Assignment 1 : DCS 511   Roll No: 2022MCS120009

1) Let A and B be events. Find the expression for the event "Exactly one of the events A and B occurs". Draw Venn diagram for this event.

Ans:
Given that A and B are the two events

Probability that exactly one of the event occurs =

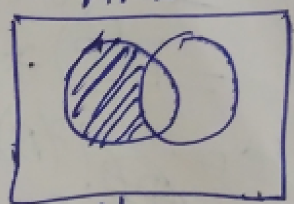
= P(A occurs and B does not occurs) or P(A does not occurs and B occurs)

$= P(A \cap B') + P(A' \cap B)$

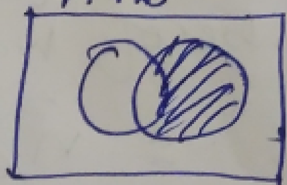
$= \{P(A) - P(A \cap B)\} + \{P(B) - P(A \cap B)\}$

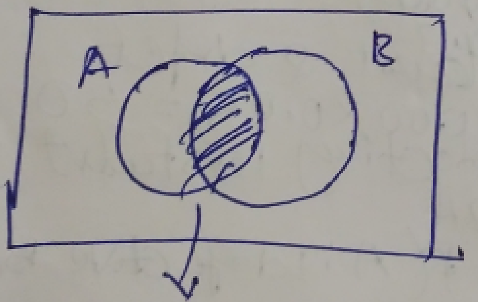
$= P(A) + P(B) - 2P(A \cap B)$

$A \cap B'$



$A' \cap B$





$P(A) + p(B) - 2p(A \cap B)$

2) How many distinct Permutation are there of 4 Red balls, 2 white balls and 3 black balls

Ans:

Given information.

Color	count
Red	4
White	2
Black	3
total	9

Possibilities $= n!$

$$= \frac{9!}{4!\ 2!\ 3!}$$

$$= \frac{9\times8\times7\times6\times5\times\cancel{4\times3\times2\times1}}{(\cancel{4\times3\times2\times1})(2\times1)(3\times2\times1)}$$

$$= \frac{9\times8\times7\times6\times5}{12}$$

$$= \frac{15120}{12}$$

$$= 1260$$

There are total of 1260 Distinct Possible ways.

(3) In How many ways can 10 students occupy 10 desks? 12 desks.

Ans: (1) No of Permutations 10 students occupy 10 desks

$\Rightarrow {}^{10}P_{10}$

$\Rightarrow$ Formula $= {}^{n}P_{r}$

$= n = 10$

$r = 10$

$= \dfrac{10!}{(10-10)!}$

$= 3628800$

(2) No of Permutations 10 students occupy 12 desks

$= {}^{n}P_{r}$

$= n = 12 \quad r = 10$

$= {}^{12}P_{10}$

$= \dfrac{12!}{(12-10)!} = \dfrac{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times\cancel{2\times1}}{\cancel{2\times1}}$

$= 239500800$

4) A lot of 100 item contains k defective items. m items are chosen at random and tested. What is the probability that r items are found defective?

Ans:

Number of ways in which m items can be selected out of 100 items $= C_m^{100}$.

There are k defective and 100-k non defective items in the lot, out of M chosen items. r are defective and m-r are non defective. Therefore, Number ways in which r items out of m selected items are defective $= C_r^k \times C_{m-r}^{100-k}$

Therefore, the probability that out of M items chosen at random, r are defective is

$$\boxed{\dfrac{C_r^k \times C_{m-r}^{100-k}}{C_m^{100}}}$$

5. A coin is tossed three times. Let us Assign equal Probability to each of the elementary events in S. What is the Probability that at least one show up in three throws?

Ans: Given information

1) Coin tossed 3 times

Sample space $= 2^3 = 8$

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Event: Probability at least one ~~or~~ Head shows up

$E = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$

Totally there 7 Possible outcome

$$P(E) = \frac{7}{8} = 0.875$$

6) A dice is tossed twice and a number of dots facing up is counted and noted in the order of occurrence.
Let A be event "total number of dots is even" and event B "both tossed had an even number of dots"
Find $P(A/B)$ and $P(B|A)$

Ans: Given Information

Dic tossed twice, so totally 36 outcome.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1.3)	(1.4)	(1.5)	(1.6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

S = the table above

A = total number of dots is even

A = {(1,1) (1,3) (1,5) (2,2) (2,4) (2,6) (3,1) (3,3) (3,5) (4,2) (4,4) (4,6) (5,1) (5,3) (5,5) (6,2) (6,4) (6,6)}

$|A| = 18$

B = Both the tosses had an even number

B = {(2,2) (2,4) (2.6) (4,2) (4,4) (4,6) (6,2) (6,4) (6,6)}

$|B| = 9$

$P(A) = \frac{18}{36}$ $P(B) = \frac{9}{36}$

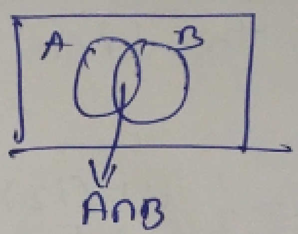
$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

→ Contd

6)

$P(A \cap B)$ = Probability of A and B

$P(A \cap B) = \frac{9}{36}$



$P(A \cap B) = \{(22)\ (24)\ (26)\ (42)\ (44)$
$(46)\ (62)\ (64)\ (66)\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{9/36}{9/36} = 1$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{9/36}{\frac{18}{36}}$$

$$= \frac{1}{2}$$

$$P(B/A) = 0.5$$

So. $P(A/B) = 1$

$P(B/A) = 0.5$

7) Two Fair dice are rolled. what is the conditional Probability that at least one lands on 6 given That the dice land on different numbers?

Ans:

S =	1	2	3	4	5	6
1	(11)	(12)	(13)	(14)	(15)	(16)
2	(21)	(22)	(23)	(24)	(25)	(26)
3	(31)	(32)	(33)	(34)	(35)	(36)
4	(41)	(42)	(43)	(44)	(45)	(46)
5	(51)	(52)	(53)	(54)	(55)	(56)
6	(61)	(62)	(63)	(64)	(65)	(66)

There are total 36 Possible outcome

A = Event of rolling the two dice would give 2 different numbers

A = {(12) (13) (14) (15) (16)
(21) (23) (24) (25) (26)
(31) (32) (34) (35) (36)
(41) (42) (43) (45) (46)
(51) (52) (53) (54) (56)
(61) (62) (63) (64) (65}

|A| = 30

$P(A) = \frac{30}{36}$

B = Rolling two dice would give atleast one 6

B = {(16) (26) (36) (46) (56)
(61) (62) (63) (64) (65) (66)}

|B| = 11

$P(B) = \frac{11}{36}$

$P(A \cap B)$ = Probability of rolling two dice would give two different numbers And would give at least 6

= {(16) (26) (36) (46) (56) (61) (62)
(63) (64) (65)

7)

$P(A \cap B) = \{(16)(26)(36)(46)(56)(61)(62)(63)(64)(65)\}$

$P(A \cap B) = \frac{10}{36}$

Therefore:

The conditional probability that at least one lands on 6 given that the dice land on different number

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{10/36}{30/36} = \frac{1}{3}$$

$$P(B/A) = 0.333$$

(8) A box contains 1000 light bulbs. The probability that there is at least 1 defective bulb in the box is 0.1 and the probability that there are at least 2 defective bulb is 0.05
Find the probability in each of the following case
(a) The box contains no defective bulbs
(b) The box contains exactly 1 defective bulbs
(c) The box contains at most 1 defective bulb.

Ans: Given information
total bulbs : 1000

(a)
P(A): Probability at least 1 defective buld 0.1
P(B): Probability that there are at least 2 defective is 0.05

Defective buld is 0.1
So probability of a bulb not being defective is
$= 1-0.1 = \underline{0.9}$
at least 1 defective $1000 \times 0.1 = 100$
Non defective buld $= 1000 - 100 = 900$
So buld being non defective $= \frac{900}{1000} = 0.9$

(b) $x^{\wedge}1000 = 0.9$
Log on both sides and $1000 \log x = \log(0.9)$
$1000 \log x = -0.00004576$
$\log x = 0.00004576$
$x = 0.9989$
Probably that a given buld is not defective
so the probability one defective is 0.09443

8)

(C) Find box Contains at most 1 defective balls

Probabilities Exactly 1 defective is

$= 1000C1 * 0.99989^{\wedge}999 * 0.0001054$

$= 0.09443$

At least 1 defective is

$= 1 - 0.09443$

$= 0.90557$