Given Joint PMF of X & Y i.e; $\underline{\underline{P_{X,Y}(x,y)}}$

We want the marginal PMF of X. ie, $\underline{\underline{P_X(x), x \in R_X}}$

$Y \backslash X$	$x=1$	$x=2$	$x=3$	$P_Y(y)$
$y=4$	.	..	.	$P_Y(4)$
$y=3$	.	.	.	$P_Y(3)$
$y=2$	.	..	.	$P_Y(2)$
$y=1$	.	.	.	$P_X(1)$
$P_X(x)$:				

$R_X = \{1, 2, 3\}$

$R_Y = \{1, 2, 3, 4\}$

$$P_X(x) = \sum_{y \in R_Y} P_{X,Y}(x, y)$$

$$P_X(1) = \sum_{y=1}^{4} P_{X,Y}(1, y) = P_{X,Y}(1,1) + P_{X,Y}(1,2) + P_{X,Y}(1,3) + P_{X,Y}(1,4)$$

$$P_X(2) = \sum_{y=1}^{4} P_{X,Y}(2, y)$$

$$P_Y(y) = \sum_{x \in R_X} P_{X,Y}(x, y) \qquad P_Y(1) = \sum_{x=1}^{3} P_{X,Y}(x, 1)$$

$$= P_{X,Y}(1,1) + P_{X,Y}(2,1) + P(3,1)$$

__Central Limit theorem:__ "Normal dist $\equiv$ Parent dist of all other prob. dists."

Let $X_1, X_2, \dots X_n$ be n iid (independent, identically distributed) r.vs. with finite mean $E(X_i) = \mu$ & variance $Var(X_i) = \sigma^2$, $i = 1, 2 \dots n$. Then the limiting distribution of

$$Z_n = \frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$$

tends to the standard normal r.v as $\underline{n \to \infty}$ $(n \geq 30)$

$$\boxed{P(Z_n \leq a) \simeq \Phi(a)}$$

To use CLT.

P1). Suppose $X_1, X_2, \ldots, X_{100}$ are iid with mean $\frac{1}{5}$ & variance $\frac{1}{9}$. Use CLT to estimate

$$P\left(\sum_{i=1}^{100} X_i < 30\right)$$

Here $n = 100$

$\mu = \frac{1}{5}$, $\sigma^2 = \frac{1}{9}$

$\Rightarrow \sigma = \frac{1}{3}$

$$\Rightarrow P\left(\frac{\sum_{i=1}^{100} X_i - 100\cdot\left(\frac{1}{5}\right)}{\frac{1}{3}\sqrt{100}} < \frac{30 - 100\cdot\left(\frac{1}{5}\right)}{\frac{1}{3}\sqrt{100}}\right)$$

Std. normal r.v

$$\Rightarrow P(Z_n < 3) \simeq \Phi(3) \simeq 0.9987$$

$$P\left(\sum X_i < \omega\right) = ?$$

$$P\left(\frac{\sum X_i - n\mu}{\sigma\sqrt{n}} < \frac{\omega - n\mu}{\sigma\sqrt{n}}\right)$$

P2). Let $X_1, X_2, \ldots X_{\underline{81}}$ be iid., each X_i with $\mu = 5$ & $\sigma^2 = 4$. Approximate $P(X_1 + X_2 + \ldots + X_{81} > 369)$. Using CLT

$n = 81$

$$P\left(\frac{X_1+X_2+\cdots+X_{81} - 81(5)}{2\sqrt{81}} > \frac{369-(81)(5)}{2\sqrt{81}} \right)$$

By CLT

$$\Rightarrow P(Z_n > -2) = 1 - P(Z_n < -2)$$

$$= 1 - \Phi(-2)$$

$$= 1 - (1 - \Phi(2))$$

$$= \Phi(2)$$

$$\approx 0.9772$$

$X \sim N(\mu, \sigma)$

$$Z = \frac{X - \mu}{\sigma}$$

$\mu = E(X)$

$\sigma = SD(X)$

$X_1, X_2, \dots X_n$ are iid

$E(X_i) = \mu$

$Var(X_i) = \sigma^2$

$\Rightarrow \cdot \; X_1 + X_2 + \dots + X_n$

$$E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n)$$

$$= n\mu$$

$$Var(X_1 + X_2 + \dots + X_n) = Var(X_1) + Var(X_2) + \dots + Var(X_n)$$

$$= n\sigma^2$$

$$SD(X_1 + \dots + X_n) = \sqrt{n\sigma^2} = \sigma\sqrt{n}$$

$$Z_n = \frac{X_1 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$$