

# LISTA DE DERIVADAS

## Reglas

1. **Constante:**  $\frac{d}{dx} c = 0$

2. **Múltiplo constante:**  $\frac{d}{dx} cf(x) = c f'(x)$

3. **Suma:**  $\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$

4. **Producto:**  $\frac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

5. **Cociente:**  $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

6. **Cadena:**  $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

7. **Potencia:**  $\frac{d}{dx} x^n = nx^{n-1}$

8. **Potencia:**  $\frac{d}{dx} [g(x)]^n = n[g(x)]^{n-1}g'(x)$

## Funciones

### Trigonométricas:

9.  $\frac{d}{dx} \operatorname{sen} x = \cos x$

10.  $\frac{d}{dx} \cos x = -\operatorname{sen} x$

11.  $\frac{d}{dx} \tan x = \sec^2 x$

12.  $\frac{d}{dx} \cot x = -\operatorname{csc}^2 x$

13.  $\frac{d}{dx} \sec x = \sec x \tan x$

14.  $\frac{d}{dx} \operatorname{csc} x = -\operatorname{csc} x \cot x$

### Trigonométricas inversas:

15.  $\frac{d}{dx} \operatorname{sen}^{-1} x = \frac{1}{\sqrt{1-x^2}}$

16.  $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

17.  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

18.  $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$

19.  $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

20.  $\frac{d}{dx} \operatorname{csc}^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

### Hiperbólicas:

21.  $\frac{d}{dx} \operatorname{senh} x = \cosh x$

22.  $\frac{d}{dx} \cosh x = \operatorname{senh} x$

23.  $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

24.  $\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$

25.  $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

26.  $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

### Hiperbólicas inversas:

27.  $\frac{d}{dx} \operatorname{senh}^{-1} x = \frac{1}{\sqrt{x^2+1}}$

28.  $\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$

29.  $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$

30.  $\frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$

31.  $\frac{d}{dx} \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$

32.  $\frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2+1}}$

### Exponencial:

33.  $\frac{d}{dx} e^x = e^x$

34.  $\frac{d}{dx} b^x = b^x (\ln b)$

### Logarítmica:

35.  $\frac{d}{dx} \ln |x| = \frac{1}{x}$

36.  $\frac{d}{dx} \log_b x = \frac{1}{x(\ln b)}$

# BREVE TABLA DE INTEGRALES

1.  $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
3.  $\int e^u du = e^u + C$
5.  $\int \operatorname{sen} u du = -\cos u + C$
7.  $\int \sec^2 u du = \tan u + C$
9.  $\int \sec u \tan u du = \sec u + C$
11.  $\int \tan u du = -\ln|\cos u| + C$
13.  $\int \sec u du = \ln|\sec u + \tan u| + C$
15.  $\int u \operatorname{sen} u du = \operatorname{sen} u - u \cos u + C$
17.  $\int \operatorname{sen}^2 u du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$
19.  $\int \tan^2 u du = \tan u - u + C$
21.  $\int \operatorname{sen}^3 u du = -\frac{1}{3}(2 + \operatorname{sen}^2 u) \cos u + C$
23.  $\int \tan^3 u du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
25.  $\int \sec^3 u du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
27.  $\int \operatorname{sen} au \cos bu du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$
29.  $\int e^{au} \operatorname{sen} bu du = \frac{e^{au}}{a^2 + b^2}(a \operatorname{sen} bu - b \cos bu) + C$
31.  $\int \operatorname{senh} u du = \cosh u + C$
33.  $\int \operatorname{sech}^2 u du = \tanh u + C$
35.  $\int \tanh u du = \ln(\cosh u) + C$
37.  $\int \ln u du = u \ln u - u + C$
39.  $\int \frac{1}{\sqrt{a^2 - u^2}} du = \operatorname{sen}^{-1} \frac{u}{a} + C$
41.  $\int \sqrt{a^2 - u^2} du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1} \frac{u}{a} + C$
43.  $\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\tan^{-1} \frac{u}{a} + C$
2.  $\int \frac{1}{u} du = \ln|u| + C$
4.  $\int a^u du = \frac{1}{\ln a} a^u + C$
6.  $\int \cos u du = \operatorname{sen} u + C$
8.  $\int \csc^2 u du = -\cot u + C$
10.  $\int \csc u \cot u du = -\csc u + C$
12.  $\int \cot u du = \ln|\operatorname{sen} u| + C$
14.  $\int \csc u du = \ln|\csc u - \cot u| + C$
16.  $\int u \cos u du = \cos u + u \operatorname{sen} u + C$
18.  $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$
20.  $\int \cot^2 u du = -\cot u - u + C$
22.  $\int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u) \operatorname{sen} u + C$
24.  $\int \cot^3 u du = -\frac{1}{2}\cot^2 u - \ln|\operatorname{sen} u| + C$
26.  $\int \csc^3 u du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$
28.  $\int \cos au \cos bu du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$
30.  $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \operatorname{sen} bu) + C$
32.  $\int \cosh u du = \operatorname{senh} u + C$
34.  $\int \operatorname{csch}^2 u du = -\coth u + C$
36.  $\int \coth u du = \ln|\operatorname{senh} u| + C$
38.  $\int u \ln u du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$
40.  $\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln|u + \sqrt{a^2 + u^2}| + C$
42.  $\int \sqrt{a^2 + u^2} du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln|u + \sqrt{a^2 + u^2}| + C$
44.  $\int \frac{1}{a^2 - u^2} du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

# TABLA DE TRANSFORMADAS DE LAPLACE

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. $t$	$\frac{1}{s^2}$
3. $t^n$	$\frac{n!}{s^{n+1}}, \quad n \text{ un entero positivo}$
4. $t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$
5. $t^{1/2}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
6. $t^\alpha$	$\frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}, \quad \alpha > -1$
7. $\text{sen } kt$	$\frac{k}{s^2 + k^2}$
8. $\cos kt$	$\frac{s}{s^2 + k^2}$
9. $\text{sen}^2 kt$	$\frac{2k^2}{s(s^2 + 4k^2)}$
10. $\cos^2 kt$	$\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
11. $e^{at}$	$\frac{1}{s - a}$
12. $\text{senh } kt$	$\frac{k}{s^2 - k^2}$
13. $\cosh kt$	$\frac{s}{s^2 - k^2}$
14. $\text{senh}^2 kt$	$\frac{2k^2}{s(s^2 - 4k^2)}$
15. $\cosh^2 kt$	$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
16. $te^{at}$	$\frac{1}{(s - a)^2}$
17. $t^n e^{at}$	$\frac{n!}{(s - a)^{n+1}}, \quad n \text{ un entero positivo}$
18. $e^{at} \text{sen } kt$	$\frac{k}{(s - a)^2 + k^2}$
19. $e^{at} \cos kt$	$\frac{s - a}{(s - a)^2 + k^2}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
20. $e^{at} \text{senh } kt$	$\frac{k}{(s - a)^2 - k^2}$
21. $e^{at} \cosh kt$	$\frac{s - a}{(s - a)^2 - k^2}$
22. $t \text{sen } kt$	$\frac{2ks}{(s^2 + k^2)^2}$
23. $t \cos kt$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$
24. $\text{sen } kt + kt \cos kt$	$\frac{2ks^2}{(s^2 + k^2)^2}$
25. $\text{sen } kt - kt \cos kt$	$\frac{2k^3}{(s^2 + k^2)^2}$
26. $t \text{senh } kt$	$\frac{2ks}{(s^2 - k^2)^2}$
27. $t \cosh kt$	$\frac{s^2 + k^2}{(s^2 - k^2)^2}$
28. $\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s - a)(s - b)}$
29. $\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s - a)(s - b)}$
30. $1 - \cos kt$	$\frac{k^2}{s(s^2 + k^2)}$
31. $kt - \text{sen } kt$	$\frac{k^3}{s^2(s^2 + k^2)}$
32. $\frac{a \text{sen} bt - b \text{sen} at}{ab(a^2 - b^2)}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$
33. $\frac{\cos bt - \cos at}{a^2 - b^2}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
34. $\text{sen } kt \text{senh } kt$	$\frac{2k^2s}{s^4 + 4k^4}$
35. $\text{sen } kt \cosh kt$	$\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
36. $\cos kt \text{sinh } kt$	$\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
37. $\cos kt \cosh kt$	$\frac{s^3}{s^4 + 4k^4}$
38. $J_0(kt)$	$\frac{1}{\sqrt{s^2 + k^2}}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
39. $\frac{e^{bt} - e^{at}}{t}$	$\ln \frac{s-a}{s-b}$
40. $\frac{2(1 - \cos kt)}{t}$	$\ln \frac{s^2 + k^2}{s^2}$
41. $\frac{2(1 - \cosh kt)}{t}$	$\ln \frac{s^2 - k^2}{s^2}$
42. $\frac{\operatorname{sen} at}{t}$	$\arctan\left(\frac{a}{s}\right)$
43. $\frac{\operatorname{sen} at \cos bt}{t}$	$\frac{1}{2} \arctan \frac{a+b}{s} + \frac{1}{2} \arctan \frac{a-b}{s}$
44. $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$
45. $\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
46. $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
47. $2\sqrt{\frac{t}{\pi}} e^{-a^2/4t} - a \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s\sqrt{s}}$
48. $e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$
49. $-e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{be^{-a\sqrt{s}}}{s(\sqrt{s} + b)}$
50. $e^{at} f(t)$	$F(s-a)$
51. $\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
52. $f(t-a) \mathcal{U}(t-a)$	$e^{-as} F(s)$
53. $g(t) \mathcal{U}(t-a)$	$e^{-as} \mathcal{L}\{g(t+a)\}$
54. $f^{(n)}(t)$	$s^n F(s) - s^{(n-1)} f(0) - \dots - f^{(n-1)}(0)$
55. $t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
56. $\int_0^t f(\tau) g(t-\tau) d\tau$	$F(s)G(s)$
57. $\delta(t)$	1
58. $\delta(t-t_0)$	$e^{-st_0}$