Note Title 09-10-2022

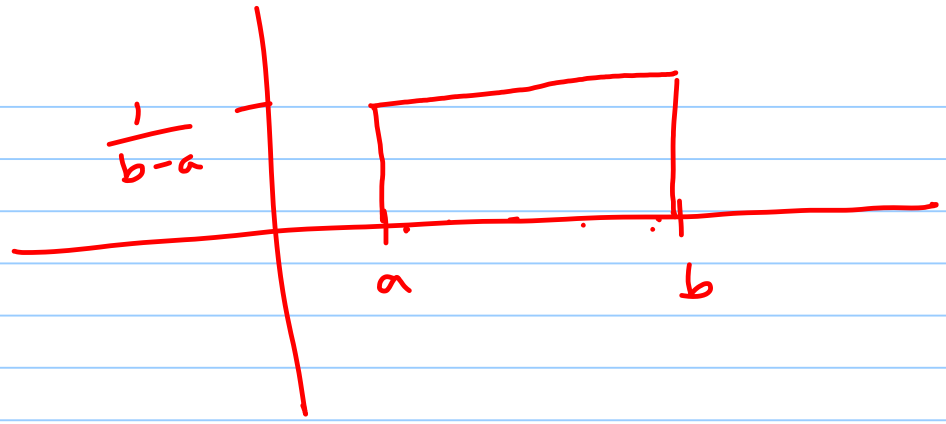
Continuous Uniform Distribution:

In CS, "Random number generation"

A continuous r.v that appears to have equally likely outcomes along its range of values.

The PDF of continuous uniform r.v on a finite interval $\underline{[a, b]}$

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$



$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

✓ Pick a real number between 0 & 1

$$R_X = [0, 1]$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx = \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{1}{b-a}\int_a^b x\, dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_{x=a}^{x=b}$$

$$= \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right]$$

$$= \frac{b+a}{2}$$

P1). If X is <u>uniformly distributed</u> over $(0, 10)$

Calculate the probabilities (i) $P(X < 3)$

(ii) $P(X > 6)$

(iii) $P(3 < X < 8)$.

The PDF of X is $f_X(x) = \begin{cases} \frac{1}{10} & ; \quad 0 < x < 10 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(i) $P(X < 3) = \int_0^3 f_X(x)\,dx = \int_0^3 \frac{1}{10}\,dx = \frac{1}{10}\int_0^3 dx$

$= \frac{1}{10}\left[x\right]_0^3$

$= \frac{3}{10}$ //

(ii) $P(X > 6) = \int_6^{10} \frac{1}{10}\,dx = \frac{1}{10}\int_6^{10} dx = \frac{1}{10}\left[x\right]_6^{10}$

$= \frac{4}{10}$

(iii) $P(3 < x < 8) = \int_3^8 f_X(x)\,dx = \int_3^8 \frac{1}{10}\,dx = \frac{5}{10} = \frac{1}{2}$

Given $X \sim N(\mu, \sigma)$ Mean

$\mu = E(X)$

$\sigma^2 = Var(X)$

Define a Standard normal r.v.

$$Z = \frac{X - \mu}{\sigma}$$

$E(ax+b) = aE(x)+b$

$a = \frac{1}{\sigma}, \; b = \frac{-\mu}{\sigma}$

$$E(Z) = E\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right) = \frac{1}{\sigma}E(X) - \frac{\mu}{\sigma} = \frac{\mu}{\sigma} - \frac{\mu}{\sigma}$$

$$= 0$$

$$\boxed{Var(ax+b) = a^2 \cdot Var(X).}$$

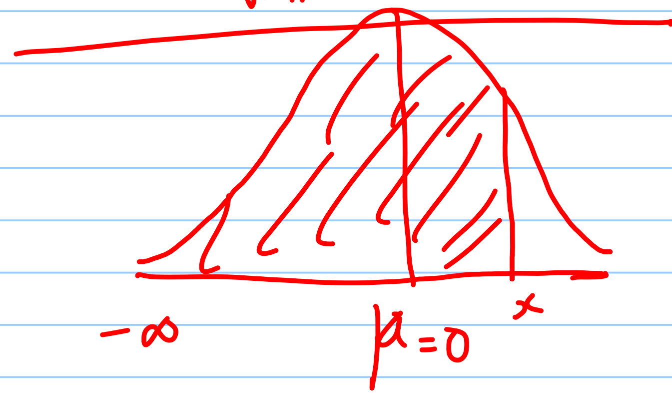
$$Var(Z) = Var\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right) = \frac{1}{\sigma^2} \cdot Var(X) = \frac{1}{\sigma^2} \cdot \sigma^2$$

$$\boxed{X \sim N(\mu, \sigma)}$$

$$\boxed{Z \sim N(0, 1)}$$

PDF of Z = ''

$$f_Z = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < \infty$$



CDF of Z:

$$\Phi(x) = P(Z \le x)$$

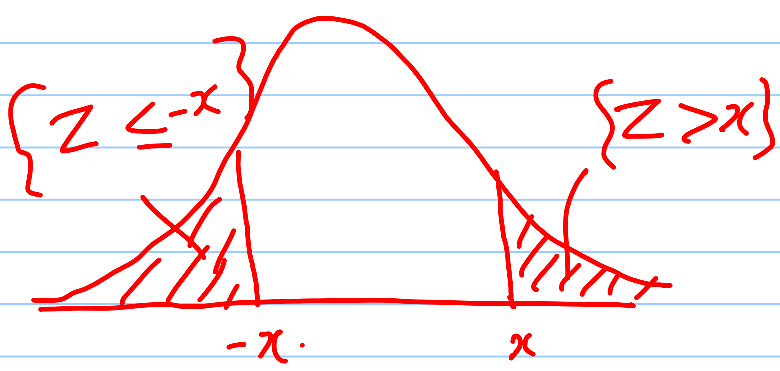
$$= \int_{-\infty}^{x} f_Z(x)\, dx$$

$$\Phi(-x) = P(Z \le -x)$$

$$= P(Z > x)$$

$$= 1 - P(Z \le x)$$

$$\boxed{\Phi(-x) = 1 - \Phi(x)}$$



P1). For a standard normal r.v Z,

Compute (i) $P(Z < 1.53)$ $\longrightarrow$ Use $\Phi(x)$

(ii) $P(Z > 1.26)$

(iii) $P(Z > -1.37)$

?

$P(Z < 1.53) = \Phi(1.53)$, To find it, we have to use the standard normal table.

$$P(Z < 1.53) = \Phi(1.53) = \underline{0.93699}$$

Standard Normal table

$$\begin{array}{cc} & 0.03 \\ & \downarrow \\ \rightarrow 1.5 \longrightarrow & (0.93699) \end{array}$$

(ii) $P(Z > 1.26) = 1 - P(Z \leq 1.26)$

$$= 1 - \Phi(1.26) = 1 - 0.89617$$

$$= 0.1038$$

0.06

$\rightarrow 1.2 \longrightarrow 0.89617$

(iii) $P(Z > -1.37) = 1 - P(Z \leq -1.37)$

$= 1 - \Phi(-1.37)$

$\Phi(-x) = 1 - \Phi(x)$

$= 1 - (1 - \Phi(1.37)$

$= \Phi(1.37)$.

0.07

1.3

Non-Standard normal r.v

If X is a normal r.v with parameters $\mu = 3$, $\sigma^2 = 9$, or $X \sim N(3, 9)$

$N(\mu, \sigma^2)$ — Variance

$X \sim N(\underset{\mu}{3}, \underset{\sigma}{3})$

σ — SD

vi). $P(2 < X < 5)$

WKT $\quad Z = \frac{X-\mu}{\sigma}$ $\quad$ Here $\mu = 3$, $\sigma^2 = 9$

$\Rightarrow \sigma = 3$

$$Z = \frac{X-3}{3}$$

$$P(2 < X < 5) = P\left(\frac{2-3}{3} < \frac{X-3}{3} < \frac{5-3}{3}\right)$$

$$= P\left(\frac{-1}{3} < Z < \frac{2}{3}\right)$$

$$P(a < Z < b) = \Phi(b) - \Phi(a)$$

$$\Phi(-x) = 1 - \Phi(x).$$

$$= \Phi\left(\tfrac{2}{3}\right) - \Phi\left(-\tfrac{1}{3}\right)$$

$$= \Phi\left(\tfrac{2}{3}\right) - \left(1 - \Phi\left(\tfrac{1}{3}\right)\right)$$

$$= \Phi(0.67) + \Phi(0.33) - 1$$

$$= 0.74857 + 0.6293 - 1$$

$$= 0.3779$$

0.07

$\rightarrow$ 0.6 $\longrightarrow$

0.03

0.3 $\rightarrow$

(ii) $P(X > 0) =$