Tarea 4

a) Tomando $h = x_n - x_{n-1}$ o como $h = (x_{n+1} - x_n)$
Se realiza la expansión de Taylor hasta orden 6

$$y(x_{n+1}) = y(x_n) + h y'(x_n) + \frac{h^2}{2} y''(x_n) + \frac{h^3}{3!} y'''(x_n) + \frac{h^4}{4!} y^{(4)}(x_n) + \frac{h^5}{5!} y^{(5)} + \frac{h^6}{6!} y^{(6)}$$

Asimismo para x_{n-1}

$$y(x_{n-1}) = y(x_n) - h y'(x_n) + \frac{h^2}{2} y''(x_n) - \frac{h^3}{3!} y'''(x_n) + \frac{h^4}{4!} y^{(4)}(x_n) - \frac{h^5}{5!} y^{(5)}(x_n) + \frac{h^6}{6!} y^{(6)}(x_n)$$

Realizando $y(x_{n+1}) + y(x_{n-1})$

$$y(x_{n+1}) + y(x_{n-1}) = 2y(x_n) + h^2 y''(x_n) + 2\frac{h^4}{4!} y^{(4)}(x_n) + O(h^6)$$

$$y(x_{n+1}) + y(x_{n-1}) - 2y(x_n) = h^2\left(y_n'' + \frac{h^2}{12} y_n^{(4)}\right) + O(h^6)$$

$$y_{n+1} - 2y_n + y_{n-1} = h^2\left(y_n'' + \frac{h^2}{12} y_n^{iv}\right) + O(h^6) \quad (1)$$

Según la ecuación diferencial (a) $y_n'' = S_n - y_n R_n$ entonces

(b) $h^2 y_n^{iv} = y_{n+1} R_{n+1} + S_{n+1} - 2R_n y_n - 2S_n + R_{n-1} y_{n-1} + S_{n-1} + O(h^4)$

Sustituyendo (a) y (b) en (1)

$$y_{n+1} - 2y_n + y_{n-1} = \left[y_{n+1} R_{n+1} + S_{n+1} - 2R_n y_n - 2S_n + R_{n-1} y_{n-1} + S_{n-1} + O(h^4)\right]\frac{h^2}{12} + h^2(S_n - R_n y_n)$$

$$y_{n+1} - \frac{h^2}{12} R_{n+1} y_{n+1} - 2y_n + \frac{2R_n h^2 y_n}{12} + R_n y_n h^2 + y_{n-1} \frac{-R_{n-1} y_{n-1} h^2}{12} = \frac{h^2}{12}(S_{n+1} + 10S_n + S_{n-1}) + O(h^6)$$

$$\left(1 - \frac{h^2}{12} R_{n+1}\right) y_{n+1} - 2\left(1 + \frac{5h^2}{12} R_n\right) y_n + \left(1 - \frac{h^2}{12} R_{n-1}\right) y_{n-1} = \frac{h^2}{12}(S_{n+1} + 10S_n + S_{n-1}) + O(h^6)$$

b) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi = E\psi$

$$\frac{d^2\psi}{dx^2}-\frac{2m}{\hbar^2}V(x)\psi = E\psi\left(-\frac{2m}{\hbar^2}\right)$$

$$\frac{d^2\psi}{dx^2}-\frac{2m}{\hbar^2}\left(V(x)-E\right)\psi = 0$$

Se deduce $R_n = \frac{2m}{\hbar^2}\left[\frac{1}{2}m\omega^2x^2-E\right]$ y $S_n=0$