

Chairing #Definition

The *Peter-Paul Inequality* is an elementary case of *Young's Inequality* about the **product** of two numbers:

$$ab \leq \frac{a^2}{2} + \frac{b^2}{2}$$

Peter-Paul's case introduces a term ϵ (or θ in our case) to this inequality; allowing for more control of the *second term* at the cost of lower control of the *first*.

"One must rob Peter to pay Paul"

$$ab \leq \frac{a^2}{2\epsilon} + \frac{\epsilon b^2}{2}, \quad \forall \epsilon > 0$$

Applying it to a vector product:

$$\mathbf{x}^T \mathbf{y} \leq \frac{\theta}{2} \|\mathbf{x}\|^2 + \frac{1}{2\theta} \|\mathbf{y}\|^2, \quad \forall \theta > 0$$