

Multi Collinearity

Data set: [mtcars.xlsx](#)

Source: Inbuilt data in R



What is Multi-Collinearity?



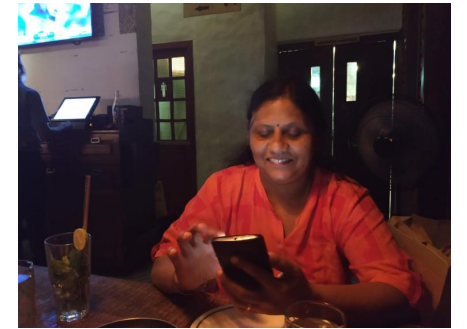
Dr Lylla is an amazing homemaker, a wonderful cook!

And we are inviting you for dinner, this weekend!!



What is Multi-Collinearity?

Menu – 1

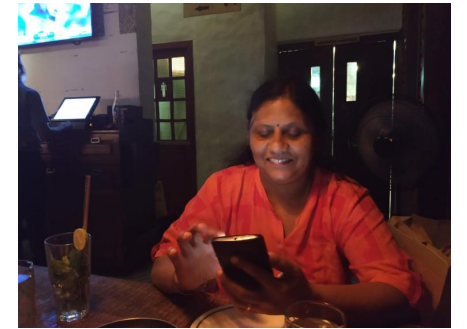


Dal Makhani, Mix Dal, Dal Tadka etc.



What is Multi-Collinearity?

Menu – 2



Dal Makhani, Paneer Butter Masala, Mix Veg etc.



What is Multi-Collinearity?

Which menu you will prefer and why?



What is Multi-Collinearity?

Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.



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What is Multi-Collinearity?

Menu – 2



Dal Makhani, Paneer Butter Masala, Mix Veg etc.



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Data: Perfect Collinearity

| | mpg | disp |
|-------------------|------|------|
| Mazda RX4 | 21 | 160 |
| Mazda RX4 Wag | 21 | 160 |
| Datsun 710 | 22.8 | 174 |
| Hornet 4 Drive | 21.4 | 163 |
| Hornet Sportabout | 18.7 | 142 |
| Valiant | 18.1 | 138 |
| Duster 360 | 14.3 | 109 |
| Merc 240D | 24.4 | 186 |
| Merc 230 | 22.8 | 174 |
| Merc 280 | 19.2 | 146 |
| Merc 280C | 17.8 | 136 |
| Merc 450SE | 16.4 | 125 |
| Merc 450SL | 17.3 | 132 |

| | mpg | disp |
|------|-----|------|
| mpg | 1 | |
| disp | 1 | 1 |

What is Multi-Collinearity?

Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.

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What is Multi-Collinearity?

Perfect Collinearity case

What is Multi-Collinearity?
Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.

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| | mpg | disp | | | | |
|------------------------------|---------------------|-----------------------|---------------------|--------------------------------------|------------------|------------------|
| mpg | 1 | | | | | |
| disp | 1 | 1 | | | | |
| SUMMARY OUTPUT | | | | | | |
| <i>Regression Statistics</i> | | | | | | |
| Multiple R | 1 | | | | | |
| R Square | 1 | | | | | |
| Adjusted R Square | 1 | | | | | |
| Standard Error | 2.79E-15 | | | | | |
| Observations | 32 | | | | | |
| ANOVA | | | | | | |
| | df | SS | MS | F | Significance F | |
| Regression | 1 | 1126.0 | 1126.0 | 1446342305551640000000000000000000.0 | 0.0 | |
| Residual | 30 | 0.0 | 0.0 | | | |
| Total | 31 | 1126.0 | | | | |
| | | | | | | |
| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | <i>Lower 95%</i> | <i>Upper 95%</i> |
| Intercept | 0.0 | 0.0 | -2.0 | 0.1 | 0.0 | 0.0 |
| disp | 0.1 | 0.0 | 12026397239205200.0 | 0.0 | 0.1 | 0.1 |

=TINV(0.05,30)

2.042272456

Data: Near Perfect Collinearity

| | mpg | disp |
|-------------------|------|-------|
| Mazda RX4 | 21 | 160 |
| Mazda RX4 Wag | 21 | 160 |
| Datsun 710 | 22.8 | 108 |
| Hornet 4 Drive | 21.4 | 258 |
| Hornet Sportabout | 18.7 | 360 |
| Valiant | 18.1 | 225 |
| Duster 360 | 14.3 | 360 |
| Merc 240D | 24.4 | 146.7 |
| Merc 230 | 22.8 | 140.8 |
| Merc 280 | 19.2 | 167.6 |
| Merc 280C | 17.8 | 167.6 |
| Merc 450SE | 16.4 | 275.8 |

| | <i>mpg</i> | <i>disp</i> |
|------|---------------------|-------------|
| mpg | 1 | |
| disp | -0.847551379 | 1 |

What is Multi-Collinearity?

Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.



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What is Multi-Collinearity?

Near Perfect case

| | | | | | | | |
|------------------------------|---------------------|-----------------------|---------------|----------------|-----------------------|------------------|--|
| | | <i>mpg</i> | <i>disp</i> | | | | |
| | mpg | 1 | | | | | |
| | disp | -0.847551379 | 1 | | | | |
| SUMMARY OUTPUT | | | | | | | |
| | | | | | | | |
| <i>Regression Statistics</i> | | | | | | | |
| Multiple R | 0.847551 | | | | | | |
| R Square | 0.718343 | | | | | | |
| Adjusted R Square | 0.708955 | | | | | | |
| Standard Error | 3.251454 | | | | | | |
| Observations | 32 | | | | | | |
| | | | | | | | |
| ANOVA | | | | | | | |
| | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> | <i>Significance F</i> | | |
| Regression | 1 | 808.9 | 808.9 | 76.5 | 0.0 | | |
| Residual | 30 | 317.2 | 10.6 | | | | |
| Total | 31 | 1126.0 | | | | | |
| | | | | | | | |
| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | <i>Lower 95%</i> | <i>Upper 95%</i> | |
| Intercept | 29.59985 | 1.2 | 24.1 | 0.0 | 27.1 | 32.1 | |
| disp | -0.04122 | 0.0 | -8.7 | 0.0 | -0.1 | 0.0 | |
| | | | | | | | |

What is Multi-Collinearity?

Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.



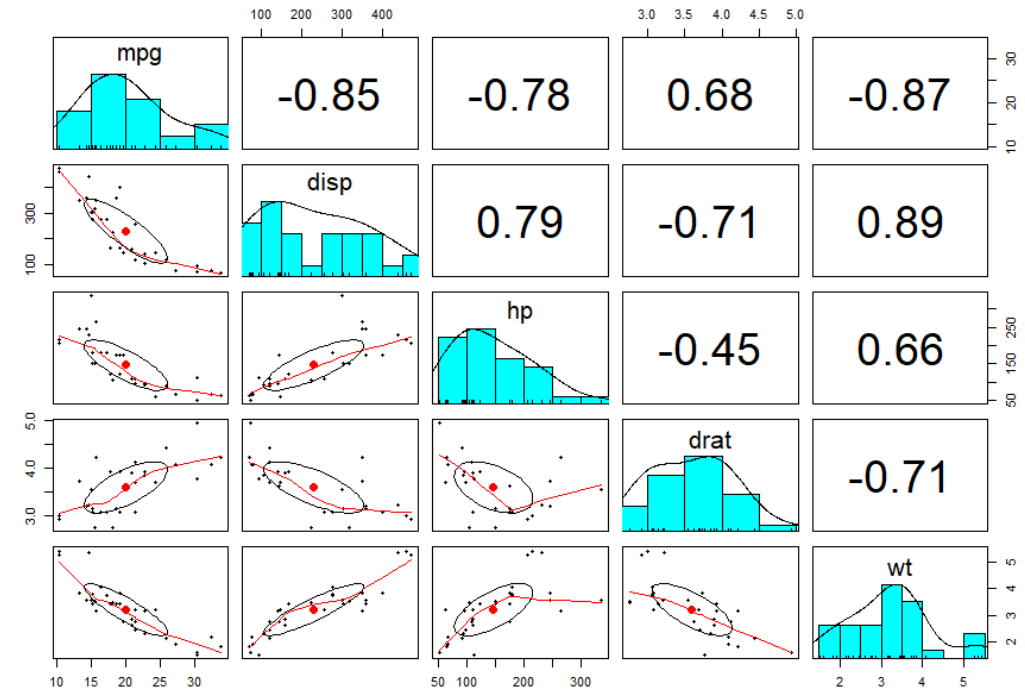
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How to detect Multi-Collinearity?

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| | mpg | disp | hp | drat | wt |
|-------------------|------|-------|-----|------|-------|
| Mazda RX4 | 21 | 160 | 110 | 3.9 | 2.62 |
| Mazda RX4 Wag | 21 | 160 | 110 | 3.9 | 2.875 |
| Datsun 710 | 22.8 | 108 | 93 | 3.85 | 2.32 |
| Hornet 4 Drive | 21.4 | 258 | 110 | 3.08 | 3.215 |
| Hornet Sportabout | 18.7 | 360 | 175 | 3.15 | 3.44 |
| Valiant | 18.1 | 225 | 105 | 2.76 | 3.46 |
| Duster 360 | 14.3 | 360 | 245 | 3.21 | 3.57 |
| Merc 240D | 24.4 | 146.7 | 62 | 3.69 | 3.19 |
| Merc 230 | 22.8 | 140.8 | 95 | 3.92 | 3.15 |
| Merc 280 | 19.2 | 167.6 | 123 | 3.92 | 3.44 |
| Merc 280C | 17.8 | 167.6 | 123 | 3.92 | 3.44 |
| Merc 450SE | 16.4 | 275.8 | 180 | 3.07 | 4.07 |



How to detect Multi-Collinearity?

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| SUMMARY OUTPUT | | | | | | |
|-----------------------|--------------|----------------|----------|----------|----------------|--------------|
| Regression Statistics | | | | | | |
| Multiple R | 0.91522066 | | | | | |
| R Square | 0.83762886 | | | | | |
| Adjusted R Square | 0.81357388 | | | | | |
| Standard Error | 2.60226088 | | | | | |
| Observations | 32 | | | | | |
| ANOVA | | | | | | |
| | df | SS | MS | F | Significance F | |
| Regression | 4 | 943.2096225 | 235.8024 | 34.82143 | 2.70431E-10 | |
| Residual | 27 | 182.837565 | 6.771762 | | | |
| Total | 31 | 1126.047188 | | | | |
| | Coefficients | Standard Error | t Stat | P-value | Lower 95% | Upper 95% |
| Intercept | 29.1487376 | 6.29358792 | 4.631498 | 8.2E-05 | 16.2353618 | 42.0621133 |
| drat | 1.76804877 | 1.319779454 | 1.339655 | 0.191525 | -0.93991499 | 4.476012527 |
| disp | 0.00381524 | 0.010804852 | 0.353104 | 0.726752 | -0.01835448 | 0.025984966 |
| wt | -3.4796675 | 1.078371371 | -3.22678 | 0.003272 | -5.69230282 | -1.267032241 |
| hp | -0.0347835 | 0.011597337 | -2.99927 | 0.005756 | -0.0585793 | -0.010987764 |

What is Multi-Collinearity?
Menu – 1



Dal Makhani, Mix Dal, Dal Tadka etc.

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3

How to detect Multi-Collinearity?

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| SUMMARY OUTPUT | | | | | | |
|-----------------------|--------------|----------------|----------|----------|----------------|--------------|
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| ANOVA | | | | | | |
| | df | SS | MS | F | Significance F | |
| Regression | 4 | 943.2096225 | 235.8024 | 34.82143 | 2.70431E-10 | |
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| Total | 31 | 1126.047188 | | | | |
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| Intercept | 29.1487376 | 6.29358792 | 4.631498 | 8.2E-05 | 16.2353618 | 42.0621133 |
| drat | 1.76804877 | 1.319779454 | 1.339655 | 0.191525 | -0.93991499 | 4.476012527 |
| disp | 0.00381524 | 0.010804852 | 0.353104 | 0.726752 | -0.01835448 | 0.025984966 |
| wt | -3.4796675 | 1.078371371 | -3.22678 | 0.003272 | -5.69230282 | -1.267032241 |
| hp | -0.0347835 | 0.011597337 | -2.99927 | 0.005756 | -0.0585793 | -0.010987764 |

High Rsqr b
few significa
predictors

High Rsqr but
few significant
predictors!

How to detect Multi-Collinearity?

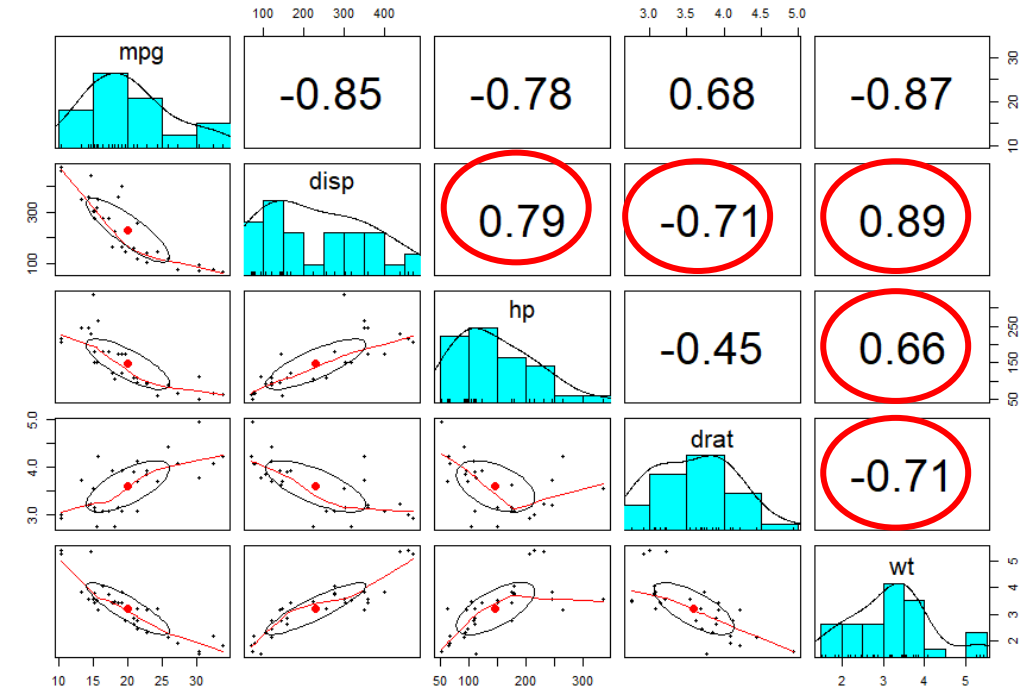
Pair-wise correlations

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| SUMMARY OUTPUT | | | | | | |
|-----------------------|--------------|----------------|----------|----------|----------------|--------------|
| | | | | | | |
| Regression Statistics | | | | | | |
| Multiple R | 0.91522066 | | | | | |
| R Square | 0.83762886 | | | | | |
| Adjusted R Square | 0.81357388 | | | | | |
| Standard Error | 2.60226088 | | | | | |
| Observations | 32 | | | | | |
| | | | | | | |
| ANOVA | | | | | | |
| | df | SS | MS | F | Significance F | |
| Regression | 4 | 943.2096225 | 235.8024 | 34.82143 | 2.70431E-10 | |
| Residual | 27 | 182.837565 | 6.771762 | | | |
| Total | 31 | 1126.047188 | | | | |
| | | | | | | |
| | Coefficients | Standard Error | t Stat | P-value | Lower 95% | Upper 95% |
| Intercept | 29.1487376 | 6.29358792 | 4.631498 | 8.2E-05 | 16.2353618 | 42.0621133 |
| drat | 1.76804877 | 1.319779454 | 1.339655 | 0.191525 | -0.93991499 | 4.476012527 |
| disp | 0.00381524 | 0.010804852 | 0.353104 | 0.726752 | -0.01835448 | 0.025984966 |
| wt | -3.4796675 | 1.078371371 | -3.22678 | 0.003272 | -5.69230282 | -1.267032241 |
| hp | -0.0347835 | 0.011597337 | -2.99927 | 0.005756 | -0.0585793 | -0.010987764 |

High pair-wise correlation

High pair-wise correlation



How to detect Multi-Collinearity?

Auxiliary Regressions

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| | Rsqr | | Num | Denom | F |
|---|-------------|--|------------------|----------------|-----------------|
| disp~wt+hp+drat | 0.88 | | 0.44 | 0.004138 | 106.3333 |
| wt~hp+disp+drat | 0.8 | | 0.4 | 0.006897 | 58 |
| hp~disp+wt+drat | 0.65 | | 0.325 | 0.012069 | 26.92857 |
| drat~hp+wt+disp | 0.56 | | 0.28 | 0.015172 | 18.45455 |
| mpg~all | 0.8376 | | | | |
| $F_{x_i} = \frac{R_{x_i}^2 / (k - 2)}{(1 - R_{x_i}^2) / (n - k + 1)}$ | | | F_Crit | 3.32765 | |
| | | | =FINV(0.05,2,29) | | |

If F Calculated is > F Critical,
multicollinearity is indicated!

OMG, all seems to be troublesome!

How to detect Multi-Collinearity?

Variance Inflation Factor

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

```
#install.packages("HH")
library(HH)

vif(mod)
```

```
> vif(mod)
      disp      hp      drat      wt
8.209402 2.894373 2.279547 5.096601
```

| Model | DV | Rsquare | VIF |
|-------|---------------------|----------|----------|
| 1 | mpg~disp+hp+wt+drat | 0.837629 | |
| 2 | disp~hp+drat+wt | 0.878188 | 8.209372 |
| 3 | hp~disp+drat+wt | 0.654502 | 2.894373 |
| 4 | drat~disp+hp+wt | 0.561316 | 2.279545 |
| 5 | wt~disp+hp+drat | 0.803791 | 5.096606 |

$$VIF = \frac{1}{1 - R_{1.2}^2}$$

What is Multi-Collinearity?

Menu – 1



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How to detect Multi-Collinearity?

Klien's rule of thumb

Data: mtcars, DV= *mpg*, IVs= *hp*, *disp*, *wt*, *drat*

| Model | DV | Rsquare | VIF |
|-------|---------------------|----------|----------|
| 1 | mpg~disp+hp+wt+drat | 0.837629 | |
| 2 | disp~hp+drat+wt | 0.878188 | 8.209372 |
| 3 | hp~disp+drat+wt | 0.654502 | 2.894373 |
| 4 | drat~disp+hp+wt | 0.561316 | 2.279545 |
| 5 | wt~disp+hp+drat | 0.803791 | 5.096606 |

If Rsqr for any predictor is > overall Rsqr, the predictor is having multicollinearity!

What is Multi-Collinearity?

Menu – 1



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**Fall down seven times
Stand up eight**

~Japanese Proverb

