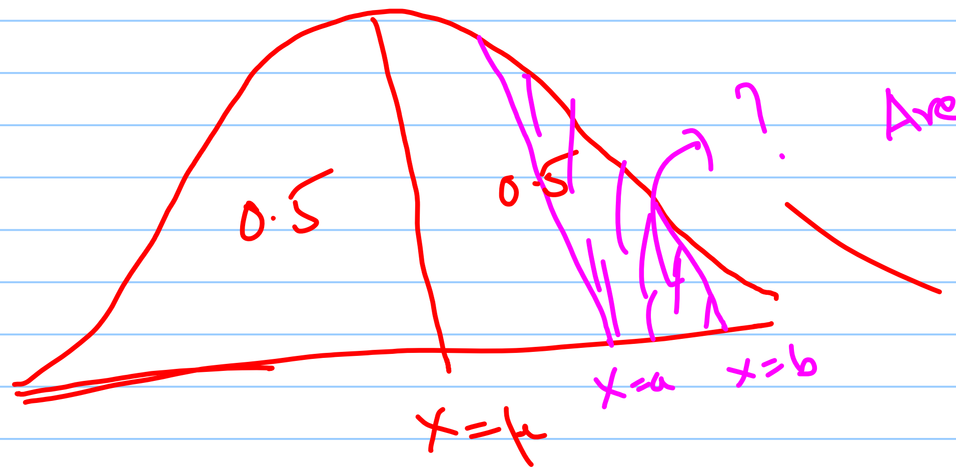
Normal Distribution :-



: Area

Std. normal

$P[X \in (a,b)]$

$P(a < X < b) =$

$X \sim N(\mu, \sigma)$

Defn. Standard Normal r.v.

$$Z = \frac{X-\mu}{\sigma}$$

$Z \sim N(0,1)$

CDF of 'Z'

$$\Phi(x) = \int_{-\infty}^{x} f_Z(x)\,dx$$

$$\Phi(-x) = 1 - \Phi(x)$$

Normal approximation to Binomial Distribution:

When n is large, a binomial r.v with parameters n, p will have approximately the normal distribution with mean np & Variance npq.

P1) Assume that in a digital communication channel, the number of bits received in error can be modelled by a binomial r.v. assume that the prob. that a bit is received in error $p = 1 \times 10^{-5}$. If 16 million bits are transmitted, what is the prob. than 150 or less errors occur?

n:

$$P(X \le 150) = \,? \; = \sum_{x=0}^{150} \binom{16{,}000{,}000}{x} (10^{-5})^{x} \cdot (1-10^{-5})^{16{,}000{,}000-x}$$

difficult

To approximate a binomial r.v by a std normal r.v
(X is)

we need to do the "continuity correction"

$$P(X \le x) = P(X \le x + 0.5)$$

$$= P\left(\frac{X - np}{\sqrt{npq}} \le \frac{x + 0.5 - np}{\sqrt{npq}} \right).$$

$P(X \geq x)$

$$P(x \leq X) \simeq P(x - 0.5 \leq X) = P\left(\frac{x-0.5-np}{\sqrt{npq}} \leq \frac{X-np}{\sqrt{npq}}\right).$$

$$P(X = x) = P(x \leq X \leq x) \simeq P(x - 0.5 \leq X \leq x + 0.5)$$

$$\simeq P\left(\frac{x-0.5-np}{\sqrt{npq}} < Z \leq \frac{x+0.5-np}{\sqrt{npq}}\right)$$

$$P(X \leq 150) \simeq P(X \leq 150.5) = P\left(\frac{X-np}{\sqrt{npq}} \leq \frac{150.5-np}{\sqrt{npq}}\right),$$

$n = 16 \times 10^6 \qquad p = 10^{-5}$

$np = \underline{160}$

$= P\left(Z \le \dfrac{150.5 - 160}{\sqrt{160(1-10^{-5})}}\right)$

$= P(Z \le -0.75)$

$= \Phi(-0.75)$

$= 1 - \Phi(0.75)$

$= 0.227$

$\Phi(-x) = 1 - \Phi(x)$

0.05

$+0.7 \longrightarrow$

Normal Approximation to a Poisson r.v X has a Parameter λ

WKT $E(X) = Var(X) = \lambda$

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

$$P(X \leq x) \simeq P(X \leq x + 0.5)$$

$$\simeq P\left(\frac{X-\lambda}{\sqrt{\lambda}} \leq \frac{x+0.5-\lambda}{\sqrt{\lambda}}\right).$$

$$\approx P(Z \leq \quad).$$

This approximation is good if $\lambda > 5$!

Exercise: Ref: Applied Statistics & prob. for engineers" Montgomery Runger

Page 118,	sec 4.5,	Problem Numbers	4.50, 51, 54
126	4.6,		4.64, 68
132	4.7,		4.95, 96, 98
137	4.8		4.112, 114, 121

Verify the answers given at the end of the book.