Exam ED23S038

Q1

Sol:



Forces on Spring

$$f_{sfl} = K_s (Z_{sfl} - Z_{ufl})$$

$$f_{sfr} = K_s (Z_{sfr} - Z_{ufr})$$

$$f_{srl} = K_s (Z_{srl} - Z_{url})$$

$$f_{srr} = K_s (Z_{srr} - Z_{urr})$$

Damping forces:-

$$f_{dfl} = C_{sfl} (\dot{Z}_{sfl} - \dot{Z}_{ufl})$$

$$f_{dfr} = C_{sfr} (\dot{Z}_{sfr} - \dot{Z}_{ufr})$$

$$f_{drl} = C_{srl} (\dot{Z}_{srl} - \dot{Z}_{url})$$

$$f_{drr} = C_{srr} (\dot{Z}_{srr} - \dot{Z}_{urr})$$

⇒ Forces on the tire:-

$$F_{tfl} = K_{tfl} (Z_{ufl} - Z_{rfl})$$

$$F_{tfr} = K_{tfr} (Z_{ufr} - Z_{rfr})$$

$$F_{trl} = K_{trl} (Z_{url} - Z_{rrl})$$

$$F_{trr} = K_{trr} (Z_{urr} - Z_{rrr})$$

Vertical forces on sprung mass:-

$$\Sigma F_v = m_s \ddot{z}_s$$

$$f_{fl} + f_{fr} + f_{rl} + f_{rr} = m_s \ddot{z}_s$$

$$(f_{sfl} + f_{dfl}) + (f_{sfr} + f_{dfr}) + (f_{srl} + f_{drl}) + (f_{srr} + f_{drr}) - m_s \ddot{z}_s = 0 \quad (1)$$

Pitching moment:-

$$(f_{srl} + f_{drl} + f_{srr} + f_{drr})b - (f_{sfl} + f_{dfl} + f_{sfr} + f_{dfr})a = I_{yy}\ddot{\theta} \quad (2)$$

rolling moment:-

$$\frac{W}{2}\left[(f_{sfr} + f_{dfr}) + (f_{srr} + f_{drr}) - (f_{sfl} + f_{dfl}) - (f_{srl} + f_{drl})\right] = I_{xx}\ddot{\phi} \quad (3)$$

Vertical forces on tyres:-

(i) Front left:- $m_{ufl} \ddot{z}_{ufl} = (f_{sfl} + f_{dfl}) - f_{tfl}$ — (4)

(ii) Front right:- $m_{ufr} \ddot{z}_{ufr} = (f_{sfr} + f_{dfr}) - f_{tfr}$ — (5)

(iii) Rear left:- $m_{url} \ddot{z}_{url} = (f_{srl} + f_{drl}) - f_{trl}$ — (6)

(iv) Rear right:- $m_{urr} \ddot{z}_{urr} = (f_{srr} + f_{drr}) - f_{trr}$ — (7)

⇒ Load on the wheels:-

$$f_{zfl} = \frac{m_s g b}{2(a+b)} + m_{ufl}\, g + f_{tfl}.$$

$$F_{zfr} = \frac{m_s g b}{2(a+b)} + m_{usfr} g + F_{tfr}$$

$$F_{zrl} = \frac{m_s g a}{2(a+b)} + m_{usrl} g + F_{trl}.$$

$$F_{zrr} = \frac{m_s g a}{2(a+b)} + m_{usrr} g + F_{trr}$$

Hence,

$$I_{yy}\ddot{\theta} = m_s a_x h + m_s g h \theta + b\left[(F_{srl}+F_{drl}) + (F_{srr}+F_{drr})\right]$$
$$-a\left[(F_{sfl}+F_{dfl}) + (F_{sfr}+F_{dfr})\right] \quad (8)$$

$$m_s a_y (h-h_{rc})\cos\phi + m_s g(h-h_{rc})\sin\phi + \frac{W}{2}\Big[(F_{sfr}+F_{dfr})$$
$$+ (F_{srr}+F_{drr}) - (F_{sfl}+F_{dfl}) - (F_{srl}+F_{drl})\Big] = I_{xx} + m_s(h-h_{rc})\ddot{\phi} \quad (9)$$

acceleration of vehicle :-

$$a_x = \dot{V}_x - V_y\dot{\psi} \rightarrow \text{longitudinal acceleration}$$

$$a_y = \dot{V}_y + V_x\dot{\psi} \rightarrow \text{lateral acceleration.} \quad (10)$$

Yaw Eqn of motion :-

$$I_{zz}\ddot{\psi} = \frac{W}{2}\Big[F_{xfr}\cos\delta - F_{xfl}\cos\delta + F_{xrr} + F_{yfl}\sin\delta - F_{yfr}\sin\delta\Big]$$

$+ l_f F_{xfl} \sin\delta + l_f F_{yfl} \cos\delta + l_f F_{xfr} \sin\delta + l_f F_{yfr} \cos\delta$

$- l_r F_{yrl} - l_r F_{yrr} + M_{zfl} + M_{zfr} + M_{zrl} + M_{zrr})$ — (11)

Now,

Stiffness of spring $(k) = \dfrac{4\pi^2 m (\omega_n)^2}{(\omega_n')^2 - (\omega_n)^2}$

For front suspension,

$$K_f = \frac{4\pi^2 m_f (\omega_{n,f})^2}{(\omega_{n,r})^2 - (\omega_{n,f})^2}$$

$$K_f = \frac{4\pi^2 (231.4)(0.9)^2}{1.5^2 - 0.9^2} = 16392.36 \text{ N/m}$$

For rear suspension:

$$K_r = \frac{4\pi^2 m_r (\omega_{n,r})^2}{(\omega_{n,f})^2 - (\omega_{n,r})^2}$$

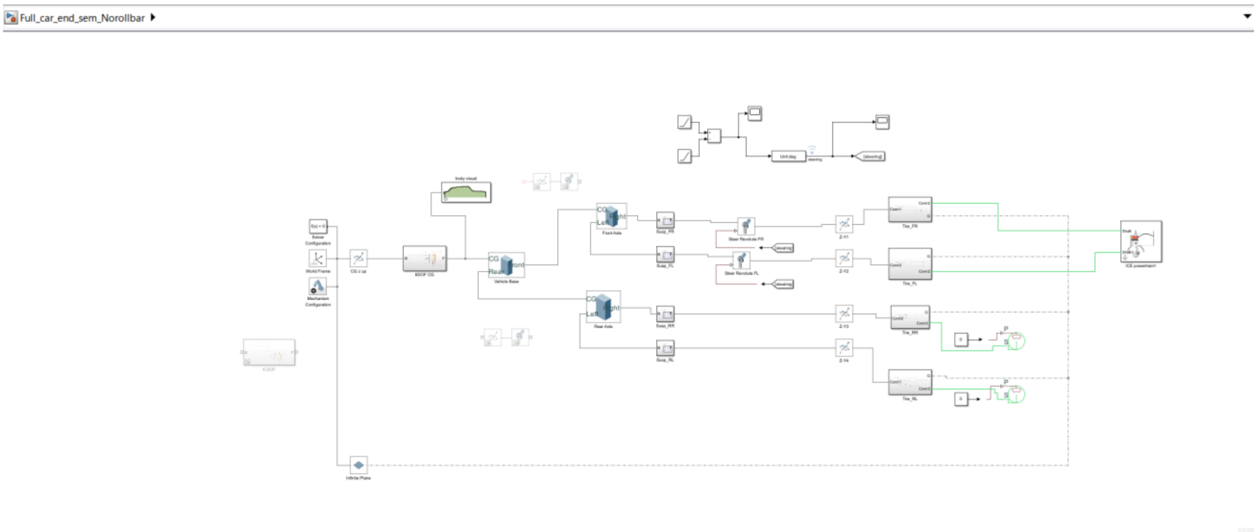
$$K_r = \frac{4\pi^2 (461.53)(0.5)^2}{0.9^2 - (1.5)^2} = -28469.49 \text{ N/m.}$$

ED5220 – Vehicle Dynamics End Sem Exam

ED23S038-Sai Santhosh Rao Kotha

Question 2:

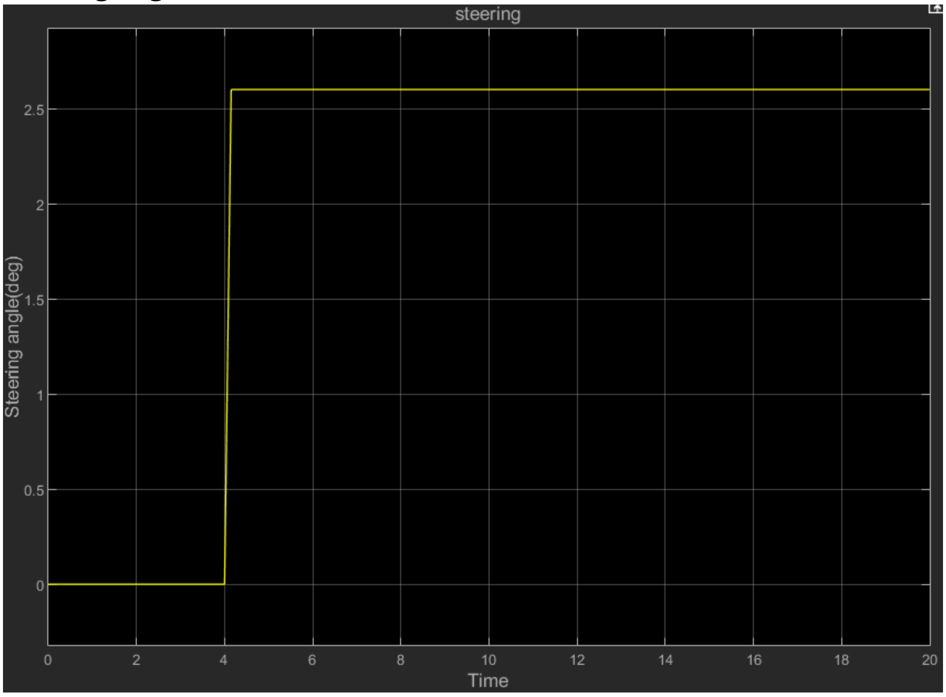
15 DOF Model development and analysis:



Behaviour analysis of 15 DOF Model under different conditions:

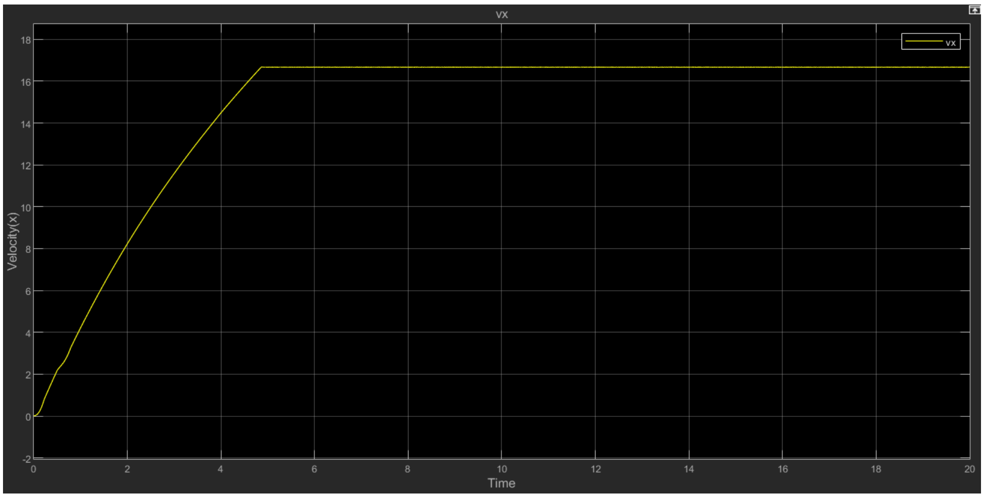
At a steering angle of 2.6 deg, the lateral acceleration of ay = 4.38 is achieved. The corresponding plots are shown below:

1) Step Steer Conditions :
   Steering angle vs Time

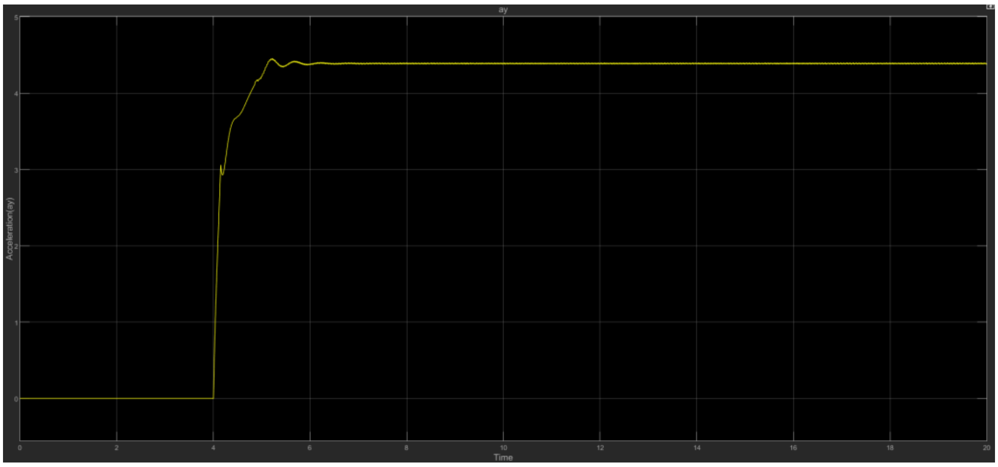


- 50% steering wheel input = 1.3 deg.
- Time taken to reach 50% steering = 4.06 sec

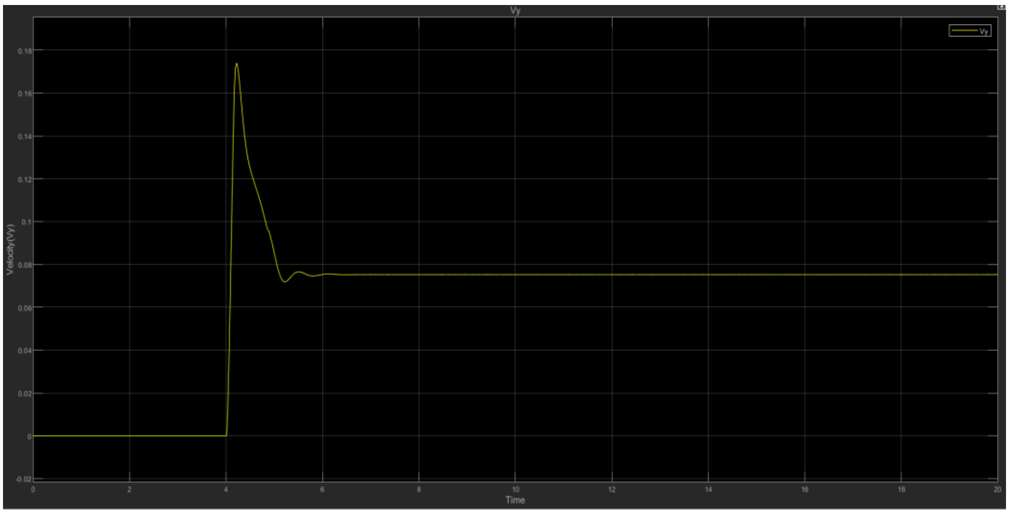
V_x vs Time:



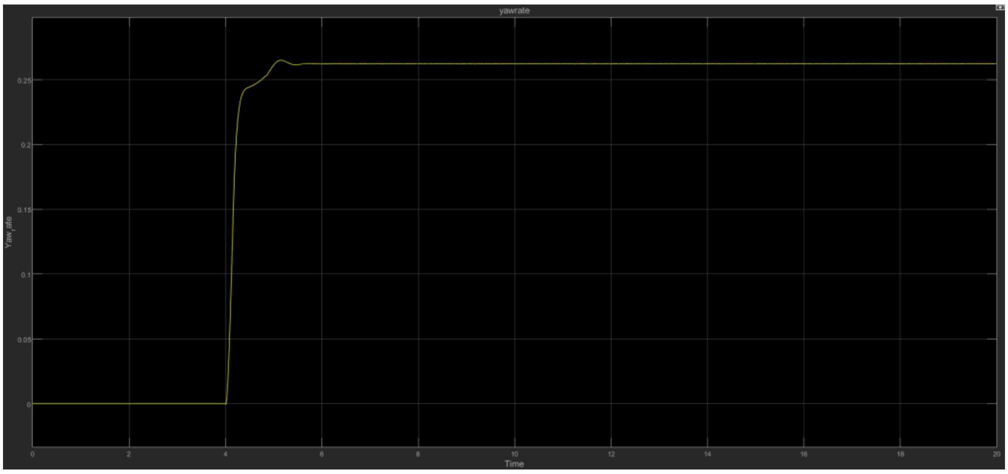
A_y vs Time:



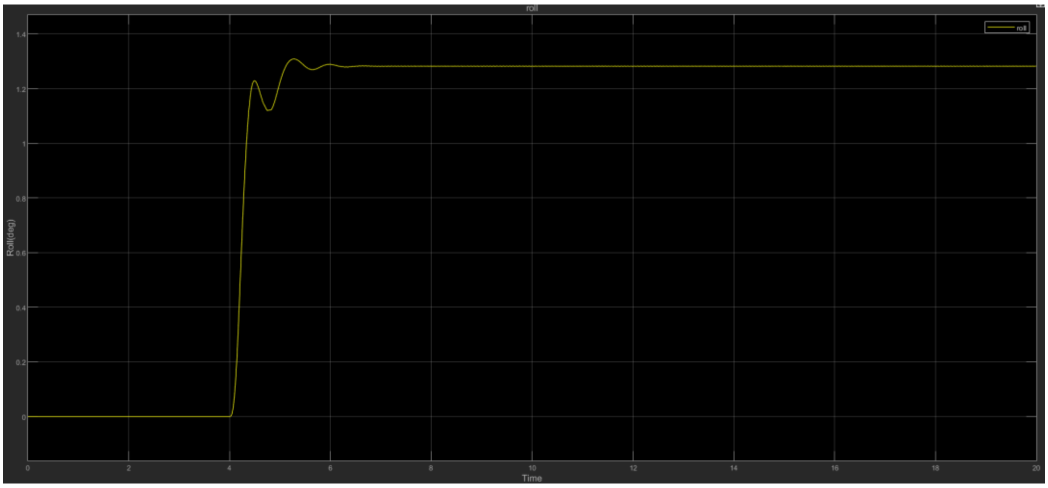
V_y vs Time:



Yaw rate vs Time



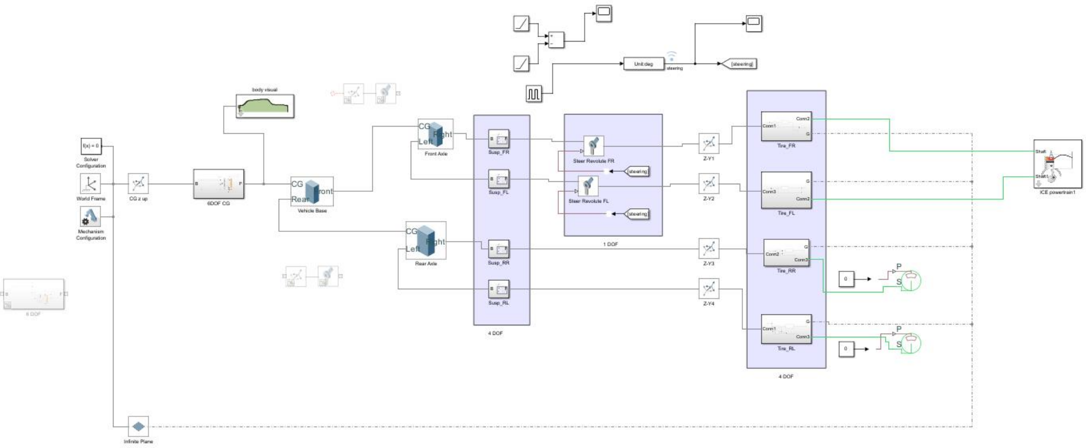
Roll vs Time:



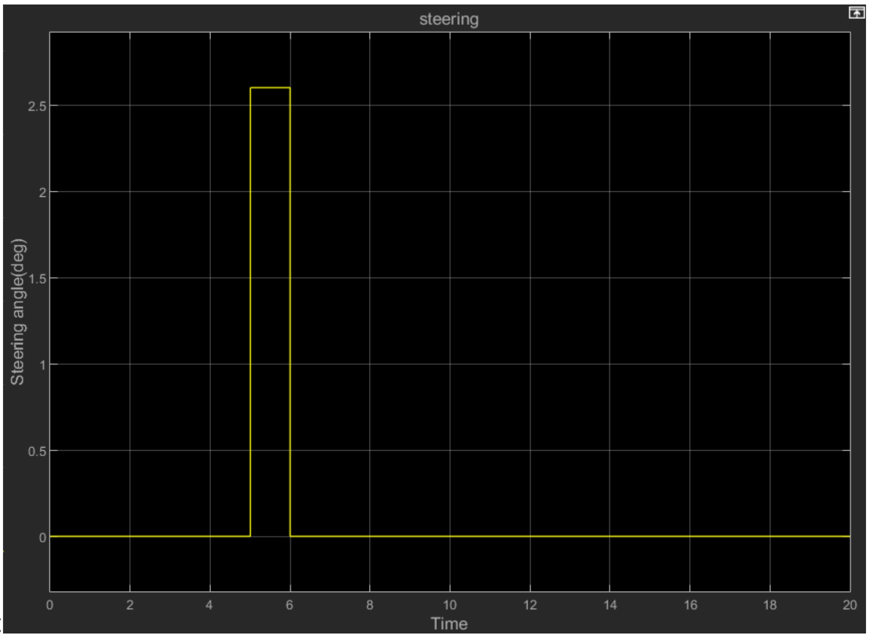
Step Input Response data:

Parameter	Value
Lateral accl response time	4.9-4.06 = 0.84sec
Lateral accl peak response time	5.25-4.06 = 1.19 sec
Lateral accl steady state gain	4.385/2.6 =1.686
Lateral accl overshoot	(4.45-4.385)/4.385 = 0.015 (1.5%)
Yaw rate response time	4.325-4.06 = 0.274 sec
Yaw rate peak response time	5.15-4.06=1.09 sec
Yaw rate steady state gain	0.262/2.6 = 0.1
Yaw rate overshoot	(0.265-0.262)/0.262 = 0.011 (1.1%)
Roll angle response time	4.4-4.06=0.34sec
Roll angle peak response time	5.25-4.06 = 1.19sec
Roll angle steady state gain	1.285/2.6 = 0.494
Roll angle overshoot	(1.308-1.285)/1.285=0.0178 (1.78%)

1) Impulse Steer Conditions :

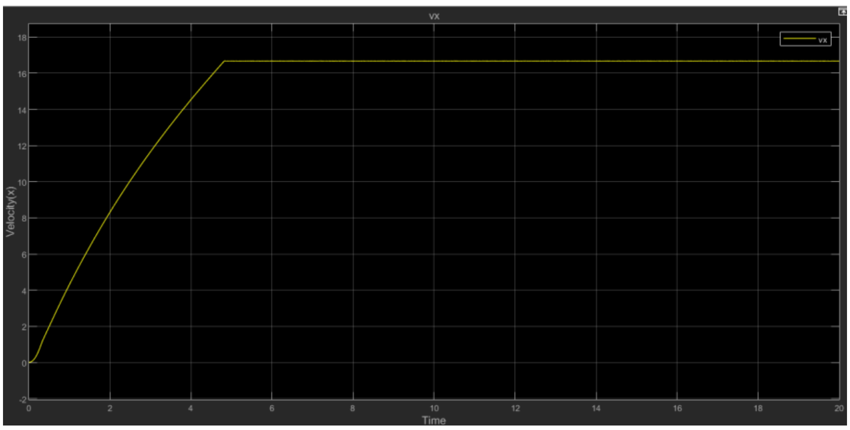


Steering angle vs Time:

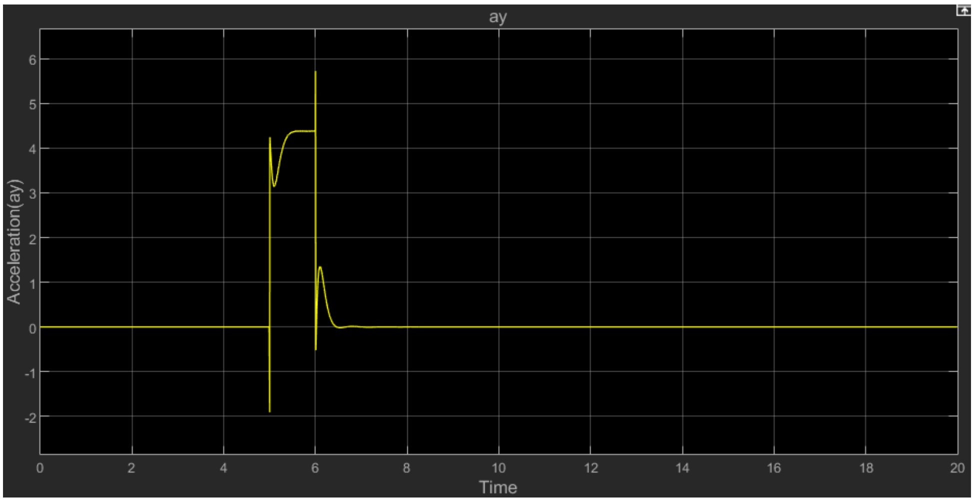


:

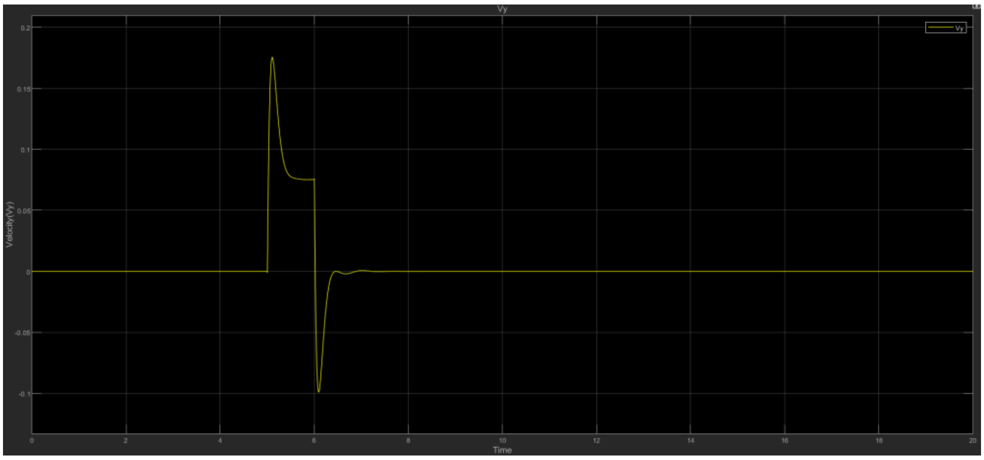
V_x vs Time:



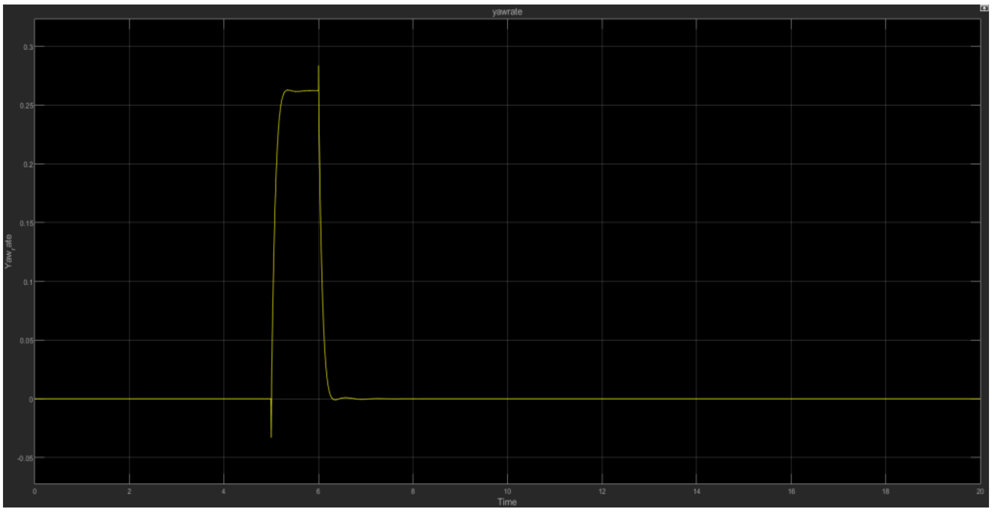
A_y vs Time:



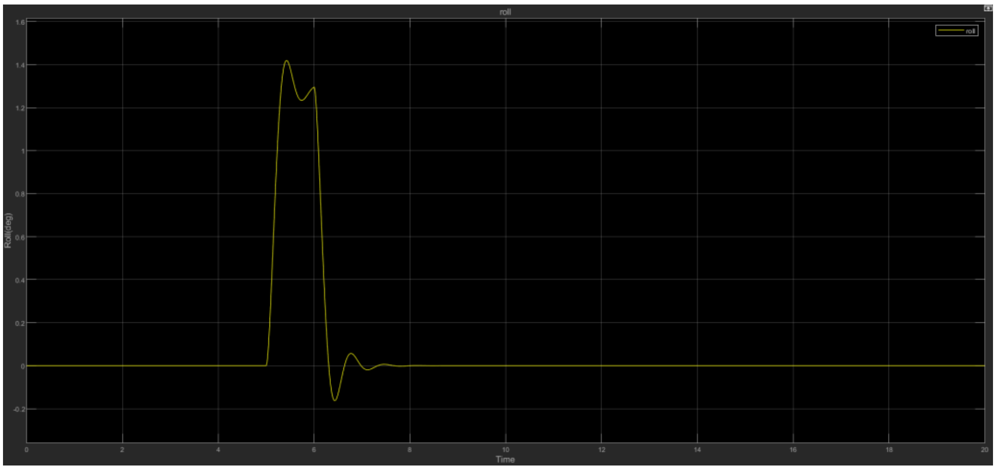
V_y vs Time:



Yaw rate vs Time



Roll vs Time:



Pulse Input Response data:

Expressions for the four parameters are:

$$a_1 = \frac{v_x}{LK_{sr}\left(1 + Kv_x^2\right)}$$

$$f_n = \frac{L}{\pi v_x} \cdot \sqrt{\frac{C_{\alpha f}C_{\alpha r}\left(1 + Kv_x^2\right)}{I_{zz}M}}$$

$$\zeta = \frac{I_{zz}\left(C_{\alpha f} + C_{\alpha r}\right) + M\left(l_f^2 C_{\alpha f} + l_r^2 C_{\alpha r}\right)}{2L\sqrt{I_{zz}MC_{\alpha f}C_{\alpha r}\left(1 + Kv_x^2\right)}}$$

$$\phi = \tan^{-1}\left(\frac{l_f L C_{\alpha r}}{\frac{LC_{\alpha r}v_x}{2\pi} - \pi I_{zz}v_x}\right) - \tan^{-1}\left(\frac{2\zeta f_n}{f_n^2 - 1}\right)$$

Parameter	Value
Steady state gain of Yaw rate response	0.25
Natural frequency of Yaw rate response	0.53
Damping of yaw velocity response	0.99 Hz
Phase delay at 1Hz of lateral acceleration	-71 deg

The Transfer functions required for Mimuro plots are:

$$\left(\frac{a_Y(s)}{\delta_H(s)}\right) = a_1 \frac{1 + b_1 s + b_2 s^2}{1 + 2\zeta \frac{s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

$$\left(\frac{\dot{\psi}(s)}{\delta_H(s)}\right) = a_2 \frac{1 + b_3 s}{1 + 2\zeta \frac{s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

Question 3:

Calculation of Understeer Gradient:

Steering angle = Ackerman steering angle + under steer Gradient * Ay

Since the vehicle is travelling straight, Ackerman steering angle is = zero.

Lateral Acceleration A_y = F_{YV} / W (in g's) = cross slope (for small angles)

Therefore, Understeer Gradient (USG) = Steering angle / cross slope.

From the above graphs, under steer gradient = 2.6/4.385 = 0.592 deg/g

Parameters that affect the understeer gradient:

Mass of the Vehicle (M = 1200 Kg)

Length of Vehicle (l = 2.6m)

Distance of Vehicle from the Front (a = 1.0m)

Distance of Vehicle from the Rear (b = 1.6m)

Cornering Stiffness of Front and Rear Tires (c (α_f), c (α_r))