Note Title 02-10-2022

Properties of PDF

(i) $f_X(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

(iii) $P(a \leq X \leq b) = \int_a^b f_X(x)\,dx = F_X(b) - F_X(a)$

(iv) $f_X(x) = \dfrac{d}{dx} F_X(x)$

P$_1$). Suppose that X is a continuous r.v whose PDF is given by

$$f_X(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

a) What is the value of c?

b). Find $P(X > 1)$.

Answer:

(a). Since $\displaystyle\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow$

$$\int_{-\infty}^{\infty} = \int_{-\infty}^{0} 0 + \int_{0}^{2} + \int_{2}^{\infty} 0$$

$$\Rightarrow \int_{0}^{2} f_X(x)\,dx = 1$$

$$\Rightarrow \int_0^2 c(4x - 2x^2)\,dx = 1 \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow c\int_0^2 (4x - 2x^2)\,dx = 1$$

$$\Rightarrow c\left[4\int_0^2 x^1\,dx - 2\int_0^2 x^2\,dx\right] = 1$$

$$\Rightarrow c\left[\overset{2}{\cancel{4}}\cdot\left[\frac{x^2}{\cancel{2}}\right]_{x=0}^{x=2} - 2\left[\frac{x^3}{3}\right]_{x=0}^{x=2}\right] = 1$$

$$\Rightarrow c\left[2(4) - \frac{2}{3}[8]\right] = 1$$

$$\Rightarrow 8c\left[1-\frac{2}{3}\right]=1$$

$$\Rightarrow 8c\left[\frac{3-2}{3}\right]=1 \Rightarrow \frac{8c}{3}=1$$

$$\Rightarrow \boxed{c = 3/8}$$

$$\Rightarrow f_X(x) = \begin{cases} \frac{3}{8}(4x-2x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(b) $$P(X>1) = \int_1^{\infty} f_X(x)\,dx = \int_1^{2} \frac{3}{8}(4x-2x^2)\,dx = 1/2$$

Verify