THOMAS

CÁLCULO

VARIAS VARIABLES

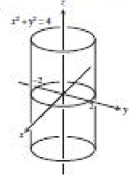
Decimosegunda edición

PEARSON

Sección 12.6, pp. 700–701

1. (d), elipsoide 3. (a), cilindro 5. (l), paraboloide hiperbólico
7. (b), cilindro 9. (k), paraboloide hiperbólico 11. (h), cono

13.



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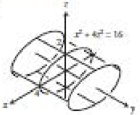

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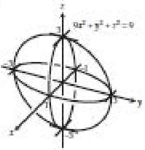

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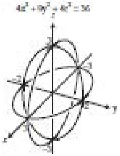

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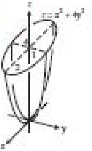

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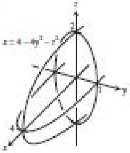

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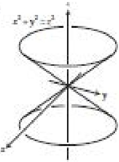

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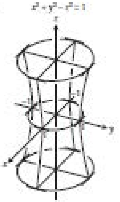

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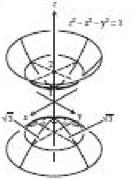

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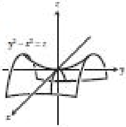

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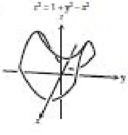

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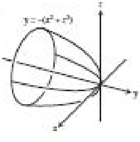
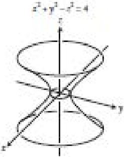


37.



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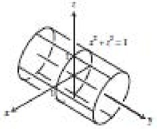
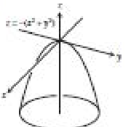
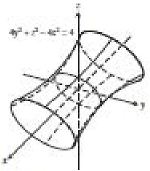


41.



43.



45. (a) $\dfrac{2\pi(9 - c^2)}{9}$ (b) 8π (c) $\dfrac{4\pi abc}{3}$

CAPÍTULO 13

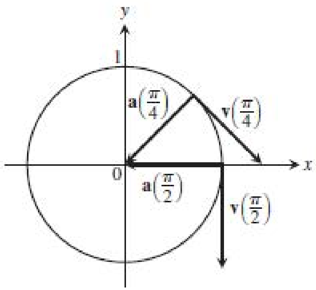
Sección 13.1, pp. 713–715

1. $y = x^2 - 2x, \quad \mathbf{v} = \mathbf{i} + 2\mathbf{j}, \quad \mathbf{a} = 2\mathbf{j}$

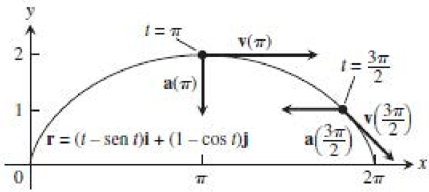
3. $y = \frac{2}{9}x^2, \quad \mathbf{v} = 3\mathbf{i} + 4\mathbf{j}, \quad \mathbf{a} = 3\mathbf{i} + 8\mathbf{j}$

5. $t = \frac{\pi}{4}$: $\mathbf{v} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}, \quad \mathbf{a} = \frac{-\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$;

$t = \pi/2$: $\mathbf{v} = -\mathbf{j}, \quad \mathbf{a} = -\mathbf{i}$



7. $t = \pi$: $\mathbf{v} = 2\mathbf{i}, \quad \mathbf{a} = -\mathbf{j}; \quad t = \frac{3\pi}{2}$: $\mathbf{v} = \mathbf{i} - \mathbf{j}, \quad \mathbf{a} = -\mathbf{i}$



9. $\mathbf{v} = \mathbf{i} + 2t\mathbf{j} + 2\mathbf{k}$; $\mathbf{a} = 2\mathbf{j}$; velocidad: 3;

dirección: $\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$; $\mathbf{v}(1) = 3\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$

11. $\mathbf{v} = (-2 \operatorname{sen} t)\mathbf{i} + (3 \cos t)\mathbf{j} + 4\mathbf{k}$;
$\mathbf{a} = (-2 \cos t)\mathbf{i} - (3 \operatorname{sen} t)\mathbf{j}$; velocidad: $2\sqrt{5}$;
dirección: $\left(-1/\sqrt{5}\right)\mathbf{i} + \left(2/\sqrt{5}\right)\mathbf{k}$;
$\mathbf{v}(\pi/2) = 2\sqrt{5}\left[\left(-1/\sqrt{5}\right)\mathbf{i} + \left(2/\sqrt{5}\right)\mathbf{k}\right]$

13. $\mathbf{v} = \left(\frac{2}{t+1}\right)\mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$; $\mathbf{a} = \left(\frac{-2}{(t+1)^2}\right)\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;
velocidad: $\sqrt{6}$; dirección: $\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}$;
$\mathbf{v}(1) = \sqrt{6}\left(\frac{1}{\sqrt{6}}\mathbf{i} + \frac{2}{\sqrt{6}}\mathbf{j} + \frac{1}{\sqrt{6}}\mathbf{k}\right)$

15. $\pi/2$ 17. $\pi/2$

19. $x = t,\ \ y = -1,\ \ z = 1 + t$ 21. $x = t,\ \ y = \frac{1}{3}t,\ \ z = t$

23. **(a)** (i): Tiene velocidad constante 1 (ii): Sí
(iii): Sentido contrario a las manecillas del reloj (iv): Sí
(b) (i): Tiene velocidad constante 2 (ii): Sí
(iii): Sentido contrario a las manecillas del reloj (iv): Sí
(c) (i): Tiene velocidad constante 1 (ii): Sí
(iii): Sentido contrario a las manecillas del reloj
(iv): Inicia en $(0, -1)$ en vez de $(1, 0)$
(d) (i): Tiene velocidad constante 1 (ii): Sí
(iii): En el sentido de las manecillas del reloj (iv): Sí
(e) (i): Tiene velocidad variable (ii): No
(iii): Sentido contrario a las manecillas del reloj (iv): Sí

25. $\mathbf{v} = 2\sqrt{5}\mathbf{i} + \sqrt{5}\mathbf{j}$

Sección 13.2, pp. 720–724

1. $(1/4)\mathbf{i} + 7\mathbf{j} + (3/2)\mathbf{k}$ 3. $\left(\frac{\pi + 2\sqrt{2}}{2}\right)\mathbf{j} + 2\mathbf{k}$

5. $(\ln 4)\mathbf{i} + (\ln 4)\mathbf{j} + (\ln 2)\mathbf{k}$

7. $\frac{e-1}{2}\mathbf{i} + \frac{e-1}{e}\mathbf{j} + \mathbf{k}$

9. $\mathbf{i} - \mathbf{j} + \frac{\pi}{4}\mathbf{k}$

11. $\mathbf{r}(t) = \left(\frac{-t^2}{2} + 1\right)\mathbf{i} + \left(\frac{-t^2}{2} + 2\right)\mathbf{j} + \left(\frac{-t^2}{2} + 3\right)\mathbf{k}$

13. $\mathbf{r}(t) = ((t+1)^{3/2} - 1)\mathbf{i} + (-e^{-t} + 1)\mathbf{j} + (\ln(t+1) + 1)\mathbf{k}$

15. $\mathbf{r}(t) = 8t\mathbf{i} + 8t\mathbf{j} + (-16t^2 + 100)\mathbf{k}$

17. $\mathbf{r}(t) = \left(\frac{3}{2}t^2 + \frac{6}{\sqrt{11}}t + 1\right)\mathbf{i} - \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t - 2\right)\mathbf{j}$
$+\left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t + 3\right)\mathbf{k} = \left(\frac{1}{2}t^2 + \frac{2t}{\sqrt{11}}\right)(3\mathbf{i} - \mathbf{j} + \mathbf{k})$
$+(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$

19. 50 seg

21. **(a)** 72.2 seg; 25,510 m **(b)** 4020 m **(c)** 6378 m

23. **(a)** $v_0 \approx 9.9$ m/seg **(b)** $\alpha \approx 18.4°$ o $71.6°$

25. 39.3° o 50.7°

31. **(b)** $\mathbf{v}_0$ bisecaría $\angle AOR$.

33. **(a)** (Suponiendo que "x" es cero en el punto de impacto)
$\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, donde $x(t) = (35 \cos 27°)t$ y
$y(t) = 4 + (35 \operatorname{sen} 27°)t - 16t^2$.
(b) En $t \approx 0.497$ seg, alcanza su máxima altura de aproximadamente 7.945 ft.
(c) Rango ≈ 37.45 ft; tiempo de vuelo ≈ 1.201 seg
(d) En $t \approx 0.254$ y $t \approx 0.740$ seg, cuando es ≈ 29.554 y ≈ 14.396 ft desde donde aterrizará
(e) Sí. Las cosas cambian porque la pelota no pasará la red.

35. 4.00 ft, 7.80 ft/seg

43. **(a)** $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$; donde

$$x(t) = \left(\frac{1}{0.08}\right)(1 - e^{-0.08t})(152\cos 20° - 17.6) \text{ y}$$

$$y(t) = 3 + \left(\frac{152}{0.08}\right)(1 - e^{-0.08t})(\operatorname{sen} 20°)$$

$$+\left(\frac{32}{0.08^2}\right)(1 - 0.08t - e^{-0.08t})$$

(b) En $t \approx 1.527$ seg alcanza la altura máxima de aproximadamente 41.893 ft.
(c) Rango ≈ 351.734 ft; tiempo de vuelo ≈ 3.181 seg
(d) En $t \approx 0.877$ y 2.190 seg, cuando está aproximadamente a 106.028 y 251.530 ft del home
(e) No

Sección 13.3, pp. 727–728

1. $\mathbf{T} = \left(-\frac{2}{3}\operatorname{sen} t\right)\mathbf{i} + \left(\frac{2}{3}\cos t\right)\mathbf{j} + \frac{\sqrt{5}}{3}\mathbf{k},\ 3\pi$

3. $\mathbf{T} = \frac{1}{\sqrt{1+t}}\mathbf{i} + \frac{\sqrt{t}}{\sqrt{1+t}}\mathbf{k},\ \frac{52}{3}$

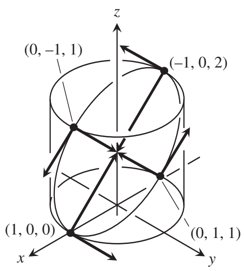
5. $\mathbf{T} = -\cos t\,\mathbf{j} + \operatorname{sen} t\,\mathbf{k},\ \frac{3}{2}$

7. $\mathbf{T} = \left(\frac{\cos t - t\operatorname{sen} t}{t+1}\right)\mathbf{i} + \left(\frac{\operatorname{sen} t + t\cos t}{t+1}\right)\mathbf{j} + \left(\frac{\sqrt{2}t^{1/2}}{t+1}\right)\mathbf{k},\ \frac{\pi^2}{2} + \pi$

9. $(0, 5, 24\pi)$ **11.** $s(t) = 5t,\quad L = \frac{5\pi}{2}$

13. $s(t) = \sqrt{3}e^t - \sqrt{3},\quad L = \frac{3\sqrt{3}}{4}$ **15.** $\sqrt{2} + \ln\left(1 + \sqrt{2}\right)$

17. **(a)** El cilindro es $x^2 + y^2 = 1$, el plano es $x + z = 1$.
(b) y **(c)**



(d) $L = \int_0^{2\pi} \sqrt{1 + \operatorname{sen}^2 t}\,dt$ **(e)** $L \approx 7.64$

Sección 13.4, pp. 733–734

1. $\mathbf{T} = (\cos t)\mathbf{i} - (\operatorname{sen} t)\mathbf{j},\quad \mathbf{N} = (-\operatorname{sen} t)\mathbf{i} - (\cos t)\mathbf{j},\quad \kappa = \cos t$

3. $\mathbf{T} = \frac{1}{\sqrt{1+t^2}}\mathbf{i} - \frac{t}{\sqrt{1+t^2}}\mathbf{j},\quad \mathbf{N} = \frac{-t}{\sqrt{1+t^2}}\mathbf{i} - \frac{1}{\sqrt{1+t^2}}\mathbf{j},\quad \kappa = \frac{1}{2\left(\sqrt{1+t^2}\right)^3}$

5. **(b)** $\cos x$

7. **(b)** $\mathbf{N} = \frac{-2e^{2t}}{\sqrt{1+4e^{4t}}}\mathbf{i} + \frac{1}{\sqrt{1+4e^{4t}}}\mathbf{j}$
(c) $\mathbf{N} = -\frac{1}{2}\left(\sqrt{4-t^2}\,\mathbf{i} + t\mathbf{j}\right)$

9. $\mathbf{T} = \frac{3\cos t}{5}\mathbf{i} - \frac{3\operatorname{sen} t}{5}\mathbf{j} + \frac{4}{5}\mathbf{k},\quad \mathbf{N} = (-\operatorname{sen} t)\mathbf{i} - (\cos t)\mathbf{j},\quad \kappa = \frac{3}{25}$

11. $\mathbf{T} = \left(\frac{\cos t - \operatorname{sen} t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{\cos t + \operatorname{sen} t}{\sqrt{2}}\right)\mathbf{j},$
$\mathbf{N} = \left(\frac{-\cos t - \operatorname{sen} t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{-\operatorname{sen} t + \cos t}{\sqrt{2}}\right)\mathbf{j},$
$\kappa = \frac{1}{e^t\sqrt{2}}$

13. $\mathbf{T} = \frac{t}{\sqrt{t^2+1}}\mathbf{i} + \frac{1}{\sqrt{t^2+1}}\mathbf{j},\quad \mathbf{N} = \frac{\mathbf{i}}{\sqrt{t^2+1}} - \frac{t\mathbf{j}}{\sqrt{t^2+1}},$
$\kappa = \frac{1}{t(t^2+1)^{3/2}}$

15. $\mathbf{T} = \left(\operatorname{sech}\frac{t}{a}\right)\mathbf{i} + \left(\tanh\frac{t}{a}\right)\mathbf{j},$
$\mathbf{N} = \left(-\tanh\frac{t}{a}\right)\mathbf{i} + \left(\operatorname{sech}\frac{t}{a}\right)\mathbf{j},$
$\kappa = \frac{1}{a}\operatorname{sech}^2\frac{t}{a}$

19. $1/(2b)$ **21.** $\left(x - \frac{\pi}{2}\right)^2 + y^2 = 1$

23. $\kappa(x) = 2/(1 + 4x^2)^{3/2}$ **25.** $\kappa(x) = |\operatorname{sen} x|/(1 + \cos^2 x)^{3/2}$

Sección 13.5, pp. 738–739

1. $\mathbf{a} = |a|\mathbf{N}$ **3.** $\mathbf{a}(1) = \frac{4}{3}\mathbf{T} + \frac{2\sqrt{5}}{3}\mathbf{N}$ **5.** $\mathbf{a}(0) = 2\mathbf{N}$

7. $\mathbf{r}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k},\ \mathbf{T}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j},$
$\mathbf{N}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j},\ \mathbf{B}\left(\frac{\pi}{4}\right) = \mathbf{k}$; plano osculador: $z = -1$; plano normal: $-x + y = 0$; plano rectificador: $x + y = \sqrt{2}$

9. $\mathbf{B} = \left(\frac{4}{5}\cos t\right)\mathbf{i} - \left(\frac{4}{5}\operatorname{sen} t\right)\mathbf{j} - \frac{3}{5}\mathbf{k},\ \tau = -\frac{4}{25}$

11. $\mathbf{B} = \mathbf{k},\ \tau = 0$ **13.** $\mathbf{B} = -\mathbf{k},\ \tau = 0$ **15.** $\mathbf{B} = \mathbf{k},\ \tau = 0$

17. Sí. Si el carro se mueve en una trayectoria curva $(\kappa \neq 0)$, entonces $a_N = \kappa|\mathbf{v}|^2 \neq 0$ y $\mathbf{a} \neq \mathbf{0}$.

23. $\kappa = \frac{1}{t},\ \rho = t$

29. Componentes de $\mathbf{v}$: -1.8701, 0.7089, 1.0000
Componentes de $\mathbf{a}$: -1.6960, -2.0307, 0
Velocidad: 2.2361; Componentes de $\mathbf{T}$: -0.8364, 0.3170, 0.4472
Componentes de $\mathbf{N}$: -0.4143, -0.8998, -0.1369
Componentes de $\mathbf{B}$: 0.3590, -0.2998, 0.8839; Curvatura: 0.5060
Torsión: 0.2813; Componente tangencial de la aceleración: 0.7746
Componente normal de la aceleración: 2.5298

31. Componentes de $\mathbf{v}$: 2.0000, 0, -0.1629
Componentes de $\mathbf{a}$: 0, -1.0000, -0.0086; Velocidad: 2.0066
Componentes de $\mathbf{T}$: 0.9967, 0, -0.0812
Componentes de $\mathbf{N}$: -0.0007, -1.0000, -0.0086
Componentes de $\mathbf{B}$: -0.0812, 0.0086, 0.9967;
Curvatura: 0.2484
Torsión: 0.0411; Componente tangencial de la aceleración: 0.0007
Componente normal de la aceleración: 1.0000

Sección 13.6, p. 742

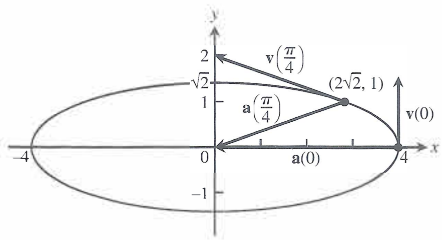
1. $\mathbf{v} = (3a \operatorname{sen} \theta)\mathbf{u}_r + 3a(1 - \cos \theta)\mathbf{u}_\theta$
$\mathbf{a} = 9a(2 \cos \theta - 1)\mathbf{u}_r + (18a \operatorname{sen} \theta)\mathbf{u}_\theta$

3. $\mathbf{v} = 2ae^{a\theta}\mathbf{u}_r + 2e^{a\theta}\mathbf{u}_\theta$
$\mathbf{a} = 4e^{a\theta}(a^2 - 1)\mathbf{u}_r + 8ae^{a\theta}\mathbf{u}_\theta$

5. $\mathbf{v} = (-8 \operatorname{sen} 4t)\mathbf{u}_r + (4 \cos 4t)\mathbf{u}_\theta$
$\mathbf{a} = (-40 \cos 4t)\mathbf{u}_r - (32 \operatorname{sen} 4t)\mathbf{u}_\theta$

Ejercicios de práctica, pp. 743–744

1. $\dfrac{x^2}{16} + \dfrac{y^2}{2} = 1$



En $t = 0$: $a_{\mathrm{T}} = 0, a_{\mathrm{N}} = 4, \kappa = 2$;

En $t = \dfrac{\pi}{4}$: $a_{\mathrm{T}} = \dfrac{7}{3}, a_{\mathrm{N}} = \dfrac{4\sqrt{2}}{3}, \kappa = \dfrac{4\sqrt{2}}{27}$

3. $|\mathbf{v}|_{\text{máx}} = 1$ **5.** $\kappa = 1/5$

7. $dy/dt = -x$; en el sentido de las manecillas del reloj

11. Sobre el suelo, aproximadamente a 66 ft 3 in del *stopboard* (el tope que ayuda al lanzador a contrarrestar la inercia que ha acumulado en el lanzamiento).

15. Longitud $= \dfrac{\pi}{4}\sqrt{1 + \dfrac{\pi^2}{16}} + \ln\left(\dfrac{\pi}{4} + \sqrt{1 + \dfrac{\pi^2}{16}}\right)$

17. $\mathbf{T}(0) = \dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$; $\mathbf{N}(0) = \dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$;

$\mathbf{B}(0) = -\dfrac{1}{3\sqrt{2}}\mathbf{i} + \dfrac{1}{3\sqrt{2}}\mathbf{j} + \dfrac{4}{3\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{\sqrt{2}}{3}$; $\tau = \dfrac{1}{6}$

19. $\mathbf{T}(\ln 2) = \dfrac{1}{\sqrt{17}}\mathbf{i} + \dfrac{4}{\sqrt{17}}\mathbf{j}$; $\mathbf{N}(\ln 2) = -\dfrac{4}{\sqrt{17}}\mathbf{i} + \dfrac{1}{\sqrt{17}}\mathbf{j}$;

$\mathbf{B}(\ln 2) = \mathbf{k}$; $\kappa = \dfrac{8}{17\sqrt{17}}$; $\tau = 0$

21. $\mathbf{a}(0) = 10\mathbf{T} + 6\mathbf{N}$

23. $\mathbf{T} = \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{i} - (\operatorname{sen} t)\mathbf{j} + \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{k}$;

$\mathbf{N} = \left(-\dfrac{1}{\sqrt{2}}\operatorname{sen} t\right)\mathbf{i} - (\cos t)\mathbf{j} - \left(\dfrac{1}{\sqrt{2}}\operatorname{sen} t\right)\mathbf{k}$;

$\mathbf{B} = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{1}{\sqrt{2}}$; $\tau = 0$

25. $\pi/3$ **27.** $x = 1 + t$, $y = t$, $z = -t$ **31.** $\kappa = 1/a$

Ejercicios adicionales y avanzados, pp. 745–746

1. (a) $\left.\dfrac{d\theta}{dt}\right|_{\theta=2\pi} = 2\sqrt{\dfrac{\pi g b}{a^2 + b^2}}$

(b) $\theta = \dfrac{gbt^2}{2(a^2 + b^2)}$, $z = \dfrac{gb^2t^2}{2(a^2 + b^2)}$

(c) $\mathbf{v}(t) = \dfrac{gbt}{\sqrt{a^2 + b^2}}\mathbf{T}$;

$\dfrac{d^2\mathbf{r}}{dt^2} = \dfrac{bg}{\sqrt{a^2 + b^2}}\mathbf{T} + a\left(\dfrac{bgt}{a^2 + b^2}\right)^2\mathbf{N}$

No existe componente en la dirección de $\mathbf{B}$.

5. (a) $\dfrac{dx}{dt} = \dot{r}\cos\theta - r\dot{\theta}\operatorname{sen}\theta, \dfrac{dy}{dt} = \dot{r}\operatorname{sen}\theta + r\dot{\theta}\cos\theta$

(b) $\dfrac{dr}{dt} = \dot{x}\cos\theta + \dot{y}\operatorname{sen}\theta, r\dfrac{d\theta}{dt} = -\dot{x}\operatorname{sen}\theta + \dot{y}\cos\theta$

7. (a) $\mathbf{a}(1) = -9\mathbf{u}_r - 6\mathbf{u}_\theta, \mathbf{v}(1) = -\mathbf{u}_r + 3\mathbf{u}_\theta$ **(b)** 6.5 in

9. (c) $\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta + \dot{z}\mathbf{k}, \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{k}$

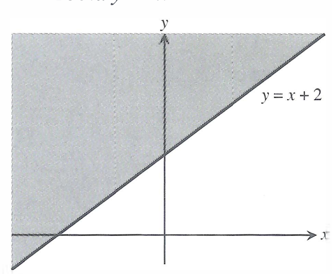
CAPÍTULO 14

Sección 14.1, pp. 753–755

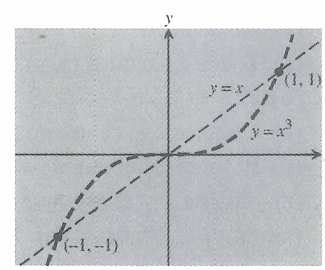
1. (a) 0 **(b)** 0 **(c)** 58 **(d)** 33

3. (a) 4/5 **(b)** 8/5 **(c)** 3 **(d)** 0

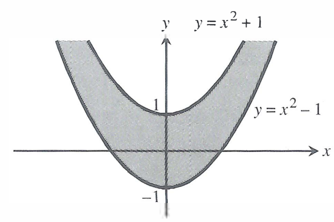
5. Dominio: todos los puntos (x, y) en o arriba de la recta $y = x + 2$



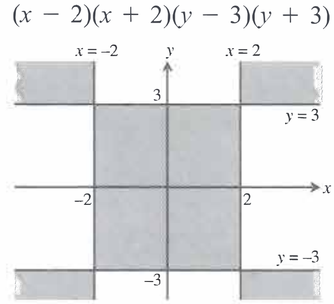
7. Dominio: todos los puntos que no se encuentran sobre la gráfica de $y = x$ o $y = x^3$



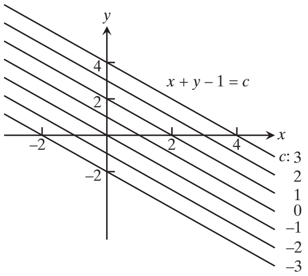
9. Dominio: todos los puntos (x, y) que satisfacen $x^2 - 1 \le y \le x^2 + 1$



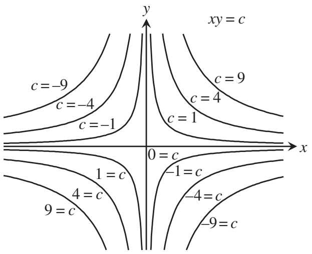
11. Dominio: todos los puntos (x, y) para los cuales $(x - 2)(x + 2)(y - 3)(y + 3) \ge 0$



13.



15.



17. **(a)** Todos los puntos en el plano xy
(b) Todos los números reales
(c) Las rectas $y - x = c$ **(d)** Sin puntos de frontera
(e) Tanto abierta como cerrada **(f)** No acotado

19. **(a)** Todos los puntos en el plano xy **(b)** $z \geq 0$
(c) Para $f(x, y) = 0$, el origen; para $f(x, y) \neq 0$, elipses con centro en (0, 0), y ejes mayores y menores a lo largo de los ejes x y y respectivamente
(d) Sin puntos de frontera **(e)** Tanto abierta como cerrada
(f) No acotado

21. **(a)** Todos los puntos en el plano xy
(b) Todos los números reales
(c) Para $f(x, y) = 0$, los ejes x y y; para $f(x, y) \neq 0$, hipérbolas con los ejes x y y como asíntotas
(d) Sin puntos de frontera **(e)** Tanto abierta como cerrada
(f) No acotado

23. **(a)** Todos los puntos (x, y) que satisfagan $x^2 + y^2 < 16$
(b) $z \geq 1/4$
(c) Círculos con centro en el origen, de radios $r < 4$
(d) El límite es el círculo $x^2 + y^2 = 16$
(e) Abierta **(f)** Acotado

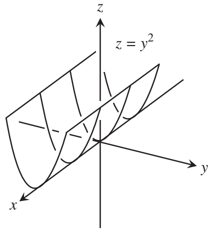
25. **(a)** $(x, y) \neq (0, 0)$ **(b)** Todos los reales
(c) Los círculos con centro en (0, 0) y radios $r > 0$
(d) El límite es el punto (0, 0)
(e) Abierta **(f)** No acotado

27. **(a)** Todos los puntos (x, y) que satisfagan $-1 \leq y - x \leq 1$
(b) $-\pi/2 \leq z \leq \pi/2$
(c) Líneas rectas de la forma $y - x = c$ donde $-1 \leq c \leq 1$
(d) La frontera está formada por las dos rectas $y = 1 + x$ y $y = -1 + x$
(e) Cerrada **(f)** No acotado

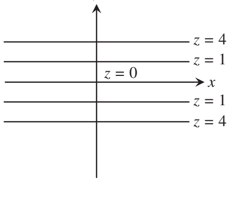
29. **(a)** Dominio: todos los puntos (x, y) fuera del círculo $x^2 + y^2 = 1$
(b) Rango: todos los reales
(c) Círculos con centro en el origen de radios $r > 1$
(d) Límite: $x^2 + y^2 = 1$
(e) Abierta **(f)** No acotado

31. (f) **33.** (a) **35.** (d)

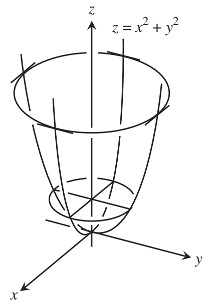
37. **(a)**



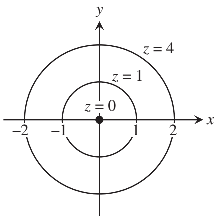
(b)



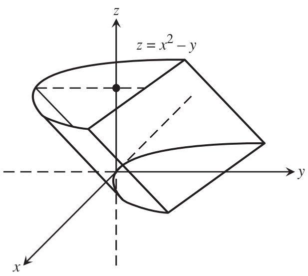
39. **(a)**



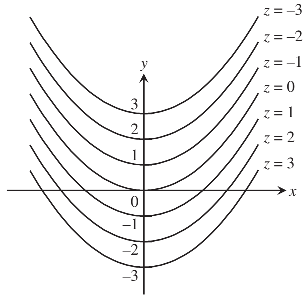
(b)



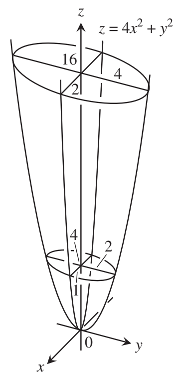
41. **(a)**



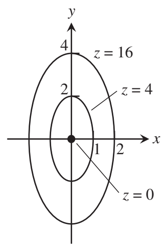
(b)



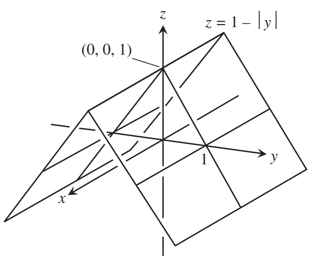
43. **(a)**



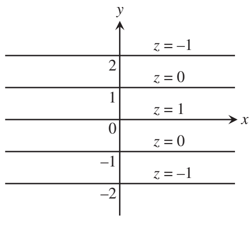
(b)



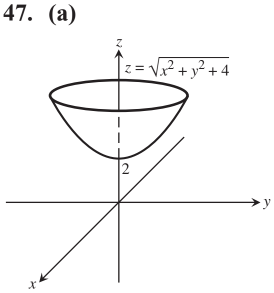
45. **(a)**



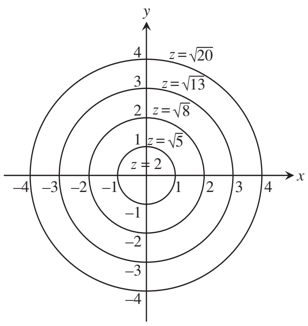
(b)



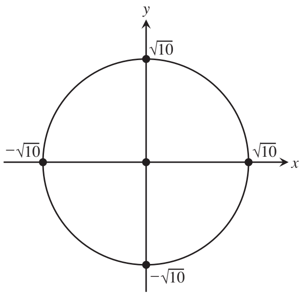
47. **(a)**



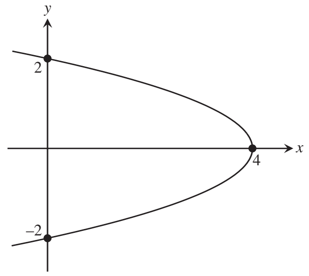
(b)



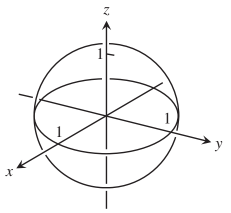
49. $x^2 + y^2 = 10$



51. $x + y^2 = 4$

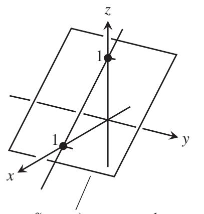


53.



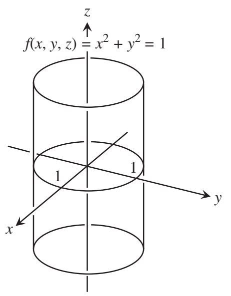
$f(x, y, z) = x^2 + y^2 + z^2 = 1$

55.

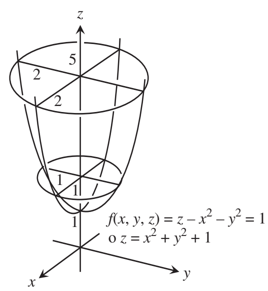


$f(x, y, z) = x + z = 1$

57.

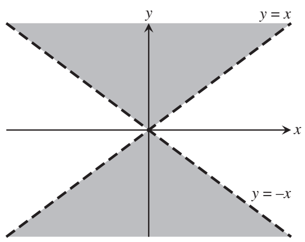


59.



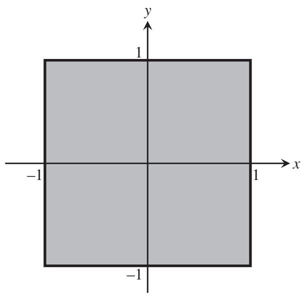
61. $\sqrt{x - y} - \ln z = 2$ **63.** $x^2 + y^2 + z^2 = 4$

65. Dominio: todos los puntos (x, y) que satisfagan $|x| < |y|$



curva de nivel: $y = 2x$

67. Dominio: todos los puntos (x, y) que satisfagan $-1 \leq x \leq 1$ y $-1 \leq y \leq 1$



curva de nivel: $\text{sen}^{-1} y - \text{sen}^{-1} x = \dfrac{\pi}{2}$

Sección 14.2, pp. 761–764

1. $5/2$ **3.** $2\sqrt{6}$ **5.** 1 **7.** $1/2$ **9.** 1 **11.** $1/4$ **13.** 0 **15.** -1 **17.** 2 **19.** $1/4$ **21.** 1 **23.** 3 **25.** $19/12$
27. 2 **29.** 3 **31.** **(a)** Todos los puntos (x, y) **(b)** Todos los puntos (x, y) excepto $(0, 0)$
33. **(a)** Todos los puntos (x, y) excepto donde $x = 0$ o $y = 0$ **(b)** Todos los puntos (x, y)
35. **(a)** Todos los puntos (x, y, z) **(b)** Todos los puntos (x, y, z), excepto el interior del cilindro $x^2 + y^2 = 1$
37. **(a)** Todos los puntos (x, y, z) con $z \neq 0$ **(b)** Todos los puntos (x, y, z) con $x^2 + z^2 \neq 1$
39. **(a)** Todos los puntos (x, y) que satisfagan $z > x^2 + y^2 + 1$
41. Considere trayectorias a lo largo de $y = x, x > 0$, y a lo largo de $y = x, x < 0$.
43. Considere las trayectorias $y = kx^2$, donde k es una constante.
45. Considere las trayectorias $y = mx$, donde m es una constante, $m \neq -1$.
47. Considere las trayectorias $y = kx^2$, donde k es una constante, $k \neq 0$.
49. Considere las trayectorias $x = 1$ y $y = x$.
51. **(a)** 1 **(b)** 0 **(c)** No existe
55. El límite es 1. **57.** El límite es 0.
59. **(a)** $f(x, y)|_{y=mx} = \text{sen}\, 2\theta$ donde $\tan\theta = m$ **61.** 0
63. No existe **65.** $\pi/2$ **67.** $f(0, 0) = \ln 3$
69. $\delta = 0.1$ **71.** $\delta = 0.005$ **73.** $\delta = 0.04$
75. $\delta = \sqrt{0.015}$ **77.** $\delta = 0.005$

Sección 14.3, pp. 772–775

1. $\dfrac{\partial f}{\partial x} = 4x, \dfrac{\partial f}{\partial y} = -3$ **3.** $\dfrac{\partial f}{\partial x} = 2x(y + 2), \dfrac{\partial f}{\partial y} = x^2 - 1$

5. $\dfrac{\partial f}{\partial x} = 2y(xy - 1), \dfrac{\partial f}{\partial y} = 2x(xy - 1)$

7. $\dfrac{\partial f}{\partial x} = \dfrac{x}{\sqrt{x^2 + y^2}}, \dfrac{\partial f}{\partial y} = \dfrac{y}{\sqrt{x^2 + y^2}}$

9. $\dfrac{\partial f}{\partial x} = \dfrac{-1}{(x + y)^2}, \dfrac{\partial f}{\partial y} = \dfrac{-1}{(x + y)^2}$

11. $\dfrac{\partial f}{\partial x} = \dfrac{-y^2 - 1}{(xy - 1)^2}, \dfrac{\partial f}{\partial y} = \dfrac{-x^2 - 1}{(xy - 1)^2}$

13. $\dfrac{\partial f}{\partial x} = e^{x+y+1}, \dfrac{\partial f}{\partial y} = e^{x+y+1}$ **15.** $\dfrac{\partial f}{\partial x} = \dfrac{1}{x + y}, \dfrac{\partial f}{\partial y} = \dfrac{1}{x + y}$

17. $\dfrac{\partial f}{\partial x} = 2\,\text{sen}\,(x - 3y)\cos(x - 3y)$,
$\dfrac{\partial f}{\partial y} = -6\,\text{sen}\,(x - 3y)\cos(x - 3y)$

19. $\dfrac{\partial f}{\partial x} = yx^{y-1}, \dfrac{\partial f}{\partial y} = x^y \ln x$ **21.** $\dfrac{\partial f}{\partial x} = -g(x), \dfrac{\partial f}{\partial y} = g(y)$

23. $f_x = y^2, f_y = 2xy, f_z = -4z$
25. $f_x = 1, f_y = -y(y^2 + z^2)^{-1/2}, f_z = -z(y^2 + z^2)^{-1/2}$
27. $f_x = \dfrac{yz}{\sqrt{1 - x^2y^2z^2}}, f_y = \dfrac{xz}{\sqrt{1 - x^2y^2z^2}}, f_z = \dfrac{xy}{\sqrt{1 - x^2y^2z^2}}$
29. $f_x = \dfrac{1}{x + 2y + 3z}, f_y = \dfrac{2}{x + 2y + 3z}, f_z = \dfrac{3}{x + 2y + 3z}$
31. $f_x = -2xe^{-(x^2+y^2+z^2)}, f_y = -2ye^{-(x^2+y^2+z^2)}, f_z = -2ze^{-(x^2+y^2+z^2)}$
33. $f_x = \text{sech}^2(x + 2y + 3z), f_y = 2\,\text{sech}^2(x + 2y + 3z)$, $f_z = 3\,\text{sech}^2(x + 2y + 3z)$
35. $\dfrac{\partial f}{\partial t} = -2\pi\,\text{sen}\,(2\pi t - \alpha), \dfrac{\partial f}{\partial \alpha} = \text{sen}\,(2\pi t - \alpha)$
37. $\dfrac{\partial h}{\partial \rho} = \text{sen}\,\phi\cos\theta, \dfrac{\partial h}{\partial \phi} = \rho\cos\phi\cos\theta, \dfrac{\partial h}{\partial \theta} = -\rho\,\text{sen}\,\phi\,\text{sen}\,\theta$
39. $W_P(P, V, \delta, v, g) = V, W_V(P, V, \delta, v, g) = P + \dfrac{\delta v^2}{2g}$,
$W_\delta(P, V, \delta, v, g) = \dfrac{Vv^2}{2g}, W_v(P, V, \delta, v, g) = \dfrac{V\delta v}{g}$,
$W_g(P, V, \delta, v, g) = -\dfrac{V\delta v^2}{2g^2}$

41. $\dfrac{\partial f}{\partial x} = 1 + y, \dfrac{\partial f}{\partial y} = 1 + x, \dfrac{\partial^2 f}{\partial x^2} = 0, \dfrac{\partial^2 f}{\partial y^2} = 0,$

$\dfrac{\partial^2 f}{\partial y\, \partial x} = \dfrac{\partial^2 f}{\partial x\, \partial y} = 1$

43. $\dfrac{\partial g}{\partial x} = 2xy + y\cos x, \dfrac{\partial g}{\partial y} = x^2 - \operatorname{sen} y + \operatorname{sen} x,$

$\dfrac{\partial^2 g}{\partial x^2} = 2y - y\operatorname{sen} x, \dfrac{\partial^2 g}{\partial y^2} = -\cos y,$

$\dfrac{\partial^2 g}{\partial y\, \partial x} = \dfrac{\partial^2 g}{\partial x\, \partial y} = 2x + \cos x$

45. $\dfrac{\partial r}{\partial x} = \dfrac{1}{x+y}, \dfrac{\partial r}{\partial y} = \dfrac{1}{x+y}, \dfrac{\partial^2 r}{\partial x^2} = \dfrac{-1}{(x+y)^2}, \dfrac{\partial^2 r}{\partial y^2} = \dfrac{-1}{(x+y)^2},$

$\dfrac{\partial^2 r}{\partial y\, \partial x} = \dfrac{\partial^2 r}{\partial x\, \partial y} = \dfrac{-1}{(x+y)^2}$

47. $\dfrac{\partial w}{\partial x} = x^2 y \sec^2(xy) + 2x\tan(xy), \dfrac{\partial w}{\partial y} = x^3\sec^2(xy),$

$\dfrac{\partial^2 w}{\partial y\, \partial x} = \dfrac{\partial^2 w}{\partial x\, \partial y} = 2x^3 y\sec^2(xy)\tan(xy) + 3x^2\sec^2(xy)$

$\dfrac{\partial^2 w}{\partial x^2} = 4xy\sec^2(xy) + 2x^2y^2\sec^2(xy)\tan(xy) + 2\tan(xy)$

$\dfrac{\partial^2 w}{\partial y^2} = 2x^4\sec^2(xy)\tan(xy)$

49. $\dfrac{\partial w}{\partial x} = \operatorname{sen}(x^2y) + 2x^2y\cos(x^2y), \dfrac{\partial w}{\partial y} = x^3\cos(x^2y),$

$\dfrac{\partial^2 w}{\partial y\, \partial x} = \dfrac{\partial^2 w}{\partial x\, \partial y} = 3x^2\cos(x^2y) - 2x^4y\operatorname{sen}(x^2y)$

$\dfrac{\partial^2 w}{\partial x^2} = 6xy\cos(x^2y) - 4x^3y^2\operatorname{sen}(x^2y)$

$\dfrac{\partial^2 w}{\partial y^2} = -x^5\operatorname{sen}(x^2y)$

51. $\dfrac{\partial w}{\partial x} = \dfrac{2}{2x+3y}, \dfrac{\partial w}{\partial y} = \dfrac{3}{2x+3y}, \dfrac{\partial^2 w}{\partial y\, \partial x} = \dfrac{\partial^2 w}{\partial x\, \partial y} = \dfrac{-6}{(2x+3y)^2}$

53. $\dfrac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4, \dfrac{\partial w}{\partial y} = 2xy + 3x^2y^2 + 4x^3y^3,$

$\dfrac{\partial^2 w}{\partial y\, \partial x} = \dfrac{\partial^2 w}{\partial x\, \partial y} = 2y + 6xy^2 + 12x^2y^3$

55. **(a)** Primero x **(b)** Primero y **(c)** Primero x
(d) Primero x **(e)** Primero y **(f)** Primero y

57. $f_x(1, 2) = -13, f_y(1, 2) = -2$

59. $f_x(-2, 3) = 1/2, f_y(-2, 3) = 3/4$ **61.** **(a)** 3 **(b)** 2

63. 12 **65.** -2 **67.** $\dfrac{\partial A}{\partial a} = \dfrac{a}{bc\operatorname{sen} A}, \dfrac{\partial A}{\partial b} = \dfrac{c\cos A - b}{bc\operatorname{sen} A}$

69. $v_x = \dfrac{\ln v}{(\ln u)(\ln v) - 1}$

71. $f_x(x, y) = 0$ para todos los puntos (x, y),

$f_y(x, y) = \begin{cases} 3y^2, & y \geq 0 \\ -2y, & y < 0 \end{cases},$

$f_{xy}(x, y) = f_{yx}(x, y) = 0$ para todos los puntos (x, y)

89. Sí

Sección 14.4, pp. 782–783

1. **(a)** $\dfrac{dw}{dt} = 0,$ **(b)** $\dfrac{dw}{dt}(\pi) = 0$

3. **(a)** $\dfrac{dw}{dt} = 1,$ **(b)** $\dfrac{dw}{dt}(3) = 1$

5. **(a)** $\dfrac{dw}{dt} = 4t\tan^{-1} t + 1,$ **(b)** $\dfrac{dw}{dt}(1) = \pi + 1$

7. **(a)** $\dfrac{\partial z}{\partial u} = 4\cos v \ln(u \operatorname{sen} v) + 4\cos v,$

$\dfrac{\partial z}{\partial v} = -4u\operatorname{sen} v\ln(u\operatorname{sen} v) + \dfrac{4u\cos^2 v}{\operatorname{sen} v}$

(b) $\dfrac{\partial z}{\partial u} = \sqrt{2}(\ln 2 + 2), \dfrac{\partial z}{\partial v} = -2\sqrt{2}(\ln 2 - 2)$

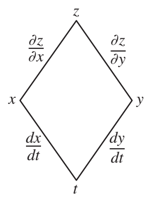
9. **(a)** $\dfrac{\partial w}{\partial u} = 2u + 4uv, \dfrac{\partial w}{\partial v} = -2v + 2u^2$

(b) $\dfrac{\partial w}{\partial u} = 3, \dfrac{\partial w}{\partial v} = -\dfrac{3}{2}$

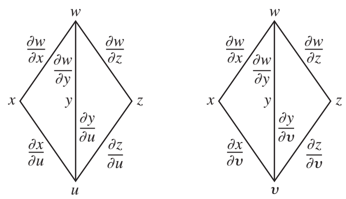
11. **(a)** $\dfrac{\partial u}{\partial x} = 0, \dfrac{\partial u}{\partial y} = \dfrac{z}{(z-y)^2}, \dfrac{\partial u}{\partial z} = \dfrac{-y}{(z-y)^2}$

(b) $\dfrac{\partial u}{\partial x} = 0, \dfrac{\partial u}{\partial y} = 1, \dfrac{\partial u}{\partial z} = -2$

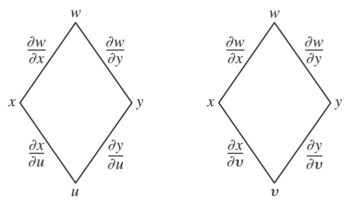
13. $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\dfrac{dy}{dt}$



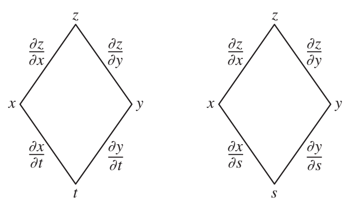
15. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial u},$
$\dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial v}$



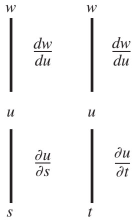
17. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}, \dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}.$



19. $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}, \dfrac{\partial z}{\partial s} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial s}$

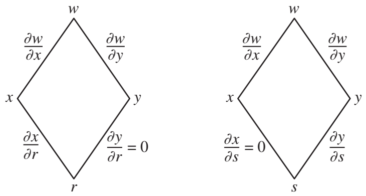


21. $\dfrac{\partial w}{\partial s} = \dfrac{dw}{du}\dfrac{\partial u}{\partial s}, \dfrac{\partial w}{\partial t} = \dfrac{dw}{du}\dfrac{\partial u}{\partial t}$



23. $\dfrac{\partial w}{\partial r} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dr} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dr} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dr}$ puesto que $\dfrac{dy}{dr} = 0,$

$\dfrac{\partial w}{\partial s} = \dfrac{\partial w}{\partial x}\dfrac{dx}{ds} + \dfrac{\partial w}{\partial y}\dfrac{dy}{ds} = \dfrac{\partial w}{\partial y}\dfrac{dy}{ds}$ puesto que $\dfrac{dx}{ds} = 0$



25. $4/3$ **27.** $-4/5$ **29.** $\dfrac{\partial z}{\partial x} = \dfrac{1}{4}, \dfrac{\partial z}{\partial y} = -\dfrac{3}{4}$

31. $\dfrac{\partial z}{\partial x} = -1, \dfrac{\partial z}{\partial y} = -1$ **33.** 12 **35.** -7

37. $\dfrac{\partial z}{\partial u} = 2, \dfrac{\partial z}{\partial v} = 1$

39. $\dfrac{\partial w}{\partial t} = 2t\, e^{s^3+t^2}, \dfrac{\partial w}{\partial s} = 3s^2\, e^{s^3+t^2}$

41. -0.00005 amps/seg

47. $(\cos 1, \operatorname{sen} 1, 1)$ y $(\cos(-2), \operatorname{sen}(-2), -2)$

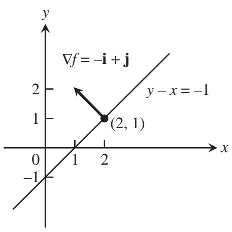
49. **(a)** Máximo en $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ y $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$; mínimo en $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ y $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

(b) Máx $= 6$, mín $= 2$

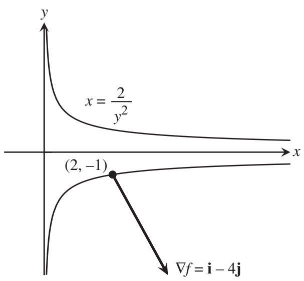
51. $2x\sqrt{x^8 + x^3} + \displaystyle\int_0^{x^2} \frac{3x^2}{2\sqrt{t^4 + x^3}}\, dt$

Sección 14.5, pp. 790–791

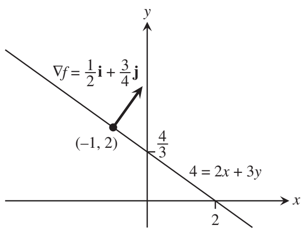
1.



3.



5.



7. $\nabla f = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ **9.** $\nabla f = -\dfrac{26}{27}\mathbf{i} + \dfrac{23}{54}\mathbf{j} - \dfrac{23}{54}\mathbf{k}$

11. -4 **13.** $21/13$ **15.** 3 **17.** 2

19. $\mathbf{u} = -\dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}, (D_{\mathbf{u}}f)_{P_0} = \sqrt{2}; -\mathbf{u} = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{j},$

$(D_{-\mathbf{u}}f)_{P_0} = -\sqrt{2}$

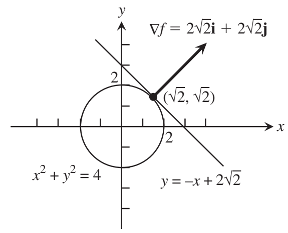
21. $\mathbf{u} = \dfrac{1}{3\sqrt{3}}\mathbf{i} - \dfrac{5}{3\sqrt{3}}\mathbf{j} - \dfrac{1}{3\sqrt{3}}\mathbf{k}, (D_{\mathbf{u}}f)_{P_0} = 3\sqrt{3};$

$-\mathbf{u} = -\dfrac{1}{3\sqrt{3}}\mathbf{i} + \dfrac{5}{3\sqrt{3}}\mathbf{j} + \dfrac{1}{3\sqrt{3}}\mathbf{k}, (D_{-\mathbf{u}}f)_{P_0} = -3\sqrt{3}$

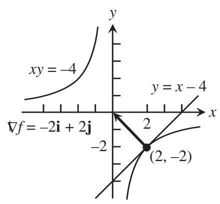
23. $\mathbf{u} = \dfrac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k}), (D_{\mathbf{u}}f)_{P_0} = 2\sqrt{3};$

$-\mathbf{u} = -\dfrac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k}), (D_{-\mathbf{u}}f)_{P_0} = -2\sqrt{3}$

25.



27.



29. **(a)** $\mathbf{u} = \dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}, D_{\mathbf{u}}\, f(1, -1) = 5$

(b) $\mathbf{u} = -\dfrac{3}{5}\mathbf{i} + \dfrac{4}{5}\mathbf{j}, D_{\mathbf{u}}\, f(1, -1) = -5$

(c) $\mathbf{u} = \dfrac{3}{5}\mathbf{i} + \dfrac{4}{5}\mathbf{j}, \mathbf{u} = -\dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}$

(d) $\mathbf{u} = -\mathbf{j}, \mathbf{u} = \dfrac{24}{25}\mathbf{i} - \dfrac{7}{25}\mathbf{j}$

(e) $\mathbf{u} = -\mathbf{i}, \mathbf{u} = \dfrac{7}{25}\mathbf{i} + \dfrac{24}{25}\mathbf{j}$

31. $\mathbf{u} = \dfrac{7}{\sqrt{53}}\mathbf{i} - \dfrac{2}{\sqrt{53}}\mathbf{j}, -\mathbf{u} = -\dfrac{7}{\sqrt{53}}\mathbf{i} + \dfrac{2}{\sqrt{53}}\mathbf{j}$

33. No, la tasa máxima de cambio es $\sqrt{185} < 14$.

35. $-7/\sqrt{5}$

Sección 14.6, pp. 799–802

1. **(a)** $x + y + z = 3$
(b) $x = 1 + 2t, y = 1 + 2t, z = 1 + 2t$

3. **(a)** $2x - z - 2 = 0$ **(b)** $x = 2 - 4t, y = 0, z = 2 + 2t$

5. **(a)** $2x + 2y + z - 4 = 0$
(b) $x = 2t, y = 1 + 2t, z = 2 + t$

7. **(a)** $x + y + z - 1 = 0$ **(b)** $x = t, y = 1 + t, z = t$

9. $2x - z - 2 = 0$ **11.** $x - y + 2z - 1 = 0$

13. $x = 1, y = 1 + 2t, z = 1 - 2t$

15. $x = 1 - 2t, y = 1, z = \dfrac{1}{2} + 2t$

17. $x = 1 + 90t, y = 1 - 90t, z = 3$

19. $df = \dfrac{9}{11{,}830} \approx 0.0008$ **21.** $dg = 0$

23. **(a)** $\dfrac{\sqrt{3}}{2}\operatorname{sen}\sqrt{3} - \dfrac{1}{2}\cos\sqrt{3} \approx 0.935°\text{C/ft}$

(b) $\sqrt{3}\operatorname{sen}\sqrt{3} - \cos\sqrt{3} \approx 1.87°\text{C/seg}$

25. **(a)** $L(x, y) = 1$ **(b)** $L(x, y) = 2x + 2y - 1$

27. **(a)** $L(x, y) = 3x - 4y + 5$ **(b)** $L(x, y) = 3x - 4y + 5$

29. **(a)** $L(x, y) = 1 + x$ **(b)** $L(x, y) = -y + \frac{\pi}{2}$

31. **(a)** $W(20, 25) = 11°F$, $W(30, -10) = -39°F$, $W(15, 15) = 0°F$
(b) $W(10, -40) \approx -65.5°F$, $W(50, -40) \approx -88°F$, $W(60, 30) \approx 10.2°F$
(c) $L(v, T) \approx -0.36\,(v - 25) + 1.337(T - 5) - 17.4088$
(d) **i)** $L(24, 6) \approx -15.7°F$
ii) $L(27, 2) \approx -22.1°F$
iii) $L(5, -10) \approx -30.2°F$

33. $L(x, y) = 7 + x - 6y$; 0.06 **35.** $L(x, y) = x + y + 1$; 0.08

37. $L(x, y) = 1 + x$; 0.0222

39. **(a)** $L(x, y, z) = 2x + 2y + 2z - 3$ **(b)** $L(x, y, z) = y + z$
(c) $L(x, y, z) = 0$

41. **(a)** $L(x, y, z) = x$ **(b)** $L(x, y, z) = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$
(c) $L(x, y, z) = \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z$

43. **(a)** $L(x, y, z) = 2 + x$
(b) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$
(c) $L(x, y, z) = x - y - z + \frac{\pi}{2} + 1$

45. $L(x, y, z) = 2x - 6y - 2z + 6$, 0.0024

47. $L(x, y, z) = x + y - z - 1$, 0.00135

49. Error máximo (estimado) ≤ 0.31 en magnitud

51. **(a)** $\pm 5\%$ **(b)** $\pm 7\%$

53. $\approx \pm 4.83\%$

55. Ponga más atención a la dimensión más pequeña. Generará la derivada parcial más grande.

57. **(a)** 0.3%

59. f es más sensible al cambio en d.

61. Q es más sensible al cambio en h.

65. En $-\frac{\pi}{4}, -\frac{\pi}{2\sqrt{2}}$; en 0, 0; en $\frac{\pi}{4}, \frac{\pi}{2\sqrt{2}}$

Sección 14.7, pp. 808–811

1. $f(-3, 3) = -5$, mínimo local **3.** $f(-2, 1)$, punto de silla

5. $f\left(3, \frac{3}{2}\right) = \frac{17}{2}$, máximo local

7. $f(2, -1) = -6$, mínimo local **9.** $f(1, 2)$, punto de silla

11. $f\left(\frac{16}{7}, 0\right) = -\frac{16}{7}$, máximo local

13. $f(0, 0)$, punto de silla; $f\left(-\frac{2}{3}, \frac{2}{3}\right) = \frac{170}{27}$, máximo local

15. $f(0, 0) = 0$, mínimo local; $f(1, -1)$, punto de silla

17. $f(0, \pm\sqrt{5})$, punto de silla; $f(-2, -1) = 30$, máximo local; $f(2, 1) = -30$, mínimo local

19. $f(0, 0)$, punto de silla; $f(1, 1) = 2$, $f(-1, -1) = 2$, máximo local

21. $f(0, 0) = -1$, máximo local

23. $f(n\pi, 0)$, puntos de silla para todo entero n

25. $f(2, 0) = e^{-4}$, mínimo local

27. $f(0, 0) = 0$, mínimo local; $f(0, 2)$, punto de silla

29. $f\left(\frac{1}{2}, 1\right) = \ln\left(\frac{1}{4}\right) - 3$, máximo local

31. Máximo absoluto: 1 en (0, 0); mínimo absoluto: -5 en (1, 2)

33. Máximo absoluto: 4 en (0, 2); mínimo absoluto: 0 en (0, 0)

35. Máximo absoluto: 11 en $(0, -3)$; mínimo absoluto: -10 en $(4, -2)$

37. Máximo absoluto: 4 en (2, 0); mínimo absoluto: $\frac{3\sqrt{2}}{2}$ en $\left(3, -\frac{\pi}{4}\right)$, $\left(3, \frac{\pi}{4}\right)$, $\left(1, -\frac{\pi}{4}\right)$, y $\left(1, \frac{\pi}{4}\right)$

39. $a = -3$, $b = 2$

41. Lo más caliente es $2\frac{1}{4}°$ en $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ y $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$; y lo más frío es $-\frac{1}{4}°$ en $\left(\frac{1}{2}, 0\right)$.

43. **(a)** $f(0, 0)$, punto de silla **(b)** $f(1, 2)$, mínimo local
(c) $f(1, -2)$, mínimo local; $f(-1, -2)$, punto de silla

49. $\left(\frac{1}{6}, \frac{1}{3}, \frac{355}{36}\right)$ **51.** $\left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$ **53.** 3, 3, 3 **55.** 12

57. $\frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}} \times \frac{4}{\sqrt{3}}$ **59.** 2 ft × 2 ft × 1 ft

61. **(a)** Sobre el semicírculo, máx $f = 2\sqrt{2}$ en $t = \pi/4$, mín $f = -2$ en $t = \pi$. Sobre el cuarto de círculo, máx en $f = 2\sqrt{2}$ en $t = \pi/4$, mín $f = 2$ en $t = 0, \pi/2$.
(b) Sobre el semicírculo, máx $g = 2$ en $t = \pi/4$, mín $g = -2$ en $t = 3\pi/4$. Sobre el cuarto de círculo, máx $g = 2$ en $t = \pi/4$, mín $g = 0$ en $t = 0, \pi/2$.
(c) Sobre el semicírculo, máx $h = 8$ en $t = 0, \pi$; mín $h = 4$ en $t = \pi/2$. Sobre el cuarto de círculo, máx $h = 8$ en $t = 0$, mín $h = 4$ en $t = \pi/2$.

63. **i)** mín $f = -1/2$ en $t = -1/2$; no hay máx
ii) máx $f = 0$ en $t = -1, 0$; mín $f = -1/2$ en $t = -1/2$
iii) máx $f = 4$ en $t = 1$; mín $f = 0$ en $t = 0$

67. $y = -\frac{20}{13}x + \frac{9}{13}$, $y|_{x=4} = -\frac{71}{13}$

Sección 14.8, pp. 818–820

1. $\left(\pm\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$, $\left(\pm\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ **3.** 39 **5.** $\left(3, \pm 3\sqrt{2}\right)$

7. **(a)** 8 **(b)** 64

9. $r = 2$ cm, $h = 4$ cm **11.** Longitud $= 4\sqrt{2}$, ancho $= 3\sqrt{2}$

13. $f(0, 0) = 0$ es mínimo, $f(2, 4) = 20$ es máximo.

15. La más baja $= 0°$, la más alta $= 125°$

17. $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ **19.** 1 **21.** (0, 0, 2), (0, 0, −2)

23. $f(1, -2, 5) = 30$ es máximo, $f(-1, 2, -5) = -30$ es mínimo.

25. 3, 3, 3 **27.** $\frac{2}{\sqrt{3}}$ por $\frac{2}{\sqrt{3}}$ por $\frac{2}{\sqrt{3}}$ unidades

29. $(\pm 4/3, -4/3, -4/3)$ **31.** $U(8, 14) = \$128$

33. $f(2/3, 4/3, -4/3) = \frac{4}{3}$ **35.** (2, 4, 4)

37. El máximo es $1 + 6\sqrt{3}$ en $\left(\pm\sqrt{6}, \sqrt{3}, 1\right)$, el mínimo es $1 - 6\sqrt{3}$ en $\left(\pm\sqrt{6}, -\sqrt{3}, 1\right)$.

39. El máximo es 4 en $(0, 0, \pm 2)$, el mínimo es 2 en $\left(\pm\sqrt{2}, \pm\sqrt{2}, 0\right)$.

Sección 14.9, p. 824

1. Cuadrática: $x + xy$; cúbica: $x + xy + \frac{1}{2}xy^2$

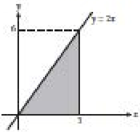
3. Cuadrática: xy; cúbica: xy

Sección 15.1, pp. 840–841

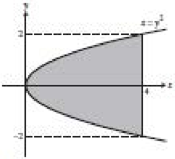
1. 24 **3.** 1 **5.** 16 **7.** $2 \ln 2 - 1$ **9.** $(3/2)(5 - e)$ **11.** $3/2$
13. 14 **15.** 0 **17.** $1/2$ **19.** $2 \ln 2$ **21.** $(\ln 2)^2$ **23.** $8/3$
25. 1 **27.** $\sqrt{2}$

Sección 15.2, pp. 847–850

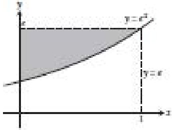
1.



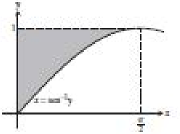
3.



5.

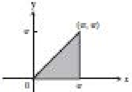


7.

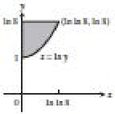


9. **(a)** $0 \le x \le 2, x^3 \le y \le 8$
(b) $0 \le y \le 8, 0 \le x \le y^{1/3}$
11. **(a)** $0 \le x \le 3, x^2 \le y \le 3x$
(b) $0 \le y \le 9, \frac{y}{3} \le x \le \sqrt{y}$
13. **(a)** $0 \le x \le 9, 0 \le y \le \sqrt{x}$
(b) $0 \le y \le 3, y^2 \le x \le 9$
15. **(a)** $0 \le x \le \ln 3, e^{-x} \le y \le 1$
(b) $\frac{1}{3} \le y \le 1, -\ln y \le x \le \ln 3$
17. **(a)** $0 \le x \le 1, x \le y \le 3 - 2x$
(b) $0 \le y \le 1, 0 \le x \le y \cup 1 \le y \le 3, 0 \le x \le \frac{3 - y}{2}$

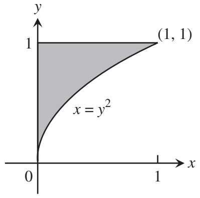
19. $\frac{\pi^2}{2} + 2$



21. $8 \ln 8 - 16 + e$

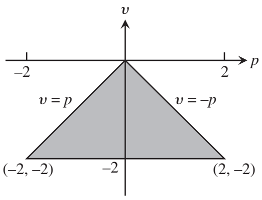


23. $e - 2$

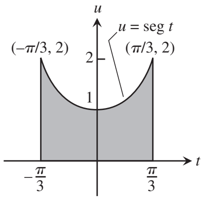


25. $\frac{3}{2}\ln 2$ **27.** $-1/10$

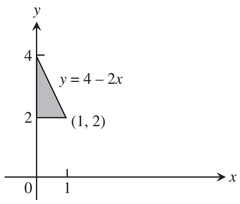
29. 8



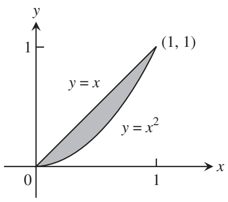
31. 2π



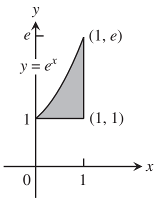
33. $\int_2^4 \int_0^{(4-y)/2} dx\, dy$



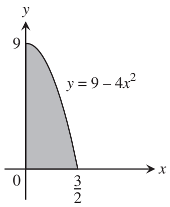
35. $\int_0^1 \int_{x^2}^{x} dy\, dx$



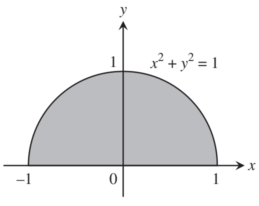
37. $\int_1^e \int_{\ln y}^{1} dx\, dy$



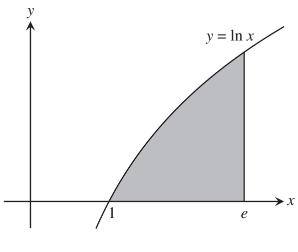
39. $\int_0^9 \int_0^{(\sqrt{9-y})/2} 16x\, dx\, dy$



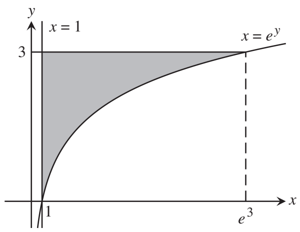
41. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 3y\, dy\, dx$



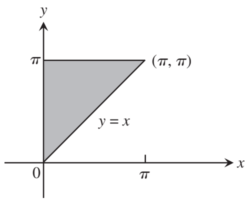
43. $\int_0^1 \int_{e^y}^{e} xy\, dx\, dy$



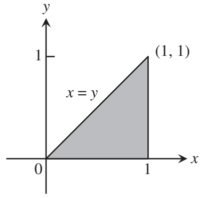
45. $\int_1^{e^3} \int_{\ln x}^{3} (x + y)\, dy\, dx$



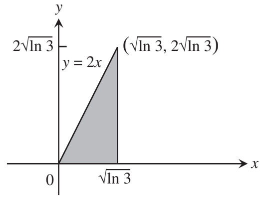
47. 2



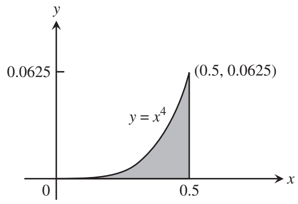
49. $\frac{e - 2}{2}$



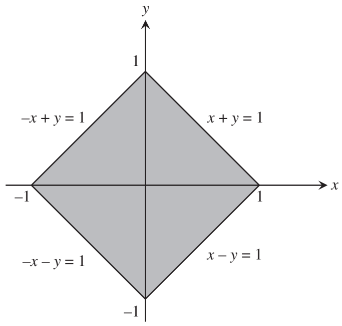
51. 2



53. $1/(80\pi)$

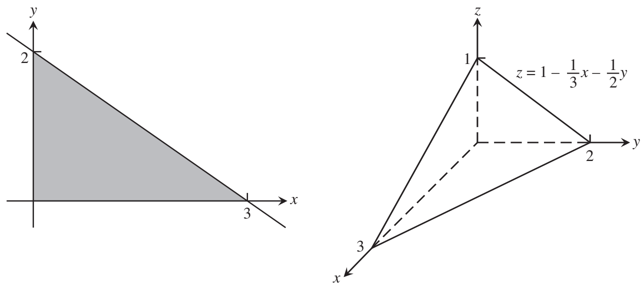


55. $-2/3$



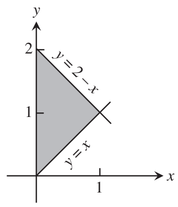
57. $4/3$ **59.** $625/12$ **61.** 16 **63.** 20 **65.** $2(1 + \ln 2)$

67.



69. 1 **71.** π^2 **73.** $-\frac{3}{32}$ **75.** $\frac{20\sqrt{3}}{9}$

77. $\int_0^1 \int_x^{2-x} (x^2 + y^2)\, dy\, dx = \frac{4}{3}$



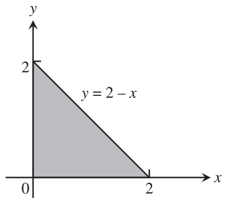
79. R es el conjunto de puntos (x, y) tales que $x^2 + 2y^2 < 4$.

81. No, por el teorema de Fubini, los dos órdenes de integración deben dar el mismo resultado.

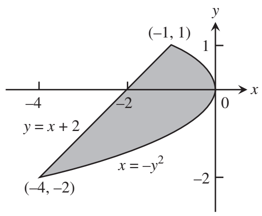
85. 0.603 **87.** 0.233

Sección 15.3, p. 852

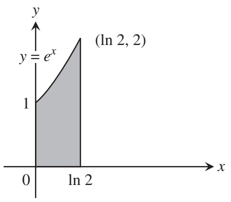
1. $\displaystyle\int_0^2\int_0^{2-x} dy\,dx = 2$ o $\displaystyle\int_0^2\int_0^{2-y} dx\,dy = 2$



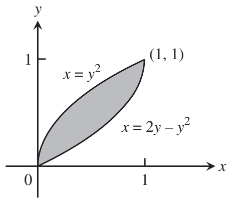
3. $\displaystyle\int_{-2}^{1}\int_{y-2}^{-y^2} dx\,dy = \frac{9}{2}$



5. $\displaystyle\int_0^{\ln 2}\int_0^{e^x} dy\,dx = 1$

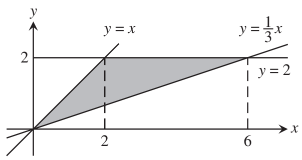


7. $\displaystyle\int_0^1\int_{y^2}^{2y-y^2} dx\,dy = \frac{1}{3}$



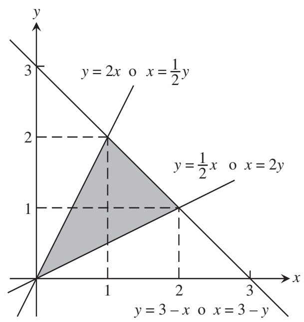
9. $\displaystyle\int_0^2\int_y^{3y} 1\,dx\,dy = 4$ o

$\displaystyle\int_0^2\int_{x/3}^{x} 1\,dy\,dx + \int_2^6\int_{x/3}^{2} 1\,dy\,dx = 4$

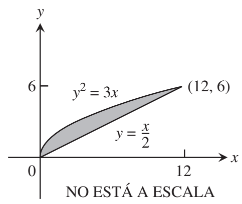


11. $\displaystyle\int_0^1\int_{x/2}^{2x} 1\,dy\,dx + \int_1^2\int_{x/2}^{3-x} 1\,dy\,dx = \frac{3}{2}$ o

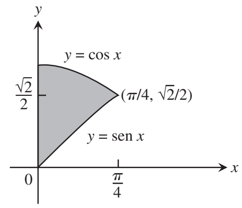
$\displaystyle\int_0^1\int_{y/2}^{2y} 1\,dx\,dy + \int_1^2\int_{y/2}^{3-y} 1\,dx\,dy = \frac{3}{2}$



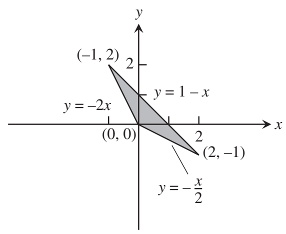
13. 12



15. $\sqrt{2} - 1$



17. $\frac{3}{2}$



19. **(a)** 0 **(b)** $4/\pi^2$ **21.** $8/3$

23. $40{,}000(1 - e^{-2})\ln(7/2) \approx 43{,}329$

Sección 15.4, pp. 857–859

1. $\frac{\pi}{2} \le \theta \le 2\pi,\ 0 \le r \le 9$ **3.** $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4},\ 0 \le r \le \csc\theta$

5. $0 \le \theta \le \frac{\pi}{6},\ 1 \le r \le 2\sqrt{3}\sec\theta$;

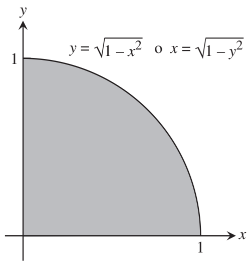
$\frac{\pi}{6} \le \theta \le \frac{\pi}{2},\ 1 \le r \le 2\csc\theta$

7. $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\ 0 \le r \le 2\cos\theta$ **9.** $\frac{\pi}{2}$

11. 2π **13.** 36 **15.** $2 - \sqrt{3}$ **17.** $(1 - \ln 2)\,\pi$

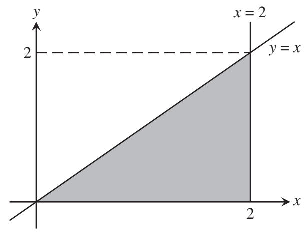
19. $(2\ln 2 - 1)(\pi/2)$ **21.** $\dfrac{2(1 + \sqrt{2})}{3}$

23.



$\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} xy\,dy\,dx$ o $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}} xy\,dx\,dy$

25.



$\displaystyle\int_0^2\int_0^x y^2(x^2 + y^2)\,dy\,dx$ o $\displaystyle\int_0^2\int_y^2 y^2(x^2 + y^2)\,dx\,dy$

27. $2(\pi - 1)$ **29.** 12π **31.** $(3\pi/8) + 1$ **33.** $\frac{2a}{3}$ **35.** $\frac{2a}{3}$

Sección 15.5, pp. 865–868

1. 1/6

3. $\int_0^1\int_0^{2-2x}\int_0^{3-3x-3y/2} dz\,dy\,dx$, $\int_0^2\int_0^{1-y/2}\int_0^{3-3x-3y/2} dz\,dx\,dy$,

$\int_0^1\int_0^{3-3x}\int_0^{2-2x-2z/3} dy\,dz\,dx$, $\int_0^3\int_0^{1-z/3}\int_0^{2-2x-2z/3} dy\,dx\,dz$,

$\int_0^2\int_0^{3-3y/2}\int_0^{1-y/2-z/3} dx\,dz\,dy$,

$\int_0^3\int_0^{2-2z/3}\int_0^{1-y/2-z/3} dx\,dy\,dz$.

El valor de las seis integrales es 1.

5. $\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy$,

$\int_{-2}^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy$,

$\int_{-2}^2\int_4^{8-y^2}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dz\,dy + \int_{-2}^2\int_{y^2}^4\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dz\,dy$,

$\int_4^8\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dy\,dz + \int_0^4\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dy\,dz$,

$\int_{-2}^2\int_4^{8-x^2}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dz\,dx + \int_{-2}^2\int_{x^2}^4\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dz\,dx$,

$\int_4^8\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dx\,dz + \int_0^4\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dx\,dz$.

El valor de las seis integrales es 16π.

7. 1 **9.** 6 **11.** $\frac{5(2-\sqrt{3})}{4}$ **13.** 18

15. 7/6 **17.** 0 **19.** $\frac{1}{2}-\frac{\pi}{8}$

21. **(a)** $\int_{-1}^1\int_0^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx$ **(b)** $\int_0^1\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$

(c) $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy\,dz$ **(d)** $\int_0^1\int_0^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$

(e) $\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{1-y} dz\,dx\,dy$

23. 2/3 **25.** 20/3 **27.** 1 **29.** 16/3 **31.** $8\pi-\frac{32}{3}$

33. 2 **35.** 4π **37.** 31/3 **39.** 1 **41.** 2 sen 4 **43.** 4

45. $a = 3$ o $a = 13/3$

47. El dominio es el conjunto de todos los puntos (x, y, z) tales que $4x^2 + 4y^2 + z^2 \leq 4$.

Sección 15.7, pp. 883–886

1. $\dfrac{4\pi\left(\sqrt{2}-1\right)}{3}$ **3.** $\dfrac{17\pi}{5}$ **5.** $\pi\left(6\sqrt{2}-8\right)$ **7.** $\dfrac{3\pi}{10}$

9. $\pi/3$

11. (a) $\displaystyle\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

(b) $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^1 r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\theta$

(c) $\displaystyle\int_0^1\int_0^{\sqrt{4-r^2}}\int_0^{2\pi} r\,d\theta\,dz\,dr$

13. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta}\int_0^{3r^2} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

15. $\displaystyle\int_0^{\pi}\int_0^{2\operatorname{sen}\theta}\int_0^{4-r\operatorname{sen}\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

17. $\displaystyle\int_{-\pi/2}^{\pi/2}\int_1^{1+\cos\theta}\int_0^{4} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

19. $\displaystyle\int_0^{\pi/4}\int_0^{\sec\theta}\int_0^{2-r\operatorname{sen}\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$ **21.** π^2 **23.** $\pi/3$

25. 5π **27.** 2π **29.** $\left(\dfrac{8-5\sqrt{2}}{2}\right)\pi$

31. (a) $\displaystyle\int_0^{2\pi}\int_0^{\pi/6}\int_0^{2}\rho^2\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta\; +$

$\displaystyle\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi}\rho^2\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$

(b) $\displaystyle\int_0^{2\pi}\int_1^{2}\int_0^{\operatorname{sen}^{-1}(1/\rho)}\rho^2\operatorname{sen}\phi\,d\phi\,d\rho\,d\theta\; +$

$\displaystyle\int_0^{2\pi}\int_0^{1}\int_0^{\pi/2}\rho^2\operatorname{sen}\phi\,d\phi\,d\rho\,d\theta$

33. $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\phi}^{2}\rho^2\operatorname{sen}\phi\,d\rho\,d\phi\,d\theta = \frac{31\pi}{6}$

35. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta = \dfrac{8\pi}{3}$

37. $\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta = \dfrac{\pi}{3}$

39. (a) $8\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$

(b) $8\int_0^{\pi/2}\int_0^{2}\int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

(c) $8\int_0^{2}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} dz \, dy \, dx$

41. (a) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$

(b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta$

(c) $\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} dz \, dy \, dx$ (d) $5\pi/3$

43. $8\pi/3$ 45. $9/4$ 47. $\dfrac{3\pi-4}{18}$ 49. $\dfrac{2\pi a^3}{3}$ 51. $5\pi/3$

53. $\pi/2$ 55. $\dfrac{4(2\sqrt{2}-1)\pi}{3}$ 57. 16π 59. $5\pi/2$

61. $\dfrac{4\pi(8-3\sqrt{3})}{3}$ 63. $2/3$ 65. $3/4$

67. $\bar{x} = \bar{y} = 0, \bar{z} = 3/8$ 69. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3/8)$

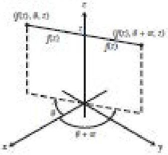
71. $\bar{x} = \bar{y} = 0, \bar{z} = 5/6$ 73. $I_z = \pi/4$ 75. $\dfrac{a^4 h\pi}{10}$

77. (a) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \dfrac{4}{5}\right), I_z = \dfrac{\pi}{12}$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \dfrac{5}{6}\right), I_z = \dfrac{\pi}{14}$

81. $\dfrac{3M}{\pi R^3}$

85. La ecuación de la superficie $r = f(z)$ nos dice que el punto $(r, \theta, z) = (f(z), \theta, z)$ permanecerá sobre la superficie para todos los valores de θ. En particular, $(f(z), \theta + \pi, z)$ permanece sobre la superficie siempre que $(f(z), \theta, z)$ se encuentre sobre la superficie, de manera que la superficie es simétrica con respecto al eje z.



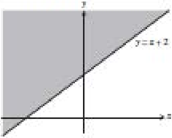
CAPÍTULO 14

Sección 14.1, pp. 753–755

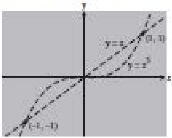
1. (a) 0 (b) 0 (c) 58 (d) 33

3. (a) 4/5 (b) 8/5 (c) 3 (d) 0

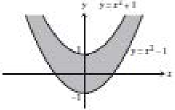
5. Dominio: todos los puntos (x, y) en o arriba de la recta $y = x + 2$



7. Dominio: todos los puntos que no se encuentran sobre la gráfica de $y = x$ o $y = x^3$



9. Dominio: todos los puntos (x, y) que satisfacen $x^2 - 1 \le y \le x^2 + 1$



11. Dominio: todos los puntos (x, y) para los cuales $(x - 2)(x + 2)(y - 3)(y + 3) \ge 0$