

## SPIN GLASS ALGORITHM

Question:

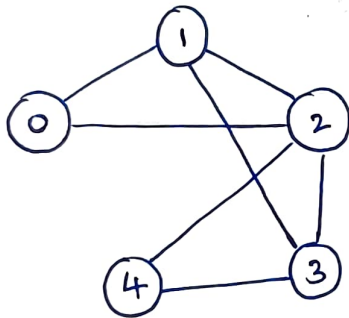
Consider a graph with the following structure. Identify the communities in the graph with the given conditions:

⇒ Initial spin:  $[+1, -1, +1, -1, +1]$

⇒ Temperature,  $T = 2.0$  (units)

⇒ Probability threshold,  $P = 0.5$  (i.e) accept the spin if  $P \leq 0.5$ ; reject otherwise.

⇒ show the steps for 5 iterations.



Answer:

Adjacency matrix,  $A =$

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	0
2	1	1	0	1	1
3	0	1	1	0	1
4	0	0	1	1	0

$$H = - \sum_{i < j} J_{ij} S_i S_j$$

where  $J_{ij} = \begin{cases} 1, & \text{if there is an edge between } i \text{ and } j \\ 0, & \text{if there is no edge between } i \text{ and } j \end{cases}$

$S_i, S_j$  are spins of  $i$  and  $j$  respectively.

Initially,  $S = [\overset{0}{+1}, \overset{1}{-1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{+1}]$

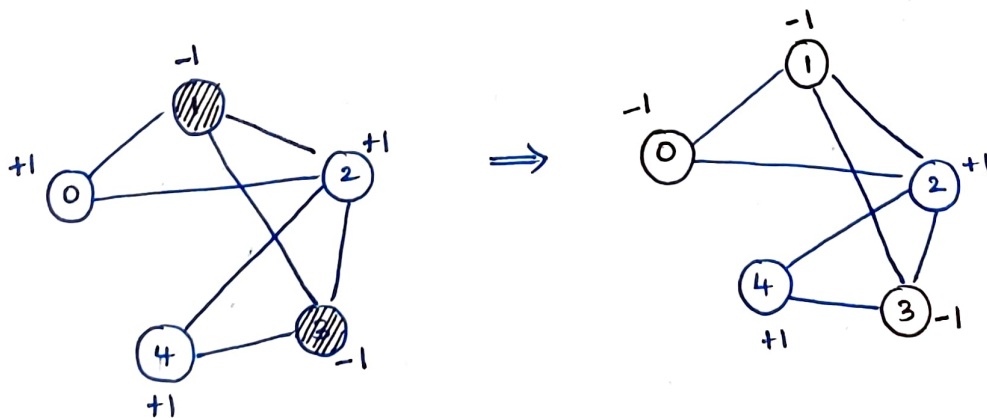
$$\begin{aligned}
 H_0 &= - (J_{01} S_0 S_1 + J_{02} S_0 S_2 + J_{12} S_1 S_2 + J_{13} S_1 S_3 + J_{24} S_2 S_4 + J_{34} S_3 S_4) \\
 &= - [(1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1)] \\
 &= - (-1 + 1 - 1 + 1 - 1 + 1 - 1) \\
 &= - (-1) \\
 &= 1
 \end{aligned}$$

Iteration 1 : Flip the spin of node 0 from +1 to -1

$$\Rightarrow S = [\overset{0}{-1}, \overset{1}{-1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{+1}]$$

$$\begin{aligned}
 H_1 &= - [(1)(-1)(-1) + (1)(-1)(1) + (1)(-1)(+1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1)] \\
 &= - (1 - 1 - 1 + 1 - 1 + 1 - 1) \\
 &= - (-1) = 1
 \end{aligned}$$

$$\Delta H = H_{\text{new}} - H_{\text{old}} = H_1 - H_0 = 1 - 1 = 0 \Rightarrow \text{accept the flip } (\because \Delta H \leq 0)$$



Iteration 2: Flip the spin of node 1 from -1 to +1

$$\Rightarrow S = [-1, +1, +1, -1, +1]$$

$$H_2 = - [ J_{01} S_0 S_1 + J_{02} S_0 S_2 + J_{12} S_1 S_2 + J_{13} S_1 S_3 + J_{23} S_2 S_3 + J_{24} S_2 S_4 + J_{34} S_3 S_4 ]$$

$$= - [ (1)(-1)(+1) + (1)(-1)(1) + (1)(1)(1) + (1)(1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) ]$$

$$= - (-1 - 1 + 1 - 1 - 1 + 1 - 1)$$

$$= - (-3) = 3$$

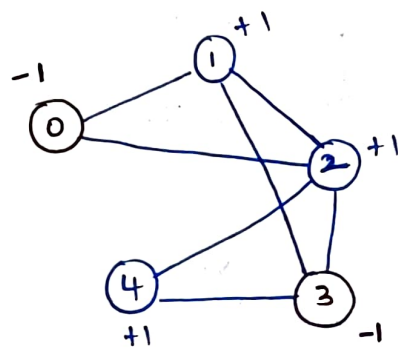
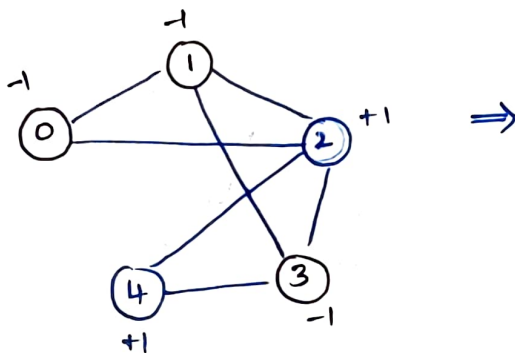
$$\Delta H = H_2 - H_1 = 3 - 1 = 2 > 0$$

$$\therefore \text{Probability, } P = \exp\left(\frac{-\Delta H}{T}\right)$$

$$= \exp\left(\frac{-2}{2}\right) = \exp(-1)$$

$$\approx 0.3679 < 0.5 \text{ (threshold)}$$

$\Rightarrow$  accept the spin



Iteration 3: Flip the spin of node 2 from +1 to -1

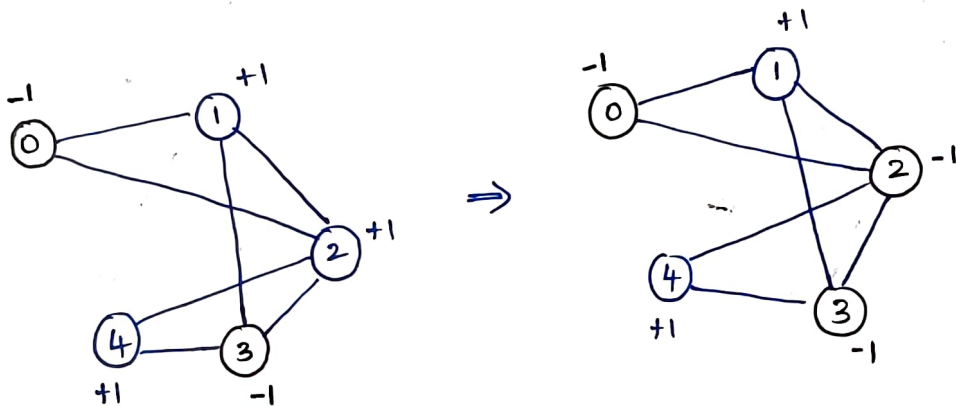
$$\Rightarrow s' = [-1, +1, -1, -1, +1]$$

$$H_3 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(-1) + (1)(-1)(-1) + (1)(-1)(1) + (1)(-1)(1)]$$

$$= -(-1 + 1 - 1 - 1 + 1 - 1 - 1)$$

$$= -(-3) = 3$$

$$\Delta H = H_3 - H_2 = 3 - 3 = 0 \Rightarrow \text{accept the flip } (\because \Delta H \leq 0)$$



Iteration 4: Flip the spin of node 3 from -1 to +1

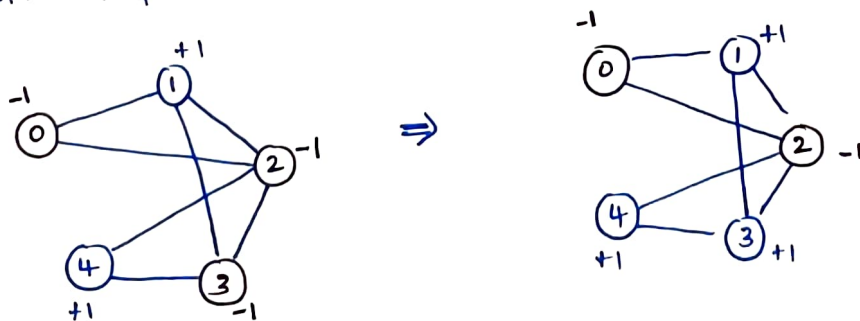
$$\Rightarrow s' = [-1, +1, -1, +1, +1]$$

$$H_4 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(1) + (1)(1)(1)]$$

$$= -(-1 + 1 - 1 + 1 - 1 - 1 + 1)$$

$$= -(-1) = 1$$

$$\Delta H = H_4 - H_3 = 0 - 1 = -1 \Rightarrow \text{accept the flip } (\because \Delta H \leq 0)$$



Iteration 5: Flip the spin of node 4 from +1 to -1

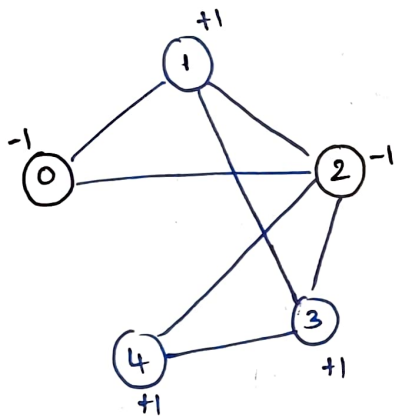
$$\Rightarrow S' = [-1, +1, -1, +1, -1]$$

$$H_5 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1)]$$

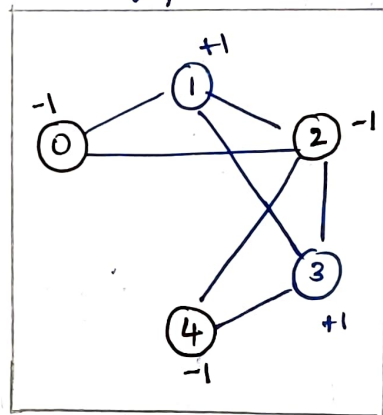
$$= -(-1 + 1 - 1 + 1 - 1 + 1 - 1)$$

$$= -(-1) = 1$$

$$\Delta H = H_5 - H_4 = 1 - 1 = 0 \Rightarrow \text{accept the flip } (\because \Delta H \leq 0)$$



$\Rightarrow$



— Final graph

Inference:

\*. Nodes that belong to community 1: {0, 2, 4}

\*. Nodes that belong to community 2: {1, 3}