Title: **Dimensional Method to Nonlinear Friction coefficient for two discs rolling contact**

Student name: **Santhosh Kumar Arroju** (U008030355)

Department: **Mechanical Engineering**

Wright state University

Contents:

1. Literature survey

Qualitative physics using dimensional analysis:

Optimum similarity analysis with applications to elastohydrodynamic lubrication: H. Moes,

This paper introduced Similarity analysis based on mathematical expressions like differential, algebraic, integral equations. Dimensional analysis is the well-known technique and modified as a vectorial dimensional analysis. Power product of quantities (PP's) will be number of original quantities and dimensionless numbers later introduced as Buckingham π's. Similarity Quantities (SQ's) arranged as a linearly independent PP's to derive suitable π's.

Jeffrey's equation is a simple model with successive steps Normalization the equation, similarity solution, Dimensional analysis, Optimum similarity analysis.

A method to Derive friction and Rolling power loss formulae for mixed elastohydrodynamic lubrication: Sheng Li and Ahmet Kahraman, A study to derive EHL based friction coefficient and rolling power loss in gear efficiency models, having dependent parameters and the results from the regression technique compared to actual EHL predictions.

2. Abstract

Friction can cause energy loss and create unwanted heat, deformation. In the class room and in laboratory friction coefficient is considered as a constant value but, in the real-world friction is nonlinear in nature. Mainly it depends up on the dependent forces, ambient conditions and profile of the specimen. The study is to get the friction coefficient relations, for number of linearly independent parameters called Power products. Select the linearly independent parameters, mechanical properties and external forces for the discs with rolling contact. There are ten independent parameters with similarity quantities are set as power product quantities.

The basic linear dependent parameters are chosen from one of the paper listed in the survey. The analysis can be simplified, for two discs with rolling contact and can be extended to gear mechanism. The Dimensional Analysis, similarity technique Buckingham π theorem was used to get the non-dimensional friction coefficient relation.

The non-trivial solutions of π can be calculated using regression techniques, the partial derivatives with linear dependent parameters gives the

3. Introduction to friction and behavior

Friction force is the component of the contact force between two rough surfaces when slides. Friction can be two types Kinetic and static friction. Kinetic friction exists when surfaces in relative motion and static friction exists when surfaces not in relative motion.

The force between two surfaces that are in contact has two components, the perpendicular component is the normal force and parallel component is friction force.

$$f = \mu N$$

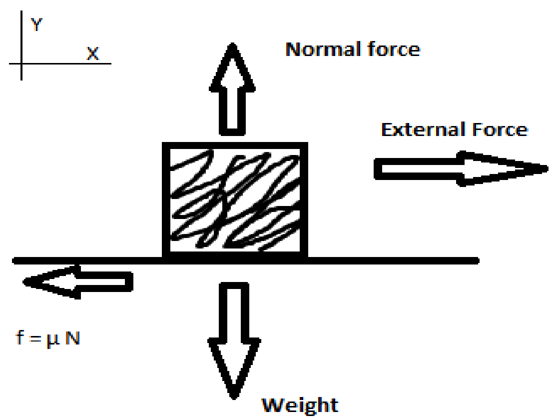


Fig 1: Free body diagram for a box with some weight

Static frictional coefficient values for some metals in air without lubrication.

Self - mated metals in Air				
Gold	2		Lead	1.5
Silver	0.8 – 1		Cadmium	0.5
Tin	1		Chromium	0.4
Aluminum	0.8 – 1.2			
Copper	0.7 – 1.4			

Solid / Fluid interaction with body can lead to friction, example: Drag on the airfoil due to combination of skin, wave, friction.

4. Introduction to dimensional analysis

The similarity analysis based on physical relations based on mathematical equations, like algebraic, differential, integral equations. Depend on the original quantities, the set of power products quantities.

Five sequence of similarity parameters:

1. Control similarity parameters
2. Dependent similarity parameters
3. Independent similarity parameters
4. Integral similarity parameters
5. Dependent similarity parameters

Dimensional analysis is using two theorems 1. Product theorem and 2. Buckingham's π theorem.

Here, Buckingham's π theorem is used to find the relation among the parameters and the ability to reason about combinations is important. The Buckingham's π theorem for a given measurement of physical quantities and some function of physical quantities is set to zero. The basic no. of dimensions need to express the variables for all π's (no. of variables - no. dimensions)

The below table gives fundamental and supplemental physical quantities.

Physical quantity	Unit	Symbol
Fundamental quantities		
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Supplementary quantities		
Plane angle	radian	rad
Solid angle	steradian	sr

Using the fundamental quantities, we can derive the quantities like pressure, force, weight, acceleration etc.

S.No.	Physical Quantity	Dimensional formula
1	Area	$M^0L^2T^0$
2	Density	$M^1L^{-3}T^0$
3	Speed	$M^0L^1T^{-1}$
4	Acceleration	$M^0L^1T^{-2}$
5	Gravitational Constant	$M^{-1}L^3T^{-2}$
6	Power	$M^1L^2T^{-3}$
7	**Strain**	**Dimensionless**
8	Frequency	$M^0L^0T^{-1}$
9	Work	$M^1L^2T^{-2}$
10	Surface Tension	$M^1L^0T^{-2}$

Dimensional conditions: all the physical equation is dimensionally homogeneous.

Such that $[\, Q_1^{a1}\, Q_2^{a2}\, Q_3^{a3}\, Q_4^{a4} \dots \dots .. Q_n^{an} \,] = [I]$ is satisfied.

Let π represents a dimensionless product,

$$\Pi = Q_1^{a1}\, Q_2^{a2}\, Q_3^{a3}\, Q_4^{a4} \dots \dots .. Q_n^{an}$$

The physical equation of the form is reducible to the form,

$[\Pi 1] = [\Pi 2] = [\Pi 3] = \cdots .. = [\Pi \mathrm{i}] = [I]$

i is the no. of independent arguments, is the maximum number of independent dimensionless products

let 'k' be the no. of arbitrary fundamental units among the 'n' units.

At least one set of 'k' which is used as fundamental units, and remaining (n-k) being derived.

Therefore (n-k) equations of the form and the number of product Π , independent variable i = n-k

So, $[\Pi 1] = [\, Q_1^{a1}\, Q_2^{b1}\, Q_3^{c1}\, Q_4^{d1} \dots \dots .. Q_n^{n1}\, P1 \,] = [I]$

$[\Pi 2] = [\, Q_1^{a2}\, Q_2^{b2}\, Q_3^{c2}\, Q_4^{d2} \dots \dots .. Q_n^{n2}\, P2] = [I]$

.

.

.

.

$[\Pi \mathrm{i}] = \left[\, Q_1^{ai}\, Q_2^{bi}\, Q_3^{ci}\, Q_4^{di} \dots \dots .. Q_n^{ni}\;\; Pi\right] = [I]$

Here, P's represents temporarily derived quantities.

In this study 4 fundamental quantities are considered mainly,

M – mass, L – length, T – time, and θ – temperature.

5. Parameters which effect the friction for two disc with rolling contact

The friction is nonlinear in real world but is taken as constant in theoretical problems. The most effective parameters for friction between two particles are been selected for the dimensional analysis are

The final parameters for the Buckingham π theorem, Dimensional analysis.

	Parameters		**Dimensional number**
1	Pressure	F/A	$M\ L^{-1}\ T^{-2}$
2	Radius of curvature	L	L
3	Velocity	m/s	$L\ T^{-1}$
4	Temperature	^{o}C	θ
5	Surface roughness 1	L	L
6	Dynamic viscosity	µ	$M\ L^{-1}\ T^{-1}$
7	Pressure-viscosity coefficient	Cp	$M^{-1}\ L\ T^{2}$
8	Density	ρ	$M\ L^{-3}$
9	Load (force)	F	$M\ L\ T^{-2}$
10	Modulus of Elasticity	E	$M\ L^{-1}\ T^{-2}$

Pressure is directly proportional to friction coefficient, because pressure = Force/Area, and force is directly proportional to friction coefficient.

Radius of curvature is also directly proportional to friction coefficient, due to contact area increases friction also increase.

Velocity is also directly proportional to friction coefficient, more velocity more the kinetic energy developed and friction also more.

Temperature is very directly proportional to friction coefficient.

Surface Roughness is directly proportional to friction coefficient, as the roughness of the surface increases the temperature then the frictional force rapidly increases.

Dynamic viscosity is not directly proportional to friction coefficient, more the viscosity less the friction value.

Pressure-viscosity coefficient is not directly proportional to friction coefficient, more the viscosity less the friction value.

Density of the material is directly proportional to friction coefficient, density is more and more the weight of the body and frictional force will be more.

Load is directly proportional to friction coefficient

6. Derive equations using Dimensional analysis for selected parameters

The dimensional analysis is done for choose parameters in case wise and found the relation.

Case. 1

Parameters

	Parameters		Dimensional number
1	Radius of curvature	L	L
2	Velocity	m/s	$L\ T^{-1}$
3	Temperature	^{o}C	θ
4	Pressure	F/A	$M\ L^{-1}\ T^{-2}$
5	Surface roughness	L	L

5 quantities and 4 dimensions (M L T θ)

So, π's = no. of quantities – no. of dimensions = 5-4 = 1

π = (radius of curvature) $(\text{pressure})^b(\text{velocity})^c(\text{Temperature})^d(\text{Surface roughness})^e$

$\pi = (L)\ (\text{pressure})^b(M\ L^{-1}\ T^{-2})^c(\theta)^d(L)^e$

Setting the equation to zero and find b, c, d, e values (b = 0, c = 0, d = 0, e = -1)

$$\boldsymbol{\pi} = \frac{\boldsymbol{Radius\ of\ curvature}}{\boldsymbol{surface\ roughness}}$$

Here, the dimensional relation doesn't affect included the temperature

Case. 2

Parameters

	Parameters		**Dimensional number**
1	Pressure	F/A	$M\ L^{-1}\ T^{-2}$
2	Radius of curvature	L	L
3	Velocity	m/s	$L\ T^{-1}$
4	Temperature	^{o}C	θ
5	Surface roughness 1	L	L
6	Surface roughness 2	L	L
7	viscosity	μ	$M\ L^{-1}\ T^{-1}$

7 quantities and 4 dimensions (M L T θ)

So, π's = no. of quantities – no. of dimensions = 7-4 = 3 (three π's, π1, π2, π3)

π1 = (radius of curvature) $(\text{pressure})^{b}(\text{velocity})^{c}(\text{Temperature})^{d}(\text{Surface roughness})^{e}$

π1 = (L) $(M\ L^{-1}\ T^{-2})^{b}(L\ T^{-1})^{c}(\theta)^{d}(L)^{e}$

Setting the equation to zero and find b, c, d, e values (b = 0, c = 0, d = 0, e = -1)

$$\boldsymbol{\pi 1 = \frac{Radius\ of\ curvature}{Surface\ roughness 1}}$$

π2 = (radius of curvature) $(\text{pressure})^{b}(\text{velocity})^{c}(\text{Surface roughness})^{d}(\text{viscosity})^{e}$

π2 = (L) $(M\ L^{-1}\ T^{-2})^{b}(L\ T^{-1})^{c}(L)^{d}(M\ L^{-1}\ T^{-1})^{e}$

Setting the equation to zero and find b, c, d, e values (b = -e, c = e, d = -1-e, e = e)

$$\boldsymbol{\pi 2 = (radius\ of\ curvature) * (\frac{velocity * viscocity}{pressure})^{e} * \frac{1}{surface\ roughness^{1+e}}}$$

$\pi 3$ = (Surface roughness 1) (pressure)^b(velocity)^c(viscosity)^d

$\pi 3$= (L) (M L^{-1} T^{-2})^b(L T^{-1})^c(M L^{-1} T^{-1})^d

Setting the equation to zero and find b, c, d values (b = 1, c = -1, d = -1)

$$\boldsymbol{\pi 3 = \frac{suraface\ roughness * pressure}{velocity * viscocity}}$$

Case. 3

Parameters

	Parameters		**Dimensional number**
1	Pressure	F/A	M L^{-1} T^{-2}
2	Radius of curvature	L	L
3	Velocity	m/s	L T^{-1}
4	Temperature	^oC	θ
5	Surface roughness 1	L	L
6	Dynamic viscosity	μ	M L^{-1} T^{-1}
7	Pressure-viscosity coefficient	Cp	M^{-1} L T^2
8	Density	ρ	M L^{-3}
9	Load (force)	F	M L T^{-2}

9 quantities and 4 dimensions (M L T θ)

So, π's = no. of quantities – no. of dimensions = 9-4 = 5 (three π's, π1, π2, π3, π4, π5)

If we not consider the temperature, to simple more. Considering only 3 dimensions (M L T)

So, 8 quantities and 3 dimensions (M L T)

So, π's = no. of quantities – no. of dimensions = 8-3 = 5 (three π's, π1, π2, π3, π4, π5)

$\pi 1$ = (radius of curvature) (pressure)^b(Pressure-viscosity coefficient)^c(Dynamic viscosity)^d

$\pi 1$ = (L) (M L^{-1} T^{-2})^b(M^{-1} L T^2)^c(M L^{-1} T^{-1})^d

Setting the equation to zero and find b, c, d, e values (b = c, d = 0)

$$\boldsymbol{\pi 1 = (pressure * pressure - viscosity\ coefficient)^{b\ or\ c} * radius\ of\ curvature}$$

$\pi2$ = (Pressure-viscosity coefficient) (density)^{b} (pressure)^{c} (load)^{d}

$\pi2 = (M\ L^{-1}\ T^{-2})\ (M\ L^{-3})^{b}(M\ L^{-1}\ T^{-2})^{c}(M\ L\ T^{-2})^{d}$

Setting the equation to zero and find b, c, d, e values (b = 0, c=1, d = 0)

$$\boldsymbol{\pi 2 = (pressure - viscosity\ coeffiecient) * pressure}$$

$\pi3$ = (load) (density)^{b} (dynamic viscosity)^{c} (velocity)^{d}

$\pi3 = (M\ L\ T^{-2})\ (M\ L^{-3})^{b}\ (M\ L^{-1}\ T^{-1})^{c}\ (L\ T^{-1})^{d}$

Setting the equation to zero and find b, c, d, e values (b = 1, c=-2, d = 0)

$$\boldsymbol{\pi 3 = \frac{load * density}{dynamic\ viscosity^{2}}}$$

$\pi4$ = (density) (load)^{b} (pressure)^{c} (dynamic viscosity)^{d}

$\pi4 = (M\ L^{-3})\ (M\ L\ T^{-2})^{b}\ (M\ L^{-1}\ T^{-2})^{c}\ (M\ L^{-1}\ T^{-1})^{d}$

Setting the equation to zero and find b, c, d, e values (b = 1, c=0, d = -2)

$$\boldsymbol{\pi 4 = \frac{load * density}{dynamic\ viscosity^{2}}}$$

$\pi5$ = (density) (load)^{b} (pressure)^{c} (dynamic viscosity)^{d}(velocity)^{e}

$\pi5 = (M\ L^{-3})\ (M\ L\ T^{-2})^{b}\ (M\ L^{-1}\ T^{-2})^{c}\ (M\ L^{-1}\ T^{-1})^{d}(L\ T^{-1})^{e}$

Setting the equation to zero and find b, c, d, e values (b = b, c=b-1, d = -2b, e = 2-2b)

$$\boldsymbol{\pi 5 = \left(\frac{density * velocity^{2}}{pressure}\right) * \left(\frac{load * pressure}{dynamic\ viscosity^{2} * velocity^{2}}\right)^{b}}$$

Improving the parameters and including material property.

Parameters:

	Parameters		**Dimensional number**
1	Pressure	F/A	$M L^{-1} T^{-2}$
2	Radius of curvature	L	L
3	Velocity	m/s	$L T^{-1}$
4	Temperature	ºC	θ
5	Surface roughness 1	L	L
6	Dynamic viscosity	µ	$M L^{-1} T^{-1}$
7	Pressure-viscosity coefficient	Cp	$M^{-1} L T^{2}$
8	Density	ρ	$M L^{-3}$
9	Load (force)	F	$M L T^{-2}$
10	Modulus of Elasticity	E	$M L^{-1} T^{-2}$

10 quantities and 4 dimensions (M L T θ)

So, π's = no. of quantities – no. of dimensions = 9-4 = 6 (three π's, π1, π2, π3, π4, π5, π6)

If we not consider the temperature, to simple more. Considering only 3 dimensions (M L T)

So, 9 quantities and 3 dimensions (M L T)

So, π's = no. of quantities – no. of dimensions = 9-3 = 6 (three π's, π1, π2, π3, π4, π5, π6)

$\pi 5 = (\text{density})\ (\text{load})^{b}\ (\text{dynamic viscosity})^{c}\ (\text{velocity})^{d} (\text{elastic modulus})^{e}$

$\pi 5 = (M L^{-3})\ (M L T^{-2})^{b}\ (M L^{-1} T^{-1})^{c}\ (L T^{-1})^{d} (M L^{-1} T^{-2})^{e}$

Setting the equation to zero and find b, c, d, e values (b = 1+e, c=-2-2e, d = -2e, e = e)

$$\boldsymbol{\pi 5} = \left(\frac{\boldsymbol{density * load}}{\boldsymbol{dynamic\ viscosity^2}}\right) * \left(\frac{\boldsymbol{load * elastic\ modulus}}{\boldsymbol{dynamic\ viscosity^2 * velocity^2}}\right)^{\boldsymbol{e}}$$

$\pi 6 = (\text{surface roughness})\ (\text{dynamic viscosity})^{b}\ (\text{velocity})^{c} (\text{elastic modulus})^{d}$

$\pi 6 = (L)\ (M L^{-1} T^{-1})^{b}\ (L T^{-1})^{c}\ (M L^{-1} T^{-2})^{d}$

Setting the equation to zero and find b, c, d, e values (b = -1, c=-1, d = 1)

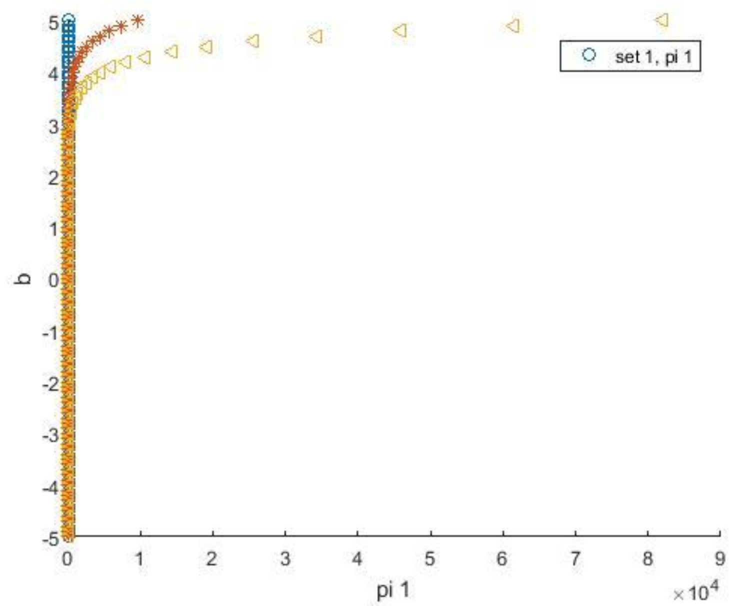
$$\pi 6 = \frac{surface\ roughness * elastic\ modulus}{Dynamic\ viscosity * velocity}$$

Parametric design values of friction coefficient from the reference paper:

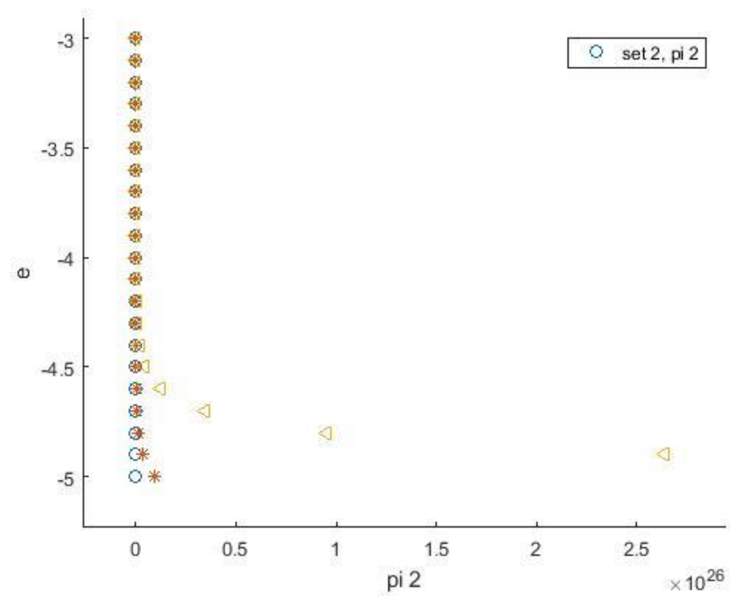
	Parameters		**Set 1**	**Set 2**	**Set 3**
1	Pressure	F/A (GPa)	0.5	1	1.5
2	Radius of curvature	L (mm)	5	20	40
3	Rolling Velocity	m/s (m/s)	1	5	10
4	Temperature	^{o}C	50	75	100
5	Surface roughness 1	L (µm)	0.1	0.4	0.7
6	Dynamic viscosity	µ (Pa s)	0.01502	0.00703	0.00398
7	Pressure-viscosity coefficient	Cp (1/GPa)	15.8	13.7	12.2
8	Density	P (Kg/m^3)	977.80	962.80	947.80
9	Load (force)	F (KN)	5	10	15
10	Modulus of Elasticity	E (GPa)	200	200	200

Plots for set no. 1, 2, 3 and used parametric design values

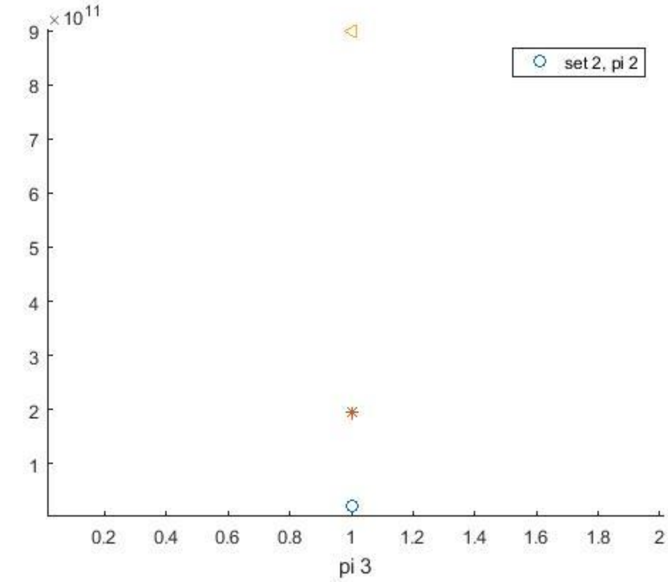
$\pi 1$



$\pi 2$

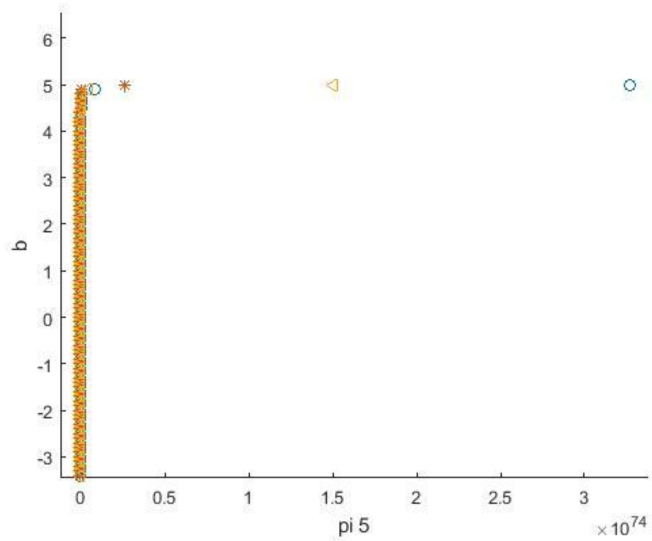


π3

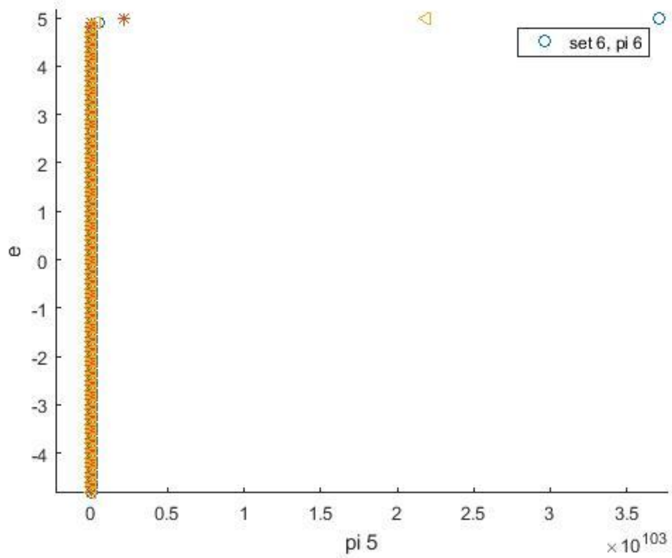


π4 is same as π3

π5



π6



7. Future work and scope

The analysis is done using one selected dimensional theorem that is Buckingham π theorem. The work can be extended to use the Product theorem also, and experimental analysis like elastoydrodynamic lubrication and regression formulae can be compared etc.,

Two discs with rolling contact is considered and the friction coefficient is calculated for selected parameters, the analysis is carried at discrete contact positions. Further detailed analysis can be extended to gears with lubrication.

The regression formulae can be used to find the non-trivial solutions for the results came using Buckingham π theorem. The analysis is experimentally done considering practical cases in variety of fields like Automobile, Manufacturing, Agricultural equipment's etc.,

In preliminary design stage of an Automobile, the calculated parameter results for friction coefficient can identify the lower energy loss, wear loss, and increases the overall efficiency, this pre-calculated results save the design period and manufacturing time.

The computational tools can be used to derive the friction and rolling power loss and efficiency calculation.

The Dimensional constant formulae like Reynolds no., Mach no., Coefficient of Friction, coefficient of Force, Prandtl no. many other in different fields like Fluid mechanics, Aerospace industry, Automobile industry, Design of wheels, gears, etc.

8. Conclusion and summary

In this study, the usage of Dimensional Analysis is predominant. The power product parameters, dimensionless parameters are calculated using Buckingham π theorem, for ten final selected parameters. The non-trivial solutions need to be calculated using regression techniques. Finally, each π's results that is friction coefficient is analyzed using partial derivatives and selected the most effective parameters. For further analysis the non-trivial solutions can be analyzed using Regression techniques.

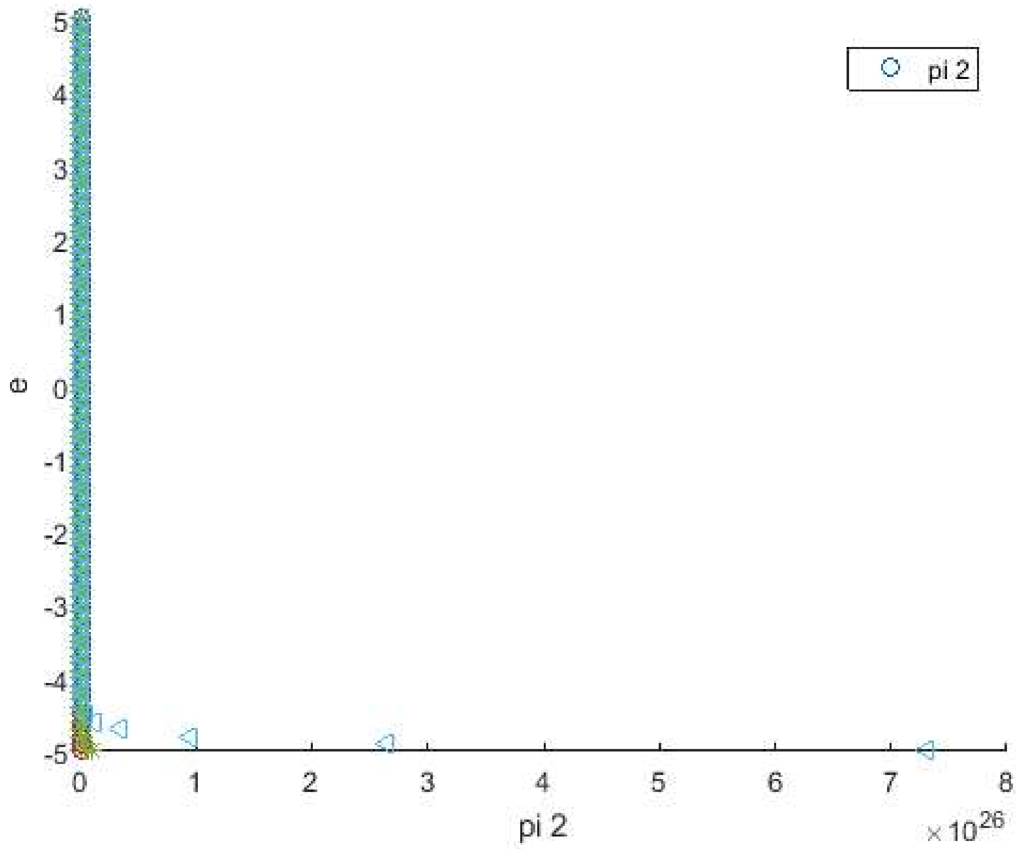
The dynamic viscosity, radius of curvature of the disc, surface roughness of the material, pressure-viscous coefficient, density, external loads, pressure, material constant, are selected as most important parameters for the Analysis. Further analysis using Regression technique can be done specially for the Non-trivial solutions of π's.

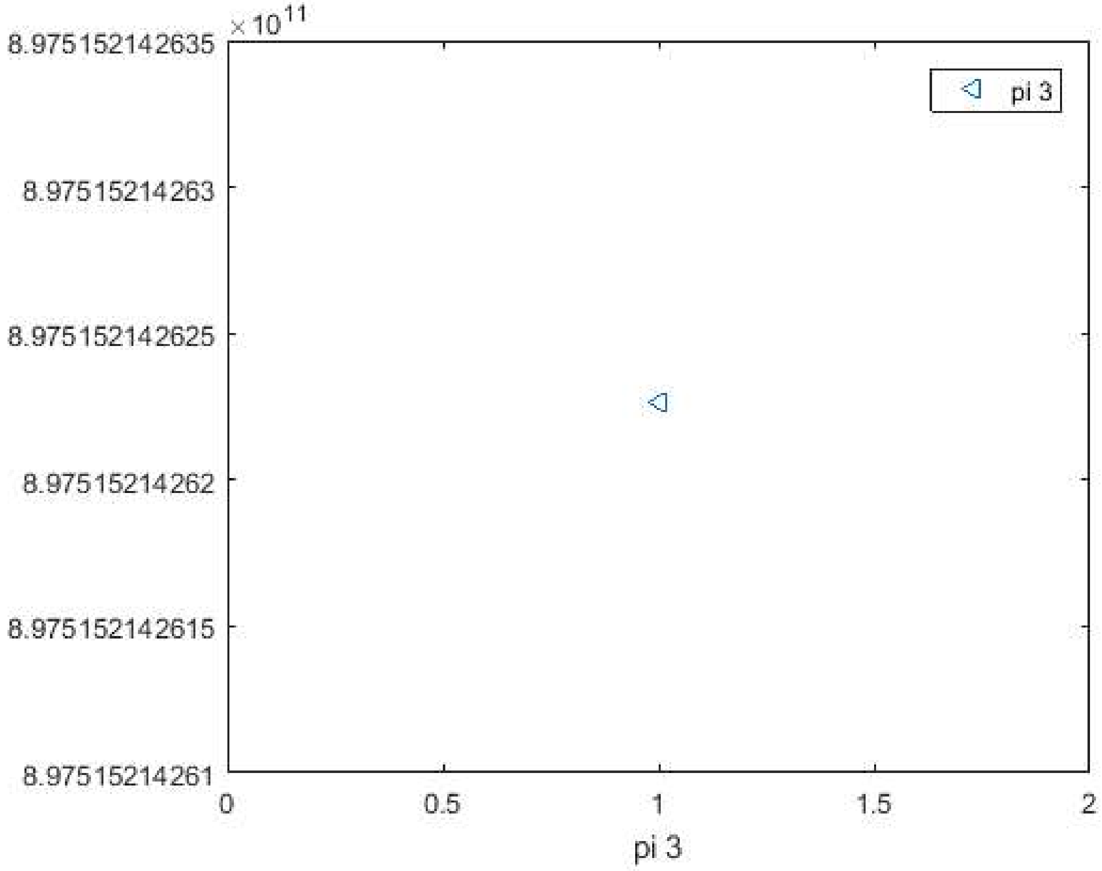
9. Bibliography

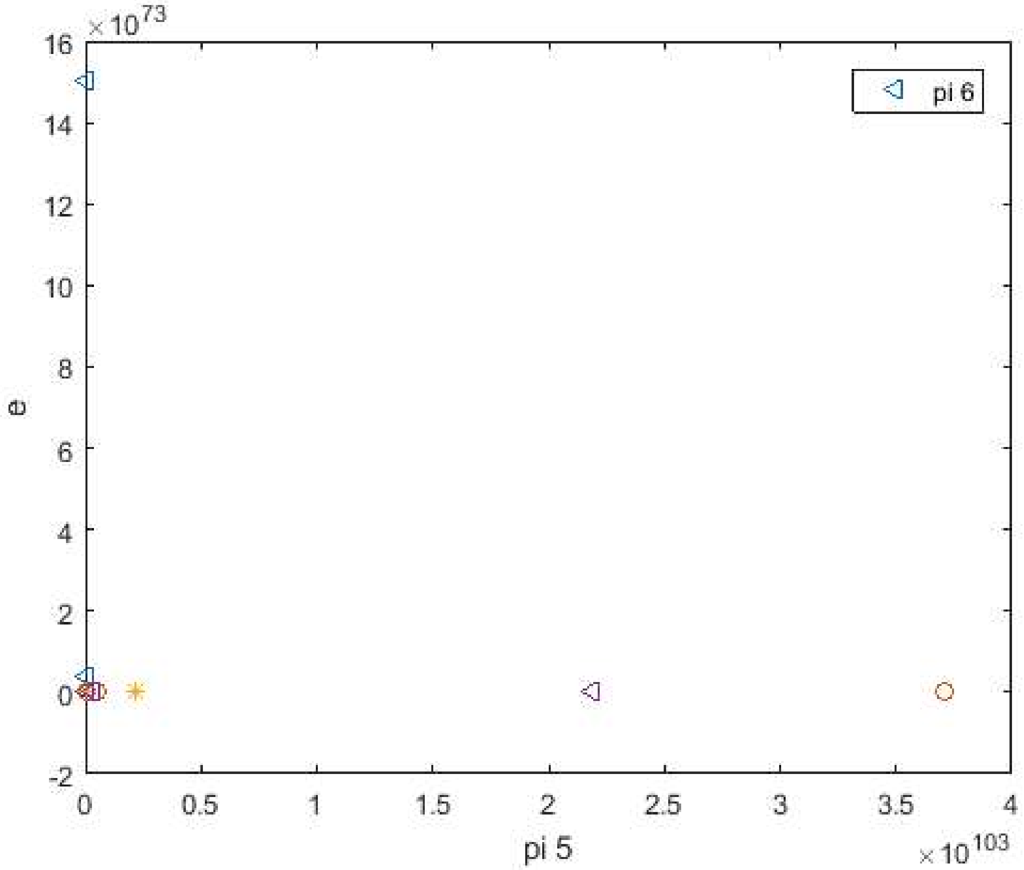
1. Qualitative physics using dimensional analysis by R. Bhaskar and Anil Nigam, IBM Thomas J. Watson Research Center, P.O. Box 704, Yorktown Heights, NY 10598, USA.
2. Optimum similarity analysis with applications to elastohydrodynamic lubrication by H. Moes University of Twente, Department of Mechanical Engineering, Tribology Group, P.O. Box 217, 7500 AE Enschede (Netherlands).
3. A method to derive friction and rolling power loss formulae for mixed elastohydrodynamic lubrication by Sheng Li and Ahmet Kahraman, Department of Mechanical and Aerospace Engineering, the ohio state university, 201 W. 19th Avenue, Columbus OH 43210, USA.

```
clc
clear all
close all

p = [0.5; 1; 1.5]*10^9;
radius = [5; 20; 40]*10^-3;
v = [1; 5; 10];
roughness = [0.1; 0.4; 0.7]*10^-6;
viscosity = [0.01502; 0.00703; 0.00398];
p_visc_coeff  = [15.8; 13.7; 12.2]*10^-9;
density = [977.80; 962.80; 947.80];
load = [5; 10; 15]*10^3;
E = [200; 200; 200]*10^9;

% set_1
b = -5:0.1:5;
pi1_1 = (p(1)* p_visc_coeff(1)).^b .* radius(1);
pi1_2 = (p(2)* p_visc_coeff(2)).^b .* radius(2);
pi1_3 = (p(3)* p_visc_coeff(3)).^b .* radius(3);
hold on
plot(pi1_1, b, 'o')
plot(pi1_2, b, '*')
plot(pi1_3, b, '<')
xlabel('pi 1')
ylabel('b')
legend('pi 1')

% set_2
e = -5:0.1:5;
pi2_1 = radius(1)*((v(1)*viscosity(1))/p(1)).^e .* (roughness(1)).^(-1-e);
pi2_2 = radius(2)*((v(2)*viscosity(2))/p(2)).^e .* (roughness(2)).^(-1-e);
pi2_3 = radius(3)*((v(3)*viscosity(3))/p(3)).^e .* (roughness(3)).^(-1-e);
hold on
plot(pi2_1, e, 'o')
plot(pi2_2, e, '*')
plot(pi2_3, e, '<')
xlabel('pi 2')
ylabel('e')
legend('pi 2')

%set_3
pi3_1 = (load(1)*density(1))/viscosity(1)^2;
pi3_2 = (load(2)*density(2))/viscosity(2)^2;
pi3_3 = (load(3)*density(3))/viscosity(3)^2;
hold on
figure
plot(pi3_1, 'o')
plot(pi3_2, '*')
plot(pi3_3, '<')
xlabel('pi 3')
legend('pi 3')

% set_5
b = -5:0.1:5;
pi5_1 = (density(1) .*v(1)^2)/p(1)* (load(1)*p(1)/(viscosity(1)^2 .* v(1)^2)).^b;
pi5_2 = (density(2) .*v(2)^2)/p(2)* (load(2)*p(1)/(viscosity(2)^2 .* v(2)^2)).^b;
pi5_3 = (density(3) .*v(3)^2)/p(3)* (load(3)*p(1)/(viscosity(3)^2 .* v(3)^2)).^b;
hold on
figure
```

```
plot(pi5_1, 'o')
plot(pi5_2, '*')
plot(pi5_3, '<')
xlabel('pi 5')
legend('pi 5')

% set_6
e = -5:0.1:5;
pi6_1 = (density(1) .*load(1))/viscosity(1)^2* (load(1)*E(1)/(viscosity(1)^2 .* v(1)^2)).^
e;
pi6_2 = (density(2) .*load(2))/viscosity(2)^2* (load(2)*E(2)/(viscosity(2)^2 .* v(2)^2)).^
e;
pi6_3 = (density(3) .*load(3))/viscosity(3)^2* (load(3)*E(3)/(viscosity(3)^2 .* v(3)^2)).^
e;

hold on
plot(pi6_1, e, 'o')
plot(pi6_2, e, '*')
plot(pi6_3, e, '<')
xlabel('pi 5')
ylabel('e')
legend('pi 6')
```







Published with MATLAB® R2016a