$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + U\frac{\partial U}{\partial y} = \nu\frac{\partial^2 U}{\partial x^2} + \nu\frac{\partial^2 U}{\partial y^2}$$

$$\frac{\partial U}{\partial t} = \frac{U_{i,j}^{l+1} - U_{i,j}^{l}}{\Delta t} - U_{i,j}^{l}$$

$$U\frac{\partial U}{\partial x} = U_{i,j}^{l}\left(\frac{U_{i+1,j}^{l} - U_{i-1,j}^{l}}{2\Delta x}\right)$$

$$U\frac{\partial U}{\partial y} = U_{i,j}^{l}\left(\frac{U_{i,j+1}^{l} - U_{i,j-1}^{l}}{2\Delta y}\right)$$

$$\nu\frac{\partial^2 U}{\partial x^2} = \nu\left(\frac{U_{i+1,j}^{l} - 2U_{i,j}^{l} + U_{i-1,j}^{l}}{(\Delta x)^2}\right)$$

$$\nu\frac{\partial^2 U}{\partial y^2} = \nu\left(\frac{U_{i,j+1}^{l} - 2U_{i,j}^{l} + U_{i,j-1}^{l}}{(\Delta y)^2}\right)$$

$$\frac{U_{i,j}^{l+1} - U_{i,j}^{l}}{\Delta t} = \frac{\nu}{(\Delta x)^2}\left(U_{i+1,j}^{l} - 2U_{i,j}^{l} + U_{i-1,j}^{l}\right) + \frac{\nu}{(\Delta y)^2}\left(U_{i,j+1}^{l} - 2U_{i,j}^{l} + U_{i,j-1}^{l}\right) - U_{i,j}^{l}\left(\frac{U_{i+1,j}^{l} - U_{i-1,j}^{l}}{2\Delta x}\right) - U_{i,j}^{l}\left(\frac{U_{i,j+1}^{l} - U_{i,j-1}^{l}}{2\Delta y}\right)$$

$$U_{i,j}^{l+1} = \frac{\nu \Delta t}{(\Delta x)^2}\left(U_{i+1,j}^{l} - 2U_{i,j}^{l} + U_{i-1,j}^{l}\right)$$

$$+ \frac{\nu \Delta t}{(\Delta y)^2}\left(U_{i,j+1}^{l} - 2U_{i,j}^{l} + U_{i,j-1}^{l}\right)$$

$$- \Delta t\, U_{i,j}^{l}\left(\frac{U_{i+1,j}^{l} - U_{i-1,j}^{l}}{2\Delta x}\right)$$

$$- \Delta t\, U_{i,j}^{l}\left(\frac{U_{i,j+1}^{l} - U_{i,j-1}^{l}}{2\Delta y}\right) + U_{i,j}^{l}$$