SPIN GLASS ALGORITHM

Question :

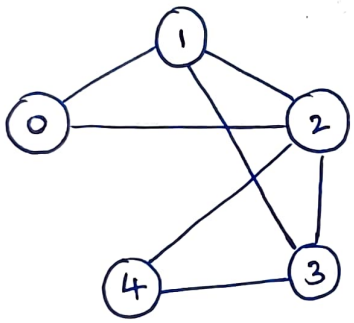
Consider a graph with the following structure. Identify the communities in the graph with the given conditions :

⇒ Initial spin : [+1, -1, +1, -1, +1]

⇒ Temperature, T = 2.0 (units)

⇒ Probability threshold, P = 0.5 (i.e) accept the spin if P<=0.5 ; reject otherwise.

⇒ Show the steps for 5 iterations.



Answer :

Adjacency matrix, A =

	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	0
2	1	1	0	1	1
3	0	1	1	0	1
4	0	0	1	1	0

$$H = -\sum_{i<j} J_{ij} S_i S_j$$

where $J_{ij} = \begin{cases} 1, & \text{if there is an edge between } i \text{ and } j \\ 0, & \text{if there is no edge between } i \text{ and } j \end{cases}$

S_i, S_j are spins of i and j respectively.

Initially, $S = [\overset{0}{+1}, \overset{1}{-1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{+1}]$

$$H_0 = -\{J_{01}S_0S_1 + J_{02}S_0S_2 + J_{12}S_1S_2 + J_{13}S_1S_3 + J_{24}S_2S_4 + J_{34}S_3S_4 + J_{23}S_2S_3\}$$

$$= -[(1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1)]$$

$$= -(-1+1-1+1-1+1-1)$$

$$= -(-1)$$

$$= 1$$

Iteration 1 : Flip the spin of node 0 from +1 to -1

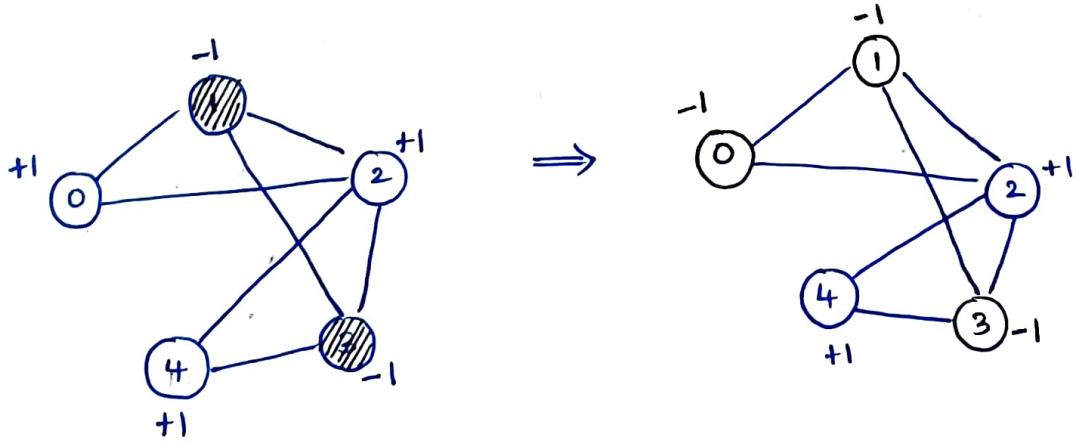
$$\Rightarrow S = [\overset{0}{-1}, \overset{1}{-1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{+1}]$$

$$H_1 = -[(1)(-1)(-1) + (1)(-1)(1) + (1)(-1)(+1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1)]$$

$$= -(1-1-1+1-1+1-1)$$

$$= -(-1) = 1$$

$\Delta H = H_{new} - H_{old} = H_1 - H_0 = 1 - 1 = 0 \Rightarrow$ accept the flip ($\because \Delta H \leq 0$)



Iteration 2 : Flip the spin of node 1 from -1 to $+1$

$$\Rightarrow S = [\overset{0}{-1}, \overset{1}{+1}, \overset{2}{+1}, \overset{3}{-1}, \overset{4}{+1}]$$

$$H_2 = -[J_{01}S_0S_1 + J_{02}S_0S_2 + J_{12}S_1S_2 + J_{13}S_1S_3 + J_{23}S_2S_3 + J_{24}S_2S_4 + J_{34}S_3S_4]$$

$$= -[(1)(-1)(+1) + (1)(-1)(1) + (1)(1)(1) + (1)(1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1)]$$

$$= -(-1-1+1-1-1+1-1)$$

$$= -(-3) = 3$$

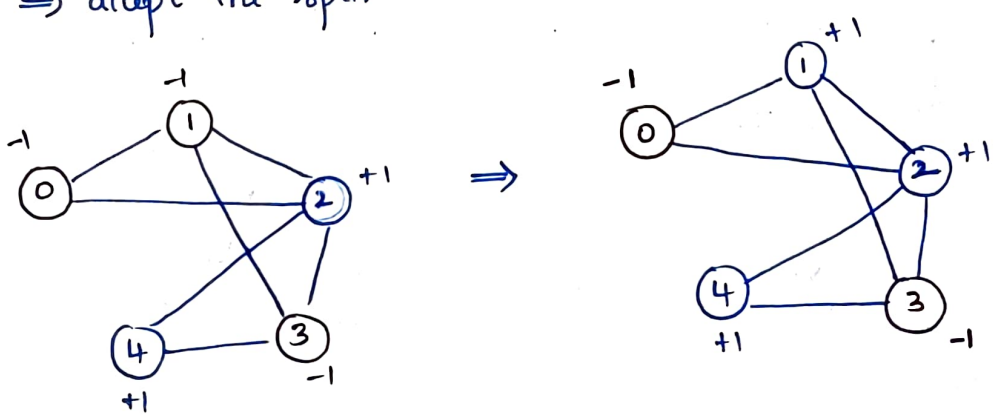
$$\Delta H = H_2 - H_1 = 3 - 1 = 2 > 0$$

$\therefore$ Probability, $P = \exp\left(\frac{-\Delta H}{T}\right)$

$$= \exp\left(\frac{-2}{2}\right) = \exp(-1)$$

$$\approx 0.3679 < 0.5 \text{ (threshold)}$$

$\Rightarrow$ accept the spin



Iteration 3 : Flip the spin of node 2 from +1 to -1

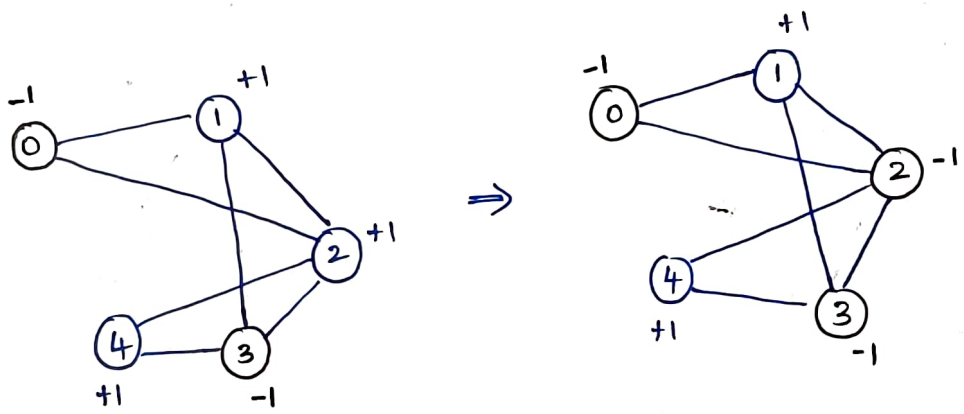
$\Rightarrow S' = [\overset{0}{-1}, \overset{1}{+1}, \overset{2}{-1}, \overset{3}{-1}, \overset{4}{+1}]$

$H_3 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(-1) + (1)(-1)(-1) + (1)(-1)(1) + (1)(-1)(1)]$

$= -(-1+1-1-1+1-1-1)$

$= -(-3) = 3$

$\Delta H = H_3 - H_2 = 3 - 3 = 0 \Rightarrow$ accept the flip ($\because \Delta H \leq 0$)



Iteration 4 : Flip the spin of node 3 from -1 to +1

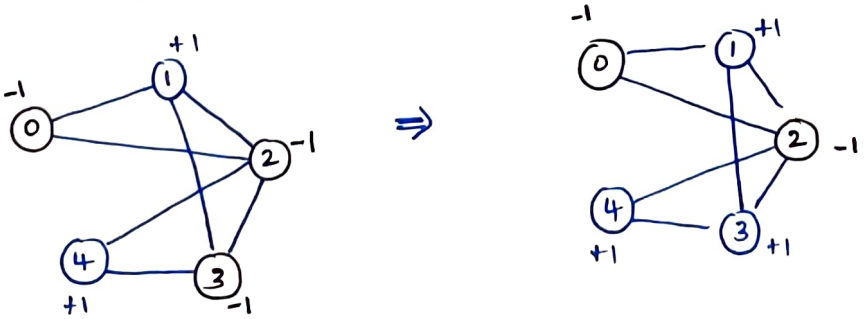
$\Rightarrow S' = [\overset{0}{-1}, \overset{1}{+1}, \overset{2}{-1}, \overset{3}{+1}, \overset{4}{+1}]$

$H_4 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(1) + (1)(1)(1)]$

$= -(-1+1-1+1-1-1+1)$

$= -(-1) = 1$

$\Delta H = H_4 - H_3 = 0 - 1 = -1 \Rightarrow$ accept the flip ($\because \Delta H \leq 0$)



Iteration 5 : Flip the spin of node 4 from +1 to −1

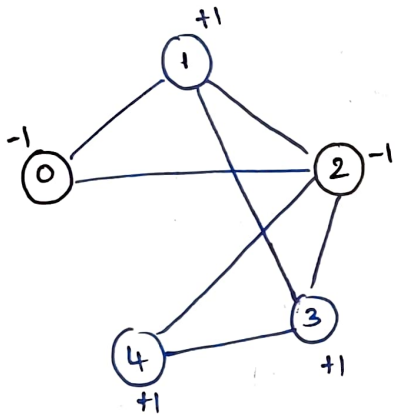
$\Rightarrow \quad S' = [\overset{0}{-1}, \overset{1}{+1}, \overset{2}{-1}, \overset{3}{+1}, \overset{4}{-1}]$

$$H_5 = -[(1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1) + (1)(1)(1) + (1)(-1)(1) + (1)(-1)(-1) + (1)(1)(-1)]$$

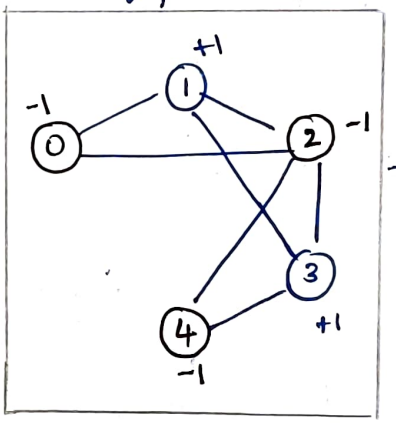
$$= -(-1+1-1+1-1+1-1)$$

$$= -(-1) = 1$$

$\Delta H = H_5 - H_4 = 1 - 1 = 0 \Rightarrow$ accept the flip $(\because \Delta H \leq 0)$



$\Longrightarrow$



— Final graph

Inference :

* Nodes that belong to community 1 : {0, 2, 4}
* Nodes that belong to community 2 : {1, 3}