MECÁNICA COMPUTACIONAL – INGENIERÍA EN INFORMÁTICA

PRIMER PARCIAL – MÉTODO DE ELEMENTOS FINITOS

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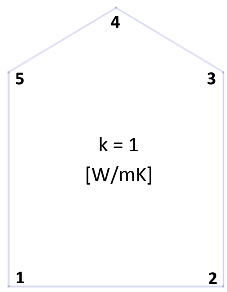
Ejercicio 1

Resolver la ecuación del calor: $k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q=0$ bajo las condiciones de contorno mostradas a continuación:



Lado 1-2: T = 100°

Lado 1-5 y 3-4: q = 0 [W/m^2]

Lado 4-5: q = 20 [W/m^2]

Lado 2-3: q = -20 [W/m^2]

Coordenadas

Nodo	X	Y
1	0.0	0.0
2	1.0	0.0
3	1.0	1.0
4	0.5	1.3
5	0.0	1.0

a) Dada la malla mostrada en la Figura 1, calcular y dejar expresado:
   - Matrices y vectores elementales
   - Matriz y vector global del sistema
   - Temperatura y flujo de calor en el punto (0.5; 0.5)

b) Dada la malla mostrada en la Figura 2, calcular y dejar expresado:
   - Matrices y vectores elementales
   - Matriz y vector global del sistema
   - Temperatura y flujo de calor en el punto (0.5; 0.5)

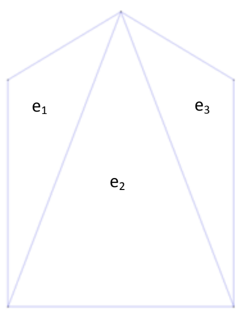


Figura 1

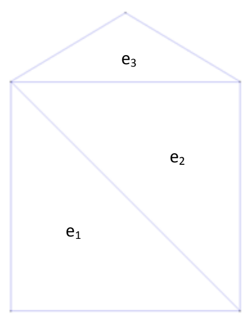


Figura 2

Ejercicio 2

Se ha resuelto por elementos finitos un problema de tensión plana utilizando funciones de forma lineales, donde se obtuvieron los siguientes resultados:

Nodo	x	y	u (mm)	v (mm)
1	0.0	0.0	0.0	0.0
2	1.0	0.0	0.7189	0.66431
3	2.0	0.0	-0.05839	0.0
4	2.0	0.5	0.55891	1.7722
5	1.25	0.5	0.40925	0.89222
6	0.0	0.5	0.0	0.0
7	0.0	1.0	0.0	0.0
8	1.0	1.0	-0.13365	0.45779
9	2.0	1.0	0.17009	3.9252

Datos:
E = 2.1e9 [Pa]
ν = 0.3
ρ = 2400 [Kg/m^3]
t = 0.005 [m]

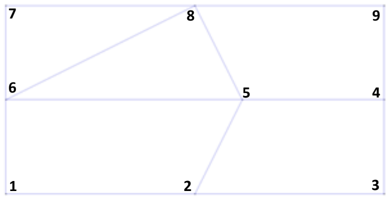


Figura 3

Dada la malla mostrada en la Figura 3, calcular en los puntos (0.75; 0.75) y (1.5; 0.75) los desplazamientos, las deformaciones y las tensiones. ¿Cuál será la magnitud de la tensión y la deformación en cualquier punto cercano al (0.75; 0.75)? Justifique.

Ejercicio 3

Resolver por el Método de Residuos Ponderados la siguiente ecuación diferencial:

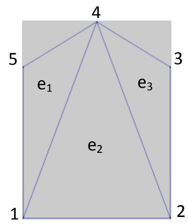
$$\frac{d}{dx}\left(\kappa\frac{d\varphi}{dx}\right)+Q=0 \qquad \forall x \in [0,1]$$

$$\varphi(x=0)=1; \qquad \frac{d\varphi}{dx}(x=1)=1$$

usando como aproximante $\varphi \approx \hat{\varphi} = \psi(x) + \sum_m a_m N_m(x)$, con N_m funciones trigonométricas. Considerar $\kappa = 1$ y $Q = -10$. Use una ponderación por colocación puntual utilizando 3 funciones base y grafique la solución. Defina una función $\psi(x)$ de forma tal que satisfaga ambas condiciones de contorno.

SOLUCION EXAMEN 1

Ejercicio 1 – Figura 1



Elemento e_1

Matriz de rigidez:

0.34	0.3	-0.64
0.3	1.0	-1.3
-0.64	-1.3	1.94

Vector de cargas:

8.333
14.164
14.164

Elemento e_2

Matriz de rigidez:

0.7462	-0.5538	-0.1923
-0.5538	0.7462	-0.1923
-0.1923	-0.1923	0.3846

Vector de cargas:

21.6667
21.6667
21.6667

Temperatura

T_1	100.0
T_2	100.0
T_3	153.9501
T_4	181.7922
T_5	162.1104

Elemento e_3

Matriz de rigidez:

0.34	-0.64	0.3
-0.64	1.94	-1.3
0.3	-1.3	1.0

Vector de cargas:

-1.667
-1.667
8.333

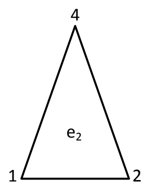
Matriz global

1.0862	-0.5538	0.0	0.1077	-0.64
-0.5538	1.0862	-0.64	0.1077	0.0
0.0	-0.64	1.94	-1.3	0.0
0.1077	0.1077	-1.3	2.3846	-1.3
-0.64	0.0	0.0	-1.3	1.94

Vector de cargas:

30.0
20.0
-1.6667
44.1643
14.1643

Temperatura y flujo de calor en (0.5; 0.5)



$$N_1(x,y) = 1 - x - 0.3846y$$

$$N_2(x,y) = x - 0.3846y$$

$$N_3(x,y) = 0.7692y$$

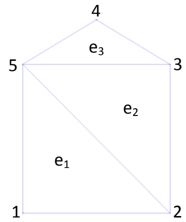
$$T(0.5{,}0.5) = N_1(0.5{,}0.5)T_1 + N_2(0.5{,}0.5)T_2 + N_3(0.5{,}0.5)T_4$$

$$T(0.5{,}0.5) = 0.3077 * 100 + 0.3077 * 100 + 0.3846 * 181.7922 = \mathbf{131.4585}$$

$$q_x = -k\left(\frac{\partial N_1}{\partial x}T_1 + \frac{\partial N_2}{\partial x}T_2 + \frac{\partial N_3}{\partial x}T_4\right) = -1(-1 * 100 + 1 * 100 + 0 * 181.7922) = \mathbf{0}$$

$$q_y = -k\left(\frac{\partial N_1}{\partial y}T_1 + \frac{\partial N_2}{\partial y}T_2 + \frac{\partial N_3}{\partial y}T_4\right) = -1(-0.3846 * 100 - 0.3846 * 100 + 0.7692 * 181.7922) = \mathbf{-62.9171}$$

Ejercicio 1 – Figura 2



Elemento e_1

Matriz de rigidez:

1.0	-0.5	-0.5
-0.5	0.5	0.0
-0.5	0.0	0.5

Vector de cargas:

16.667
16.667
16.667

Elemento e_2

Matriz de rigidez:

0.5	-0.5	0
-0.5	1.0	-0.5
0	-0.5	0.5

Vector de cargas:

6.667
6.667
16.667

Temperatura

T_1	100.0
T_2	100.0
T_3	157.6348
T_4	173.1605
T_5	175.6890

Elemento e_3

Matriz de rigidez:

0.5667	-0.8333	0.2667
-0.8333	1.6667	-0.8333
0.2667	-0.8333	0.5667

Vector de cargas:

5.0
10.8310
10.8310

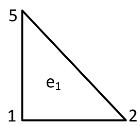
Matriz global

1.0	-0.5	0.0	0.0	-0.5
-0.5	1.0	-0.5	0.0	0.0
0.0	-0.5	1.5667	-0.833	-0.233
0.0	0.0	-0.833	1.667	-0.833
-0.5	0.0	-0.233	-0.833	1.5667

Vector de cargas:

16.667
23.333
11.667
10.8310
44.1643

Temperatura y flujo de calor en (0.5; 0.5)



$$N_1(x,y) = 1 - x - y$$

$$N_2(x,y) = x$$

$$N_3(x,y) = y$$

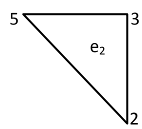
$$T(0.5{,}0.5) = N_1(0.5{,}0.5)T_1 + N_2(0.5{,}0.5)T_2 + N_3(0.5{,}0.5)T_5$$

$$T(0.5{,}0.5) = 0 * 100 + 0.5 * 100 + 0.5 * 175.6890 = \mathbf{137.8445}$$

$$q_x = -k\left(\frac{\partial N_1}{\partial x}T_1 + \frac{\partial N_2}{\partial x}T_2 + \frac{\partial N_3}{\partial x}T_4\right) = -1(-1 * 100 + 1 * 100 + 0 * 181.7922) = \mathbf{0}$$

$$q_y = -k\left(\frac{\partial N_1}{\partial y}T_1 + \frac{\partial N_2}{\partial y}T_2 + \frac{\partial N_3}{\partial y}T_4\right) = -1(-0.3846 * 100 - 0.3846 * 100 + 0.7692 * 181.7922) = -\mathbf{62.9171}$$

Corroboración elemento e_2: (misma temperatura en el nodo, distinto flujo por discontinuidad)



$$N_1(x,y) = 1 - y$$

$$N_2(x,y) = -1 + x + y$$

$$N_3(x,y) = 1 - x$$

$$T(0.5{,}0.5) = N_1(0.5{,}0.5)T_2 + N_2(0.5{,}0.5)T_3 + N_3(0.5{,}0.5)T_5$$

$$T(0.5{,}0.5) = 0.5 * 100 + 0 * 157.6348 + 0.5 * 175.6890 = \mathbf{137.8445}$$

$$q_x = -k\left(\frac{\partial N_1}{\partial x}T_2 + \frac{\partial N_2}{\partial x}T_3 + \frac{\partial N_3}{\partial x}T_5\right) = -1(0 * 100 + 1 * 157.6348 - 1 * 175.6890) = \mathbf{18.0542}$$

$$q_y = -k\left(\frac{\partial N_1}{\partial y}T_2 + \frac{\partial N_2}{\partial y}T_3 + \frac{\partial N_3}{\partial y}T_5\right) = -1(-1 * 100 + 1 * 157.6348 + 0 * 181.7922) = -\mathbf{57.6348}$$

Ejercicio 2

Punto (0.75; 0.75) asociado al elemento triangular (6 5 8)

$$N_1(x,y) = 1.2 - 0.8x - 0.4y$$

$$N_2(x,y) = 0.8 + 0.8x - 1.6y$$

$$N_3(x,y) = -1 + 2y$$

$$u(0.75{,}0.75) = N_1(0.75{,}0.75)u_6 + N_2(0.75{,}0.75)u_5 + N_3(0.75{,}0.75)u_8 = 0.3 * 0.0 + 0.2 * 0.40925 - 0.5 * 0.13365 = \mathbf{0.015}$$

$$v(0.75{,}0.75) = N_1(0.75{,}0.75)v_6 + N_2(0.75{,}0.75)v_5 + N_3(0.75{,}0.75)v_8 = 0.3 * 0.0 + 0.2 * 0.8922 + 0.5 * 0.45779 = \mathbf{0.4073}$$

$$\varepsilon_x = b_1u_6 + b_2u_5 + b_3u_8 = -0.8 * 0.0 + 0.8 * 0.40925 + 0.0 * 0.13365 = \mathbf{0.3274}$$

$$\varepsilon_y = c_1v_6 + c_2v_5 + c_3v_8 = -0.4 * 0.0 - 1.6 * 0.8922 + 2 * 0.4578 = \mathbf{-0.5120}$$

$$\varepsilon_{xy} = c_1u_6 + c_2u_5 + c_3u_8 + b_1v_6 + b_2v_5 + v_3b_8 = \mathbf{-0.2083}$$

$$\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_{xy}\end{bmatrix} = \overline{\overline{D_{3x3}}}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{bmatrix} = \mathbf{1e8}\begin{bmatrix}\mathbf{4.0110}\\ \mathbf{-9.5481}\\ \mathbf{-1.6826}\end{bmatrix}$$

Punto (1.5; 0.75) asociado al elemento cuadrangular (5 4 9 8)

$$N_1(x,y) = 5.333 - 2.666x - 5.333y + 2.666xy$$

$$N_2(x,y) = -3.333 + 2.666x + 3.333y - 2.666xy$$

$$N_3(x,y) = 1 - x - 2y + 2xy$$

$$N_4(x,y) = -2 + x + 4y - 2xy$$

$$u(1.5{,}0.75) = N_1(1.5{,}0.75)u_5 + N_2(1.5{,}0.75)u_4 + N_3(1.5{,}0.75)u_9 + N_4(1.5{,}0.75)u_8$$

$$u(0.5{,}0.75) = 0.333 * 0.4093 + 0.1667 * 0.5589 + 0.25 * 0.1701 - 0.25 * 0.1337 = \mathbf{0.2387}$$

$$v(1.5{,}0.75) = N_1(1.5{,}0.75)v_5 + N_2(1.5{,}0.75)v_4 + N_3(1.5{,}0.75)v_9 + N_4(1.5{,}0.75)v_8$$

$$v(0.5{,}0.75) = 0.333 * 0.8922 + 0.1667 * 1.7722 + 0.25 * 3.9252 + 0.25 * 0.4578 = \mathbf{1.6885}$$

$$\varepsilon_x = b_1u_5 + b_2u_4 + b_3u_9 + b_4u_8 + (d_1u_5 + d_2u_4 + d_3u_9 + d_4u_8)y|_{0.75} = \mathbf{0.2516}$$

$$\varepsilon_y = c_1u_5 + c_2u_4 + c_3u_9 + c_4u_8 + (d_1v_5 + d_2v_4 + d_3v_9 + d_4v_8)x|_{1.5} = \mathbf{2.0119}$$

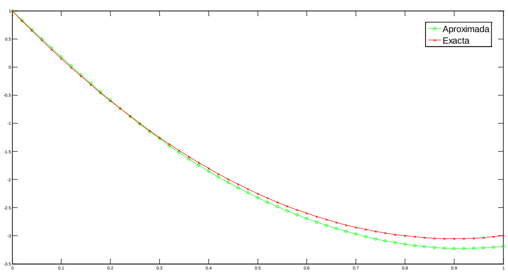
$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \mathbf{1.4385}$$

$$\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_{xy}\end{bmatrix} = \overline{\overline{D_{3x3}}}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{bmatrix} = \mathbf{1e9}\begin{bmatrix}\mathbf{1.9736}\\ \mathbf{4.8171}\\ \mathbf{1.1619}\end{bmatrix}$$

Ejercicio 3

$\widehat{\emptyset} = \psi + \sum_{m=1}^{3} a_m N_m;$ $\qquad \psi = x + 1;\ N_m = \sin\left(\frac{(2*m-1)\pi x}{2}\right)$

$$K_{lm} = -\left(\frac{(2*m-1)\pi}{2}\right)^2 \sin\left(\frac{(2*m-1)\pi x}{2}\right)\Bigg|_{x_l}$$



$$\begin{bmatrix}-0.9442 & -20.5162 & -56.9895\\ -1.7447 & -15.7024 & 43.6179\\ -2.2796 & 8.4981 & 23.6058\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix} = \begin{bmatrix}10\\ 10\\ 10\end{bmatrix} \quad \Rightarrow \quad \underline{a} = \begin{bmatrix}-5.2953\\ -0.1461\\ -0.0351\end{bmatrix}$$

$$\widehat{\emptyset} = x + 1 - 5.2953\sin\left(\frac{\pi x}{2}\right) - 0.1461\sin\left(\frac{3\pi x}{2}\right) - 0.0351\sin\left(\frac{5\pi x}{2}\right)$$