Tarea Mét. Comp 2 Santiago Mongua Niño 202012703 | 02 | 02 | 24

1. Mostrar que los operadores diferenciales son consistentes

$$f'(x)=\frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$$

$$f(x)=x^2$$

$$f'(x)=\frac{-(x+2h)^2+4(x+h)^2-3x^2}{2h}$$

$$f'(x)=\frac{\cancel{-x^2}-4xh-\cancel{4h^2}+\cancel{4x^2}+8xh+\cancel{4h^2}-\cancel{3x^2}}{2h}$$

$$f'(x)=\frac{4x\cancel{h}}{2\cancel{h}} \Rightarrow f'(x)=2x \quad \text{correcto}$$

$$f''(x)=\frac{f(x+h)-2f(x)+f(x-h)}{h^2}$$

$$f(x)=x^2$$

$$f''(x)=\frac{(x+h)^2-2x^2+(x-h)^2}{h^2}$$

$$f''(x)=\frac{\cancel{x^2}+\cancel{2xh}+h^2-\cancel{2x^2}+\cancel{x^2}-\cancel{2xh}+h^2}{h^2}$$

$$f''(x)=\frac{2\cancel{h^2}}{\cancel{h^2}} \Rightarrow f''(x)=2 \quad \text{correcto}$$

$$f(x)=\sin(x)$$

$$f'(x)=\frac{-\sin(x+2h)+4\sin(x+h)-3\sin(x)}{2h}$$

$$f'(x)=\frac{1}{2h}\left[-\sin(x)\cos(2h)-\cos(x)\sin(2h)+4\sin(x)\cos(h)+4\sin(h)\cos(x)-3\sin x\right]$$ Aplico regla de L'Hôpital

$$f'(x)=\frac{1}{2}\left[\sin(x)\cdot 2\sin(2h)-2\cos(x)\cos(2h)-4\sin(x)\sin(h)+4\cos(x)\cos(h)\right]$$

$$f'(x)=\frac{1}{2}\left[2\cos(x)\left[2\cos(h)-\cos(2h)\right]+2\sin(x)\left[\sin(2h)-2\sin(h)\right]\right]$$

$$f'(x)=\cos(x)\left[2\cos(h)-\cos(2h)\right]+\sin(x)\left[\sin(2h)-2\sin(h)\right]$$ Tomando el límite cuando $h\to 0$

$$f'(x)=\lim_{h\to 0}\left\{\cos(x)\left[2\cos(h)-\cos(2h)\right]+\sin(x)\left[\sin(2h)-2\sin(h)\right]\right\}$$

$$f'(x)=\cos(x)\left[2\cos(0)-\cos(2(0))\right]+\cancel{\sin(x)\left[\sin(2(0))-2\sin(0)\right]}$$

$$f'(x)=\cos(x)\left[2-1\right] \Rightarrow f'(x)=\cos x \quad \text{correcto}$$

$$f(x)=\sin(x)$$

$$f''(x)=\frac{\sin(x+h)-2\sin(x)+\sin(x-h)}{h^2}$$

$$f''(x) = \frac{\sin(x)\cos(h) - \cancel{\cos(x)\sin(h)} - 2\sin(x) + \sin(x)\cos(h) + \cancel{\cos(x)\sin(h)}}{h^2}$$

$$f''(x) = \frac{2\sin(x)\cos(h) - 2\sin(x)}{h^2}$$

$$f''(x) = \frac{2\sin(x)[\cos(h) - 1]}{h^2}$$ Aplicando L'Hôpital 2 veces.

$$f''(x) = \frac{\cancel{2}\sin(x)[-\sin(h)]}{\cancel{2}h}$$

$$f''(x) = -\sin(x)\cos(h)$$ Evaluando límite cuando $h \to 0$

$$f''(x) = \lim_{h \to 0} [-\sin(x)\cos(h)] = -\sin(x)\cos(0) \Rightarrow f''(x) = -\sin(x)$$ correcto

Se mostró que ambos operadores son consistentes.

3. Velocidad luz en unidades au/año

$$3 \times 10^8 \frac{\cancel{m}}{\cancel{s}} \times \frac{86400 \cancel{s}}{1 \cancel{día}} \times \frac{360 \cancel{día}}{1 año} \times \frac{6,68 \times 10^{-12} au}{1 \cancel{m}} \Rightarrow c = 62375 \frac{au}{año}$$

5. Empezamos con el caso base

$$\int_0^{u_1} du = \alpha u_0 \, dt$$

$$u_1 - u_0 = \alpha u_0 \Delta t$$

$$u_1 = u_0 + \alpha u_0 \Delta t$$

$$u_1 = (1 + \alpha \Delta t) u_0$$

Ahora bien, esto se puede generalizar mediante la expansión de serie de Taylor y tomando el paso $x_k = x_0 + k\Delta t$

$u(x_{k+1}) = u(x_k) + \Delta t \, u'(x_k)$ despejando u' y restando αu

$$\frac{u(x_{k+1}) - u(x_k)}{\Delta t} - \alpha u(x_k) = u'(x_k) - \alpha u(x_k)$$

$$u(x_{k+1}) - u(x_k) = \alpha u(x_k) \Delta t$$

$$u(x_{k+1}) = (1 + \alpha \Delta t) \, u(x_k)$$

Como se observa el $(k+1)$ requiere de los k anteriores que son como el caso base y con $k=2$

$$u(x_2) = (1 + \alpha \Delta t) \, u(x_1)$$

$$u(x_2) = (1 + \alpha \Delta t)^2 u_0$$ Por inducción en k se tiene que

$$\boxed{u_k = (1 + \alpha \Delta t)^k u_0}$$