Tarea 2 Parte teórica Santiago Monguá

2. Algoritmo de Verlet

(a) El paso x_{n+1} y el paso x_{n-1} están dados por las expansiones en serie de Taylor como sigue

$$x_{n+1} = x_n + h v_n + \frac{1}{2} h^2 a_n + \frac{1}{6} h^3 a_n'$$

$$x_{n-1} = x_n - h v_n + \frac{1}{2} h^2 a_n - \frac{1}{6} h^3 a_n'$$ Sumando ambas

$$x_{n+1} + x_{n-1} = 2x_n + h^2 a_n$$ Restando el valor exacto $x(h_i)$

$$\epsilon_{n+1} + \epsilon_{n-1} = 2\epsilon_n + h^2 \ddot{\varepsilon}_i \epsilon_n$$

$$\epsilon_{n+1} - (2 + h^2 a_n') + \epsilon_{n-1} = 0 \quad a)$$

b) Oscilador armónico $m\ddot{x} = -kx$ $\qquad \omega^2 = \frac{k}{m}$

$$\ddot{x} = -\frac{k}{m} x$$

$$\ddot{x} = -\omega^2 x$$

$$a' = \frac{\partial \ddot{x}}{\partial x} = -\omega^2$$ Reemplazando en a)

$$\epsilon_{n+1} - 2(1-R)\epsilon_n + \epsilon_{n-1} = 0 \qquad 2R = h^2\omega^2$$

c) $\epsilon_n = \epsilon_0 \lambda^n$

$$\cancel{\epsilon_0}\lambda^{n+1} - 2(1-R)\cancel{\epsilon_0}\lambda^n + \cancel{\epsilon_0}\lambda^{n-1} = 0$$ Divide entre λ

$$\lambda - 2(1-R) + \lambda^{-1} = 0$$

$$\frac{\lambda^2+1}{\lambda} - 2(1-R) = 0$$

$$\frac{\lambda^2 + 1 - 2\lambda(1-R)}{\lambda} = 0 \Rightarrow \lambda^2 - 2(1-R)\lambda + 1 = 0$$

$$\lambda_\pm = \frac{-(-2(1-R)) \pm \sqrt{(-2(1-R))^2 - 4(1)(1)}}{2}$$

$$\lambda_\pm = (1-R) \pm \sqrt{R^2 - 2R}$$

$$|\lambda_\pm| \leq 1$$

$$\lambda_\pm \leq 1$$

$$(1-R) \pm \sqrt{R(R-2)} \leq 1$$

$$\pm\sqrt{R(R-2)} \leq 1-(1-R)$$

$$R(R-2) \leq R^2$$

$$(R-2) \leq R$$

$$R \leq 2$$

$$\frac{h^2\omega^2}{2} \leq 2$$

$$h\omega \leq 2 \Rightarrow \boxed{h \leq \frac{2}{\omega}}$$