# Breve tabla de integrales

## Formas básicas

1. $\int k\,dx = kx + C \quad (\text{cualquier número } k)$

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$

6. $\int \operatorname{sen} x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \operatorname{sen} x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\operatorname{sen} x| + C$

14. $\int \operatorname{senh} x\,dx = \cosh x + C$

15. $\int \cosh x\,dx = \operatorname{senh} x + C$

16. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{sen}^{-1} \frac{x}{a} + C$

17. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

18. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left|\frac{x}{a}\right| + C$

19. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \operatorname{senh}^{-1} \frac{x}{a} + C \quad (a > 0)$

20. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C \quad (x > a > 0)$

## Formas que incluyen $ax + b$

21. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$

22. $\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left[\frac{ax+b}{n+2} - \frac{b}{n+1}\right] + C, \quad n \neq -1, -2$

23. $\int (ax+b)^{-1}\,dx = \frac{1}{a} \ln|ax+b| + C$

24. $\int x(ax+b)^{-1}\,dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$

25. $\int x(ax+b)^{-2}\,dx = \frac{1}{a^2}\left[\ln|ax+b| + \frac{b}{ax+b}\right] + C$

26. $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln\left|\frac{x}{ax+b}\right| + C$

27. $\int \left(\sqrt{ax+b}\right)^n\,dx = \frac{2}{a} \frac{\left(\sqrt{ax+b}\right)^{n+2}}{n+2} + C, \quad n \neq -2$

28. $\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$

**29.** **(a)** $\displaystyle\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C$ **(b)** $\displaystyle\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \tan^{-1}\sqrt{\frac{ax-b}{b}} + C$

**30.** $\displaystyle\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}} + C$ **31.** $\displaystyle\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} + C$

## Formas que incluyen $a^2 + x^2$

**32.** $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ **33.** $\displaystyle\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\tan^{-1}\frac{x}{a} + C$

**34.** $\displaystyle\int \frac{dx}{\sqrt{a^2+x^2}} = \operatorname{senh}^{-1}\frac{x}{a} + C = \ln\left(x+\sqrt{a^2+x^2}\right) + C$

**35.** $\displaystyle\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

**36.** $\displaystyle\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}(a^2+2x^2)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

**37.** $\displaystyle\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

**38.** $\displaystyle\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = \ln\left(x+\sqrt{a^2+x^2}\right) - \frac{\sqrt{a^2+x^2}}{x} + C$

**39.** $\displaystyle\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = -\frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + \frac{x\sqrt{a^2+x^2}}{2} + C$

**40.** $\displaystyle\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$ **41.** $\displaystyle\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$

## Formas que incluyen $a^2 - x^2$

**42.** $\displaystyle\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$ **43.** $\displaystyle\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right| + C$

**44.** $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a} + C$ **45.** $\displaystyle\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a} + C$

**46.** $\displaystyle\int x^2\sqrt{a^2-x^2}\,dx = \frac{a^4}{8}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{8}x\sqrt{a^2-x^2}\,(a^2-2x^2) + C$

**47.** $\displaystyle\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$ **48.** $\displaystyle\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\operatorname{sen}^{-1}\frac{x}{a} - \frac{\sqrt{a^2-x^2}}{x} + C$

**49.** $\displaystyle\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + C$ **50.** $\displaystyle\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

**51.** $\displaystyle\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

## Formas que incluyen $x^2 - a^2$

**52.** $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

**53.** $\displaystyle\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

54. $\displaystyle\int\left(\sqrt{x^2-a^2}\right)^n dx = \frac{x\left(\sqrt{x^2-a^2}\right)^n}{n+1} - \frac{na^2}{n+1}\int\left(\sqrt{x^2-a^2}\right)^{n-2} dx, \quad n \neq -1$

55. $\displaystyle\int \frac{dx}{\left(\sqrt{x^2-a^2}\right)^n} = \frac{x\left(\sqrt{x^2-a^2}\right)^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2}\int\frac{dx}{\left(\sqrt{x^2-a^2}\right)^{n-2}}, \quad n \neq 2$

56. $\displaystyle\int x\left(\sqrt{x^2-a^2}\right)^n dx = \frac{\left(\sqrt{x^2-a^2}\right)^{n+2}}{n+2} + C, \quad n \neq -2$

57. $\displaystyle\int x^2\sqrt{x^2-a^2}\, dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

58. $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x}\, dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right| + C$

59. $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^2}\, dx = \ln\left|x+\sqrt{x^2-a^2}\right| - \frac{\sqrt{x^2-a^2}}{x} + C$

60. $\displaystyle\int \frac{x^2}{\sqrt{x^2-a^2}}\, dx = \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + \frac{x}{2}\sqrt{x^2-a^2} + C$

61. $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C$

62. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$

## Formas trigonométricas

63. $\displaystyle\int \operatorname{sen} ax\, dx = -\frac{1}{a}\cos ax + C$

64. $\displaystyle\int \cos ax\, dx = \frac{1}{a}\operatorname{sen} ax + C$

65. $\displaystyle\int \operatorname{sen}^2 ax\, dx = \frac{x}{2} - \frac{\operatorname{sen} 2ax}{4a} + C$

66. $\displaystyle\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a} + C$

67. $\displaystyle\int \operatorname{sen}^n ax\, dx = -\frac{\operatorname{sen}^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int \operatorname{sen}^{n-2} ax\, dx$

68. $\displaystyle\int \cos^n ax\, dx = \frac{\cos^{n-1} ax\operatorname{sen} ax}{na} + \frac{n-1}{n}\int \cos^{n-2} ax\, dx$

69. (a) $\displaystyle\int \operatorname{sen} ax\cos bx\, dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$

(b) $\displaystyle\int \operatorname{sen} ax\operatorname{sen} bx\, dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$

(c) $\displaystyle\int \cos ax\cos bx\, dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} + \frac{\operatorname{sen}(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$

70. $\displaystyle\int \operatorname{sen} ax\cos ax\, dx = -\frac{\cos 2ax}{4a} + C$

71. $\displaystyle\int \operatorname{sen}^n ax\cos ax\, dx = \frac{\operatorname{sen}^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

72. $\displaystyle\int \frac{\cos ax}{\operatorname{sen} ax}\, dx = \frac{1}{a}\ln|\operatorname{sen} ax| + C$

73. $\displaystyle\int \cos^n ax\operatorname{sen} ax\, dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

74. $\displaystyle\int \frac{\operatorname{sen} ax}{\cos ax}\, dx = -\frac{1}{a}\ln|\cos ax| + C$

75. $\displaystyle\int \operatorname{sen}^n ax\cos^m ax\, dx = -\frac{\operatorname{sen}^{n-1} ax\cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n}\int \operatorname{sen}^{n-2} ax\cos^m ax\, dx, \quad n \neq -m \quad (\text{reduce } \operatorname{sen}^n ax)$

76. $\displaystyle\int \operatorname{sen}^n ax\cos^m ax\, dx = \frac{\operatorname{sen}^{n+1} ax\cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n}\int \operatorname{sen}^n ax\cos^{m-2} ax\, dx, \quad m \neq -n \quad (\text{reduce } \cos^m ax)$

77. $\displaystyle\int \frac{dx}{b + c \operatorname{sen} ax} = \frac{-2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\frac{b - c}{b + c}} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right] + C, \quad b^2 > c^2$

78. $\displaystyle\int \frac{dx}{b + c \operatorname{sen} ax} = \frac{-1}{a\sqrt{c^2 - b^2}} \ln\left|\frac{c + b \operatorname{sen} ax + \sqrt{c^2 - b^2}\cos ax}{b + c \operatorname{sen} ax}\right| + C, \quad b^2 < c^2$

79. $\displaystyle\int \frac{dx}{1 + \operatorname{sen} ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

80. $\displaystyle\int \frac{dx}{1 - \operatorname{sen} ax} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

81. $\displaystyle\int \frac{dx}{b + c \cos ax} = \frac{2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\frac{b - c}{b + c}} \tan\frac{ax}{2}\right] + C, \quad b^2 > c^2$

82. $\displaystyle\int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \ln\left|\frac{c + b \cos ax + \sqrt{c^2 - b^2}\operatorname{sen} ax}{b + c \cos ax}\right| + C, \quad b^2 < c^2$

83. $\displaystyle\int \frac{dx}{1 + \cos ax} = \frac{1}{a}\tan\frac{ax}{2} + C$

84. $\displaystyle\int \frac{dx}{1 - \cos ax} = -\frac{1}{a}\cot\frac{ax}{2} + C$

85. $\displaystyle\int x \operatorname{sen} ax\, dx = \frac{1}{a^2}\operatorname{sen} ax - \frac{x}{a}\cos ax + C$

86. $\displaystyle\int x \cos ax\, dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\operatorname{sen} ax + C$

87. $\displaystyle\int x^n \operatorname{sen} ax\, dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx$

88. $\displaystyle\int x^n \cos ax\, dx = \frac{x^n}{a}\operatorname{sen} ax - \frac{n}{a}\int x^{n-1}\operatorname{sen} ax\, dx$

89. $\displaystyle\int \tan ax\, dx = \frac{1}{a}\ln|\sec ax| + C$

90. $\displaystyle\int \cot ax\, dx = \frac{1}{a}\ln|\operatorname{sen} ax| + C$

91. $\displaystyle\int \tan^2 ax\, dx = \frac{1}{a}\tan ax - x + C$

92. $\displaystyle\int \cot^2 ax\, dx = -\frac{1}{a}\cot ax - x + C$

93. $\displaystyle\int \tan^n ax\, dx = \frac{\tan^{n-1} ax}{a(n - 1)} - \int \tan^{n-2} ax\, dx, \quad n \neq 1$

94. $\displaystyle\int \cot^n ax\, dx = -\frac{\cot^{n-1} ax}{a(n - 1)} - \int \cot^{n-2} ax\, dx, \quad n \neq 1$

95. $\displaystyle\int \sec ax\, dx = \frac{1}{a}\ln|\sec ax + \tan ax| + C$

96. $\displaystyle\int \csc ax\, dx = -\frac{1}{a}\ln|\csc ax + \cot ax| + C$

97. $\displaystyle\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + C$

98. $\displaystyle\int \csc^2 ax\, dx = -\frac{1}{a}\cot ax + C$

99. $\displaystyle\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \tan ax}{a(n - 1)} + \frac{n - 2}{n - 1}\int \sec^{n-2} ax\, dx, \quad n \neq 1$

100. $\displaystyle\int \csc^n ax\, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n - 1)} + \frac{n - 2}{n - 1}\int \csc^{n-2} ax\, dx, \quad n \neq 1$

101. $\displaystyle\int \sec^n ax \tan ax\, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$

102. $\displaystyle\int \csc^n ax \cot ax\, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$

## Formas trigonométricas inversas

103. $\displaystyle\int \operatorname{sen}^{-1} ax\, dx = x \operatorname{sen}^{-1} ax + \frac{1}{a}\sqrt{1 - a^2x^2} + C$

104. $\displaystyle\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \frac{1}{a}\sqrt{1 - a^2x^2} + C$

105. $\displaystyle\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \frac{1}{2a}\ln(1 + a^2x^2) + C$

106. $\displaystyle\int x^n \operatorname{sen}^{-1} ax\, dx = \frac{x^{n+1}}{n + 1}\operatorname{sen}^{-1} ax - \frac{a}{n + 1}\int \frac{x^{n+1}\, dx}{\sqrt{1 - a^2x^2}}, \quad n \neq -1$

107. $\displaystyle\int x^n \cos^{-1} ax\, dx = \frac{x^{n+1}}{n + 1}\cos^{-1} ax + \frac{a}{n + 1}\int \frac{x^{n+1}\, dx}{\sqrt{1 - a^2x^2}}, \quad n \neq -1$

108. $\displaystyle\int x^n \tan^{-1} ax\, dx = \frac{x^{n+1}}{n + 1}\tan^{-1} ax - \frac{a}{n + 1}\int \frac{x^{n+1}\, dx}{1 + a^2x^2}, \quad n \neq -1$

## Formas exponenciales y logarítmicas

**109.** $\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

**110.** $\displaystyle\int b^{ax}\,dx = \frac{1}{a}\frac{b^{ax}}{\ln b} + C, \quad b > 0, b \neq 1$

**111.** $\displaystyle\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1) + C$

**112.** $\displaystyle\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$

**113.** $\displaystyle\int x^n b^{ax}\,dx = \frac{x^n b^{ax}}{a\ln b} - \frac{n}{a\ln b}\int x^{n-1}b^{ax}\,dx, \quad b > 0, b \neq 1$

**114.** $\displaystyle\int e^{ax}\operatorname{sen} bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\operatorname{sen} bx - b\cos bx) + C$

**115.** $\displaystyle\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\operatorname{sen} bx) + C$

**116.** $\displaystyle\int \ln ax\,dx = x\ln ax - x + C$

**117.** $\displaystyle\int x^n(\ln ax)^m\,dx = \frac{x^{n+1}(\ln ax)^m}{n + 1} - \frac{m}{n + 1}\int x^n(\ln ax)^{m-1}\,dx, \quad n \neq -1$

**118.** $\displaystyle\int x^{-1}(\ln ax)^m\,dx = \frac{(\ln ax)^{m+1}}{m + 1} + C, \quad m \neq -1$

**119.** $\displaystyle\int \frac{dx}{x\ln ax} = \ln|\ln ax| + C$

## Formas que incluyen $\sqrt{2ax - x^2}$, $a > 0$

**120.** $\displaystyle\int \frac{dx}{\sqrt{2ax - x^2}} = \operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) + C$

**121.** $\displaystyle\int \sqrt{2ax - x^2}\,dx = \frac{x - a}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) + C$

**122.** $\displaystyle\int \left(\sqrt{2ax - x^2}\right)^n dx = \frac{(x - a)\left(\sqrt{2ax - x^2}\right)^n}{n + 1} + \frac{na^2}{n + 1}\int \left(\sqrt{2ax - x^2}\right)^{n-2} dx$

**123.** $\displaystyle\int \frac{dx}{\left(\sqrt{2ax - x^2}\right)^n} = \frac{(x - a)\left(\sqrt{2ax - x^2}\right)^{2-n}}{(n - 2)a^2} + \frac{n - 3}{(n - 2)a^2}\int \frac{dx}{\left(\sqrt{2ax - x^2}\right)^{n-2}}$

**124.** $\displaystyle\int x\sqrt{2ax - x^2}\,dx = \frac{(x + a)(2x - 3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2}\operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) + C$

**125.** $\displaystyle\int \frac{\sqrt{2ax - x^2}}{x}\,dx = \sqrt{2ax - x^2} + a\operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) + C$

**126.** $\displaystyle\int \frac{\sqrt{2ax - x^2}}{x^2}\,dx = -2\sqrt{\frac{2a - x}{x}} - \operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) + C$

**127.** $\displaystyle\int \frac{x\,dx}{\sqrt{2ax - x^2}} = a\operatorname{sen}^{-1}\left(\frac{x - a}{a}\right) - \sqrt{2ax - x^2} + C$

**128.** $\displaystyle\int \frac{dx}{x\sqrt{2ax - x^2}} = -\frac{1}{a}\sqrt{\frac{2a - x}{x}} + C$

## Formas hiperbólicas

**129.** $\displaystyle\int \operatorname{senh} ax\,dx = \frac{1}{a}\cosh ax + C$

**130.** $\displaystyle\int \cosh ax\,dx = \frac{1}{a}\operatorname{senh} ax + C$

**131.** $\displaystyle\int \operatorname{senh}^2 ax\,dx = \frac{\operatorname{senh} 2ax}{4a} - \frac{x}{2} + C$

**132.** $\displaystyle\int \cosh^2 ax\,dx = \frac{\operatorname{senh} 2ax}{4a} + \frac{x}{2} + C$

**133.** $\displaystyle\int \operatorname{senh}^n ax\,dx = \frac{\operatorname{senh}^{n-1} ax\cosh ax}{na} - \frac{n - 1}{n}\int \operatorname{senh}^{n-2} ax\,dx, \quad n \neq 0$

**134.** $\displaystyle\int \cosh^n ax\,dx = \frac{\cosh^{n-1} ax \operatorname{senh} ax}{na} + \frac{n-1}{n}\int \cosh^{n-2} ax\,dx, \quad n \neq 0$

**135.** $\displaystyle\int x \operatorname{senh} ax\,dx = \frac{x}{a}\cosh ax - \frac{1}{a^2}\operatorname{senh} ax + C$

**136.** $\displaystyle\int x \cosh ax\,dx = \frac{x}{a}\operatorname{senh} ax - \frac{1}{a^2}\cosh ax + C$

**137.** $\displaystyle\int x^n \operatorname{senh} ax\,dx = \frac{x^n}{a}\cosh ax - \frac{n}{a}\int x^{n-1}\cosh ax\,dx$

**138.** $\displaystyle\int x^n \cosh ax\,dx = \frac{x^n}{a}\operatorname{senh} ax - \frac{n}{a}\int x^{n-1}\operatorname{senh} ax\,dx$

**139.** $\displaystyle\int \tanh ax\,dx = \frac{1}{a}\ln(\cosh ax) + C$

**140.** $\displaystyle\int \coth ax\,dx = \frac{1}{a}\ln|\operatorname{senh} ax| + C$

**141.** $\displaystyle\int \tanh^2 ax\,dx = x - \frac{1}{a}\tanh ax + C$

**142.** $\displaystyle\int \coth^2 ax\,dx = x - \frac{1}{a}\coth ax + C$

**143.** $\displaystyle\int \tanh^n ax\,dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax\,dx, \quad n \neq 1$

**144.** $\displaystyle\int \coth^n ax\,dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax\,dx, \quad n \neq 1$

**145.** $\displaystyle\int \operatorname{sech} ax\,dx = \frac{1}{a}\operatorname{sen}^{-1}(\tanh ax) + C$

**146.** $\displaystyle\int \operatorname{csch} ax\,dx = \frac{1}{a}\ln\left|\tanh\frac{ax}{2}\right| + C$

**147.** $\displaystyle\int \operatorname{sech}^2 ax\,dx = \frac{1}{a}\tanh ax + C$

**148.** $\displaystyle\int \operatorname{csch}^2 ax\,dx = -\frac{1}{a}\coth ax + C$

**149.** $\displaystyle\int \operatorname{sech}^n ax\,dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2} ax\,dx, \quad n \neq 1$

**150.** $\displaystyle\int \operatorname{csch}^n ax\,dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1}\int \operatorname{csch}^{n-2} ax\,dx, \quad n \neq 1$

**151.** $\displaystyle\int \operatorname{sech}^n ax \tanh ax\,dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$

**152.** $\displaystyle\int \operatorname{csch}^n ax \coth ax\,dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$

**153.** $\displaystyle\int e^{ax}\operatorname{senh} bx\,dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b}\right] + C, \quad a^2 \neq b^2$

**154.** $\displaystyle\int e^{ax}\cosh bx\,dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b}\right] + C, \quad a^2 \neq b^2$

## Algunas integrales definidas

**155.** $\displaystyle\int_0^{\infty} x^{n-1}e^{-x}\,dx = \Gamma(n) = (n-1)!, \quad n > 0$

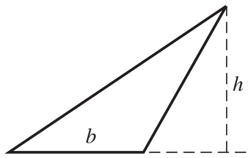
**156.** $\displaystyle\int_0^{\infty} e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \quad a > 0$

**157.** $$\int_0^{\pi/2} \operatorname{sen}^n x\,dx = \int_0^{\pi/2}\cos^n x\,dx = \begin{cases} \dfrac{1\cdot 3\cdot 5\cdot \cdots \cdot (n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot n}\cdot\dfrac{\pi}{2}, & \text{si } n \text{ es un entero par} \geq 2 \\[2ex] \dfrac{2\cdot 4\cdot 6\cdot \cdots \cdot (n-1)}{3\cdot 5\cdot 7\cdot \cdots \cdot n}, & \text{si } n \text{ es un entero impar} \geq 3 \end{cases}$$

## FÓRMULAS DE GEOMETRÍA

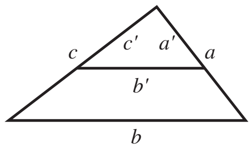
$A =$ área, $B =$ área de la base, $C =$ circunferencia,
$S =$ área lateral o área de la superficie, $V =$ volumen

### Triángulo



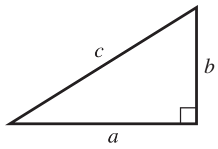

$$A = \frac{1}{2}bh$$

### Triángulos semejantes



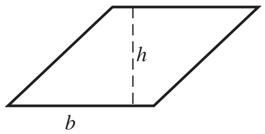

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$$

### Teorema de Pitágoras



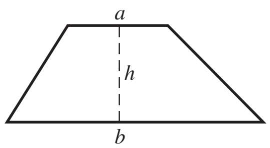

$$a^2 + b^2 = c^2$$

### Paralelogramo



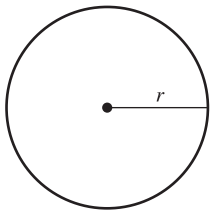

$$A = bh$$

### Trapecio



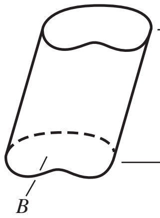

$$A = \frac{1}{2}(a + b)h$$

### Círculo

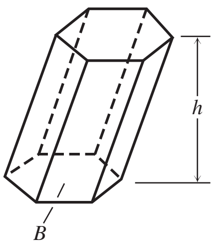



$A = \pi r^2$,
$C = 2\pi r$

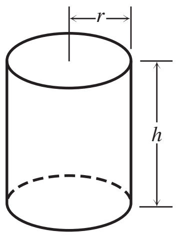
### Cualquier cilindro o prisma con bases paralelas




$V = Bh$

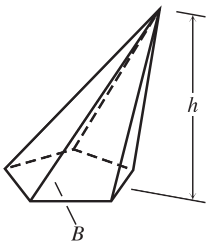



### Cilindro circular recto

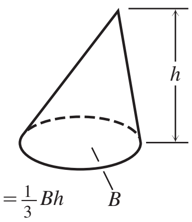



$V = \pi r^2 h$
$S = 2\pi rh =$ Área lateral

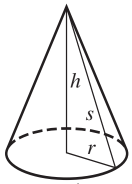
### Cualquier cono o pirámide




$V = \frac{1}{3}Bh$

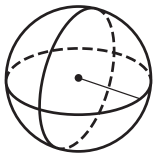



### Cono circular recto




$V = \frac{1}{3}\pi r^2 h$
$S = \pi rs =$ Área lateral

### Esfera



$V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$

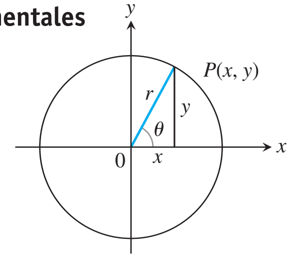
## Fórmulas de trigonometría

### 1. Definiciones e identidades fundamentales

Seno: $\operatorname{sen}\theta = \dfrac{y}{r} = \dfrac{1}{\csc\theta}$

Coseno: $\cos\theta = \dfrac{x}{r} = \dfrac{1}{\sec\theta}$

Tangente: $\tan\theta = \dfrac{y}{x} = \dfrac{1}{\cot\theta}$




### 2. Identidades

$$\operatorname{sen}(-\theta) = -\operatorname{sen}\theta, \quad \cos(-\theta) = \cos\theta$$

$$\operatorname{sen}^2\theta + \cos^2\theta = 1, \quad \sec^2\theta = 1 + \tan^2\theta, \quad \csc^2\theta = 1 + \cot^2\theta$$

$$\operatorname{sen} 2\theta = 2\operatorname{sen}\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \operatorname{sen}^2\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}, \quad \operatorname{sen}^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\operatorname{sen}(A + B) = \operatorname{sen} A\cos B + \cos A\operatorname{sen} B$$

$$\operatorname{sen}(A - B) = \operatorname{sen} A\cos B - \cos A\operatorname{sen} B$$

$$\cos(A + B) = \cos A\cos B - \operatorname{sen} A\operatorname{sen} B$$

$$\cos(A - B) = \cos A\cos B + \operatorname{sen} A\operatorname{sen} B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\operatorname{sen}\left(A - \frac{\pi}{2}\right) = -\cos A, \quad \cos\left(A - \frac{\pi}{2}\right) = \operatorname{sen} A$$

$$\operatorname{sen}\left(A + \frac{\pi}{2}\right) = \cos A, \quad \cos\left(A + \frac{\pi}{2}\right) = -\operatorname{sen} A$$

$$\operatorname{sen} A\operatorname{sen} B = \frac{1}{2}\cos(A - B) - \frac{1}{2}\cos(A + B)$$

$$\cos A\cos B = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B)$$

$$\operatorname{sen} A\cos B = \frac{1}{2}\operatorname{sen}(A - B) + \frac{1}{2}\operatorname{sen}(A + B)$$

$$\operatorname{sen} A + \operatorname{sen} B = 2\operatorname{sen}\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$$

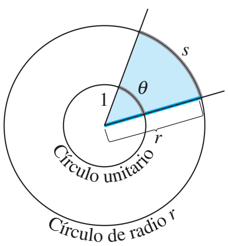
$$\operatorname{sen} A - \operatorname{sen} B = 2\cos\frac{1}{2}(A + B)\operatorname{sen}\frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2\operatorname{sen}\frac{1}{2}(A + B)\operatorname{sen}\frac{1}{2}(A - B)$$

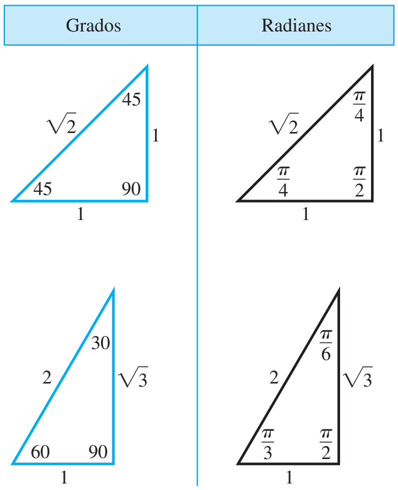
## Funciones trigonométricas

### Medida en radianes

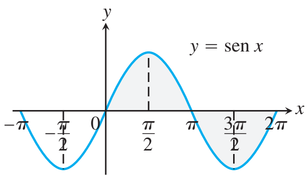



$\dfrac{s}{r} = \dfrac{\theta}{1} = \theta$ o $\theta = \dfrac{s}{r}$,

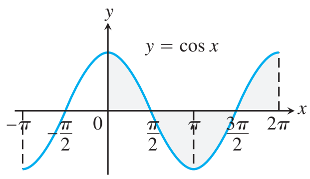
$180° = \pi$ radianes.



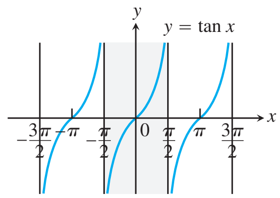

Los ángulos de dos triángulos comunes, en grados y en radianes.



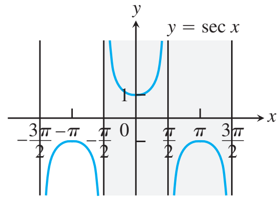

Dominio: $(-\infty, \infty)$
Rango: $[-1, 1]$



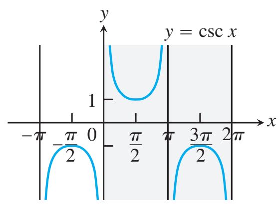

Dominio: $(-\infty, \infty)$
Rango: $[-1, 1]$



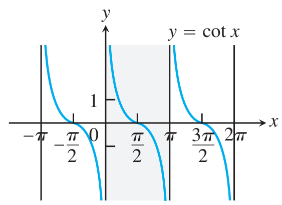

Dominio: Todos los números reales, excepto múltiplos enteros impares de $\pi/2$
Rango: $(-\infty, \infty)$




Dominio: Todos los números reales, excepto múltiplos enteros impares de $\pi/2$
Rango: $(-\infty, -1] \cup [1, \infty)$




Dominio: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Rango: $(-\infty, -1] \cup [1, \infty)$




Dominio: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Rango: $(-\infty, \infty)$

## SERIES

### Criterios para la convergencia de series infinitas

1. **El criterio del término *n*-ésimo:** A menos que $a_n \to 0$, la serie diverge.
2. **Serie geométrica:** $\sum ar^n$ converge si $|r| < 1$; de otra forma diverge.
3. **Serie *p*:** $\sum 1/n^p$ converge si $p > 1$; de otra forma diverge.
4. **Serie con términos no negativos:** Intente con el criterio de la integral, el criterio de la razón o el criterio de la raíz. Intente comparar con una serie conocida con el criterio de la comparación o el criterio de comparación del límite.
5. **Serie con algunos términos negativos:** ¿La serie $\sum |a_n|$ converge? Si la respuesta es sí, entonces también lo hace $\sum a_n$ ya que convergencia absoluta implica convergencia.
6. **Serie alternante:** La serie $\sum a_n$ converge si la serie satisface las condiciones del criterio de las series alternantes.

### Serie de Taylor

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \qquad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty$$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \qquad |x| < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \qquad -1 < x \le 1$$

$$\ln \frac{1+x}{1-x} = 2 \tanh^{-1} x = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + \cdots\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \qquad |x| < 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| \le 1$$

### Serie binomial

$$(1+x)^m = 1 + mx + \frac{m(m-1)x^2}{2!} + \frac{m(m-1)(m-2)x^3}{3!} + \cdots + \frac{m(m-1)(m-2)\cdots(m-k+1)x^k}{k!} + \cdots$$

$$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \qquad |x| < 1,$$

donde

$$\binom{m}{1} = m, \qquad \binom{m}{2} = \frac{m(m-1)}{2!}, \qquad \binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k!} \qquad \text{para } k \ge 3.$$

# FÓRMULAS DE OPERADORES VECTORIALES (FORMA CARTESIANA)

Fórmulas para Grad, Div, Rot y el laplaciano

| | **Cartesianas** $(x, y, z)$<br>**i**, **j**, y **k** son vectores unitarios en las direcciones en que aumentan $x$, $y$ y $z$.<br>$M$, $N$, y $P$ son los componentes escalares de $\mathbf{F}(x, y, z)$ en estas direcciones. |
|---|---|
| **Gradiente** | $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$ |
| **Divergencia** | $\nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$ |
| **Rotacional** | $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$ |
| **Laplaciano** | $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ |

### Triples productos escalares

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u} = (\mathbf{w} \times \mathbf{u}) \cdot \mathbf{v}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$$

### El teorema fundamental de las integrales de línea

1. Sea $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ un campo vectorial cuyos componentes son continuos en toda una región abierta y conexa $D$ en el espacio. Entonces existe una función derivable $f$ tal que

$$\mathbf{F} = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

si y sólo si para todos los puntos $A$ y $B$ en $D$ el valor de $\int_A^B \mathbf{F} \cdot d\mathbf{r}$ es independiente de la trayectoria que une a $A$ con $B$ en $D$.

2. Si la integral es independiente de la trayectoria de $A$ a $B$, su valor es

$$\int_A^B \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A).$$

### Teorema de Green y su generalización a tres dimensiones

Forma normal del teorema de Green: $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds = \iint_R \nabla \cdot \mathbf{F}\, dA$

Teorema de la divergencia: $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_D \nabla \cdot \mathbf{F}\, dV$

Forma tangencial del teorema de Green: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \nabla \times \mathbf{F} \cdot \mathbf{k}\, dA$

Teorema de Stokes: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$

### Identidades vectoriales

En las siguientes identidades, $f$ y $g$ son funciones escalares derivables, $\mathbf{F}$, $\mathbf{F}_1$, y $\mathbf{F}_2$ son campos vectoriales derivables, y $a$ y $b$ son constantes reales.

$$\nabla \times (\nabla f) = \mathbf{0}$$
$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla \cdot (g\mathbf{F}) = g\nabla \cdot \mathbf{F} + \nabla g \cdot \mathbf{F}$$
$$\nabla \times (g\mathbf{F}) = g\nabla \times \mathbf{F} + \nabla g \times \mathbf{F}$$

$$\nabla \cdot (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \cdot \mathbf{F}_1 + b\nabla \cdot \mathbf{F}_2$$
$$\nabla \times (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \times \mathbf{F}_1 + b\nabla \times \mathbf{F}_2$$

$$\nabla(\mathbf{F}_1 \cdot \mathbf{F}_2) = (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 + (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 + \mathbf{F}_1 \times (\nabla \times \mathbf{F}_2) + \mathbf{F}_2 \times (\nabla \times \mathbf{F}_1)$$

$$\nabla \cdot (\mathbf{F}_1 \times \mathbf{F}_2) = \mathbf{F}_2 \cdot \nabla \times \mathbf{F}_1 - \mathbf{F}_1 \cdot \nabla \times \mathbf{F}_2$$

$$\nabla \times (\mathbf{F}_1 \times \mathbf{F}_2) = (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 - (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 + (\nabla \cdot \mathbf{F}_2)\mathbf{F}_1 - (\nabla \cdot \mathbf{F}_1)\mathbf{F}_2$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - (\nabla \cdot \nabla)\mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$$
$$(\nabla \times \mathbf{F}) \times \mathbf{F} = (\mathbf{F} \cdot \nabla)\mathbf{F} - \frac{1}{2}\nabla(\mathbf{F} \cdot \mathbf{F})$$

# LÍMITES

## Leyes generales

Si $L$, $M$, $c$, y $k$ son números reales y

$$\lim_{x\to c} f(x) = L \qquad \text{y} \qquad \lim_{x\to c} g(x) = M, \quad \text{entonces}$$

*Regla de la suma:* $\lim_{x\to c}(f(x) + g(x)) = L + M$

*Regla de la diferencia:* $\lim_{x\to c}(f(x) - g(x)) = L - M$

*Regla del producto:* $\lim_{x\to c}(f(x) \cdot g(x)) = L \cdot M$

*Regla del múltiplo constante:* $\lim_{x\to c}(k \cdot f(x)) = k \cdot L$

*Regla del cociente:* $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$

## El teorema de la compresión o del sándwich

Si $g(x) \leq f(x) \leq h(x)$ en un intervalo abierto que contiene a $c$, excepto posiblemente en $x = c$, y si

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L,$$

entonces $\lim_{x\to c} f(x) = L$.

## Desigualdades

Si $f(x) \leq g(x)$ en un intervalo abierto que contiene a $c$, excepto posiblemente en $x = c$, y ambos límites existen, entonces

$$\lim_{x\to c} f(x) \leq \lim_{x\to c} g(x).$$

## Continuidad

Si $g$ es continua en $L$ y $\lim_{x\to c} f(x) = L$, entonces

$$\lim_{x\to c} g(f(x)) = g(L).$$

## Fórmulas específicas

Si $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, entonces

$$\lim_{x\to c} P(x) = P(c) = a_n c^n + a_{n-1}c^{n-1} + \cdots + a_0.$$

Si $P(x)$ y $Q(x)$ son polinomios y $Q(c) \neq 0$, entonces

$$\lim_{x\to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

Si $f(x)$ es continua en $x = c$, entonces

$$\lim_{x\to c} f(x) = f(c).$$

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1 \qquad \text{y} \qquad \lim_{x\to 0} \frac{1 - \cos x}{x} = 0$$

## Regla de L'Hôpital

Si $f(a) = g(a) = 0$, y existen $f'$ y $g'$ en un intervalo abierto $I$ que contiene a $a$, y $g'(x) \neq 0$ en $I$ si $x \neq a$, entonces

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

suponiendo que existe el límite de la derecha.

# REGLAS DE DERIVACIÓN

## Fórmulas generales

Suponga que $u$ y $v$ son funciones derivables de $x$.

*Constante:* $\frac{d}{dx}(c) = 0$

*Suma:* $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

*Diferencia:* $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

*Múltiplo constante:* $\frac{d}{dx}(cu) = c\frac{du}{dx}$

*Producto:* $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

*Cociente:* $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

*Potencia:* $\frac{d}{dx}x^n = nx^{n-1}$

*Regla de la cadena:* $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$

## Funciones trigonométricas

$\frac{d}{dx}(\operatorname{sen} x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\operatorname{sen} x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$ $\qquad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

## Funciones exponenciales y logarítmicas

$\frac{d}{dx}e^x = e^x$ $\qquad$ $\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}a^x = a^x \ln a$ $\qquad$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

## Funciones trigonométricas inversas

$\frac{d}{dx}(\operatorname{sen}^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$ $\qquad$ $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$ $\qquad$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2}$ $\qquad$ $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$

## Funciones hiperbólicas

$\frac{d}{dx}(\operatorname{senh} x) = \cosh x$ $\qquad$ $\frac{d}{dx}(\cosh x) = \operatorname{senh} x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

## Funciones hiperbólicas inversas

$\frac{d}{dx}(\operatorname{senh}^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$ $\qquad$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}$ $\qquad$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1 - x^2}}$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2}$ $\qquad$ $\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1 + x^2}}$

## Funciones paramétricas

Si $x = f(t)$ y $y = g(t)$ son derivables, entonces

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{y} \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}.$$