USN | | | | | | | | | | MATDIP301

Third Semester B.E. Degree Examination, June/July 2016
Advanced Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Express the complex number
   $\frac{(1+i)(1+3i)}{(1+5i)}$ in the form a + ib.   (06 Marks)

   b. Find the modulus and amplitude of $1 + \cos\theta + i\sin\theta$.   (07 Marks)
   c. Find the cube root of $1 - i$.   (07 Marks)

2 a. Find the nth derivative of $e^{ax}\cos(bx + c)$.   (06 Marks)

   b. Find the nth derivative of $\frac{6x}{(x-2)(x+2)(x-1)}$.   (07 Marks)

   c. If $y = \sin^{-1}x$, prove that $(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0$.   (07 Marks)

3 a. Find the angle of intersection of the curves $r^2\sin 2\theta = a^2$, $r^2\cos 2\theta = b^2$.   (06 Marks)
   b. Find the nodal equation of the curve $r(1 - \cos\theta) = 2a$.   (07 Marks)
   c. Expand $\log(\sec x)$ upto the term containing x^4 using Maclaurin's series.   (07 Marks)

4 a. If $u = x^3 - 3xy^2 + x + e^x\cos y + 1$, show that $u_{xx} + u_{yy} = 0$.   (06 Marks)

   b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $xu_x + yu_y + zu_z = 0$.   (07 Marks)

   c. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$, where $u = x + y + z$, $v = y + z$, $w = z$.   (07 Marks)

5 a. Obtain reduction formula for $\int \cos^n x\, dx$, where n is positive integer.   (06 Marks)

   b. Evaluate $\int_0^2 \frac{x^4}{\sqrt{4 - x^2}}dx$.   (07 Marks)

   c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dz\,dy\,dx$.   (07 Marks)

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6 a. Prove that: i) $\Gamma(n+1) = n\,\Gamma(n)$ and   ii) $\Gamma(n+1) = n!$ for a positive integer n.   (06 Marks)

   b. Prove that $\beta(m, n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.   (07 Marks)

   c. Show that $\displaystyle\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \cdot \int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta = \pi$.   (07 Marks)

7 a. Solve $\dfrac{dy}{dx} = (9x + y + 1)^2$.   (06 Marks)

   b. Solve $ye^{xy}\,dx + (xe^{xy} + 2y)\,dy = 0$.   (07 Marks)

   c. Solve $\dfrac{dy}{dx} + y\cot x = \cos x$.   (07 Marks)

8 a. Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 5e^{-2x}$.   (06 Marks)

   b. Solve $(D^2 - 4D + 13)y = \cos 2x$.   (07 Marks)

   c. Solve $(D^2 + 2D + 1)y = x^2 + 2x$.   (07 Marks)

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