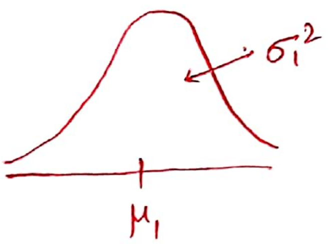
P2. An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a P-value in your answer:

Hypothesis Testing on the Difference in Means of Two normal Populations, with Variances are Known.

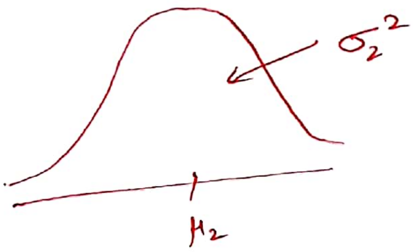
Population 1



Sample 1

$x_{11}, x_{12}, \ldots x_{1n_1}$

Population 2



Sample 2

$x_{21}, x_{22}, \ldots, x_{2n_2}$

Assumptions for Two Sample inference:

(i) $X_{11}, X_{12}, \ldots X_{1n_1}$ is a random sample of size n_1 from population 1

(ii) $X_{21}, X_{22}, \ldots X_{2n_2}$ is a random sample of size n_2 from population 2

(iii) The two populations represented by X_1 & X_2 are independent

(iv) Both are Normal populations.

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Recall.

$$E(\bar{X}_1 - \bar{X}_2) = E(\bar{X}_1) - E(\bar{X}_2) = \mu_1 - \mu_2$$

$$Var(\bar{X}_1 - \bar{X}_2) = Var(\bar{X}_1) + Var(\bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\Rightarrow (\bar{X}_1 - \bar{X}_2) \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$\Rightarrow \quad Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

Null hypothesis $H_0: \mu_1 - \mu_2 = \Delta_0$

Test statistic: $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

Alternative Hypothesis	Level α Z-test.
$H_1: \mu_1 - \mu_2 \neq \Delta_0$	Reject H_0 if $Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$ Accept H_0 if $-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	Reject H_0 if $Z > Z_\alpha$ Accept H_0 if $Z \leq +Z_\alpha$
$H_1: \mu_1 - \mu_2 < \Delta_0$	Reject H_0 if $Z < -Z_\alpha$ Accept H_0 if $Z \geq -Z_\alpha$

(A). Paint Drying Time Problem.

A Product developer is interested in reducing the drying time of a Primer paint. Two formulations of the Paint are tested; formulation 1 is the Standard Chemistry; and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the Standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using $\alpha = 0.05$?

Soln:

Null Hypothesis $H_0: \mu_1 - \mu_2 = 0$ ie, $\mu_1 = \mu_2$

Alternative Hypothesis $H_1: \mu_1 > \mu_2$. We want to reject H_0 if the new ingredient reduces mean drying time.

Test Statistic: $$Z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Given $\alpha = 0.05$, $\sigma_1^2 = \sigma_2^2 = 8^2$, $n_1 = n_2 = 10$,

$\bar{x}_1 = 121$ & $\bar{x}_2 = 112$

Computation of the test statistic:

$$Z = \frac{(121-112)}{\sqrt{\frac{8^2}{10} + \frac{8^2}{10}}} = 2.52$$

Critical value $Z_\alpha = Z_{0.05} = 1.65$

Right tail Z-test

Reject H_0 if $Z > Z_\alpha$

Accept H_0 if $Z \leq Z_\alpha$.

Decision: Since $Z = 2.52 > 1.65 = Z_\alpha$

we reject H_0 and we conclude that adding the new ingredient to the paint significantly reduces the drying time.