MATDIP301

Third Semester B.E. Degree Examination, June/July 2014
Advanced Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions***

1 a Find the modulus and amplitude of $\dfrac{5+3i}{4-2i}$ **(06 Marks)**

b Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\dfrac{n\pi}{4}$ **(07 Marks)**

c Prove that $\left(\dfrac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$ **(07 Marks)**

2 a Obtain the n^{th} derivative of $e^{ax}\sin(bx+c)$ **(06 Marks)**

b Find the n^{th} derivative of $\dfrac{x+3}{(x-1)(x+2)}$ **(07 Marks)**

c If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ **(07 Marks)**

3 a Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$. **(06 Marks)**

b. Find the pedal equation of the curve $r^n = a^n \cos n\theta$ **(07 Marks)**

c Using Maclaurin's series expand $\log(1+\sin x)$ upto the term containing x^4. **(07 Marks)**

4 a. If $z = \dfrac{x^2+y^2}{x+y}$, then show that $\left(\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)$ **(07 Marks)**

b. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

5 a. Obtain the reduction formula for $I_n = \int_0^{\pi/2} \sin^n x \, dx$ **(06 Marks)**

b. Evaluate $\int_0^{\pi}\int_{2\sin\theta}^{4\sin\theta} r^3 \, dr \, d\theta$ **(07 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z) \, dx \, dy \, dz$ **(07 Marks)**

6 a. With usual notations, prove that

$$\beta(m,n) = \frac{\Gamma(m)\ \Gamma(n)}{\Gamma(m+n)}$$ (06 Marks)

b. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ (07 Marks)

c. Prove that $\beta(m, ½) = 2^{2m-1}\ \beta(m, m)$ (07 Marks)

7 a. Solve $\frac{dy}{dx} = (4x+y+1)^2$, if $y(0) = 1$. (06 Marks)

b. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ (07 Marks)

c. Solve $\left\{y\left(1+\frac{1}{x}\right) + \cos y\right\}dx + (x + \log x - x\sin y)dy = 0$ (07 Marks)

8 a. Solve: $(D^3 + D^2 + 4D + 4)y = 0$ (06 Marks)

b. Solve: $(D^2 - 5D + 1)y = 1 + x^2$ (07 Marks)

c. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin x$ (07 Marks)

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