ME 7080 – Multidisciplinary Structural Optimization

Project: FEA Analysis of 2D warren Wing Structure with Beam elements

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Contents:

1. Abstract

This Analysis is based on custom FEA analysis of 2D warren truss wing structure using Beam element.The Structure is modelled start from node points, elements, with Dof using FEA Analysis. The material used is Al2024 T3, with density 2780Kg/m^3, Exx 73 GPa, poisons raio 0.33, analytical static FEA analysis and Modal analysis is done to get the global Stiffness matrix, mass matrix, the deflections, stresses, and frequency values. Total 17 elements with Area as a design variable, with 7 constraints are chosen for the sensitivity analysis get the sensitivity data for the given constraints with respect to each design variable. The single point function approximation methods are used like linear, reciprocal methods to get the next design point. The weight function, constraints, gradients on the concentrated elements are compared.

Finite Element Analysis of a 2D warren Truss Aircraft Wing Structure

Materials: Aluminum

Design variables: Elemental area for all members

$$W = \rho * \sum_{i=1}^{25} A_i * L_i$$

$$A_i : Area\ of\ memeber, L_i : Length\ of\ memeber$$

Objective: Minimum Weight optimization, frequency/Sensitivity analysis for Eigen value problem

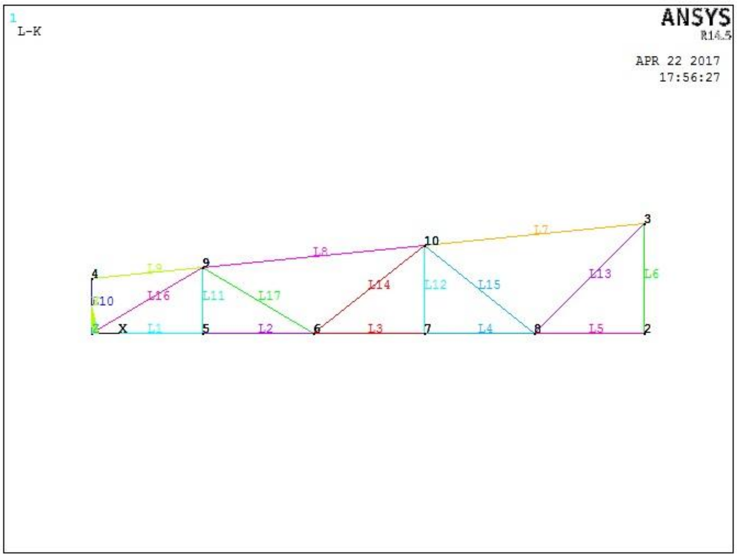
Goal of Objective: To have high stiffness, high strength, and low weight

Constraints: Tip deflections, fundamental Frequency.

Loads: Considered concentrated loads, on the selected location of the wing structure, engine load of 6800N at node_5, and fuel tank load at node_8 are applied.

Boundary conditions: Ux, Uy, θz at node_6, Ux, Uy at node_7

Check Stability of the structure: m = 2j-3, the structure is stable.



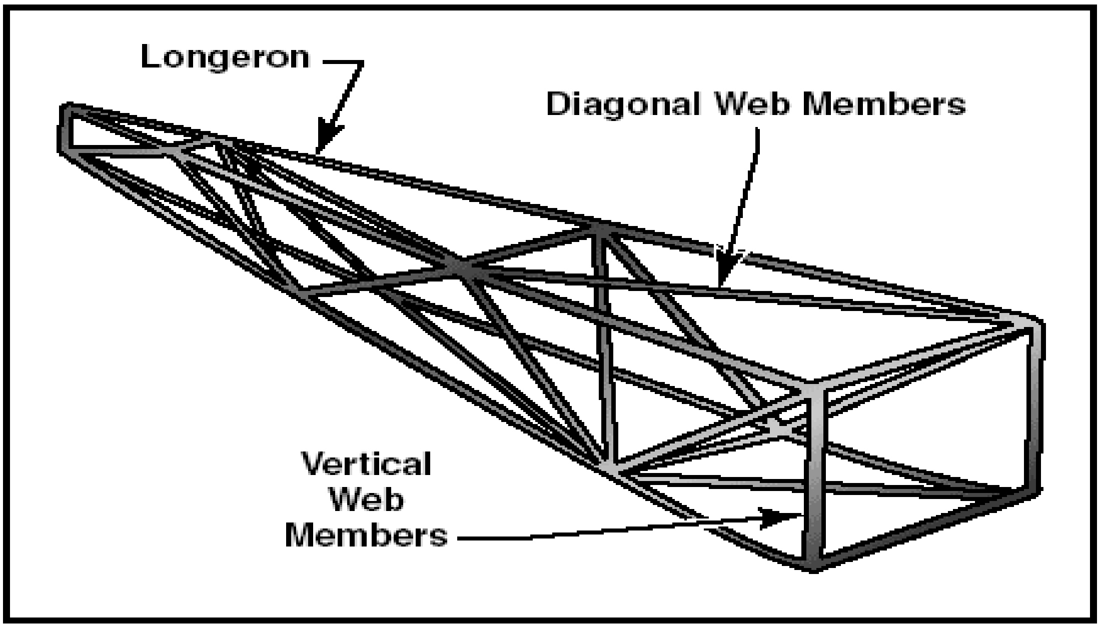
Nodes 1:10

Elements 1:17

Dof: 1:30

Loads: -F14 = -6800N, -F23 = -1360N

Bounadry conditions: U16,17,18,19,20 = 0



2. Scope of Project

The minimum weight optimization is very important in the initial design point that increase the fuel economy, performance, efficiency of the vehicle. By including all the primary design parameters, design variables like maximum deflection, stresses, vibration analysis, frequency, modes shapes at high and low frequency values can be optimized to the minimum points at the preliminary design stage.

The initial stage to learn the sensitivity with one or more design variables for the given linear or nonlinear function with appropriate constraints and bounds, with using the simplified model and apply the concepts.

Wing structure is modelled as a simple structure using beam elements withvariable areas. The dynamic, gust, loads are neglected for the analysis.

The deflection, stresses, and strains due to the static loads like self weight, engine weight, fuel and auxiliary parts in the wing causes dynamic loads behaviour, but for academic understanding customize the design problem with simple static loads as a point load at the engine location. Develop the model using Ansys static FEA analysis and Model analysis to get the deflections, stresses at nodal points and elemental stresses, and fundamental frequency at zero initial conditions.

Selected three Areas for upper, lower and diagonal elements and weight optimization with Area as a design variable and defined constrains with factor of safety.

3. Design Goals

Objective is to do the Weight optimization, with elemental Areas as design variables for a 2D Warren wing structure modelled used FEA simulation package (ANSYS) and MATLAB to optimize and run the iterations.

Objective:weight of the total structure, is product of density, area, length of each element. The selected variables are at tip node points of the lower members these are selected due to deflection is maximum at those nodes a trail run is simulated with some initial data points in ANSYS software. The loading on the wing structure considered in this case mainly due to engine weight, fuel weight for simplifying the problem applied concentrated loads at particular nodes.

Constraints: the tip displacements 1, 2, 3, 4, 5, 8 nodal points in the negative downward direction should be less than 20mm, the loading on the nodes 5, 8 gives maximum displacement and at tip 1. The stress is maximum in the element 5, 17 due to the concentrated loads.

Design variables: Elemental area's for all members

$$W = \rho * \sum_{i=1}^{25} A_i * L_i$$

$$A_i: Area\ of\ memeber, L_i: Length\ of\ memeber$$

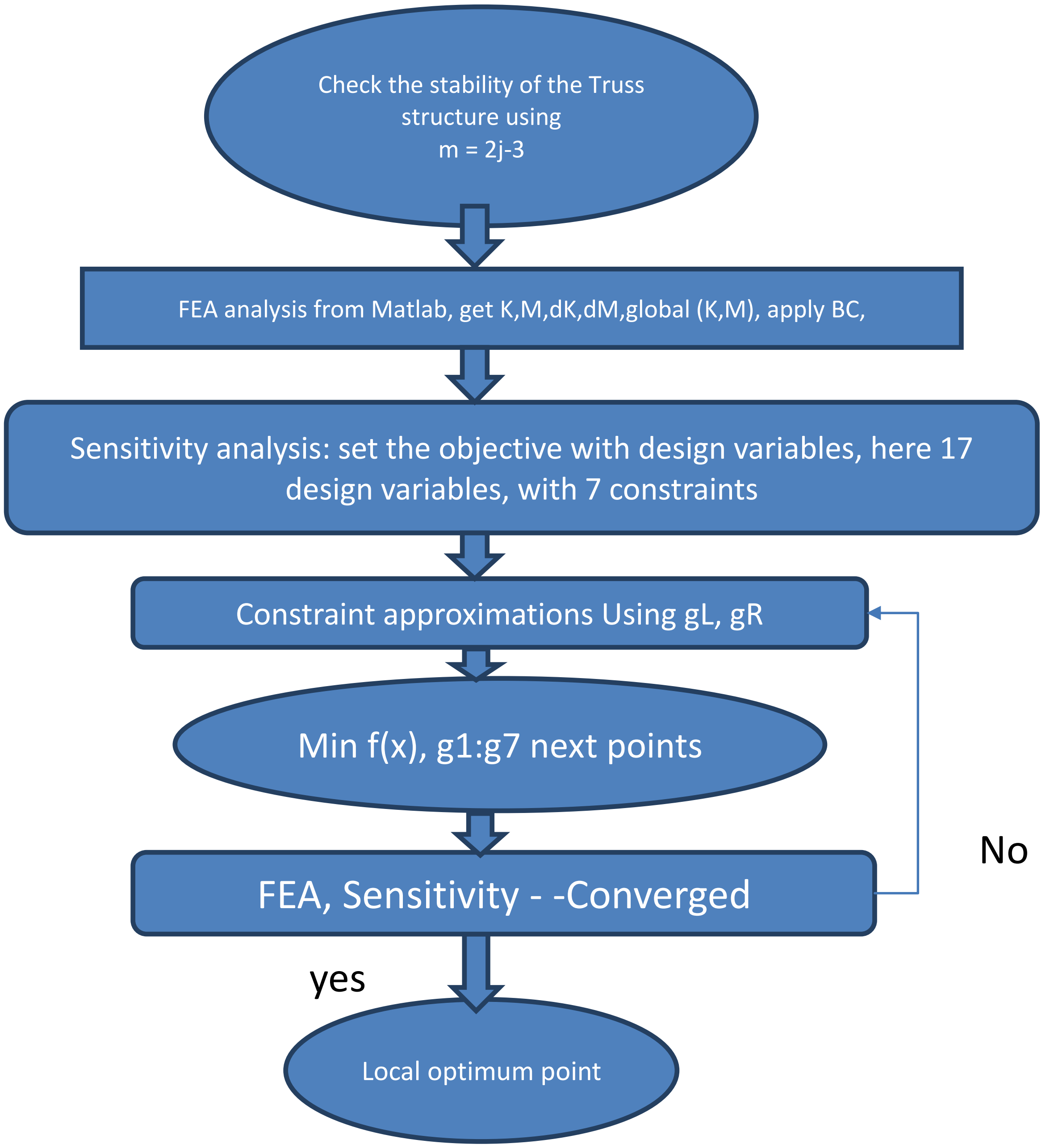
Considering FOS = 2

To find the minimum weight, for a given loading conditions and by sensitive analysis at selected nodal points like displacement values and fundamental frequency.

Goal of the analysis is to get the optimum point for the given loading conditions, boundary consitions and constraints.

4. Problem Formulation

Design optimization formulation, flow chart.

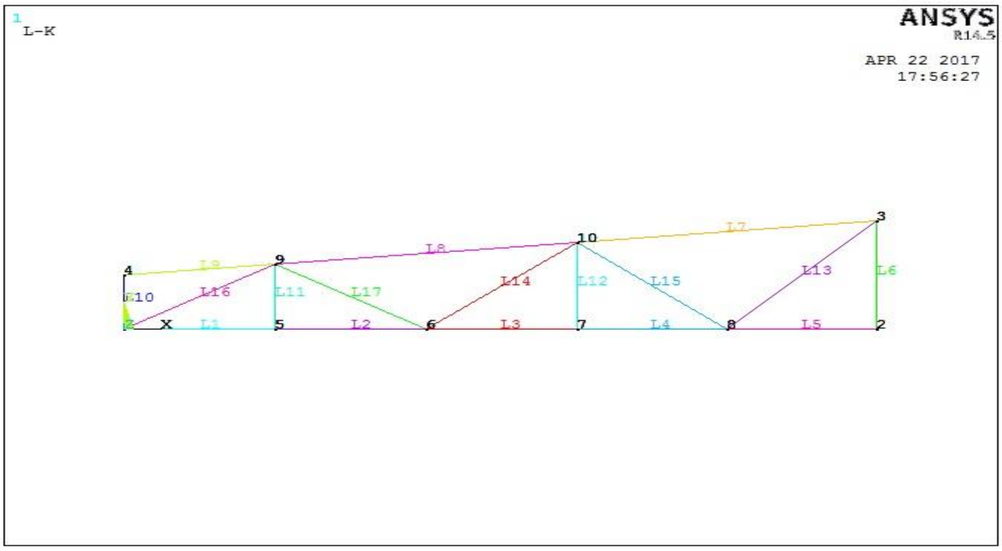


node points	x	y
1	0	0
2	0.8	0
3	1.6	0
4	2.4	0
5	3.2	0
6	4	0
7	4	0.8
8	2.4	0.64
9	0.8	0.48
10	0	0.4

Material Properties For Al2024 T3	S.I units
Exx	7.30E+10
density	2780
μ	0.33

ELEMENT CONNECTIVITY					DOF
ELEMENT NO.	NODE i	NODE j	Area	initial A	
1	1	2	A1	0.001257	1 2 3 4 5 6
2	2	3	A1	0.001257	4 5 6 7 8 9
3	3	4	A1	0.001257	7 8 9 10 11 12
4	4	5	A1	0.001257	10 11 12 13 14 15
5	5	6	A1	0.001257	13 14 15 16 17 18
6	7	8	A2	0.000707	16 17 18 19 20 21
7	8	9	A2	0.00181	19 20 21 22 23 24
8	9	10	A2	0.00181	22 23 24 25 26 27
9	10	1	A3	0.00181	25 26 27 28 29 30
10	1	9	A3	0.000707	28 29 30 1 2 3
11	2	9	A3	0.000707	1 2 3 25 26 27
12	9	3	A3	0.000707	25 26 27 4 5 6
13	3	8	A3	0.000707	25 26 27 7 8 9
14	4	8	A3	0.000707	7 8 9 22 23 24
15	8	5	A3	0.000707	22 23 24 10 11 12
16	5	7	A3	0.000707	22 23 24 13 14 15
17	6	7	A3	0.000707	13 14 15 19 20 21

Loading: F(14) = -68000; F(23) = -1360



Linear Static FEA:

The force vector is multiple of Stiffness matrix and displacement vector.

F = K U

2D Plane Frame (Beam): Axial, Shear, Bending

$$K = \frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

$$similarly \frac{dK}{dAi}$$

$$T = \begin{bmatrix} cos\theta & sin\theta & 00 & 0 & 0 \\ -sin\theta & cos\theta & 00 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0\ cos\theta & sin\theta & 0 \\ 0 & 0 & 0 - sin\theta & cos\theta & 0 \\ 0 & 0 & 0\ \ 0 & 0 & 1 \end{bmatrix}$$

K_loacal = T’ K T

$$\text{Mass Matrix} = \rho AL \begin{bmatrix} 2/6 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 156/420 & 22L/420 & 0 & 54/420 & -13L/420 \\ 0 & 22L/420 & 4L^2/420 & 0 & 13L/420 & -3L^2/420 \\ 1/6 & 0 & 0 & 2/6 & 0 & 0 \\ 0 & 54/420 & 13L/420 & 0 & 156/420 & -22L/420 \\ 0 & -13L/420 & -3L^2/420 & 0 & -22L/420 & 4L^2/420 \end{bmatrix}$$

$similarly\ dM/dAi$

M_local = T’ M T

Global Stiffness and Mass matrix:

Dof = 1:30

K_global(Dof,Dof)

M_global(Dof,Dof)

Boundary conditions: U16:20 = 0;

<u>Modal Analysis:</u>

Modal analysis is used to determine structure vibration characteristics – natural frequency and mode shapes.

$M\ddot{X} + KX = F(t)$ Equation of motion for Un-damped forced vibration

Eigen value theory

$M\ddot{X} + C\dot{X} + KX = F(t)$,

Where: M: Global Mass matrix

C: Global damping matrix

K: Global stiffness matrix

F: Global load vector

For undamped free vibration: $M\ddot{X} + KX = 0$

$x = A\ e^{-\omega it}, \ddot{x} = -\omega^2 Ae^{-\omega ti}$

$|K - \lambda M|X = 0$

Where: $\lambda = \omega^2, eigen\ value\ for\ each\ mode\ with\ natural\ frequency$

$X = eigen\ vector\ for\ each\ mode$

Sensitivity Analysis:

Sensitivity analysis of weight of a structure with respect to Design variables (Ai)

Weight = density * L*A

dW/dA(i) = density*L*

Direct method(x<g):

$$\frac{dg}{dx_i} = \frac{\partial g}{\partial x_i} + \left(\frac{\partial g}{\partial u}\right)' * \frac{du}{dx_i}$$

$$g: Constraint/objective$$

$$x_i: Design\ varoable$$

For Constraint_1; g1 = Ug(12) < 0.15 m (displacement Dof_12)

From Finite element analysis $\frac{dU}{dx_i} = K^{-1}\left[\frac{dF}{dx_i} - \frac{dK}{dx_i}U\right] x: A\ Design\ variable$

$$\frac{dg}{dx_i} = \frac{\partial g}{\partial x_i} + \left(\frac{\partial g}{\partial u}\right)' * K^{-1} * \left[\frac{dF}{dx_i} - \frac{dK}{dx_i}U\right]$$

$\frac{dUg}{du} =$ [0 1 zeros(28,1)] Total derivative Uglobal w.r.t U(1:30)

⇨ $\frac{\mathrm{dU}}{\mathrm{dA}1} = \left(-\frac{1}{20\mathrm{e}-03}\right) * \left(\frac{\mathrm{dUg}}{\mathrm{du}2}\right) * (-\mathrm{inv(Kg\_global)}) * \left(\frac{\mathrm{dKg}_{\mathrm{global}}}{dA1}\right) * \mathrm{Ug}$

For undamped free vibration: $M\ddot{X} + KX = 0$

$$x = A\,e^{-\omega it}, \ddot{x} = -\omega^2 Ae^{-\omega ti}$$

$$|K - \lambda M|X = 0$$

Where: $\lambda = \omega^2, eigen\ value\ for\ each\ mode\ with\ natural\ frequency$

$$X = eigen\ vector\ for\ each\ mode$$

Constraints:

g1 : f1 > 20 Hz

Sensitivity w.r.t t(i) $\frac{dg1}{dti} = \frac{\partial g1}{\partial ti} + \frac{\partial g1}{\partial f1} * [\frac{\partial f1}{\partial ti} + \frac{\partial f1}{\partial \lambda 1}\ (V'(\frac{dK}{dti} - \lambda * \frac{dM}{dti})V\ )]$

5. Problem statement

Objective function: W = r*A*L
Objective: Minimum Weight optimization, frequency/Sensitivity analysis for Eigen value problem
Constraints: g1(x): U2 < 20E-03
g2(x): U5 < 20E-03
g3(x): U8 < 20E-03
g4(x): U11 < 20E-03
g5(x): U14 < 20E-03
g6(x): U23 < 20E-03
g7(x): f1 > 20

Design variables: Elemental diameters for all members

$$W = \rho * \sum_{i=1}^{25} A_i * L_i$$

$$A_i : Area\ of\ memeber, L_i : Length\ of\ memeber$$

Considering FOS = 2
To find the minimum weight, for a given loading conditions and by sensitive analysis at selected nodal points like displacement values and fundamental frequency.

F = [F(14), F(23)] = [-68000, -1360]
BC's = U16,17,18,19,20 = 0

6. Solution approach

FEA analysis of 2D structure with 17 design variables and 7 constraints are considered for the sensitivity analysis. Weight sensitivity is also done with respect to the design variables.

Design variables: 17 {A1: A17}

Constraints: g1:

Constraint_1; g1 = Ug(2) < -20e-03 m (tip displacement Dof_2)

Constraint_2; g2 = Ug(5) < -20e-03 m (displacement Dof_5)

Constraint_3; g3 = Ug(8) < -20e-03 m (displacement Dof_8)

Constraint_4; g4 = Ug(11) < -20e-03 m (displacement Dof_11)

Constraint_5; g5 = Ug(14) < -10e-03 m (displacement Dof_14)

Constraint_6; g6 = Ug(23) < -10e-03 m (displacement Dof_23)

Constraint_7; g7 = freq(1) > 20 Hz (1st Natural Frequency > 20 Hz)

Initial design points in the weight function and constraint values gives, with free vibration, zero initial frequency.

weight(X0)	493.8933
g1	1.0255
g2	1.0248
g3	1.0249
g4	1.0271
g5	1.0203
g6	1.0243
g7	1

	A(initial)	A(next point)	% Error
weight(X0)	493.8933	-8.72802E+14	176.66
g1	1.0255	1	2.55
g2	1.0248	1	2.48
g3	1.0249	1	2.49
g4	1.0271	1	2.71
g5	1.0203	1	2.03
g6	1.0243	1	2.43
g7	1	1	0

```
r=2780, g=9.81
L =
[0.800000000000000,0.800000000000000,0.800000000000000,0.800000000000000,0.800000000000000,0.894427190999916,1.60000000000000,1.62185079461706,0.899777750336160,0.632455532033676,1.05830052442584,0.692820323027551,1.05830052442584,0.480000000000000,0.800000000000000,1.13137084989848,1.20000000000000];
s = [-1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
A =
[0.00125714285714286,0.00125714285714286,0.00125714285714286,0.00125714285714286,0.00125714285714286,0.000707142857142857,0.00181028571428571,0.00181028571428571,0.00181028571428571,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857];
alpha = -1:0.1:1;
x1 = A(1) + alpha*s(1);
x2
.
.
.

g = r*g*(x1*L(1)+x2*L(2)+x3*L(3)+x4*L(4) . . .

linear approx.
gL = g_x0 + (x1 - A(1)).*dg_x1 + (x2 - A(2)).*dg_x2 . . .

Reciprocal approx..
gR = g_x0 + (x1 - A(1)).*(A(1)./x1).*dg_x1 + (x2 - A(2)).*(A(2)./x2). . .
```

7. Discussion of Results

K_global after applying Boundary conditions:

2.24E+08	24124136	-10183.5	-1.1E+08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2.8E+07	-2.4E+07	-10183.5	-8.2E+07	0	0
24124136	21274336	54239.04	0	-215089.3	86035.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2.4E+07	-2.1E+07	11758.9	0	-137735	-43555.6
-10183.5	54239.04	75225.33	0	-86035.71	22942.86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10183.51	-11758.9	5487.487	0	43555.58	9182.323
-1.1E+08	0	0	3.04E+08	0	0	-1.1E+08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7.5E+07	0	0	0	0	0
0	-215089	-86035.7	0	534957.01	36296.32	0	-215089	86035.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-104778	36296.32	0	0	0
0	86035.71	22942.86	0	36296.317	108536	0	-86035.7	22942.86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-36296.3	8382.275	0	0	0
0	0	0	-1.1E+08	0	0	3.11E+08	29490649	-43286	-1.1E+08	0	0	0	0	0	0	0	0	0	0	0	-5.4E+07	-5.4E+07	-53469.5	-2.8E+07	24124136	10183.51	0	0	0
0	0	0	0	-215089.3	-86035.7	29490649	75281548	65228.43	0	-215089	86035.71	0	0	0	0	0	0	0	0	0	-5.4E+07	-5.4E+07	53469.52	24124136	-2.1E+07	11758.9	0	0	0
0	0	0	0	86035.714	22942.86	-43286	65228.43	126943.9	0	-86035.7	22942.86	0	0	0	0	0	0	0	0	0	53469.52	-53469.5	12098.77	-10183.5	-11758.9	5487.487	0	0	0
0	0	0	0	0	0	-1.1E+08	0	0	2.94E+08	0	0	-1.1E+08	0	0	0	0	0	0	0	0	-6.5E+07	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-215089	-86035.7	0	498234.2	27222.24	0	-215089	86035.71	0	0	0	0	0	0	0	-68055.6	27222.24	0	0	0	0	0	0
0	0	0	0	0	0	0	86035.71	22942.86	0	27222.24	106290	0	-86035.7	22942.86	0	0	0	0	0	0	0	-27222.2	7259.263	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-1.1E+08	0	0	2.71E+08	-1435908	606.622	0	0	0	0	0	-9017.89	-2.3E+07	22801633	9624.514	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-215089	-86035.7	-1435908	47163645	17690.36	0	0	0	0	0	8065.848	22801633	-2.3E+07	9624.514	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	86035.71	22942.86	606.622	17690.36	111716.6	0	0	0	0	0	4839.509	-9624.51	-9624.51	5133.074	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-9017.89	8065.848	4839.509	0	0	0	0	0	70239.09	-4437.96	44379.57	23787.15	0	0	0	0	0	0
0	0	0	0	0	0	-5.4E+07	-5.4E+07	53469.52	-6.5E+07	0	0	-2.3E+07	22801633	-9624.51	0	0	0	0	0	-4437.96	3.04E+08	47047461	43726.23	-8.1E+07	-8062177	4319.179	0	0	0
0	0	0	0	0	0	-5.4E+07	-5.4E+07	-53469.5	0	-68055.6	-27222.2	22801633	-2.3E+07	-9624.51	0	0	0	0	0	44379.57	47047461	78556318	-89128.5	-8062177	-859746	-43191.8	0	0	0
0	0	0	0	0	0	-53469.5	53469.52	12098.77	0	27222.24	7259.263	9624.514	9624.514	5133.074	0	0	0	0	0	23787.15	43726.23	-89128.5	143489.9	-4319.18	43191.79	23466.68	0	0	0
-2.8E+07	-2.4E+07	10183.51	-7.5E+07	0	0	-2.8E+07	24124136	-10183.5	0	0	0	0	0	0	0	0	0	0	0	0	-8.1E+07	-8062177	-4319.18	3.56E+08	22572778	9713.886	-1.5E+08	-1.5E+07	14033.07
-2.4E+07	-2.1E+07	-11758.9	0	-104778.4	-36296.3	24124136	-2.1E+07	-11758.9	0	0	0	0	0	0	0	0	0	0	0	0	-8062177	-859746	43191.79	22572778	44572087	-156953	-1.5E+07	-1764539	-140331
-10183.5	11758.9	5487.487	0	36296.317	8382.275	10183.51	11758.9	5487.487	0	0	0	0	0	0	0	0	0	0	0	0	4319.179	-43191.8	23466.68	9713.886	-156953	170245.3	-14033.1	140330.7	42298.72
-8.2E+07	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1.5E+08	-1.5E+07	-14033.1	2.27E+08	14510602	-14033.1
0	-137735	43555.58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1.5E+07	-1764539	140330.7	14510602	1902274	183886.2
0	-43555.6	9182.323	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14033.07	-140331	42298.72	-14033.1	183886.2	102962.1

M_global after applying Boundary conditions:

2.073857	-0.03922	-0.0755	0.465981	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.312778	0.039222	0.044614	0.20722	0	0
-0.03922	2.239054	0.163153	0	0.359471	-0.06923	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.039222	0.301456	-0.05152	0	0.159855	0.024339
-0.0755	0.163153	0.04397	0	0.0692315	-0.01278	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.04461	0.051516	-0.01664	0	-0.02434	-0.00355
0.465981	0	0	2.317919	0	0	0.465981	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.226998	0	0	0	0	0
0	0.359471	0.069231	0	2.5828241	0.049427	0	0.359471	-0.06923	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.175112	-0.02921	0	0	0
0	-0.06923	-0.01278	0	0.0494273	0.040309	0	0.069231	-0.01278	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.029207	-0.00467	0	0	0
0	0	0	0.465981	0	0	2.923891	0.021248	0.058725	0.465981	0	0	0	0	0	0	0	0	0	0	0	0.139295	0.017974	0.009913	0.312778	-0.03922	-0.04461	0	0	0
0	0	0	0	0.359471	0.069231	0.021248	3.148233	0.103958	0	0.359471	-0.06923	0	0	0	0	0	0	0	0	0	0.017974	0.139295	-0.00991	-0.03922	0.301456	-0.05152	0	0	0
0	0	0	0	-0.069231	-0.01278	0.058725	0.103958	0.058345	0	0.069231	-0.01278	0	0	0	0	0	0	0	0	0	-0.00991	0.009913	-0.00155	0.044614	0.051516	-0.01664	0	0	0
0	0	0	0	0	0	0.465981	0	0	2.388152	0	0	0.465981	0	0	0	0	0	0	0	0	0.262114	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.359471	0.069231	0	2.661084	0.065903	0	0.359471	-0.06923	0	0	0	0	0	0	0	0.202202	-0.03894	0	0	0	0	0	0
0	0	0	0	0	0	0	-0.06923	-0.01278	0	0.065903	0.043669	0	0.069231	-0.01278	0	0	0	0	0	0	0	0.038943	-0.00719	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.465981	0	0	3.483928	-0.00229	-0.01732	0	0	0	0	0	0.065309	0.328322	-0.04236	-0.05507	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.359471	0.069231	-0.00229	3.686963	0.192055	0	0	0	0	0	-0.05841	-0.04236	0.328322	-0.05507	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.06923	-0.01278	-0.01732	0.192055	0.093549	0	0	0	0	0	-0.02426	0.055073	0.055073	-0.02033	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.065309	-0.05841	-0.02426	0	0	0	0	0	0.242068	0.039679	-0.39679	-0.14724	0	0	0	0	0	0
0	0	0	0	0	0	0.139295	0.017974	-0.00991	0.262114	0	0	0.328322	-0.04236	0.055073	0	0	0	0	0	0.039679	7.051346	-0.03677	-0.07458	1.357274	0.030786	-0.04077	0	0	0
0	0	0	0	0	0	0.017974	0.139295	0.009913	0	0.202202	0.038943	-0.04236	0.328322	0.055073	0	0	0	0	0	-0.39679	-0.03677	7.716713	-0.19435	0.030786	1.052494	0.407706	0	0	0
0	0	0	0	0	0	0.009913	-0.00991	-0.00155	0	-0.03894	-0.00719	-0.05507	-0.05507	-0.02033	0	0	0	0	0	-0.14724	-0.07458	-0.19435	0.439561	0.040771	-0.40771	-0.15335	0	0	0
0.312778	0.039222	-0.04461	0.226998	0	0	0.312778	-0.03922	0.044614	0	0	0	0	0	0	0	0	0	0	0	0	1.357274	0.030786	0.040771	6.143805	-0.04787	-0.04776	0.752995	0.01708	-0.01255
0.039222	0.301456	0.051516	0	0.1751125	0.029207	-0.03922	0.301456	0.051516	0	0	0	0	0	0	0	0	0	0	0	0	0.030786	1.052494	-0.40771	-0.04787	6.692203	0.253813	0.01708	0.583907	0.125486
0.044614	-0.05152	-0.01664	0	-0.029207	-0.00467	-0.04461	-0.05152	-0.01664	0	0	0	0	0	0	0	0	0	0	0	0	-0.04077	0.407706	-0.15335	-0.04776	0.253813	0.289997	0.012549	-0.12549	-0.02619
0.20722	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.752995	0.01708	0.012549	1.925552	-0.01708	-0.02124
0	0.159855	-0.02434	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.01708	0.583907	-0.12549	-0.01708	2.142005	0.253551
0	0.024339	-0.00355	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.01255	0.125486	-0.02619	-0.02124	0.253551	0.039651

Natural frequency and sensitivity:

The objective function sensitivity and constraints sensitivity at the initial design points:

element no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
L	0.8	0.8	0.8	0.8	0.8	0.894427	1.6	1.621851	0.899778	0.632456	1.058301	0.69282	1.058301	0.48	0.8	1.131371	1.2
A	0.001257	0.001257	0.001257	0.001257	0.0012571	0.000707	0.00181	0.00181	0.00181	0.000707	0.000707	0.000707	0.000707	0.000707	0.000707	0.000707	0.000707
Natural Freq 1:30	2798.498	2396.299	2010.144	1701.2	1378.336	1209.494	948.2047	799.9746	661.4489	609.4053	530.3492	408.1928	334.121	284.8937	265.1098	228.5831	189.7413
				170.8944	150.02083	138.3191	84.51267	46.97778	51.00804	59.39755	66.53148	0.159155	0.159155	0.159155	0.159155	0.159155	
Weight sensitivity	279.5886	279.5886	279.5886	279.5886	279.58857	234.4421	671.0126	680.1764	377.3514	165.7756	277.3957	181.5981	277.3957	125.8149	209.6914	296.5485	314.5371
dg1/da	-0.0089	0.0554	-0.1074	-0.0629	4.9425	1.06E-05	0.1008	-0.0437	7.17E-04	0.0033	6.18E-05	0.3837	0.0858	0.0437	1.1536	3.6449	21.7262
dg2/da	-0.0019	-0.0295	0.0102	0.021	-0.1256	1.65E-05	-0.0029	0.0276	-0.0059	-0.0271	-1.29E-05	-0.0157	-0.0481	0.0057	-0.0714	0.092	-0.3973
dg3/da	2.14E-04	0.0392	-0.0492	0.0442	4.2866	4.2866	1.06E-05	0.0751	0.031	1.36E-04	2.87E-07	0.0918	0.0033	0.0477	0.7113	3.7559	22.3899
dg4/da	4.24E-04	0.0165	-0.0108	0.0715	3.7305	-6.54E-05	0.0326	0.0142	-3.97E-05	-2.22E-04	3.91E-07	0.0293	-0.0016	0.0193	0.321	2.3598	28.4264
dg5/da	3.68E-05	0.0037	0.002	0.0497	4.4242	1.20E-05	-0.012	0.0029	1.92E-05	8.69E-05	1.46E-08	0.0082	3.52E-04	2.82E-04	0.1	0.0506	48.9295
dg6/da	1.36E-04	2.09E-02	0.0127	0.2927	7.3194	2.36E-05	0.1013	0.0164	8.23E-05	3.73E-04	1.33E-07	0.0482	0.0018	9.41E-04	0.5792	7.7262	45.9963
dg7/da	8072.827	-1113.75	13528.04	-9452.57	-1031.359	-22889.8	-1466.48	-3065.89	-848.089	-384.111	-35.2504	-29.4232	-1171.04	300.2866	-2507.13	-1595.11	-327.311

The weight sensitivity on the element_7, _8 are more for the given conditions around 670-680

The static FEA, displacement sensitivity is more prone to element_17, for U2,U5 and much more for U8, U11, U5, U8 around 28-46

The vibration sensitivity is more for element_6 is about -22889

From the above analysis results more chance failure can occur is on the element_17, for the given loading conditions and Boundary conditions.

for next design point, for concentrated element_17

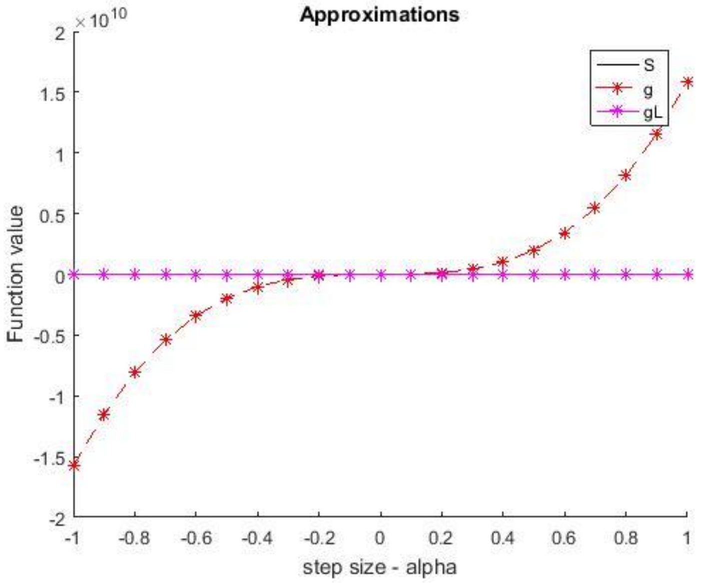
dg1/da	1.48E-24
dg2/da	-2.83E-18
dg3/da	1.30E-24
dg4/da	5.81E-17
dg5/da	2.24E-24
dg6/da	2.41E-24
dg7/da	0.557333623928322 - 0.0262797143207266i

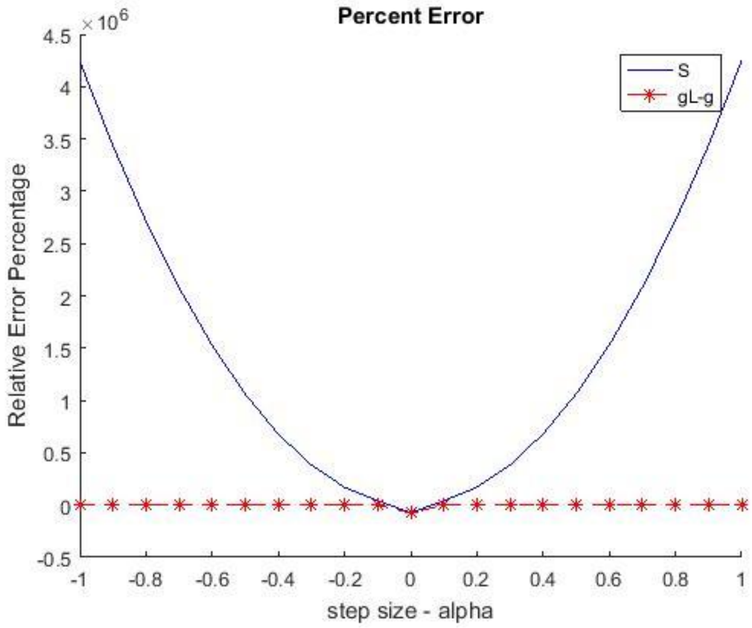
Function Approximations

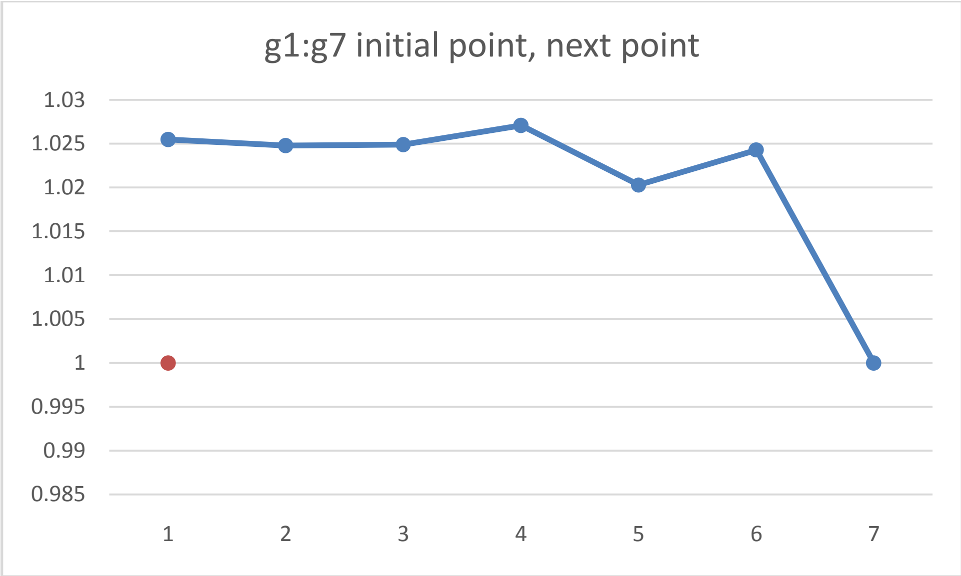
Linear, Reciprocal Function Approximation methods are used to find the next design point for the weight function, gradients of constraints are calculated from the above table gradient values and weight value at next design point is -872801977054907, means the search direction and design point is in infeasible direction.

The error % is 176.66 for the weight value.

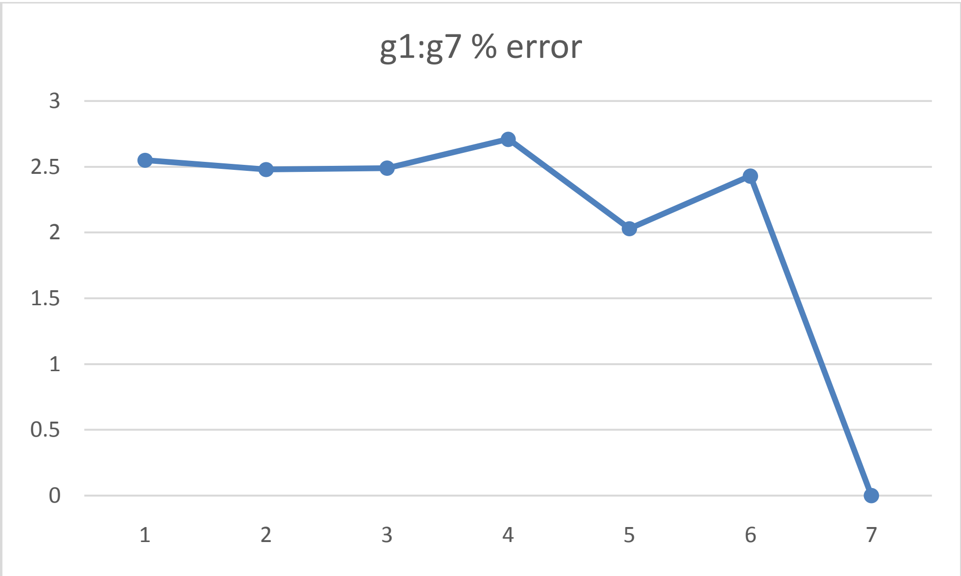
	A(initial)	A(next point)	% Error
weight(X0)	493.8933	-8.72802E+14	176.66
g1	1.0255	1	2.55
g2	1.0248	1	2.48
g3	1.0249	1	2.49
g4	1.0271	1	2.71
g5	1.0203	1	2.03
g6	1.0243	1	2.43
g7	1	1	0


Approximations
× 10^10
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Function value
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
step size - alpha
S
g
gL


Percent Error
× 10^6
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
Relative Error Percentage
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
step size - alpha
S
gL-g

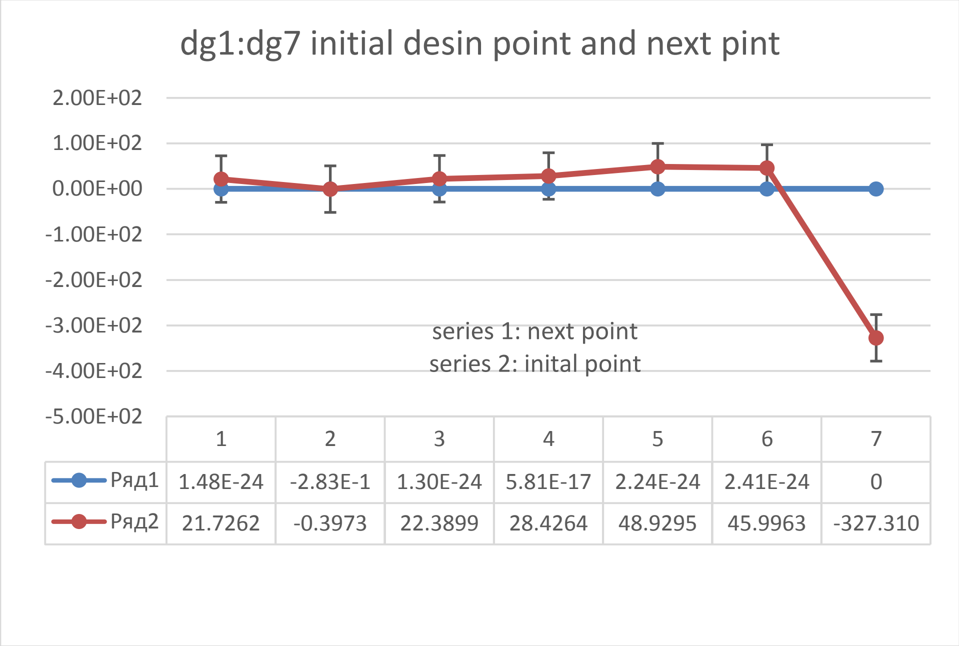


The graph shows blue colour is initial point g1:g7 values and for next point g1:g7 are constant at ‘1’.



The above graph shows the error% for g1:g7 for two design points initial, and next point.

Design variables have varying effects on the constraint responses.



	1	2	3	4	5	6	7
Ряд1	1.48E-24	-2.83E-1	1.30E-24	5.81E-17	2.24E-24	2.41E-24	0
Ряд2	21.7262	-0.3973	22.3899	28.4264	48.9295	45.9963	-327.310

The above graph shows gradient information only for selected element_17, it is more effected due to stresses. The sensitivity at dg7/df frequency is changed from zero to –ve value and the static FEA displacement sensitivity not much differed.

8. Summary and Recommendations

Summary:

The next design point is in infeasible domain, the error percentage for the approximation and actual data for weight function is more 176.66%.

	A(initial)	A(next point)	% Error
weight(Ai)	493.8933	-8.72802E+14	176.66

For a small perturbation value for alpha, the gradients are moved jumped to infeasible direction.

Recommendations:

Penalty methods are to be used to get the minimum optimum design point.

More detailed analysis can be done using penalty methods, different search directions to get the optimum point and to converge the solution.

The constraints can be extended broadly including stresses, Buckling factor, forced vibration analysis for more realistic manner.

Contents


```
clear all
close all
clc
```

FEA Analysis %%

```
       E = 73e9, r=2780
       n1=[0,0];
       n2=[0.8,0];
       n3=[1.6,0];
       n4=[2.4,0];
       n5=[3.2,0];
       n6=[4,0];
       n7=[4,0.8];
       n8=[2.4,0.64];
       n9=[0.8,0.48];
       n10=[0,0.4];

       L1 = sqrt(((n2(1)-n1(1))^2+(n2(2)-n1(2)^2)));
       L2 = sqrt(((n3(1)-n2(1))^2+(n3(2)-n2(2)^2)));
       L3 = sqrt(((n4(1)-n3(1))^2+(n4(2)-n3(2)^2)));
       L4 = sqrt(((n5(1)-n4(1))^2+(n5(2)-n4(2)^2)));
       L5 = sqrt(((n6(1)-n5(1))^2+(n6(2)-n5(2)^2)));
       L6 = sqrt(((n7(1)-n6(1))^2+(n7(2)-n6(2)^2)));
       L7 = sqrt(((n8(1)-n7(1))^2+(n8(2)-n7(2)^2)));
       L8 = sqrt(((n9(1)-n8(1))^2+(n9(2)-n8(2)^2)));
       L9 = sqrt(((n10(1)-n9(1))^2+(n10(2)-n9(2)^2)));
       L10 = sqrt(((n10(1)-n1(1))^2+(n10(2)-n1(2)^2)));
       L11 = sqrt(((n9(1)-n1(1))^2+(n9(2)-n1(2)^2)));
       L12 = sqrt(((n9(1)-n2(1))^2+(n9(2)-n2(2)^2)));
       L13 = sqrt(((n9(1)-n3(1))^2+(n9(2)-n3(2)^2)));
       L14 = sqrt(((n3(1)-n8(1))^2+(n3(2)-n8(2)^2)));
       L15 = sqrt(((n8(1)-n4(1))^2+(n8(2)-n4(2)^2)));
       L16 = sqrt(((n8(1)-n5(1))^2+(n8(2)-n5(2)^2)));
       L17 = sqrt(((n7(1)-n5(1))^2+(n7(2)-n5(2)^2)));
       tetha1 = 0;
       tetha2 = 0;
       tetha3 = 0;
```

```
       tetha4 = 0;
       tetha5 = 0;
       tetha6 = 90;
       tetha7 = 180 + atand(0.16/(L4+L5));
       tetha8 = 180 + atand(0.16/(L2+L3));
       tetha9 = 180 + atand(0.08/L1);
       tetha10 = 0;
       tetha11 = atand(L12/L1);
       tetha12 = 0;
       tetha13 = 180 - atand(L12/L2);
       tetha14 = atand(L15/L3);
       tetha15 = 0;
       tetha16 = 180 - atand(L15/L4);
       tetha17 = atand(L6/L5);

       L = [L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14 L15 L16 L17]
       thet = [tetha1 tetha2 tetha3 tetha4 tetha5 tetha6 tetha7 tetha8 tetha9 tetha10 tetha11
 tetha12 tetha13 tetha14 tetha15 tetha16 tetha17];
       R = [0.020 0.020 0.020 0.020 0.020 0.015 0.024 0.024 0.024 0.015 0.015 0.015 0.015 0.0
15 0.015 0.015 0.015]
       A = (22/7)*R.^2
       I = (22/7)*R.^4 /4

       for i=1:17;  % Beam elements %
            k_localbeam(:,:,i) = [(E*A(i))/L(i),                  0,                  0, -(E*
A(i))/L(i),                  0,                  0;
                                                                      0,  (12*E*I(i))/L(i)^3, (6*E
*I(i))/L(i)^2,                                  0, -(12*E*I(i))/L(i)^3,   (6*E*I(i))/L(i)
^2;
                                                                      0,   (6*E*I(i))/L(i)^2,   (4
*E*I(i))/L(i),                                  0,  -(6*E*I(i))/L(i)^2,     (2*E*I(i))/L(
i);
                            -(E*A(i))/L(i),                  0,                  0, (E*A(i))
/L(i),                  0,                  0;
                                                                      0, -(12*E*I(i))/L(i)^3, -(6*
E*I(i))/L(i)^2,                                  0,  (12*E*I(i))/L(i)^3, -(6*E*I(i))/L(i)
^2;
                                                                      0,   (6*E*I(i))/L(i)^2,    (
2*E*I(i))/L(i),                                  0,  -(6*E*I(i))/L(i)^2,    (4*E*I(i))/L(
i)];


             M_localbeam(:,:,i)=(r*A(i)*L(i))*[2/6 0 0 1/6 0 0;
                                 0 156/420 (22*L(i))/420 0 54/420 (-13*L(i))/420;
                            0 (22*L(i))/420 (4*L(i)*L(i))/420 0 (13*L(i))/420 (-3*L(i)*L(i))/
420;
                                                                     1/6 0 0 2/6 0 0;
                                       0 54/420 (13*L(i))/420 0 156/420 (-22*L(i))/420;
                            0 (-13*L(i))/420 (-3*L(i)*L(i))/420 0 (-22*L(i))/420 (4*L(i)*L(i)
)/420];

            dk_da_localbeam(:,:,i) = [(E)/L(i),                  0,                  0, -(E)/
L(i),                  0,                  0;
                                                                      0,  (12*E*I(i))/L(i)^3, (6*E
*I(i))/L(i)^2,                                  0, -(12*E*I(i))/L(i)^3,   (6*E*I(i))/L(i)
^2;
                                                                      0,   (6*E*I(i))/L(i)^2,   (4
```

```
*E*I(i))/L(i),                                          0,  -(6*E*I(i))/L(i)^2,     (2*E*I(i))/L(
i);
                                   -(E)/L(i),                    0,                   0, (E)/L
(i),                    0,                    0;
                                                                 0, -(12*E*I(i))/L(i)^3, -(6*
E*I(i))/L(i)^2,                                     0,  (12*E*I(i))/L(i)^3, -(6*E*I(i))/L(i)
^2;
                                                                 0,   (6*E*I(i))/L(i)^2,    (
2*E*I(i))/L(i),                                     0,  -(6*E*I(i))/L(i)^2,    (4*E*I(i))/L(
i)];

      dM_da_localbeam(:,:,i)= (r*L(i))*[2/6 0 0 1/6 0 0;
                                 0 156/420 (22*L(i))/420 0 54/420 (-13*L(i))/420;
                            0 (22*L(i))/420 (4*L(i)*L(i))/420 0 (13*L(i))/420 (-3*L(i)*L(i))/
420;
                                                                1/6 0 0 2/6 0 0;
                                       0 54/420 (13*L(i))/420 0 156/420 (-22*L(i))/420;
                            0 (-13*L(i))/420 (-3*L(i)*L(i))/420 0 (-22*L(i))/420 (4*L(i)*L(i)
)/420];

       T_beam(:,:,i)=[cosd(thet(i)) sind(thet(i)) 0 0 0 0;
                     -sind(thet(i)) cosd(thet(i)) 0 0 0 0;
                                                   0 0 1 0 0 0;
                      0 0 0 cosd(thet(i)) sind(thet(i)) 0;
                     0 0 0 -sind(thet(i)) cosd(thet(i)) 0;
                                                   0 0 0 0 0 1];
       k_beam(:,:,i) = T_beam(:,:,i)'*k_localbeam(:,:,i)*T_beam(:,:,i);    % k global for Bea
m elements %
       M_beam(:,:,i) = T_beam(:,:,i)'*M_localbeam(:,:,i)*T_beam(:,:,i);
       dk_da_beam(:,:,i) = T_beam(:,:,i)'*dk_da_localbeam(:,:,i)*T_beam(:,:,i) ;   % dk/dt  g
lobal Beam elements %
       dM_da_beam(:,:,i) = T_beam(:,:,i)'*dM_da_localbeam(:,:,i)*T_beam(:,:,i) ;

       end
```

```
E =

   7.3000e+10


r =

        2780


L =

  Columns 1 through 7

    0.8000    0.8000    0.8000    0.8000    0.8000    0.8944    1.6000

  Columns 8 through 14

    1.6219    0.8998    0.6325    1.0583    0.6928    1.0583    0.4800
```

```
  Columns 15 through 17

    0.8000    1.1314    1.2000


R =

  Columns 1 through 7

    0.0200    0.0200    0.0200    0.0200    0.0200    0.0150    0.0240

  Columns 8 through 14

    0.0240    0.0240    0.0150    0.0150    0.0150    0.0150    0.0150

  Columns 15 through 17

    0.0150    0.0150    0.0150


A =

  Columns 1 through 7

    0.0013    0.0013    0.0013    0.0013    0.0013    0.0007    0.0018

  Columns 8 through 14

    0.0018    0.0018    0.0007    0.0007    0.0007    0.0007    0.0007

  Columns 15 through 17

    0.0007    0.0007    0.0007


I =

   1.0e-06 *

  Columns 1 through 7

    0.1257    0.1257    0.1257    0.1257    0.1257    0.0398    0.2607

  Columns 8 through 14

    0.2607    0.2607    0.0398    0.0398    0.0398    0.0398    0.0398

  Columns 15 through 17

    0.0398    0.0398    0.0398
```

Global Stiffness Matrix (kg_global = kg_truss + kg_beam) %%

```
format short
kg_global=zeros(30,30,1); dkg_global = zeros(30,30,17);
```

```
     Mg_global=zeros(30,30,1); dMg_global = zeros(30,30,17);
for i = 1:17;
     Dof = zeros(17,17);
Dof = [1 2 3 4 5 6;4 5 6 7 8 9; 7 8 9 10 11 12; 10 11 12 13 14 15; 13 14 15 16 17 18; 16 17 1
8 19 20 21; 19 20 21 22 23 24; 22 23 24 25 26 27; 25 26 27 28 29 30;28 29 30 1 2 3;1 2 3 25 2
6 27; 4 5 6 25 26 27; 25 26 27 7 8 9; 7 8 9 22 23 24; 10 11 12 22 23 24; 22 23 24 13 14 15; 1
3 14 15 19 20 21];
     kg_global(Dof(i,:),Dof(i,:),1)= kg_global(Dof(i,:),Dof(i,:),1)+ k_beam(:,:,i);
     dkg_global(Dof(i,:),Dof(i,:),i)= dkg_global(Dof(i,:),Dof(i,:),i)+ dk_da_beam(:,:,i);
     Mg_global(Dof(i,:),Dof(i,:),1)= Mg_global(Dof(i,:),Dof(i,:),1)+M_beam(:,:,i);
     dMg_global(Dof(i,:),Dof(i,:),i)= dMg_global(Dof(i,:),Dof(i,:),i)+dM_da_beam(:,:,i);

end
     kg_beforeBC = kg_global;
     Mg_beforeBC = Mg_global;
     kg_beforeBC, Mg_beforeBC
```

```
kg_beforeBC =

   1.0e+08 *

  Columns 1 through 7

    2.2422    0.2412   -0.0001   -1.1471         0         0         0
    0.2412    0.2127    0.0005         0   -0.0022    0.0009         0
   -0.0001    0.0005    0.0008         0   -0.0009    0.0002         0
   -1.1471         0         0    3.0394         0         0   -1.1471
         0   -0.0022   -0.0009         0    0.0053    0.0004         0
         0    0.0009    0.0002         0    0.0004    0.0011         0
         0         0         0   -1.1471         0         0    3.1124
         0         0         0         0   -0.0022   -0.0009    0.2949
         0         0         0         0    0.0009    0.0002   -0.0004
         0         0         0         0         0         0   -1.1471
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0   -0.5393
         0         0         0         0         0         0   -0.5361
         0         0         0         0         0         0   -0.0005
   -0.2789   -0.2412    0.0001   -0.7451         0         0   -0.2789
   -0.2412   -0.2092   -0.0001         0   -0.0010   -0.0004    0.2412
   -0.0001    0.0001    0.0001         0    0.0004    0.0001    0.0001
   -0.8162         0         0         0         0         0         0
         0   -0.0014    0.0004         0         0         0         0
         0   -0.0004    0.0001         0         0         0         0

  Columns 8 through 14
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0022	0.0009	0	0	0	0	0
-0.0009	0.0002	0	0	0	0	0
0.2949	-0.0004	-1.1471	0	0	0	0
0.7528	0.0007	0	-0.0022	0.0009	0	0
0.0007	0.0013	0	-0.0009	0.0002	0	0
0	0	2.9396	0	0	-1.1471	0
-0.0022	-0.0009	0	0.0050	0.0003	0	-0.0022
0.0009	0.0002	0	0.0003	0.0011	0	-0.0009
0	0	-1.1471	0	0	2.7138	-0.0144
0	0	0	-0.0022	-0.0009	-0.0144	0.4716
0	0	0	0.0009	0.0002	0.0000	0.0002
0	0	0	0	0	-1.1471	0
0	0	0	0	0	0	-0.0022
0	0	0	0	0	0	0.0009
0	0	0	0	0	-0.1913	-0.2137
0	0	0	0	0	-0.2137	-0.2391
0	0	0	0	0	-0.0001	0.0001
-0.5361	0.0005	-0.6453	0	0	-0.2283	0.2280
-0.5393	-0.0005	0	-0.0007	-0.0003	0.2280	-0.2283
0.0005	0.0001	0	0.0003	0.0001	0.0001	0.0001
0.2412	-0.0001	0	0	0	0	0
-0.2092	-0.0001	0	0	0	0	0
0.0001	0.0001	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 21

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0009	0	0	0	0	0	0
0.0002	0	0	0	0	0	0
0.0000	-1.1471	0	0	-0.1913	-0.2137	-0.0001
0.0002	0	-0.0022	0.0009	-0.2137	-0.2391	0.0001
0.0011	0	-0.0009	0.0002	0.0001	-0.0001	0.0000
0	1.1476	0	-0.0002	-0.0005	0	-0.0002
-0.0009	0	0.5793	-0.0009	0	-0.5771	0
0.0002	-0.0002	-0.0009	0.0006	0.0002	0	0.0001
0.0001	-0.0005	0	0.0002	1.0096	0.2954	0.0004
-0.0001	0	-0.5771	0	0.2954	0.8250	-0.0005
0.0000	-0.0002	0	0.0001	0.0004	-0.0005	0.0007
-0.0001	0	0	0	-0.8178	-0.0817	-0.0000
-0.0001	0	0	0	-0.0817	-0.0087	0.0004

```
    0.0001         0         0         0    0.0000   -0.0004    0.0002
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 22 through 28

         0         0         0   -0.2789   -0.2412   -0.0001   -0.8162
         0         0         0   -0.2412   -0.2092    0.0001         0
         0         0         0    0.0001   -0.0001    0.0001         0
         0         0         0   -0.7451         0         0         0
         0         0         0         0   -0.0010    0.0004         0
         0         0         0         0   -0.0004    0.0001         0
   -0.5393   -0.5361   -0.0005   -0.2789    0.2412    0.0001         0
   -0.5361   -0.5393    0.0005    0.2412   -0.2092    0.0001         0
    0.0005   -0.0005    0.0001   -0.0001   -0.0001    0.0001         0
   -0.6453         0         0         0         0         0         0
         0   -0.0007    0.0003         0         0         0         0
         0   -0.0003    0.0001         0         0         0         0
   -0.2283    0.2280    0.0001         0         0         0         0
    0.2280   -0.2283    0.0001         0         0         0         0
   -0.0001   -0.0001    0.0001         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.8178   -0.0817    0.0000         0         0         0         0
   -0.0817   -0.0087   -0.0004         0         0         0         0
   -0.0000    0.0004    0.0002         0         0         0         0
    3.0373    0.4705    0.0004   -0.8068   -0.0806    0.0000         0
    0.4705    0.7856   -0.0009   -0.0806   -0.0086   -0.0004         0
    0.0004   -0.0009    0.0014   -0.0000    0.0004    0.0002         0
   -0.8068   -0.0806   -0.0000    3.5638    0.2257    0.0001   -1.4542
   -0.0806   -0.0086    0.0004    0.2257    0.4457   -0.0016   -0.1451
    0.0000   -0.0004    0.0002    0.0001   -0.0016    0.0017   -0.0001
         0         0         0   -1.4542   -0.1451   -0.0001    2.2704
         0         0         0   -0.1451   -0.0176    0.0014    0.1451
         0         0         0    0.0001   -0.0014    0.0004   -0.0001

  Columns 29 through 30

         0         0
   -0.0014   -0.0004
    0.0004    0.0001
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
```

0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
-0.1451	0.0001
-0.0176	-0.0014
0.0014	0.0004
0.1451	-0.0001
0.0190	0.0018
0.0018	0.0010

Mg_beforeBC =

Columns 1 through 7

2.0739	-0.0392	-0.0755	0.4660	0	0	0
-0.0392	2.2391	0.1632	0	0.3595	-0.0692	0
-0.0755	0.1632	0.0440	0	0.0692	-0.0128	0
0.4660	0	0	2.3179	0	0	0.4660
0	0.3595	0.0692	0	2.5828	0.0494	0
0	-0.0692	-0.0128	0	0.0494	0.0403	0
0	0	0	0.4660	0	0	2.9239
0	0	0	0	0.3595	0.0692	0.0212
0	0	0	0	-0.0692	-0.0128	0.0587
0	0	0	0	0	0	0.4660
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0.1393
0	0	0	0	0	0	0.0180
0	0	0	0	0	0	0.0099
0.3128	0.0392	-0.0446	0.2270	0	0	0.3128
0.0392	0.3015	0.0515	0	0.1751	0.0292	-0.0392
0.0446	-0.0515	-0.0166	0	-0.0292	-0.0047	-0.0446
0.2072	0	0	0	0	0	0
0	0.1599	-0.0243	0	0	0	0
0	0.0243	-0.0036	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.3595	-0.0692	0	0	0	0	0
0.0692	-0.0128	0	0	0	0	0
0.0212	0.0587	0.4660	0	0	0	0
3.1482	0.1040	0	0.3595	-0.0692	0	0
0.1040	0.0583	0	0.0692	-0.0128	0	0
0	0	2.3882	0	0	0.4660	0
0.3595	0.0692	0	2.6611	0.0659	0	0.3595
-0.0692	-0.0128	0	0.0659	0.0437	0	0.0692
0	0	0.4660	0	0	3.4839	-0.0023
0	0	0	0.3595	0.0692	-0.0023	3.6870
0	0	0	-0.0692	-0.0128	-0.0173	0.1921
0	0	0	0	0	0.4660	0
0	0	0	0	0	0	0.3595
0	0	0	0	0	0	-0.0692
0	0	0	0	0	0.3432	0.0447
0	0	0	0	0	0.0447	0.3532
0	0	0	0	0	0.0653	-0.0584
0.0180	-0.0099	0.2621	0	0	0.3283	-0.0424
0.1393	0.0099	0	0.2022	0.0389	-0.0424	0.3283
-0.0099	-0.0016	0	-0.0389	-0.0072	-0.0551	-0.0551
-0.0392	0.0446	0	0	0	0	0
0.3015	0.0515	0	0	0	0	0
-0.0515	-0.0166	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 21

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0692	0	0	0	0	0	0
-0.0128	0	0	0	0	0	0
-0.0173	0.4660	0	0	0.3432	0.0447	0.0653
0.1921	0	0.3595	-0.0692	0.0447	0.3532	-0.0584
0.0935	0	0.0692	-0.0128	-0.0653	0.0584	-0.0243
0	1.5851	0	-0.0824	0.2261	0	0.0487
0.0692	0	1.6246	-0.1172	0	0.2931	0
-0.0128	-0.0824	-0.1172	0.0304	-0.0487	0	-0.0100
-0.0653	0.2261	0	-0.0487	4.1764	-0.0750	0.2601
0.0584	0	0.2931	0	-0.0750	4.4002	-0.7704
-0.0243	0.0487	0	-0.0100	0.2601	-0.7704	0.2421
0.0551	0	0	0	1.3390	0.0304	0.0397
0.0551	0	0	0	0.0304	1.0383	-0.3968
-0.0203	0	0	0	-0.0397	0.3968	-0.1472
0	0	0	0	0	0	0
0	0	0	0	0	0	0

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 22 through 28

0	0	0	0.3128	0.0392	0.0446	0.2072
0	0	0	0.0392	0.3015	-0.0515	0
0	0	0	-0.0446	0.0515	-0.0166	0
0	0	0	0.2270	0	0	0
0	0	0	0	0.1751	-0.0292	0
0	0	0	0	0.0292	-0.0047	0
0.1393	0.0180	0.0099	0.3128	-0.0392	-0.0446	0
0.0180	0.1393	-0.0099	-0.0392	0.3015	-0.0515	0
-0.0099	0.0099	-0.0016	0.0446	0.0515	-0.0166	0
0.2621	0	0	0	0	0	0
0	0.2022	-0.0389	0	0	0	0
0	0.0389	-0.0072	0	0	0	0
0.3283	-0.0424	-0.0551	0	0	0	0
-0.0424	0.3283	-0.0551	0	0	0	0
0.0551	0.0551	-0.0203	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
1.3390	0.0304	-0.0397	0	0	0	0
0.0304	1.0383	0.3968	0	0	0	0
0.0397	-0.3968	-0.1472	0	0	0	0
7.0513	-0.0368	-0.0746	1.3573	0.0308	-0.0408	0
-0.0368	7.7167	-0.1943	0.0308	1.0525	0.4077	0
-0.0746	-0.1943	0.4396	0.0408	-0.4077	-0.1534	0
1.3573	0.0308	0.0408	6.1438	-0.0479	-0.0478	0.7530
0.0308	1.0525	-0.4077	-0.0479	6.6922	0.2538	0.0171
-0.0408	0.4077	-0.1534	-0.0478	0.2538	0.2900	0.0125
0	0	0	0.7530	0.0171	0.0125	1.9256
0	0	0	0.0171	0.5839	-0.1255	-0.0171
0	0	0	-0.0125	0.1255	-0.0262	-0.0212

Columns 29 through 30

0	0
0.1599	0.0243
-0.0243	-0.0036
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

```
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0171   -0.0125
    0.5839    0.1255
   -0.1255   -0.0262
   -0.0171   -0.0212
    2.1420    0.2536
    0.2536    0.0397
```

Force Vector (Fg) %%

```
Fg=zeros(30,1); Fg(14)= -7000; Fg(23) = -1360;     % considering Engine concentrated loa
ds at 14, fuel weight at 23 %
```

Apply Boundary conditions %% U[11,12,13,14]=0

```
BC = 16; kg_global(BC,:)=0; kg_global(:,BC)=0; kg_global(BC,BC)=1; Fg(BC)=0;
BC = 17; kg_global(BC,:)=0; kg_global(:,BC)=0; kg_global(BC,BC)=1; Fg(BC)=0;
BC = 18; kg_global(BC,:)=0; kg_global(:,BC)=0; kg_global(BC,BC)=1; Fg(BC)=0;
BC = 19; kg_global(BC,:)=0; kg_global(:,BC)=0; kg_global(BC,BC)=1; Fg(BC)=0;
BC = 20; kg_global(BC,:)=0; kg_global(:,BC)=0; kg_global(BC,BC)=1; Fg(BC)=0;

BC = 16; Mg_global(BC,:)=0; Mg_global(:,BC)=0; Mg_global(BC,BC)=1; Fg(BC)=0;
BC = 17; Mg_global(BC,:)=0; Mg_global(:,BC)=0; Mg_global(BC,BC)=1; Fg(BC)=0;
BC = 18; Mg_global(BC,:)=0; Mg_global(:,BC)=0; Mg_global(BC,BC)=1; Fg(BC)=0;
BC = 19; Mg_global(BC,:)=0; Mg_global(:,BC)=0; Mg_global(BC,BC)=1; Fg(BC)=0;
BC = 20; Mg_global(BC,:)=0; Mg_global(:,BC)=0; Mg_global(BC,BC)=1; Fg(BC)=0;


     kg_afterBC = kg_global;
     Mg_afterBC = Mg_global;
     kg_afterBC, Mg_afterBC
```

```
kg_afterBC =

   1.0e+08 *

  Columns 1 through 7

    2.2422    0.2412   -0.0001   -1.1471         0         0         0
    0.2412    0.2127    0.0005         0   -0.0022    0.0009         0
   -0.0001    0.0005    0.0008         0   -0.0009    0.0002         0
   -1.1471         0         0    3.0394         0         0   -1.1471
         0   -0.0022   -0.0009         0    0.0053    0.0004         0
         0    0.0009    0.0002         0    0.0004    0.0011         0
         0         0         0   -1.1471         0         0    3.1124
         0         0         0         0   -0.0022   -0.0009    0.2949
```

0	0	0	0	0.0009	0.0002	-0.0004
0	0	0	0	0	0	-1.1471
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	-0.5393
0	0	0	0	0	0	-0.5361
0	0	0	0	0	0	-0.0005
-0.2789	-0.2412	0.0001	-0.7451	0	0	-0.2789
-0.2412	-0.2092	-0.0001	0	-0.0010	-0.0004	0.2412
-0.0001	0.0001	0.0001	0	0.0004	0.0001	0.0001
-0.8162	0	0	0	0	0	0
0	-0.0014	0.0004	0	0	0	0
0	-0.0004	0.0001	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0022	0.0009	0	0	0	0	0
-0.0009	0.0002	0	0	0	0	0
0.2949	-0.0004	-1.1471	0	0	0	0
0.7528	0.0007	0	-0.0022	0.0009	0	0
0.0007	0.0013	0	-0.0009	0.0002	0	0
0	0	2.9396	0	0	-1.1471	0
-0.0022	-0.0009	0	0.0050	0.0003	0	-0.0022
0.0009	0.0002	0	0.0003	0.0011	0	-0.0009
0	0	-1.1471	0	0	2.7138	-0.0144
0	0	0	-0.0022	-0.0009	-0.0144	0.4716
0	0	0	0.0009	0.0002	0.0000	0.0002
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	-0.0001	0.0001
-0.5361	0.0005	-0.6453	0	0	-0.2283	0.2280
-0.5393	-0.0005	0	-0.0007	-0.0003	0.2280	-0.2283
0.0005	0.0001	0	0.0003	0.0001	0.0001	0.0001
0.2412	-0.0001	0	0	0	0	0
-0.2092	-0.0001	0	0	0	0	0
0.0001	0.0001	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 21

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0009	0	0	0	0	0	0
0.0002	0	0	0	0	0	0
0.0000	0	0	0	0	0	-0.0001
0.0002	0	0	0	0	0	0.0001
0.0011	0	0	0	0	0	0.0000
0	0.0000	0	0	0	0	0
0	0	0.0000	0	0	0	0
0	0	0	0.0000	0	0	0
0	0	0	0	0.0000	0	0
0	0	0	0	0	0.0000	0
0.0000	0	0	0	0	0	0.0007
-0.0001	0	0	0	0	0	-0.0000
-0.0001	0	0	0	0	0	0.0004
0.0001	0	0	0	0	0	0.0002
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 22 through 28

0	0	0	-0.2789	-0.2412	-0.0001	-0.8162
0	0	0	-0.2412	-0.2092	0.0001	0
0	0	0	0.0001	-0.0001	0.0001	0
0	0	0	-0.7451	0	0	0
0	0	0	0	-0.0010	0.0004	0
0	0	0	0	-0.0004	0.0001	0
-0.5393	-0.5361	-0.0005	-0.2789	0.2412	0.0001	0
-0.5361	-0.5393	0.0005	0.2412	-0.2092	0.0001	0
0.0005	-0.0005	0.0001	-0.0001	-0.0001	0.0001	0
-0.6453	0	0	0	0	0	0
0	-0.0007	0.0003	0	0	0	0
0	-0.0003	0.0001	0	0	0	0
-0.2283	0.2280	0.0001	0	0	0	0
0.2280	-0.2283	0.0001	0	0	0	0
-0.0001	-0.0001	0.0001	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0000	0.0004	0.0002	0	0	0	0
3.0373	0.4705	0.0004	-0.8068	-0.0806	0.0000	0
0.4705	0.7856	-0.0009	-0.0806	-0.0086	-0.0004	0

```
    0.0004   -0.0009    0.0014   -0.0000    0.0004    0.0002         0
   -0.8068   -0.0806   -0.0000    3.5638    0.2257    0.0001   -1.4542
   -0.0806   -0.0086    0.0004    0.2257    0.4457   -0.0016   -0.1451
    0.0000   -0.0004    0.0002    0.0001   -0.0016    0.0017   -0.0001
         0         0         0   -1.4542   -0.1451   -0.0001    2.2704
         0         0         0   -0.1451   -0.0176    0.0014    0.1451
         0         0         0    0.0001   -0.0014    0.0004   -0.0001

  Columns 29 through 30

         0         0
   -0.0014   -0.0004
    0.0004    0.0001
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
   -0.1451    0.0001
   -0.0176   -0.0014
    0.0014    0.0004
    0.1451   -0.0001
    0.0190    0.0018
    0.0018    0.0010


Mg_afterBC =

  Columns 1 through 7

    2.0739   -0.0392   -0.0755    0.4660         0         0         0
   -0.0392    2.2391    0.1632         0    0.3595   -0.0692         0
   -0.0755    0.1632    0.0440         0    0.0692   -0.0128         0
    0.4660         0         0    2.3179         0         0    0.4660
         0    0.3595    0.0692         0    2.5828    0.0494         0
         0   -0.0692   -0.0128         0    0.0494    0.0403         0
         0         0         0    0.4660         0         0    2.9239
         0         0         0         0    0.3595    0.0692    0.0212
         0         0         0         0   -0.0692   -0.0128    0.0587
         0         0         0         0         0         0    0.4660
         0         0         0         0         0         0         0
```

```
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0    0.1393
         0         0         0         0         0         0    0.0180
         0         0         0         0         0         0    0.0099
    0.3128    0.0392   -0.0446    0.2270         0         0    0.3128
    0.0392    0.3015    0.0515         0    0.1751    0.0292   -0.0392
    0.0446   -0.0515   -0.0166         0   -0.0292   -0.0047   -0.0446
    0.2072         0         0         0         0         0         0
         0    0.1599   -0.0243         0         0         0         0
         0    0.0243   -0.0036         0         0         0         0

  Columns 8 through 14

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.3595   -0.0692         0         0         0         0         0
    0.0692   -0.0128         0         0         0         0         0
    0.0212    0.0587    0.4660         0         0         0         0
    3.1482    0.1040         0    0.3595   -0.0692         0         0
    0.1040    0.0583         0    0.0692   -0.0128         0         0
         0         0    2.3882         0         0    0.4660         0
    0.3595    0.0692         0    2.6611    0.0659         0    0.3595
   -0.0692   -0.0128         0    0.0659    0.0437         0    0.0692
         0         0    0.4660         0         0    3.4839   -0.0023
         0         0         0    0.3595    0.0692   -0.0023    3.6870
         0         0         0   -0.0692   -0.0128   -0.0173    0.1921
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0    0.0653   -0.0584
    0.0180   -0.0099    0.2621         0         0    0.3283   -0.0424
    0.1393    0.0099         0    0.2022    0.0389   -0.0424    0.3283
   -0.0099   -0.0016         0   -0.0389   -0.0072   -0.0551   -0.0551
   -0.0392    0.0446         0         0         0         0         0
    0.3015    0.0515         0         0         0         0         0
   -0.0515   -0.0166         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 15 through 21

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0692	0	0	0	0	0	0
-0.0128	0	0	0	0	0	0
-0.0173	0	0	0	0	0	0.0653
0.1921	0	0	0	0	0	-0.0584
0.0935	0	0	0	0	0	-0.0243
0	1.0000	0	0	0	0	0
0	0	1.0000	0	0	0	0
0	0	0	1.0000	0	0	0
0	0	0	0	1.0000	0	0
0	0	0	0	0	1.0000	0
-0.0243	0	0	0	0	0	0.2421
0.0551	0	0	0	0	0	0.0397
0.0551	0	0	0	0	0	-0.3968
-0.0203	0	0	0	0	0	-0.1472
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 22 through 28

0	0	0	0.3128	0.0392	0.0446	0.2072
0	0	0	0.0392	0.3015	-0.0515	0
0	0	0	-0.0446	0.0515	-0.0166	0
0	0	0	0.2270	0	0	0
0	0	0	0	0.1751	-0.0292	0
0	0	0	0	0.0292	-0.0047	0
0.1393	0.0180	0.0099	0.3128	-0.0392	-0.0446	0
0.0180	0.1393	-0.0099	-0.0392	0.3015	-0.0515	0
-0.0099	0.0099	-0.0016	0.0446	0.0515	-0.0166	0
0.2621	0	0	0	0	0	0
0	0.2022	-0.0389	0	0	0	0
0	0.0389	-0.0072	0	0	0	0
0.3283	-0.0424	-0.0551	0	0	0	0
-0.0424	0.3283	-0.0551	0	0	0	0
0.0551	0.0551	-0.0203	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0397	-0.3968	-0.1472	0	0	0	0
7.0513	-0.0368	-0.0746	1.3573	0.0308	-0.0408	0
-0.0368	7.7167	-0.1943	0.0308	1.0525	0.4077	0
-0.0746	-0.1943	0.4396	0.0408	-0.4077	-0.1534	0
1.3573	0.0308	0.0408	6.1438	-0.0479	-0.0478	0.7530
0.0308	1.0525	-0.4077	-0.0479	6.6922	0.2538	0.0171

```
   -0.0408    0.4077   -0.1534   -0.0478    0.2538    0.2900    0.0125
         0         0         0    0.7530    0.0171    0.0125    1.9256
         0         0         0    0.0171    0.5839   -0.1255   -0.0171
         0         0         0   -0.0125    0.1255   -0.0262   -0.0212

  Columns 29 through 30

         0         0
    0.1599    0.0243
   -0.0243   -0.0036
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0171   -0.0125
    0.5839    0.1255
   -0.1255   -0.0262
   -0.0171   -0.0212
    2.1420    0.2536
    0.2536    0.0397
```

Displacement

```
format short
Ug = inv(kg_afterBC)*Fg
```

```
Ug =

   1.0e-03 *

    0.0534
   -0.5092
    0.0227
    0.0536
   -0.4951
    0.0084
```

```
    0.0561
   -0.4975
   -0.0345
    0.0579
   -0.5417
   -0.0036
    0.0670
   -0.4061
    0.4259
         0
         0
         0
         0
         0
    0.2991
    0.0452
   -0.4859
    0.1084
    0.0502
   -0.5056
   -0.0013
    0.0531
   -0.5327
    0.0459
```

Eigen Values and Eigen Vectors

```
[V,D]=eig(kg_afterBC,Mg_afterBC)        % V: eigen vectors, D: eigen values %
```

```
V =

  Columns 1 through 7

   -0.3714    0.2745   -0.3888   -0.4172   -0.0442   -0.4635    0.0521
    0.0478   -0.0511   -0.0908   -0.0665   -0.0759   -0.1952   -0.0086
   -1.0000    1.0000   -0.1165   -0.6128    0.5179    0.0944    0.4063
    0.4286   -0.0622    0.6026   -0.1150   -0.3113   -0.4685   -0.0025
    0.0365   -0.0164   -0.0037   -0.0078   -0.0017    0.0999   -0.0268
   -0.1118    0.3830   -0.5597   -0.9479    0.0705    0.6893   -0.0983
   -0.2781   -0.3111   -0.0908    0.2793   -0.3355   -0.2395   -0.0937
   -0.0269   -0.0117    0.0760    0.1531    0.0428   -0.4797    0.1639
    0.4125    0.2993   -0.5967   -1.0000    0.2274    0.9354   -0.0696
    0.1931    0.4820   -0.6090    0.1745    0.1206   -0.1957   -0.1170
   -0.0104   -0.0083    0.0147    0.0139   -0.0027    0.0295   -0.0290
    0.0803    0.0840    0.0432    0.0362    0.3608   -1.0000    0.3035
   -0.0656   -0.1949    0.4441   -0.1623    0.5541   -0.2005   -0.0950
    0.0006   -0.0047   -0.0213   -0.0129   -0.0703    0.0875    0.0191
   -0.0166    0.0189    0.2185    0.1247    0.4172   -0.5317   -0.0453
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

0.0412	0.0561	-0.0816	-0.0599	-0.0023	0.3871	-0.0932
0.0216	-0.0625	-0.1299	-0.1691	-0.2242	0.3989	0.0040
0.0124	0.0075	-0.0156	-0.0661	0.0400	0.1734	-0.0511
0.0135	-0.0328	0.0428	-0.0530	0.0329	0.2361	-0.0549
-0.0610	0.1733	0.4606	0.1858	0.1784	0.2268	0.0594
0.0030	-0.0148	0.0601	0.0533	-0.0377	0.0664	-0.0644
-0.0476	0.0030	0.0550	0.0403	0.0286	-0.0720	0.1907
0.2092	-0.4281	-1.0000	0.0370	0.3724	0.0130	0.0907
-0.0101	-0.0126	-0.0961	-0.0664	-0.0749	-0.1324	-0.0756
-0.0039	0.0789	0.1178	0.3510	1.0000	0.8224	1.0000

Columns 8 through 14

-0.1272	-0.0386	0.0121	-0.0102	0.0082	-0.0138	-0.0002
-0.0066	0.0911	-0.1389	0.0504	-0.0193	0.0507	-0.0961
-0.3356	-0.2182	0.2200	-0.4737	-0.2219	-1.0000	-0.3023
-0.0708	-0.0153	-0.0037	-0.0058	0.0050	-0.0190	0.0045
-0.0300	-0.0054	0.0010	0.0012	-0.0005	0.0488	0.0681
-0.9111	0.1692	-0.5694	-0.2698	-0.6948	-0.6500	-0.8665
0.0339	0.0107	-0.0099	-0.0042	0.0046	-0.0243	0.0172
0.3227	0.0421	0.0687	0.0095	0.0081	-0.0187	0.0409
-0.6134	0.1748	-0.2904	-0.2178	-0.6451	0.2760	0.6891
0.1963	0.0151	0.0025	-0.0027	0.0006	-0.0123	-0.0025
-0.0844	-0.0005	0.0111	0.0049	0.0245	-0.0045	-0.0770
-0.3096	0.4396	0.2858	-0.1311	-0.5714	0.6411	1.0000
0.1981	0.0079	0.0030	-0.0018	-0.0018	-0.0023	-0.0118
0.3444	-0.1014	-0.0867	0.0017	0.0084	-0.0174	0.0893
-1.0000	0.2927	0.2464	-0.0529	-0.1449	0.2671	0.0392
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.4412	0.0707	0.0199	-0.0342	-0.0046	-0.0159	0.3351
0.2322	0.0224	0.0218	0.0002	-0.0024	-0.0057	-0.0203
-0.1658	0.0272	0.0028	0.0060	0.0219	-0.0462	0.1077
-0.0879	0.0413	0.0250	-0.0627	-0.0463	0.0595	0.1388
-0.0693	-0.0077	-0.0163	0.0015	-0.0009	-0.0150	-0.0084
-0.0367	-0.0588	0.0470	0.0076	-0.0064	0.0417	-0.0633
0.3065	0.1246	-0.1154	-0.1514	-0.0722	0.0363	-0.0143
-0.1227	-0.0212	-0.0112	-0.0079	0.0093	-0.0051	-0.0144
-0.0989	-0.1076	0.1128	0.0893	-0.1142	-0.1169	0.0698
0.9306	1.0000	-1.0000	-1.0000	1.0000	0.7246	-0.2954

Columns 15 through 21

-0.1480	0.0314	-0.0007	0.0074	-0.0005	-0.0062	0.0014
-0.0587	-0.0190	-0.0077	-0.0050	0.0785	-0.1101	-0.1010
-0.0606	-0.7747	0.4978	-0.9597	-0.2626	-1.0000	0.2274
-0.1384	0.0268	-0.0016	0.0021	-0.0056	0.0042	0.0063
0.0234	0.0850	-0.0575	0.0133	-0.0660	0.1257	0.0993
0.4837	0.0460	-0.0257	1.0000	0.3386	0.6414	-0.0497
-0.1139	0.0218	-0.0012	0.0005	-0.0097	0.0109	0.0104
0.0209	-0.0035	-0.0090	0.0342	0.0657	-0.0926	-0.0916
0.4597	1.0000	-0.3279	-0.3377	-0.4384	-0.9593	-0.0030
-0.0829	0.0161	-0.0012	-0.0015	-0.0071	0.0083	0.0087
-0.0362	-0.0766	-0.0443	0.0278	-0.0684	0.0602	0.0960

-1.0000	-0.6041	1.0000	0.0082	0.5471	0.8221	0.0950
-0.0434	0.0089	-0.0003	-0.0035	-0.0074	0.0095	0.0104
0.0289	-0.0240	0.0019	0.0211	0.0238	-0.0353	-0.0399
-0.6851	-0.9314	-0.5223	0.4011	-1.0000	-0.6719	-0.3559
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.2321	-0.3854	-0.5991	0.0780	0.5398	-0.5514	1.0000
-0.0854	0.0170	-0.0025	-0.0014	-0.0018	0.0013	0.0025
-0.0097	-0.0042	-0.0045	0.0335	0.0513	-0.0712	-0.0784
-0.2359	-0.3199	-0.6054	-0.0808	0.5318	-0.1450	0.2408
-0.1437	0.0248	-0.0034	-0.0037	-0.0067	0.0099	0.0073
-0.0398	-0.0015	-0.0082	0.0208	0.0847	-0.1153	-0.1091
-0.0885	-0.1839	-0.5658	-0.3595	0.1841	0.8056	-0.7443
-0.1532	0.0314	0.0027	0.0102	0.0067	-0.0257	-0.0048
0.0022	-0.0573	-0.0891	-0.1331	-0.0890	0.3499	0.0458
-0.0552	0.3190	0.4087	0.5229	0.2079	-0.9531	0.0553

Columns 22 through 28

0.0133	-0.0313	-0.0204	-0.0239	0	0	0
-0.0989	0.2446	0.0580	0.0949	0	0	0
-1.0000	0.3101	-0.7626	-0.6981	0	0	0
0.0094	-0.0239	-0.0089	-0.0126	0	0	0
-0.8166	0.4347	-0.3056	-0.1944	0	0	0
0.2295	-0.3238	0.2431	0.2264	0	0	0
0.0106	-0.0271	-0.0055	-0.0115	0	0	0
-0.0516	0.1616	0.0187	0.0298	0	0	0
0.9845	0.6024	0.2448	-0.1179	0	0	0
0.0059	-0.0176	-0.0030	-0.0049	0	0	0
0.4667	1.0000	0.0730	-0.3550	0	0	0
-0.1810	-0.1465	-0.1430	0.0363	0	0	0
0.0052	-0.0172	-0.0025	-0.0026	0	0	0
-0.0134	0.0638	0.0065	0.0018	0	0	0
-0.3775	-0.8440	-0.0125	0.3229	0	0	0
0	0	0	0	1.0000	0	0
0	0	0	0	0	1.0000	0
0	0	0	0	0	0	1.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.4500	0.4274	-1.0000	1.0000	0	0	0
-0.0012	-0.0012	0.0005	0.0029	0	0	0
-0.0333	0.1199	0.0136	0.0110	0	0	0
-0.4539	-0.6319	0.7929	-0.6498	0	0	0
0.0014	-0.0076	0.0038	0.0035	0	0	0
-0.0753	0.2090	0.0302	0.0624	0	0	0
0.4358	-0.1608	-0.6384	-0.0933	0	0	0
0.0160	-0.0390	-0.0364	-0.0395	0	0	0
-0.2355	0.5661	0.5213	0.5790	0	0	0
0.1249	-0.4262	-0.3460	-0.5770	0	0	0

Columns 29 through 30

0	0
0	0

```
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    1.0000         0
         0    1.0000
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0


D =

   1.0e+08 *

  Columns 1 through 7

    3.0918         0         0         0         0         0         0
         0    2.2669         0         0         0         0         0
         0         0    1.5952         0         0         0         0
         0         0         0    1.1425         0         0         0
         0         0         0         0    0.7500         0         0
         0         0         0         0         0    0.5775         0
         0         0         0         0         0         0    0.3549
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.2526	0	0	0	0	0	0
0	0.1727	0	0	0	0	0
0	0	0.1466	0	0	0	0
0	0	0	0.1110	0	0	0
0	0	0	0	0.0658	0	0
0	0	0	0	0	0.0441	0
0	0	0	0	0	0	0.0320
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 21

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
   0.0277         0         0         0         0         0         0
        0    0.0206         0         0         0         0         0
        0         0    0.0142         0         0         0         0
        0         0         0    0.0115         0         0         0
        0         0         0         0    0.0089         0         0
        0         0         0         0         0    0.0076         0
        0         0         0         0         0         0    0.0028
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0

  Columns 22 through 28

        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0
   0.0009         0         0         0         0         0         0
        0    0.0010         0         0         0         0         0
        0         0    0.0014         0         0         0         0
        0         0         0    0.0017         0         0         0
        0         0         0         0    0.0000         0         0
        0         0         0         0         0    0.0000         0
        0         0         0         0         0         0    0.0000
        0         0         0         0         0         0         0
        0         0         0         0         0         0         0

  Columns 29 through 30

        0         0
        0         0
        0         0
```

```
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0000         0
         0    0.0000
```

Natural Frequency and Mode shapes

```
Natural_frequency= sqrt(D)/(2*pi)
Mode_shapes = V
```

```
Natural_frequency =

   1.0e+03 *

  Columns 1 through 7

    2.7985         0         0         0         0         0         0
         0    2.3963         0         0         0         0         0
         0         0    2.0101         0         0         0         0
         0         0         0    1.7012         0         0         0
         0         0         0         0    1.3783         0         0
         0         0         0         0         0    1.2095         0
         0         0         0         0         0         0    0.9482
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.8000	0	0	0	0	0	0
0	0.6614	0	0	0	0	0
0	0	0.6094	0	0	0	0
0	0	0	0.5303	0	0	0
0	0	0	0	0.4082	0	0
0	0	0	0	0	0.3341	0
0	0	0	0	0	0	0.2849
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 21

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

```
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.2651         0         0         0         0         0         0
         0    0.2286         0         0         0         0         0
         0         0    0.1897         0         0         0         0
         0         0         0    0.1709         0         0         0
         0         0         0         0    0.1500         0         0
         0         0         0         0         0    0.1383         0
         0         0         0         0         0         0    0.0845
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 22 through 28

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0470         0         0         0         0         0         0
         0    0.0510         0         0         0         0         0
         0         0    0.0594         0         0         0         0
         0         0         0    0.0665         0         0         0
         0         0         0         0    0.0002         0         0
         0         0         0         0         0    0.0002         0
         0         0         0         0         0         0    0.0002
         0         0         0         0         0         0         0
```

```
         0         0         0         0         0         0         0

  Columns 29 through 30

         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    0.0002         0
         0    0.0002


Mode_shapes =

  Columns 1 through 7

   -0.3714    0.2745   -0.3888   -0.4172   -0.0442   -0.4635    0.0521
    0.0478   -0.0511   -0.0908   -0.0665   -0.0759   -0.1952   -0.0086
   -1.0000    1.0000   -0.1165   -0.6128    0.5179    0.0944    0.4063
    0.4286   -0.0622    0.6026   -0.1150   -0.3113   -0.4685   -0.0025
    0.0365   -0.0164   -0.0037   -0.0078   -0.0017    0.0999   -0.0268
   -0.1118    0.3830   -0.5597   -0.9479    0.0705    0.6893   -0.0983
   -0.2781   -0.3111   -0.0908    0.2793   -0.3355   -0.2395   -0.0937
   -0.0269   -0.0117    0.0760    0.1531    0.0428   -0.4797    0.1639
    0.4125    0.2993   -0.5967   -1.0000    0.2274    0.9354   -0.0696
    0.1931    0.4820   -0.6090    0.1745    0.1206   -0.1957   -0.1170
   -0.0104   -0.0083    0.0147    0.0139   -0.0027    0.0295   -0.0290
    0.0803    0.0840    0.0432    0.0362    0.3608   -1.0000    0.3035
   -0.0656   -0.1949    0.4441   -0.1623    0.5541   -0.2005   -0.0950
    0.0006   -0.0047   -0.0213   -0.0129   -0.0703    0.0875    0.0191
   -0.0166    0.0189    0.2185    0.1247    0.4172   -0.5317   -0.0453
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0412	0.0561	-0.0816	-0.0599	-0.0023	0.3871	-0.0932
0.0216	-0.0625	-0.1299	-0.1691	-0.2242	0.3989	0.0040
0.0124	0.0075	-0.0156	-0.0661	0.0400	0.1734	-0.0511
0.0135	-0.0328	0.0428	-0.0530	0.0329	0.2361	-0.0549
-0.0610	0.1733	0.4606	0.1858	0.1784	0.2268	0.0594
0.0030	-0.0148	0.0601	0.0533	-0.0377	0.0664	-0.0644
-0.0476	0.0030	0.0550	0.0403	0.0286	-0.0720	0.1907
0.2092	-0.4281	-1.0000	0.0370	0.3724	0.0130	0.0907
-0.0101	-0.0126	-0.0961	-0.0664	-0.0749	-0.1324	-0.0756
-0.0039	0.0789	0.1178	0.3510	1.0000	0.8224	1.0000

Columns 8 through 14

-0.1272	-0.0386	0.0121	-0.0102	0.0082	-0.0138	-0.0002
-0.0066	0.0911	-0.1389	0.0504	-0.0193	0.0507	-0.0961
-0.3356	-0.2182	0.2200	-0.4737	-0.2219	-1.0000	-0.3023
-0.0708	-0.0153	-0.0037	-0.0058	0.0050	-0.0190	0.0045
-0.0300	-0.0054	0.0010	0.0012	-0.0005	0.0488	0.0681
-0.9111	0.1692	-0.5694	-0.2698	-0.6948	-0.6500	-0.8665
0.0339	0.0107	-0.0099	-0.0042	0.0046	-0.0243	0.0172
0.3227	0.0421	0.0687	0.0095	0.0081	-0.0187	0.0409
-0.6134	0.1748	-0.2904	-0.2178	-0.6451	0.2760	0.6891
0.1963	0.0151	0.0025	-0.0027	0.0006	-0.0123	-0.0025
-0.0844	-0.0005	0.0111	0.0049	0.0245	-0.0045	-0.0770
-0.3096	0.4396	0.2858	-0.1311	-0.5714	0.6411	1.0000
0.1981	0.0079	0.0030	-0.0018	-0.0018	-0.0023	-0.0118
0.3444	-0.1014	-0.0867	0.0017	0.0084	-0.0174	0.0893
-1.0000	0.2927	0.2464	-0.0529	-0.1449	0.2671	0.0392
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.4412	0.0707	0.0199	-0.0342	-0.0046	-0.0159	0.3351
0.2322	0.0224	0.0218	0.0002	-0.0024	-0.0057	-0.0203
-0.1658	0.0272	0.0028	0.0060	0.0219	-0.0462	0.1077
-0.0879	0.0413	0.0250	-0.0627	-0.0463	0.0595	0.1388
-0.0693	-0.0077	-0.0163	0.0015	-0.0009	-0.0150	-0.0084
-0.0367	-0.0588	0.0470	0.0076	-0.0064	0.0417	-0.0633
0.3065	0.1246	-0.1154	-0.1514	-0.0722	0.0363	-0.0143
-0.1227	-0.0212	-0.0112	-0.0079	0.0093	-0.0051	-0.0144
-0.0989	-0.1076	0.1128	0.0893	-0.1142	-0.1169	0.0698
0.9306	1.0000	-1.0000	-1.0000	1.0000	0.7246	-0.2954

Columns 15 through 21

-0.1480	0.0314	-0.0007	0.0074	-0.0005	-0.0062	0.0014
-0.0587	-0.0190	-0.0077	-0.0050	0.0785	-0.1101	-0.1010
-0.0606	-0.7747	0.4978	-0.9597	-0.2626	-1.0000	0.2274
-0.1384	0.0268	-0.0016	0.0021	-0.0056	0.0042	0.0063
0.0234	0.0850	-0.0575	0.0133	-0.0660	0.1257	0.0993
0.4837	0.0460	-0.0257	1.0000	0.3386	0.6414	-0.0497
-0.1139	0.0218	-0.0012	0.0005	-0.0097	0.0109	0.0104
0.0209	-0.0035	-0.0090	0.0342	0.0657	-0.0926	-0.0916

0.4597	1.0000	-0.3279	-0.3377	-0.4384	-0.9593	-0.0030
-0.0829	0.0161	-0.0012	-0.0015	-0.0071	0.0083	0.0087
-0.0362	-0.0766	-0.0443	0.0278	-0.0684	0.0602	0.0960
-1.0000	-0.6041	1.0000	0.0082	0.5471	0.8221	0.0950
-0.0434	0.0089	-0.0003	-0.0035	-0.0074	0.0095	0.0104
0.0289	-0.0240	0.0019	0.0211	0.0238	-0.0353	-0.0399
-0.6851	-0.9314	-0.5223	0.4011	-1.0000	-0.6719	-0.3559
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.2321	-0.3854	-0.5991	0.0780	0.5398	-0.5514	1.0000
-0.0854	0.0170	-0.0025	-0.0014	-0.0018	0.0013	0.0025
-0.0097	-0.0042	-0.0045	0.0335	0.0513	-0.0712	-0.0784
-0.2359	-0.3199	-0.6054	-0.0808	0.5318	-0.1450	0.2408
-0.1437	0.0248	-0.0034	-0.0037	-0.0067	0.0099	0.0073
-0.0398	-0.0015	-0.0082	0.0208	0.0847	-0.1153	-0.1091
-0.0885	-0.1839	-0.5658	-0.3595	0.1841	0.8056	-0.7443
-0.1532	0.0314	0.0027	0.0102	0.0067	-0.0257	-0.0048
0.0022	-0.0573	-0.0891	-0.1331	-0.0890	0.3499	0.0458
-0.0552	0.3190	0.4087	0.5229	0.2079	-0.9531	0.0553

Columns 22 through 28

0.0133	-0.0313	-0.0204	-0.0239	0	0	0
-0.0989	0.2446	0.0580	0.0949	0	0	0
-1.0000	0.3101	-0.7626	-0.6981	0	0	0
0.0094	-0.0239	-0.0089	-0.0126	0	0	0
-0.8166	0.4347	-0.3056	-0.1944	0	0	0
0.2295	-0.3238	0.2431	0.2264	0	0	0
0.0106	-0.0271	-0.0055	-0.0115	0	0	0
-0.0516	0.1616	0.0187	0.0298	0	0	0
0.9845	0.6024	0.2448	-0.1179	0	0	0
0.0059	-0.0176	-0.0030	-0.0049	0	0	0
0.4667	1.0000	0.0730	-0.3550	0	0	0
-0.1810	-0.1465	-0.1430	0.0363	0	0	0
0.0052	-0.0172	-0.0025	-0.0026	0	0	0
-0.0134	0.0638	0.0065	0.0018	0	0	0
-0.3775	-0.8440	-0.0125	0.3229	0	0	0
0	0	0	0	1.0000	0	0
0	0	0	0	0	1.0000	0
0	0	0	0	0	0	1.0000
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.4500	0.4274	-1.0000	1.0000	0	0	0
-0.0012	-0.0012	0.0005	0.0029	0	0	0
-0.0333	0.1199	0.0136	0.0110	0	0	0
-0.4539	-0.6319	0.7929	-0.6498	0	0	0
0.0014	-0.0076	0.0038	0.0035	0	0	0
-0.0753	0.2090	0.0302	0.0624	0	0	0
0.4358	-0.1608	-0.6384	-0.0933	0	0	0
0.0160	-0.0390	-0.0364	-0.0395	0	0	0
-0.2355	0.5661	0.5213	0.5790	0	0	0
0.1249	-0.4262	-0.3460	-0.5770	0	0	0

Columns 29 through 30

```
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
    1.0000         0
         0    1.0000
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
         0         0
```

Sensitivity Analysis of weight of structure w.r.t. Design Variable 'R: radius'

```
W = r*A.*L(i);
for i=1:17
Weight_Sensitivity(i) =r*(22/7)*2*R(i)*L(i);
end
Weight_Sensitivity
```

```
Weight_Sensitivity =

  Columns 1 through 7

  279.5886  279.5886  279.5886  279.5886  279.5886  234.4421  671.0126

  Columns 8 through 14

  680.1764  377.3514  165.7756  277.3957  181.5981  277.3957  125.8149

  Columns 15 through 17

  209.6914  296.5485  314.5371
```

Sensitivity Analysis

```
%Constraint_1; g1 = Ug(2) < -20e-03 m (tip displacement Dof_2)
for i=1:17
    dUg_dU2 = [ 0;1 ;zeros(28,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(2)
    dg1_da(:,:,i) = -(-1/20e-03)*(dUg_dU2)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg1_da
%Constraint_2; g2 = Ug(5) < -20e-03 m (displacement Dof_5)
for i=1:17
    dUg_dU5 = [ 0;0;0;0;0;1 ;zeros(24,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(6)
    dg2_da(:,:,i) = -(-1/20e-03)*(dUg_dU5)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg2_da
%Constraint_3; g3 = Ug(8) < -20e-03 m (displacement Dof_8)
for i=1:17
    dUg_dU8 = [ zeros(7,1);1 ;zeros(22,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(11)
    dg3_da(:,:,i) = -(-1/20e-03)*(dUg_dU8)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg3_da
%Constraint_4; g4 = Ug(11) < -20e-03 m (displacement Dof_11)
for i=1:17
    dUg_dU11 = [ zeros(10,1);1 ;zeros(19,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(1)
    dg4_da(:,:,i) = -(-1/20e-03)*(dUg_dU11)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg4_da
%Constraint_5; g5 = Ug(14) < -10e-03 m (displacement Dof_14)
for i=1:17
    dUg_dU14 = [ zeros(13,1);1 ;zeros(16,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(1)
    dg5_da(:,:,i) = -(-1/10e-03)*(dUg_dU14)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg5_da
%Constraint_6; g6 = Ug(23) < -10e-03 m (displacement Dof_23)
for i=1:17
    dUg_dU23 = [zeros(22,1);1 ;zeros(7,1)];  % partial diff g1 w.r.t Ug,  dUg/dUg(23)
    dg6_da(:,:,i) = -(-1/10e-03)*(dUg_dU23)'*(inv(kg_afterBC))*(-dkg_global(:,:,i)*Ug);
end
dg6_da
%Constraint_7; g7 = freq(1) > 20 Hz (1st Natural Frequency > 20 Hz)
% V = eigen_vectors
df_dD1 = 1/(4*pi*sqrt(D(1)));
df_dD2 = 1/(4*pi*sqrt(D(2)));

for i=1:17       % beam %
 dg7_da(i) =  1/20 * df_dD1*(V(1:6,i)'*(dk_da_beam(:,:,i) - D(1:6,1:6)*dM_da_beam(:,:,i)))*V(
1:6,i);
end
dg7_da
```

```
dg1_da(:,:,1) =

   -0.0089
```

```
dg1_da(:,:,2) =

    0.0554


dg1_da(:,:,3) =

   -0.1074


dg1_da(:,:,4) =

   -0.0629


dg1_da(:,:,5) =

    4.9425


dg1_da(:,:,6) =

   1.0605e-05


dg1_da(:,:,7) =

    0.1008


dg1_da(:,:,8) =

   -0.0437


dg1_da(:,:,9) =

   7.1698e-04


dg1_da(:,:,10) =

    0.0033


dg1_da(:,:,11) =

   6.1849e-05


dg1_da(:,:,12) =

    0.3837
```

```
dg1_da(:,:,13) =

    0.0858


dg1_da(:,:,14) =

    0.0437


dg1_da(:,:,15) =

    1.1536


dg1_da(:,:,16) =

    3.6449


dg1_da(:,:,17) =

   21.7262


dg2_da(:,:,1) =

   -0.0019


dg2_da(:,:,2) =

   -0.0295


dg2_da(:,:,3) =

    0.0102


dg2_da(:,:,4) =

    0.0210


dg2_da(:,:,5) =

   -0.1256


dg2_da(:,:,6) =

   1.6547e-05


dg2_da(:,:,7) =
```

```
   -0.0029


dg2_da(:,:,8) =

    0.0276


dg2_da(:,:,9) =

   -0.0059


dg2_da(:,:,10) =

   -0.0271


dg2_da(:,:,11) =

  -1.2913e-05


dg2_da(:,:,12) =

   -0.0157


dg2_da(:,:,13) =

   -0.0481


dg2_da(:,:,14) =

    0.0057


dg2_da(:,:,15) =

   -0.0714


dg2_da(:,:,16) =

    0.0920


dg2_da(:,:,17) =

   -0.3973


dg3_da(:,:,1) =

   2.1358e-04
```

```
dg3_da(:,:,2) =

    0.0392


dg3_da(:,:,3) =

   -0.0492


dg3_da(:,:,4) =

    0.0442


dg3_da(:,:,5) =

    4.2866


dg3_da(:,:,6) =

   1.0602e-05


dg3_da(:,:,7) =

    0.0751


dg3_da(:,:,8) =

    0.0310


dg3_da(:,:,9) =

   1.3621e-04


dg3_da(:,:,10) =

   6.1894e-04


dg3_da(:,:,11) =

   2.8696e-07


dg3_da(:,:,12) =

    0.0918


dg3_da(:,:,13) =
```

```
    0.0033

dg3_da(:,:,14) =

    0.0477

dg3_da(:,:,15) =

    0.7113

dg3_da(:,:,16) =

    3.7559

dg3_da(:,:,17) =

   22.3899

dg4_da(:,:,1) =

   4.2374e-04

dg4_da(:,:,2) =

    0.0165

dg4_da(:,:,3) =

   -0.0108

dg4_da(:,:,4) =

    0.0715

dg4_da(:,:,5) =

    3.7305

dg4_da(:,:,6) =

  -6.5359e-05

dg4_da(:,:,7) =

    0.0326
```

```
dg4_da(:,:,8) =

    0.0142


dg4_da(:,:,9) =

  -3.9661e-05


dg4_da(:,:,10) =

  -2.2155e-04


dg4_da(:,:,11) =

   3.9060e-07


dg4_da(:,:,12) =

    0.0293


dg4_da(:,:,13) =

   -0.0016


dg4_da(:,:,14) =

    0.0193


dg4_da(:,:,15) =

    0.3210


dg4_da(:,:,16) =

    2.3598


dg4_da(:,:,17) =

   28.4264


dg5_da(:,:,1) =

   3.6778e-05
```

```
dg5_da(:,:,2) =

    0.0037


dg5_da(:,:,3) =

    0.0020


dg5_da(:,:,4) =

    0.0497


dg5_da(:,:,5) =

    4.4242


dg5_da(:,:,6) =

   1.2003e-05


dg5_da(:,:,7) =

   -0.0120


dg5_da(:,:,8) =

    0.0029


dg5_da(:,:,9) =

   1.9249e-05


dg5_da(:,:,10) =

   8.6900e-05


dg5_da(:,:,11) =

   1.4646e-08


dg5_da(:,:,12) =

    0.0082


dg5_da(:,:,13) =
```

```
   3.5160e-04


dg5_da(:,:,14) =

   2.8197e-04


dg5_da(:,:,15) =

    0.1000


dg5_da(:,:,16) =

    0.0506


dg5_da(:,:,17) =

   48.9295


dg6_da(:,:,1) =

   1.3555e-04


dg6_da(:,:,2) =

    0.0209


dg6_da(:,:,3) =

    0.0127


dg6_da(:,:,4) =

    0.2927


dg6_da(:,:,5) =

    7.3194


dg6_da(:,:,6) =

   2.3618e-05


dg6_da(:,:,7) =

    0.1013
```

```
dg6_da(:,:,8) =

    0.0164


dg6_da(:,:,9) =

   8.2291e-05


dg6_da(:,:,10) =

   3.7338e-04


dg6_da(:,:,11) =

   1.3288e-07


dg6_da(:,:,12) =

    0.0482


dg6_da(:,:,13) =

    0.0018


dg6_da(:,:,14) =

   9.4082e-04


dg6_da(:,:,15) =

    0.5792


dg6_da(:,:,16) =

    7.7262


dg6_da(:,:,17) =

   45.9963


dg7_da =

   1.0e+04 *

  Columns 1 through 7
```

```
    0.8073   -0.1114    1.3528   -0.9453   -0.1031   -2.2890   -0.1466

  Columns 8 through 14

   -0.3066   -0.0848   -0.0384   -0.0035   -0.0029   -0.1171    0.0300

  Columns 15 through 17

   -0.2507   -0.1595   -0.0327
```

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Contents


```
clear all
clc
```

Function Approx gL, gR

```
r=2780, g=9.81
L = [0.800000000000000,0.800000000000000,0.800000000000000,0.800000000000000,0.80000000000000
0,0.894427190999916,1.60000000000000,1.62185079461706,0.899777750336160,0.632455532033676,1.0
5830052442584,0.692820323027551,1.05830052442584,0.480000000000000,0.800000000000000,1.131370
84989848,1.20000000000000];
s = [-1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
A = [0.00125714285714286,0.00125714285714286,0.00125714285714286,0.00125714285714286,0.001257
14285714286,0.000707142857142857,0.00181028571428571,0.00181028571428571,0.00181028571428571,
0.000707142857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857,0.0007071
42857142857,0.000707142857142857,0.000707142857142857,0.000707142857142857];
alpha = -1:0.1:1;
x1 = A(1) + alpha*s(1);
x2 = A(2) + alpha*s(2);
x3 = A(3) + alpha*s(3);
x4 = A(4) + alpha*s(4);
x5 = A(5) + alpha*s(5);
x6 = A(6) + alpha*s(6);
x7 = A(7) + alpha*s(7);
x8 = A(8) + alpha*s(8);
x9 = A(9) + alpha*s(9);
x10 = A(10) + alpha*s(10);
x11 = A(11) + alpha*s(11);
x12= A(12) + alpha*s(12);
x13 = A(13) + alpha*s(13);
x14= A(14) + alpha*s(14);
x15 = A(15) + alpha*s(15);
x16 = A(16) + alpha*s(16);
x17 = A(17) + alpha*s(17);

g = r*g*(x1*L(1)+x2*L(2)+x3*L(3)+x4*L(4)+x5*L(5)+x6*L(6)+x7*L(7)+x8*L(8)+x9*L(9)+x10*L(10)+x1
1*L(11)+x12*L(12)+x13*L(13)+x14*L(14)+x15*L(15)+x16*L(16)+x17*L(17))

g_x0 = g(1);

dg_x1 = r*g.*x1*L(1);
dg_x2 = r*g.*x2*L(2);
dg_x3 = r*g.*x3*L(3);
dg_x4 = r*g.*x4*L(4);
dg_x5 = r*g.*x5*L(5);
dg_x6 = r*g.*x6*L(6);
dg_x7 = r*g.*x7*L(7);
dg_x8 = r*g.*x8*L(8);
```

```
dg_x9 = r*g.*x9*L(9);
dg_x10 = r*g.*x10*L(10);
dg_x11 = r*g.*x11*L(11);
dg_x12 = r*g.*x12*L(12);
dg_x13 = r*g.*x13*L(13);
dg_x14 = r*g.*x14*L(14);
dg_x15 = r*g.*x15*L(15);
dg_x16 = r*g.*x16*L(16);
dg_x17 = r*g.*x17*L(17);
```

```
r =

        2780


g =

    9.8100


s =

  Columns 1 through 13

    -1     0     1     1     1     1     1     1     1     1     1     1     1

  Columns 14 through 17

     1     1     1     1


g =

   1.0e+05 *

  Columns 1 through 7

   -3.7229   -3.3501   -2.9774   -2.6046   -2.2318   -1.8590   -1.4862

  Columns 8 through 14

   -1.1134   -0.7406   -0.3678    0.0049    0.3777    0.7505    1.1233

  Columns 15 through 21

    1.4961    1.8689    2.2417    2.6144    2.9872    3.3600    3.7328
```

Linear Approximation

```
gL = g_x0 + (x1 - A(1)).*dg_x1 + (x2 - A(2)).*dg_x2 + (x3 - A(3)).*dg_x3+(x4 - A(4)).*dg_x4 +
 (x5 - A(5)).*dg_x5 + (x6 - A(6)).*dg_x6+(x7 - A(7)).*dg_x7 + (x8 - A(8)).*dg_x8 + (x9 - A(9)
).*dg_x9+(x10 - A(10)).*dg_x10 + (x11 - A(11)).*dg_x11 + (x12 - A(12)).*dg_x12+(x13 - A(13)).
*dg_x13 + (x14 - A(14)).*dg_x14 + (x15 - A(15)).*dg_x15+(x16 - A(16)).*dg_x16 + (x17 - A(17))
```

```
.*dg_x17
```

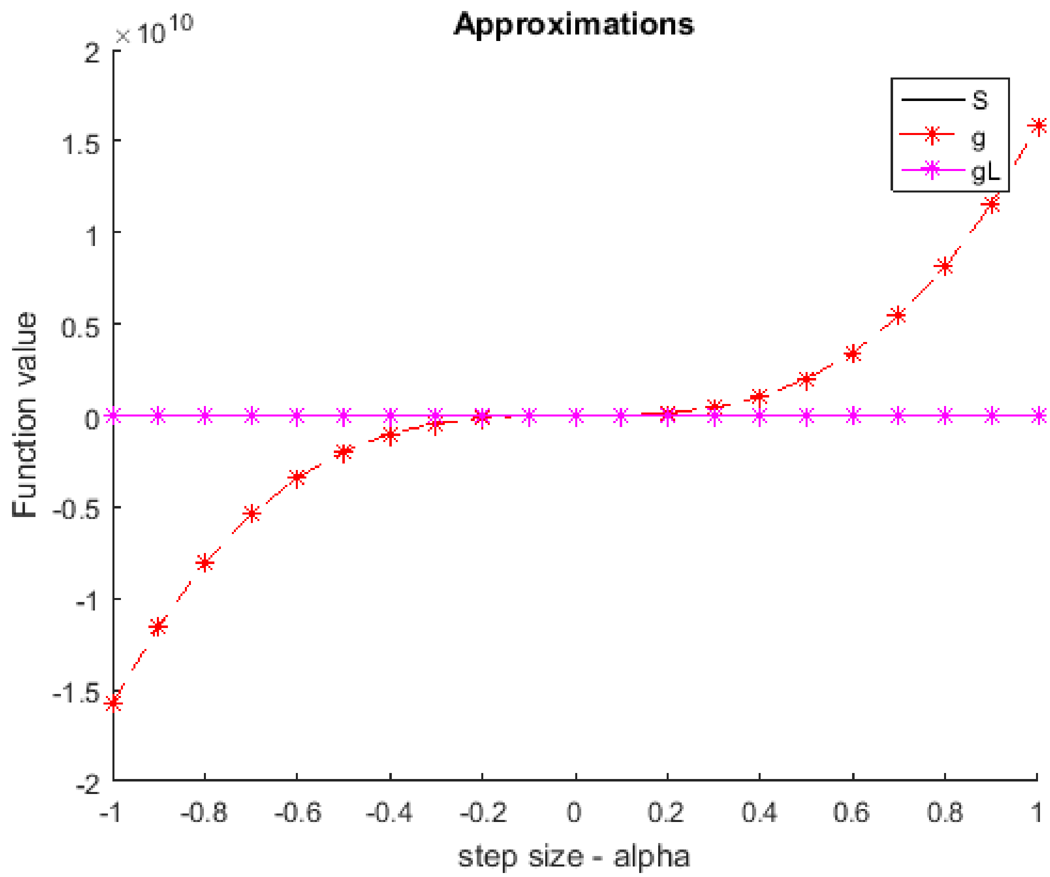
```
gL =

   1.0e+10 *

  Columns 1 through 7

   -1.5788   -1.1507   -0.8079   -0.5410   -0.3405   -0.1969   -0.1007

  Columns 8 through 14

   -0.0424   -0.0126   -0.0016   -0.0000    0.0016    0.0128    0.0430

  Columns 15 through 21

    0.1018    0.1987    0.3431    0.5445    0.8125    1.1565    1.5861
```

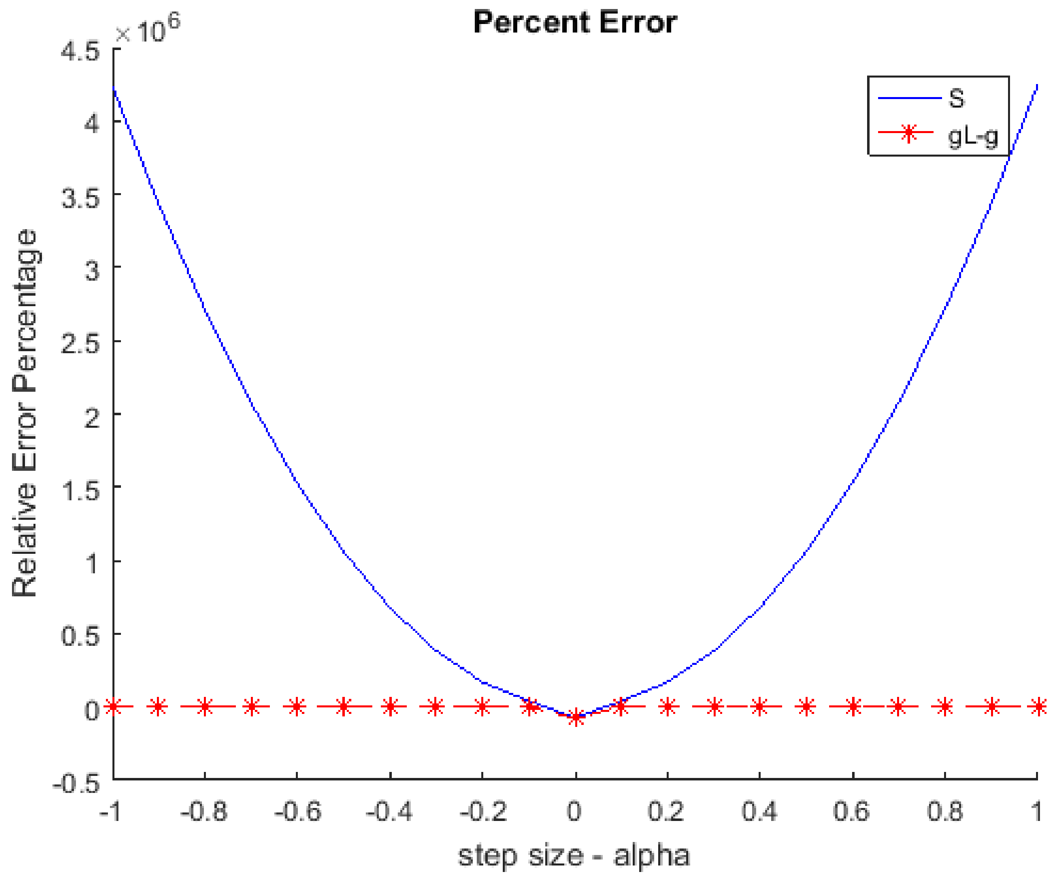
Reciprocal Approximaiton

```
gR = g_x0 + (x1 - A(1)).*(A(1)./x1).*dg_x1 + (x2 - A(2)).*(A(2)./x2).*dg_x2 + (x3 - A(3)).*(A
(3)./x3).*dg_x3 +        (x4 - A(4)).*(A(4)./x4).*dg_x4 + (x5 - A(5)).*(A(5)./x5).*dg_x5 + (x6
 - A(6)).*(A(6)./x6).*dg_x6 + (x7 - A(7)).*(A(7)./x7).*dg_x7 + (x8 - A(8)).*(A(8)./x8).*dg_x8
 + (x9 - A(9)).*(A(9)./x9).*dg_x9 + (x10 - A(10)).*(A(10)./x10).*dg_x10 + (x11 - A(11)).*(A(1
1)./x11).*dg_x11 + (x12 - A(12)).*(A(12)./x12).*dg_x12 + (x13 - A(13)).*(A(13)./x13).*dg_x13
+ (x14 - A(14)).*(A(14)./x14).*dg_x14 + (x15 - A(15)).*(A(15)./x15).*dg_x15 +(x16 - A(16)).*(
A(16)./x16).*dg_x16 + (x17 - A(17)).*(A(17)./x17).*dg_x17

figure; hold on
plot(alpha, g,'k')
plot(alpha, gL,'r--*')
plot(alpha, gR,'m-*')
legend('S','g','gL')
title('Approximations')
xlabel('step size - alpha')
ylabel('Function value')

figure; hold on
plot(alpha,(gL-g)./g*100,'b')
plot(alpha,(gR-g)./g*100,'r--*')
legend('S','gL-g', 'gR-g')
title('Percent Error')
xlabel('step size - alpha')
ylabel('Relative Error Percentage')
```

```
gR =

   1.0e+07 *

  Columns 1 through 7

    1.5248    1.2279    0.9622    0.7278    0.5246    0.3528    0.2122
```

```
  Columns 8 through 14

    0.1029    0.0249   -0.0218   -0.0372   -0.0214    0.0258    0.1042

  Columns 15 through 21

    0.2139    0.3548    0.5271    0.7307    0.9655    1.2316    1.5290

Warning: Ignoring extra legend entries.
```





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