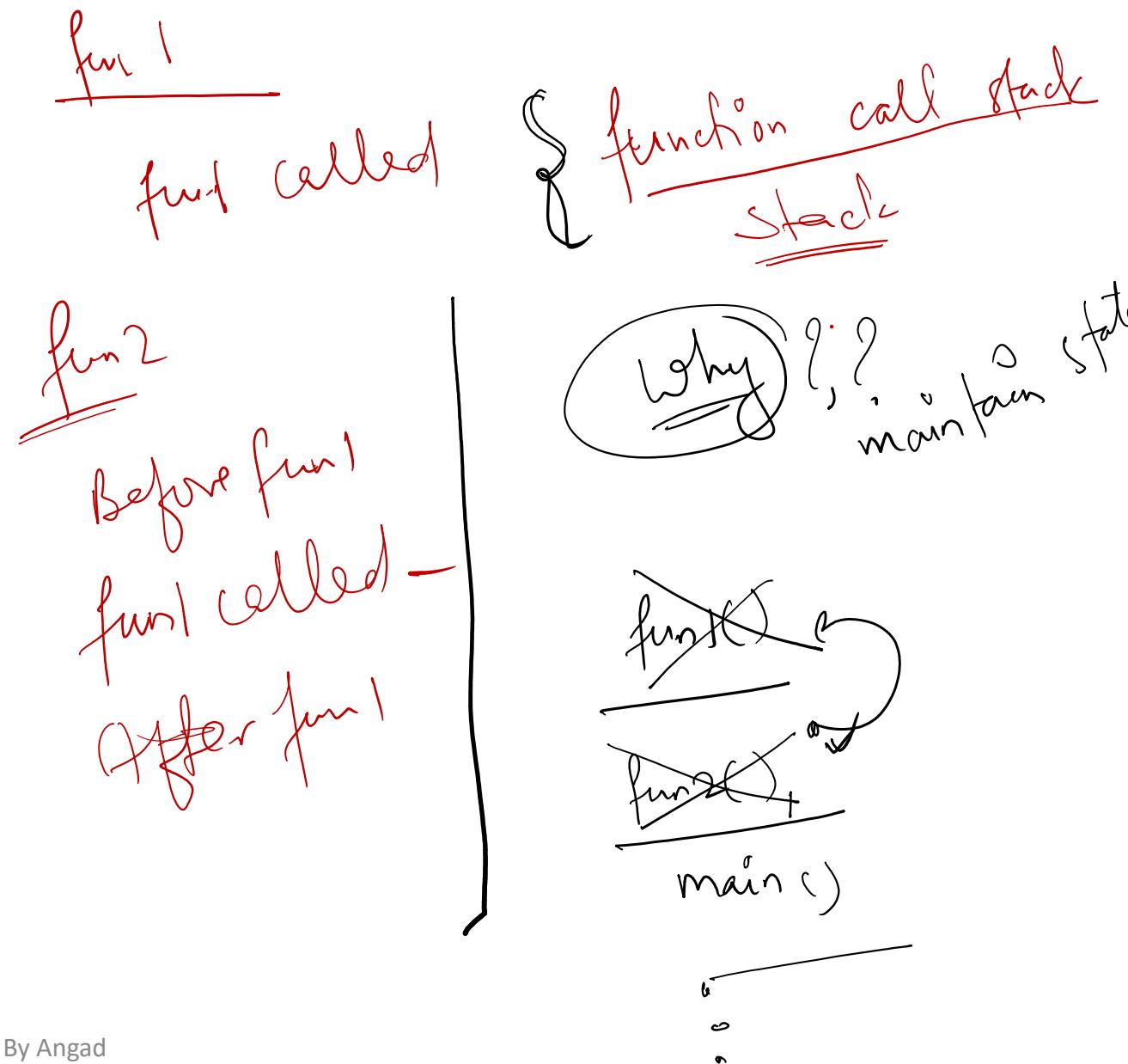


What will be the output of this code?

```
# First Basic Example
def fun1():
    print("fun1() Called")
    :
    :
def fun2():
    print("Before fun1()")
    fun1()
    print("After fun1()")
```

main → fun2()



Recursion: A function calls
itself Directly or Indirectly

Direct Recursion

def fun1():

_____✓

fun1()

_____✓
_____✓

Indirect Recursion

def fun1():

fun2()

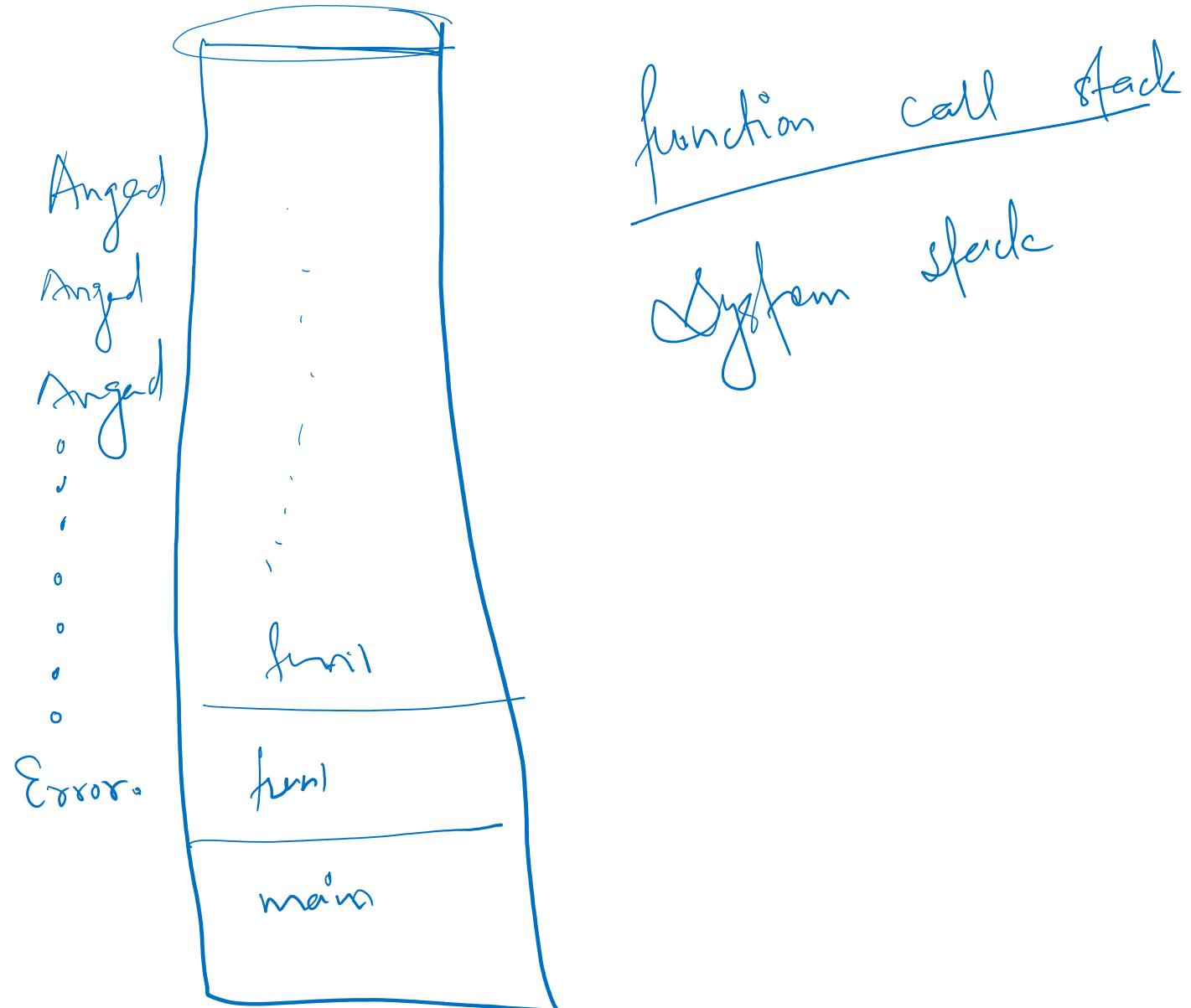
def fun2():

fun1()

What will be the output of this code?

Recursion

```
def fun1():
    print("Angad")
    fun1()
```



Write a function which prints “Angad” n times.

Recursion

```
def fun1():
    print("Angad")
    fun1()
```

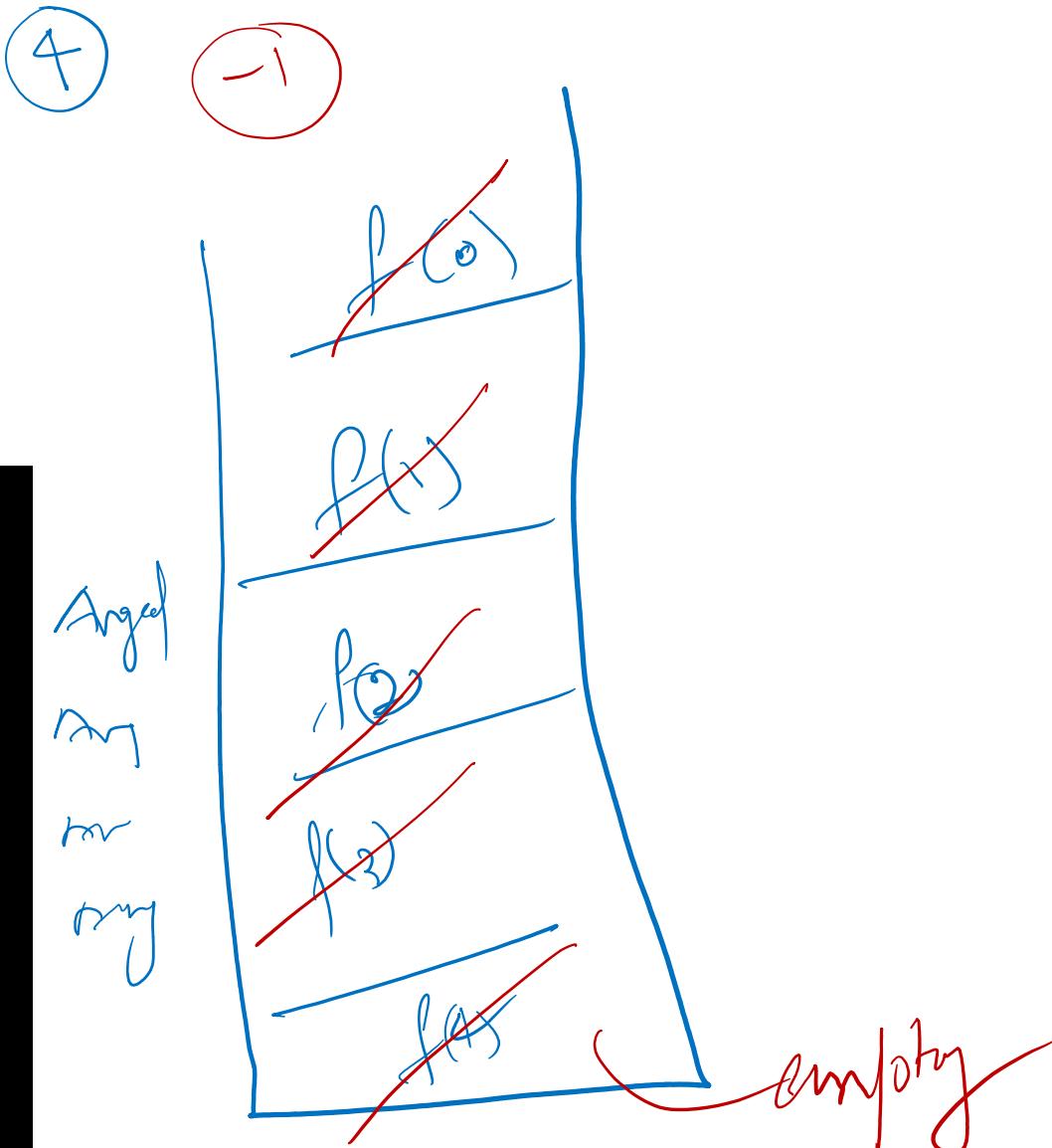
Modify it

S

parameter
(n)
if $n = 0$
return
print(Angad)
fun(1)

Different Cases for inputs like Zero, Negative

```
def fun1(n):
    # Base Case
    if n == 0:
        return
    print("Angad")
    # Recursive Case
    fun1(n-1)
```



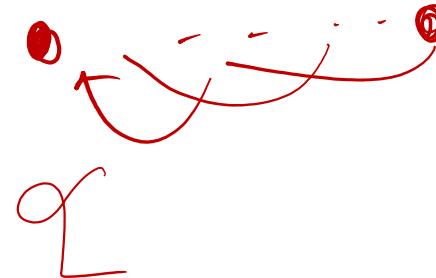
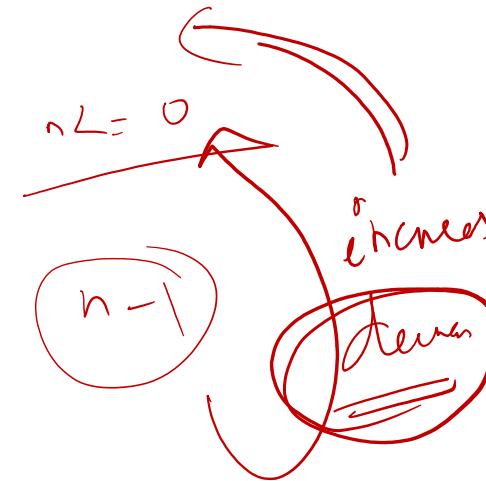
Typical Structure of Recursive Functions

Typical Recursive Function

```
def funX(parameters):  
    # Base Case  
    # _____  
    # _____
```

```
#Recursive Call  
funX(.....)  
# _____  
# _____
```

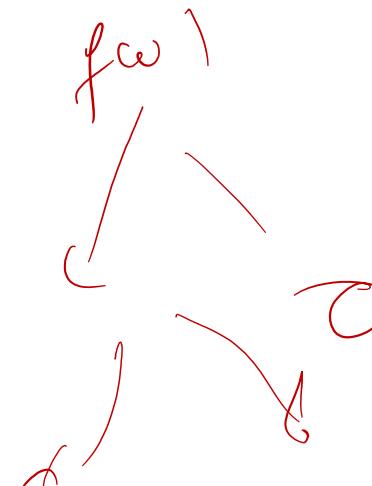
With at least one change to the parameter so that call approaches towards base case.



Applications of Recursion

- Many Alogs are base on recursion
 - DP → dynamic programming ✓
 - Backtracking ✓
 - Diving & Conquer ✓
- Many problems are inherently recursive
 - Tower of Hanoi ✓
 - DFS Based Traversals
 - DFS of Graph ✓
 - Tree DFS Traversals ✓

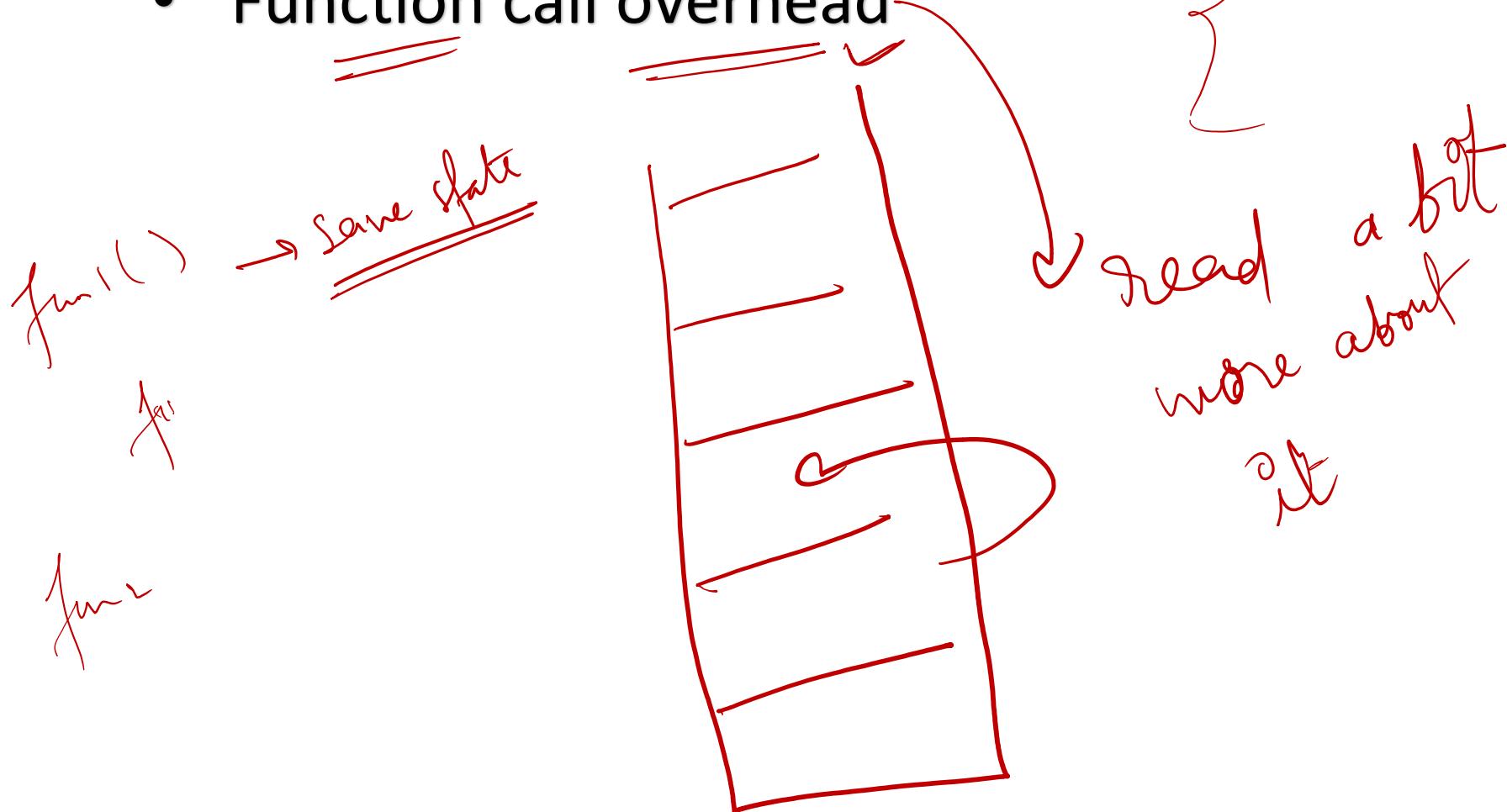
Solid on
Recursion
Easy for
you



Problems with recursion

- Extra stack space ✓

- Function call overhead



Writing Base Cases in Recursion

~~Recursiv~~
Write recursive solution for it

Factorial n where $n \geq 0$

- I/P: $n=4$
- O/P: 24
- I/P: $n=0$
- O/P: 1

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

\hookrightarrow return ~~not~~ fact(n-1)
fact(4)

~~factorial~~

~~$4!$ or $n!$~~

$$n! = n \times (n-1) \times (n-2) \times (n-3) \cdots 1$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 120$$

$$0! = 1$$

~~Iterative~~

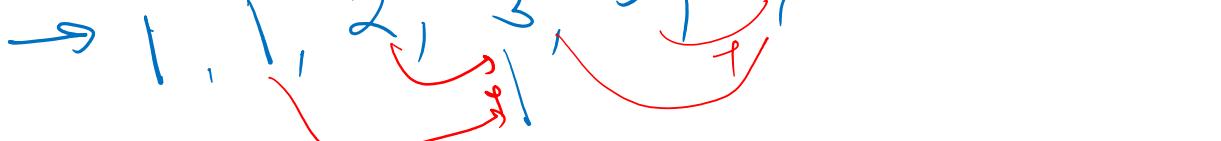
N-th Fibonacci Number where $n \geq 0$

- I/P: $n=4$
- O/P: 3
- I/P: $n=0$
- O/P: 0

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

~~fibonacci~~

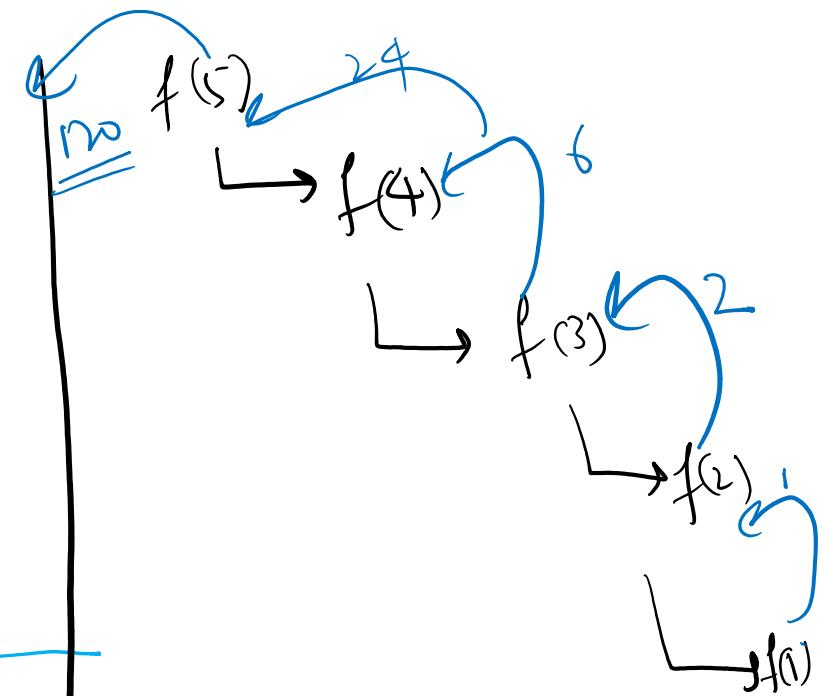
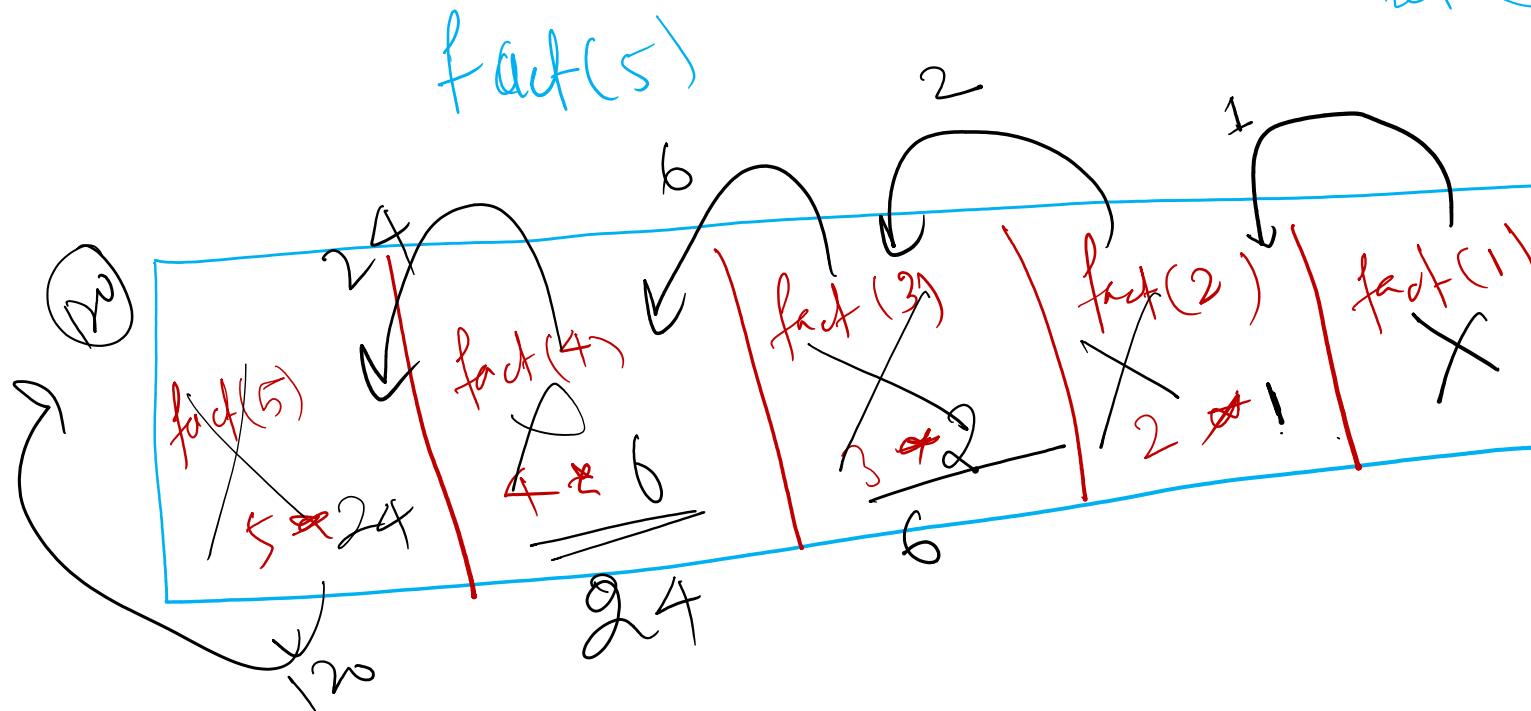
\rightarrow



```

def factorialN(n):
# ..... if (n == 0 || n == 1)
# ..... return 1
return n * factorialN(n-1)
    
```

$\text{if } (n \geq 1)$
 $\text{return } 1$
 $\text{if } (n < 0)$
 $\text{return } -1$

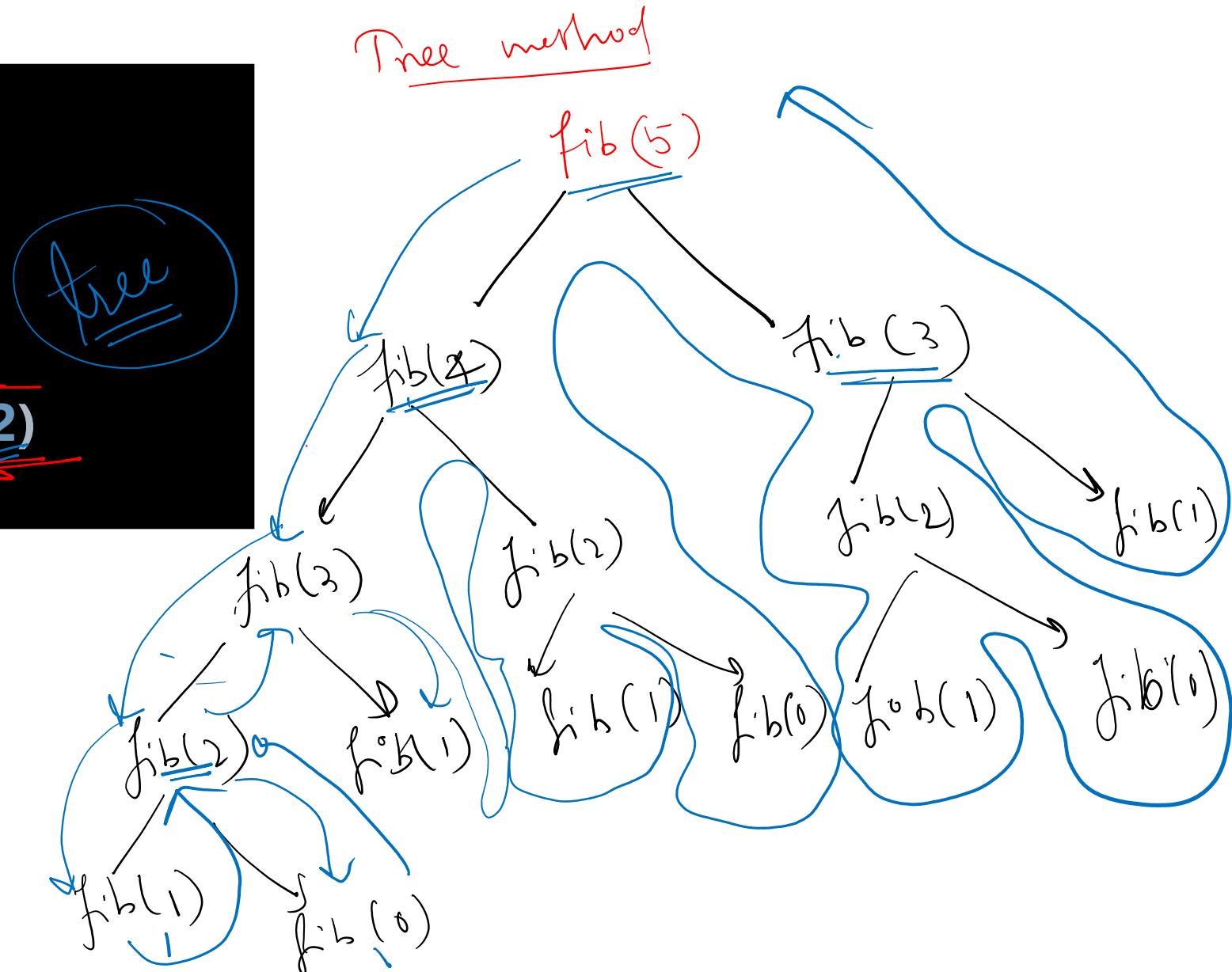


```

def fib(n):
# .....
# if(n <= 1)
#     return n
# .....
return fib(n-1) + fib(n-2)

```

Draw recursion tree
for $\underline{\underline{\text{fib}(5)}}$



Tail Recursion



A recursive function is called tail recursive if the function does not do anything after the last recursive call.

```
def fun1(n):
    if n == 0:
        return
    print("Angad")
    fun1(n-1) →
```

① Tail Recur^{inv}

```
def fun2(n):
    # .....
    # .....
    fun2(n-1) →
    print(n, end="")
```

② Not tail Rec

```
def factorialN(n):
    # .....
    # .....
    return n * factorialN(n-1)
```

③ ↗ multiplication
of last call

Tail Call Elimination

```
def fun1(n):
    if n == 0:
        return
    print(n, end="")
    fun1(n-1)
```

✓ eliminate recursion
Recursive to non recursive

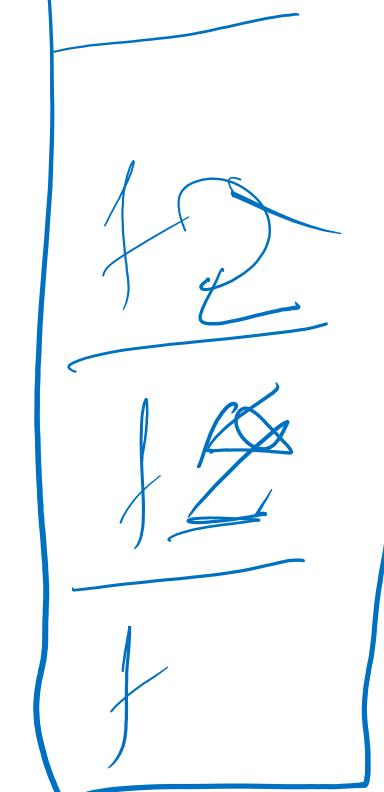
→ C++, Java

Python does not do it

Extra space

func stack

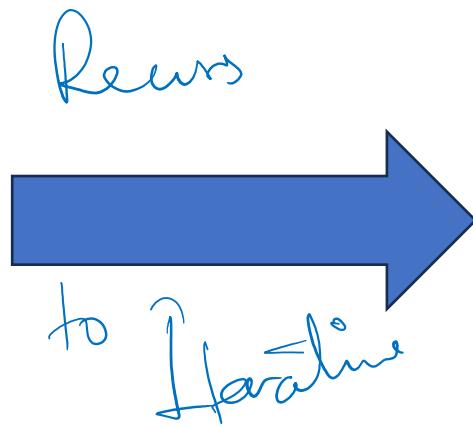
non-tail Recur
we can't do it



Conversion to Loop

See if → Tail Recursion

```
def fun1(n):
    if n == 0:
        return
    print(n, end="")
    fun1(n-1)
```



```
def fun1(n):
    while n != 0:
        print(n, end="")
        n = n-1
```

- ✓ Replace If with while
- ✓ Changes values of Parameters at the end

Examples of Tail Recursion

Quick Sort ✓

Post Order Tree Traversal ✓

- Merge Sort is not Tail recursive

~~inhardt~~

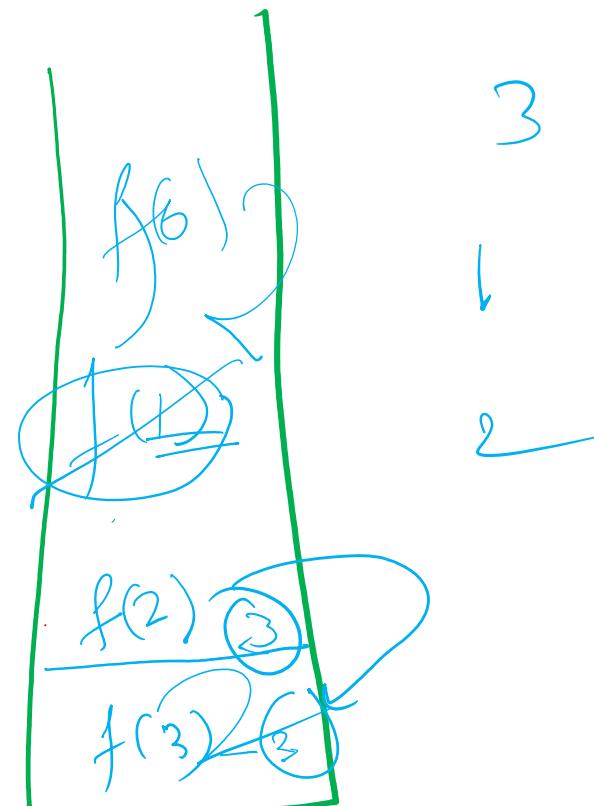
~~it~~
Concept wise we can
done here solving
problem ahead

Recursion Practice : Finding Output

Find Output of the below program

```
def prinFun(n):
    if n == 0:
        return
    print(n)
    prinFun(n - 1)
    print(n)

prinFun(3)
```

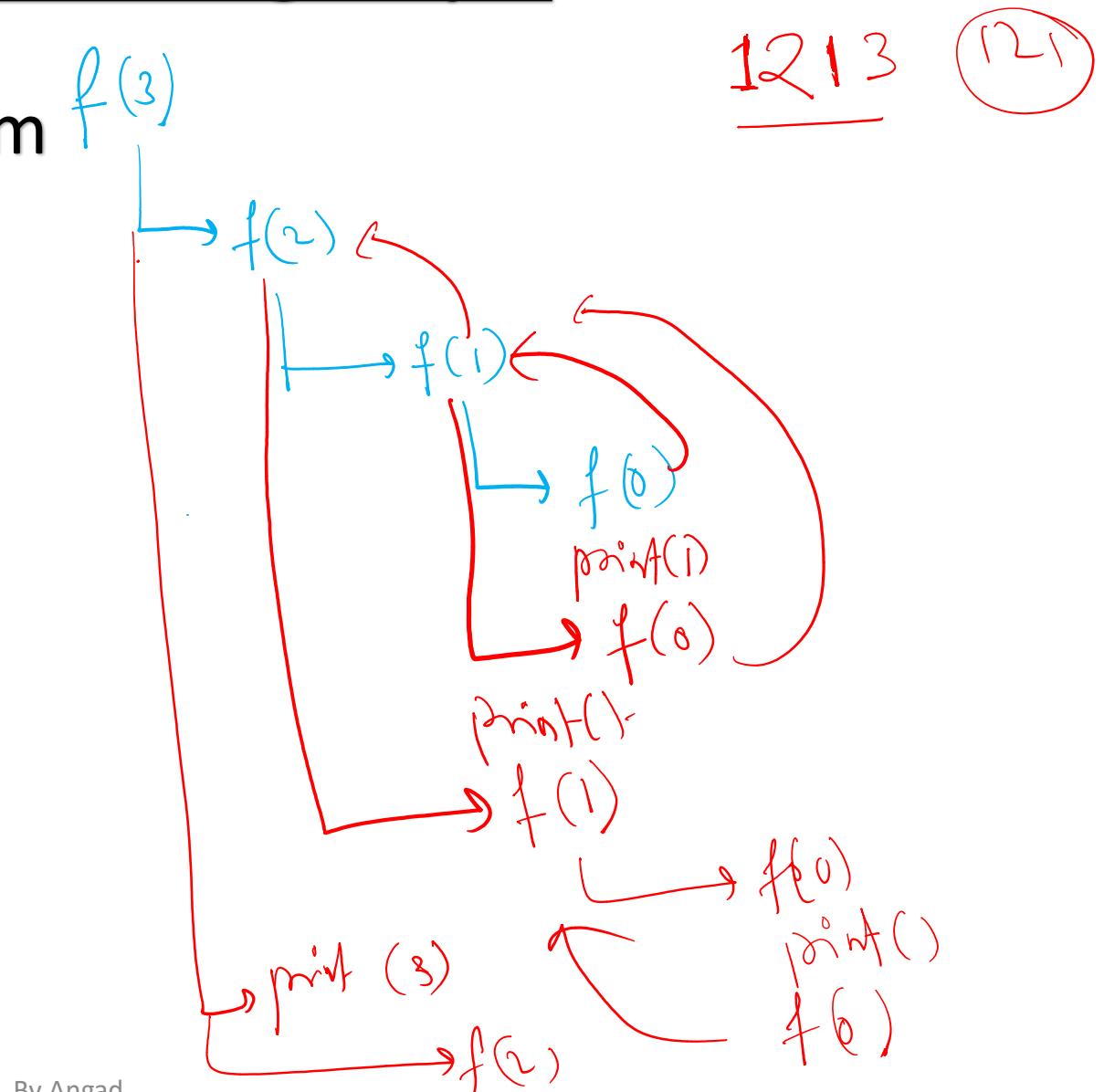


Recursion Practice : Finding Output

Find Output of the below program

```
def printFun2(n):
    if n == 0:
        return
    printFun2(n - 1)
    print(n)
    printFun2(n - 1)
```

print(3)



Recursion Practice : Finding Output

Find Output of the below program

```
def printFun3(n):
    if n <= 1:
        return 0
    else:
        return 1 + printFun3(n/2)
```

point($f(16)$) :

$f(16)$

A

$\log_2 N$

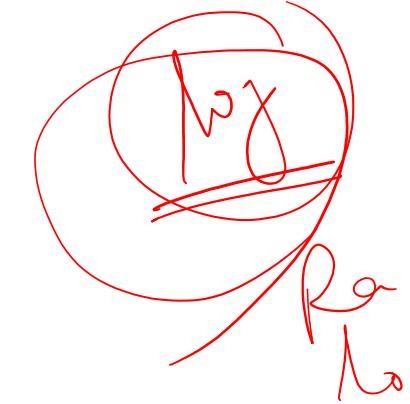
$f(16) = 4$

$\rightarrow 1 + f(8) = 3$

$\rightarrow 1 + f(4) = 2$

$\rightarrow 1 + f(2) = 1$

$\rightarrow 1 + f(1) = 0$



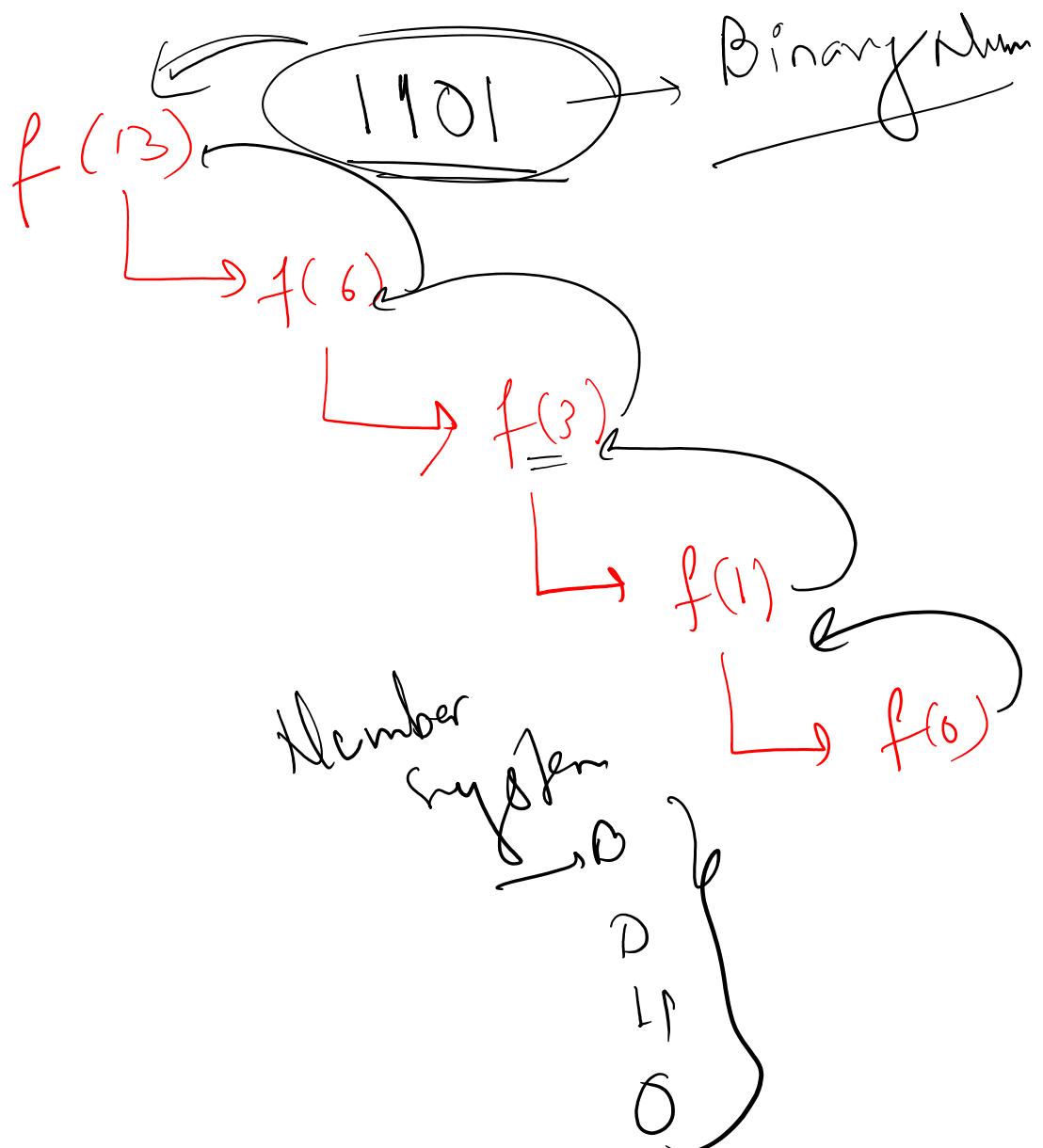
Recursion Practice : Finding Output

Find Output of the below program

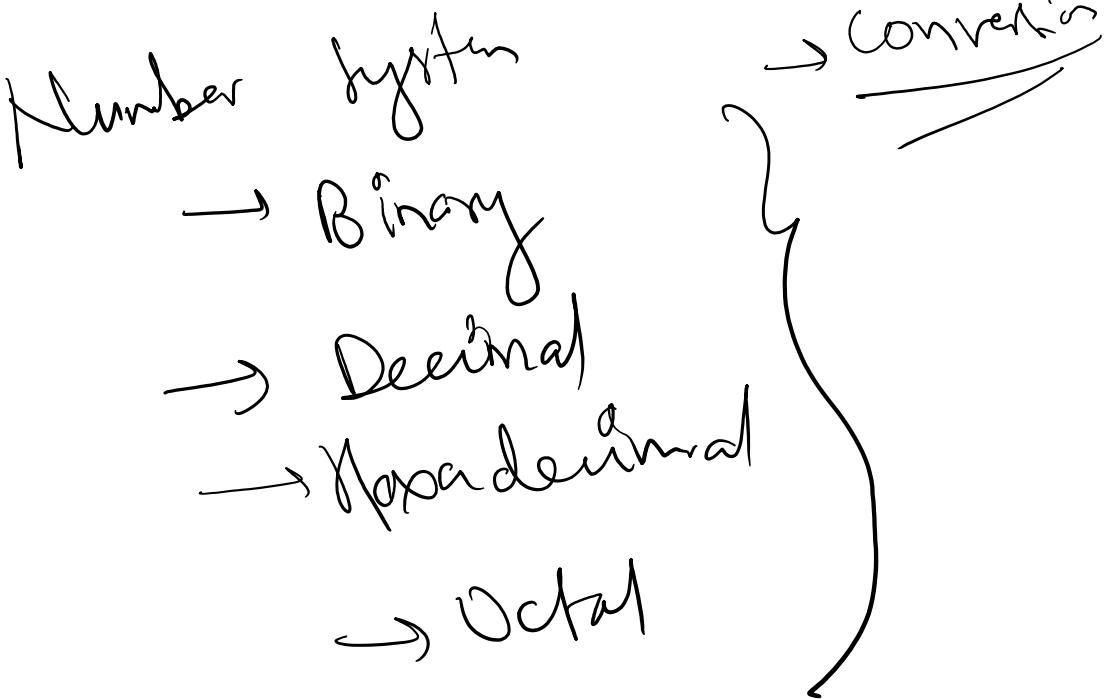
```
def printFun4(n):
    if n == 0:
        return
    → printFun4(n // 2)
    print(n % 2)
```

$n = 13$

IS



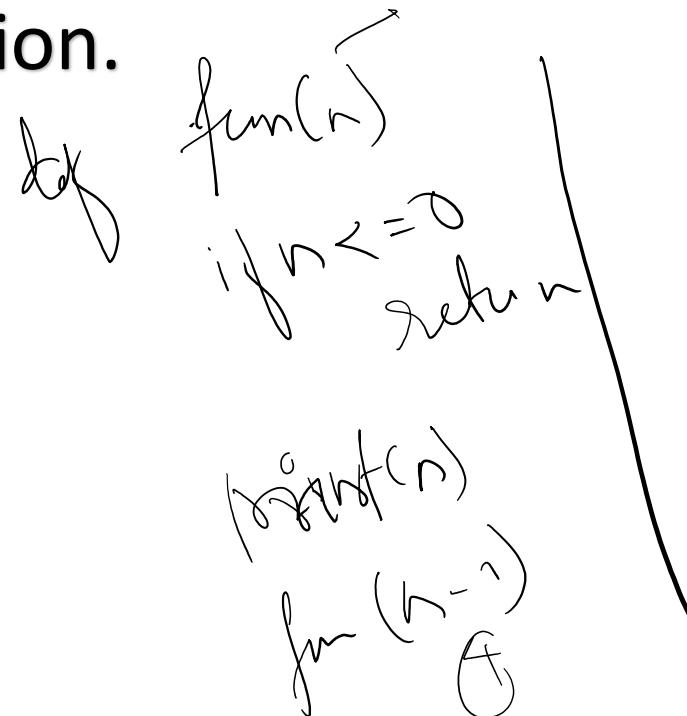
Recursion Practice : Finding Output



Recursion Practice : Writing Programs

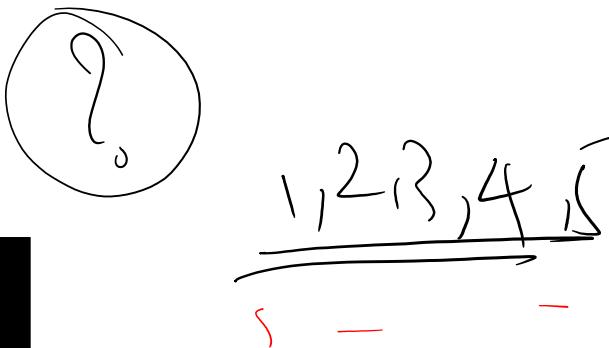
Print n to 1 using Recursion.

- I/P: n=5
- O/P: 5 4 3 2 1
- I/P: n=2
- O/P: 2 1



Recursion Practice : Writing Programs

```
def print1toN(n):  
    if n == 0:  
        return  
    print1toN(n - 1)  
    print(n)
```



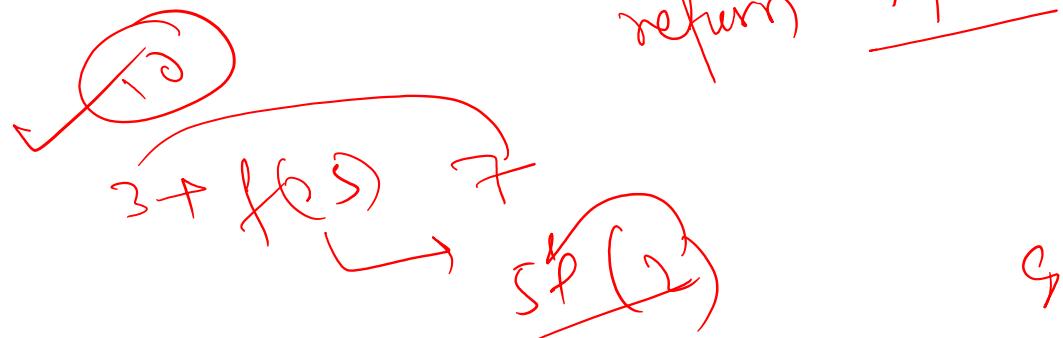
Recursion Practice : Writing Programs

Sum of Digits using Recursion.

- I/P: n=253
- O/P: 10
- I/P: n=984
- O/P: 21

def sumD(n)
Base
if ($n < 10$)
return n

Recursive
return $n \% 10 + \text{sum}(n // 10)$



Recursion Practice : Writing Programs

Palindrome Check using Recursion.

*Smaller subproblem
to find overall
solution*

- I/P: ~~abbcbbba~~
 - O/P: YES
-
- I/P: abba
 - O/P: YES
-
- I/P: geeks
 - O/P: NO

Recursion Practice : Writing Programs

```
def pal(str, start, end):  
    if start > end:  
        return True  
    return (str[start] == str[end]) and  
           pal(str, start+1, end-1)
```

