t-Tests on the mean of a Normal Population, When Variance σ^2 is Unknown:-

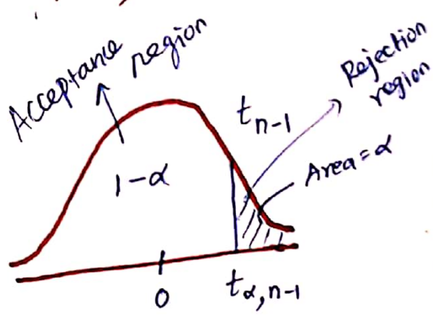
If $x_1, x_2, \ldots x_n$ is a random sample of size 'n' from a normal population with mean μ & variance σ^2, then we know that the test statistic $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ has a Student's t distribution with n-1 degrees of freedom.

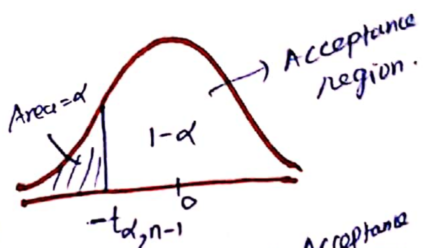
Now we frame the t-test based on Student t-distribution with n-1 degrees of freedom. The acceptance & rejection regions according to the direction of H_1.

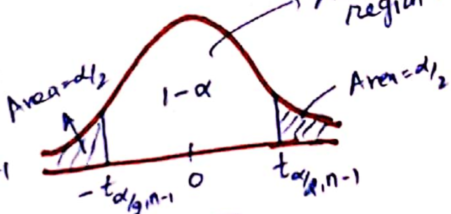
Null hypothesis $H_0: \mu = \mu_0$

Test Statistic $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ (small samples)

Alternative Hypothesis	Level α 't'-test
(i) **Right Tail Alternative** $H_1: \mu > \mu_0$	reject H_0 if $t > t_{\alpha, n-1}$ accept H_0 if $t \leq t_{\alpha, n-1}$
(ii) **Left Tail Alternative** $H_1: \mu < \mu_0$	reject H_0 if $t < -t_{\alpha, n-1}$ Accept H_0 if $t \geq -t_{\alpha, n-1}$
(iii) **Two Sided Alternative** $H_1: \mu \neq \mu_0$	reject H_0 if $\lvert t \rvert > t_{\alpha/2, n-1}$ accept H_0 if $\lvert t \rvert \leq t_{\alpha/2, n-1}$







Acceptance region $A = \left[-t_{\alpha/2, n-1}, t_{\alpha/2, n-1}\right]$

Rejection region $R = (-\infty, -t_{\alpha/2, n-1}) \cup (t_{\alpha/2, n-1}, +\infty)$.

(Pi). (Unauthorized use of a computer account)

[If an unauthorized person accesses a computer account with the correct user name and password (stolen or cracked).

Can this intrusion be detected?"

One Method: The time between keystrokes, the time a key is depressed, the frequency of various keywords are measured and compared with the account owner. If there are noticeable differences, an intruder is detected.]

(P1). The following times between keystrokes were recorded when a user typed the username and password:

0.46, 0.38, 0.31, 0.24, 0.20, 0.31, 0.34, 0.42, 0.09, 0.18, 0.46, 0.21 seconds. A long time authorized user of the account makes 0.2 seconds between keystrokes. At a 1% level of significance, is this an evidence of an unathorized attempt?

Soln: $H_0: \mu = 0.2$

$H_1: \mu \neq 0.2$

Given $\alpha = 0.01$, $n = 12$, we found the sample statistics

$\bar{x} = 0.3$, $s = 0.1183$.

Compute the t-statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{0.3 - 0.2}{(0.1183)/\sqrt{12}} = 5.8565.$

For two sided alternative,

The acceptance region is $[-t_{\alpha/2,n-1}, t_{\alpha/2,n-1}]$

Here $t_{\alpha/2,n-1} = t_{\frac{0.01}{2},12-1} = t_{0.005,11} = 3.106$ (From t-distribution table)

$\Rightarrow$ The acceptance region is $[-3.106, 3.106]$

$\Rightarrow$ Our test-statistic does not belong to the acceptance region

$\Rightarrow$ Therefore, we reject the null hypothesis, and conclude that there is a significant evidence of an unauthorized use of that account.

(P2). The Edison Electric institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal.

Soln:

H_0: $\mu = 46$ kilowatt hours

H_1: $\mu < 46$ kilowatt hours.

Given $\alpha = 0.05$, $n = 12$, $\bar{x} = 42$ kilowatt hours

$s = 11.9$ kilowatt hours

Test Statistic : $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{42 - 46}{11.9/\sqrt{12}} = -1.16$.

For Left tail alternative, Level α t-test

reject H_0 if $t < -t_{\alpha, n-1}$

accept H_0 if $t \geq -t_{\alpha, n-1}$

$t_{\alpha, n-1} = t_{0.05, 12-1} = t_{0.05, 11} = 1.796$.

The acceptance region is $t \geq -1.796$. $[-1.796, +\infty)$

∴ Our test statistic value belongs to the acceptance region.

⇒ Accept H_0. and conclude that the average number of kilowatt hours used annually by home vacuum cleaners is not significantly less than 46.

Tests on the Variance of a Normal Distribution :-

Suppose that we wish to test the hypothesis that the variance of a normal population σ^2 equals a specified value, say σ_0^2.

Let $x_1, x_2, \ldots, x_n$ be a random sample of size 'n' from this normal population. To test

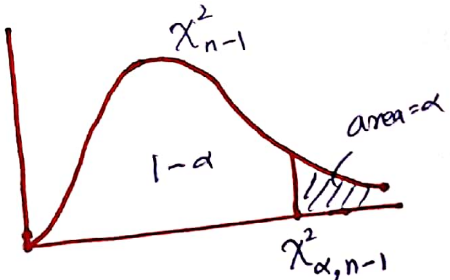
$$H_0 : \sigma^2 = \sigma_0^2$$

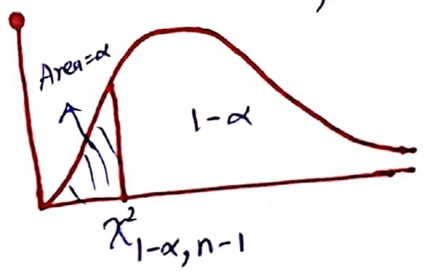
$$H_1 : \sigma^2 \neq \sigma_0^2$$

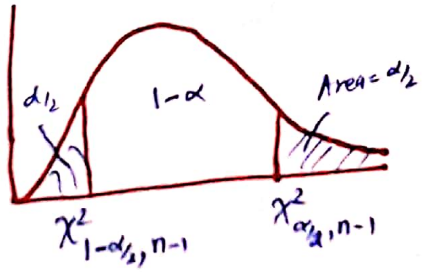
We will use the test statistic $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

We know that the test statistic χ^2 ~~$\sim \chi^2$~~ follows a Chi-squared distribution with '$n-1$' degrees of freedom.

Alternative Hypothesis	Level α test
(i). $H_1 : \sigma^2 > \sigma_0^2$	accept H_0 if $\chi^2 \leq \chi^2_{\alpha, n-1}$ Reject H_0 if $\chi^2 > \chi^2_{\alpha, n-1}$
(ii) $H_1 : \sigma^2 < \sigma_0^2$	Reject H_0 if $\chi^2 < \chi^2_{1-\alpha, n-1}$ Accept H_0 if $\chi^2 \geq \chi^2_{1-\alpha, n-1}$
(iii) $H_1 : \sigma^2 \neq \sigma_0^2$	Reject H_0 if $\chi^2 < \chi^2_{1-\alpha/2, n-1}$ or $\chi^2 > \chi^2_{\alpha/2, n-1}$ Accept H_0 if $\chi^2_{1-\alpha/2, n-1} < \chi^2 < \chi^2_{\alpha/2, n-1}$







(P1) Automated Filling:

An automated filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume exceeds 0.01 (fluid ounces)^2, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use $\alpha = 0.05$, and assume that fill volume has a normal distribution.

Solution:

$H_0: \sigma^2 = 0.01$

$H_1: \sigma^2 > 0.01$

Right Tail alternative.

Reject H_0 if $\chi^2 > \chi^2_{\alpha, n-1}$

Accept H_0 if $\chi^2 \leq \chi^2_{\alpha, n-1}$

Test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

$\therefore$ Given $\alpha = 0.05$, $n = 20$, $s^2 = 0.0153$

$\chi^2_{\alpha, n-1} = \chi^2_{0.05, 20-1} = \chi^2_{0.05, 19} = 30.14.$

$\therefore$ Compute the value of the test statistic $\chi^2 = \frac{(20-1)(0.0153)}{0.01} = 29.07$

Because $\chi^2 = 29.07 < 30.14 = \chi^2_{0.05, 19}$

we accep H_0 and we conclude that there is no strong evidence that the variance of fill volume exceeds 0.01.

So there is no evidence of a problem with incorrectly filled bottles.

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