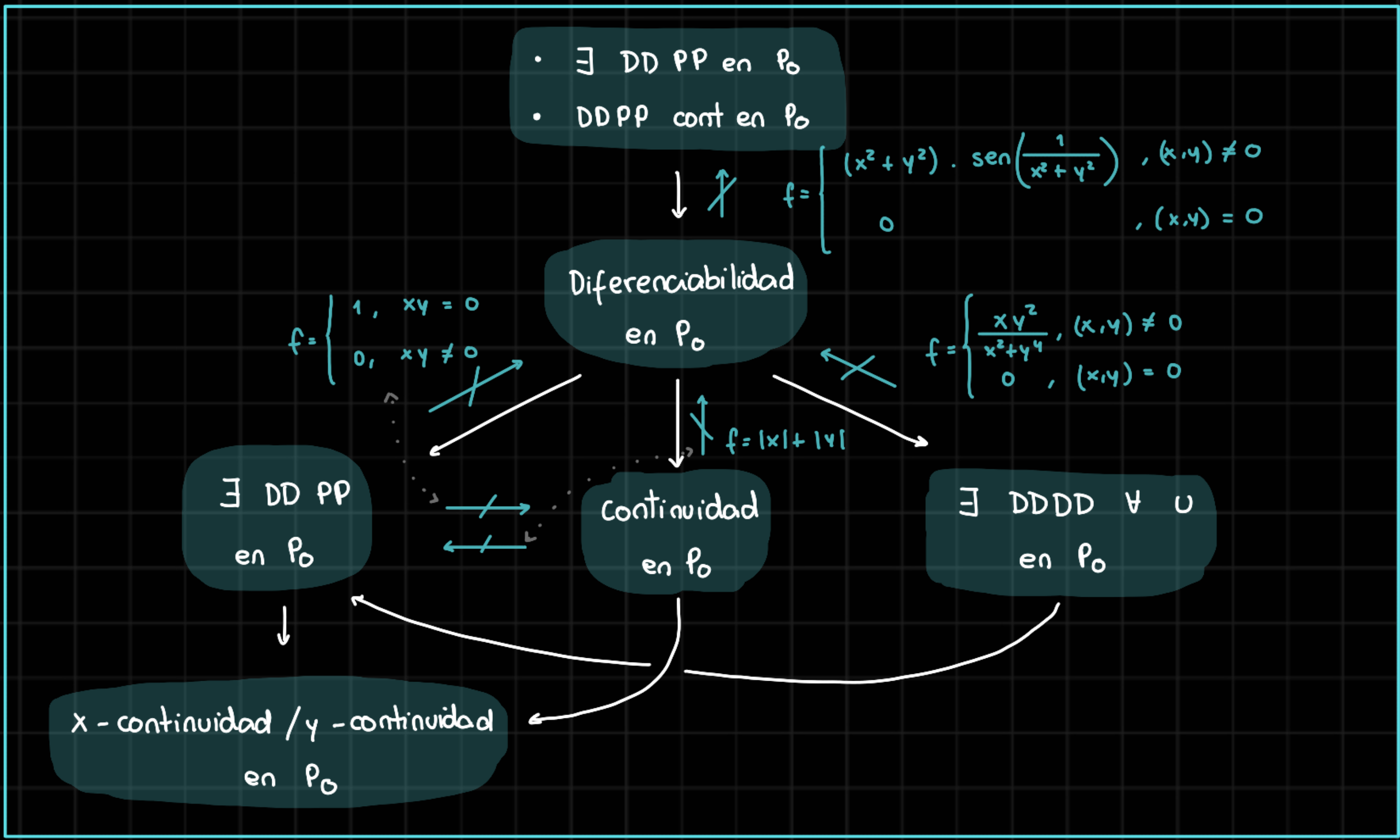

$$f(x,y) = |x| + |y|$$

- es contínua en $(0,0)$

$$f(0,0) = |0| + |0| = 0 \quad y \quad \lim_{(x,y)\to(0,0)} |x| + |y| = |0| + |0| = 0$$

- $\nexists$ f_x en $(0,0)$

$$\lim_{h\to 0} \frac{|0+h| + |0| - |0| - |0|}{h} = \lim_{h\to 0} \frac{|h|}{h} \begin{cases} \lim\limits_{h\to 0^+} \frac{|h|}{h} = 1 \\ \lim\limits_{h\to 0^-} \frac{|h|}{h} = -1 \end{cases} \Rightarrow \lim \nexists$$

$$f = \begin{cases} 1, & xy = 0 \\ 0, & xy \neq 0 \end{cases}$$

- ∃ f_x y f_y en $(0,0)$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,0+h) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

- f no es contínua en $(0,0)$

$$\lim_{\substack{(x,0)\to(0,0)\\ \underline{y=0}}} 1 = 1 \neq \lim_{\substack{x\to 0\\ \underline{y=x}}} 0 = 0$$

$$f = \begin{cases} \dfrac{xy^2}{x^2+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

- $\exists$ DDDD $\forall$ u en $(0,0)$:

$$\lim_{s\to 0} \frac{f(x+s.u_1,\, y+s.u_1) - \overbrace{f(x,y)}^{0}}{s}$$

$$\lim_{s\to 0} \frac{\frac{(s.u_1)\,(s^2.u_2^2)}{(s^2.u_1^2)+(s^4.u_2^4)}}{s} = \lim_{s\to 0} \frac{s\,u_1 \;.\; s^2\,u_2^2}{(s^3\,u_1^2) + (s^5.u_2^4)}$$

$$\lim_{s\to 0} \frac{\cancel{s^3}\, u_1 u_2^2}{\cancel{s^3}\,(u_1^2 + s^2.u_2^4)}$$

$$\lim_{s\to 0} \frac{u_1 u_2^2}{u_1^2 + s^2.u_2^4} = \frac{\cancel{u_1}.u_2^2}{u_1^{\cancel{2}}} = \boxed{\frac{u_2^2}{u_1}}$$

- f no es contínua en $(0,0)$:

$$\lim_{\substack{(x,0)\to(0,0)\\ \underline{y=0}}} \frac{xy^2}{x^2+y^4} = \frac{x.0^2}{x^2+0^4} = 0$$

$$\lim_{\substack{y\to 0\\ \underline{x=y^2}}} \frac{x\,y^2}{x^2+y^4} = \frac{y^4}{y^4+y^4} = \frac{\cancel{y^4}}{2\cancel{y^4}} = 1/2$$

$$f = \begin{cases} (x^2+y^2)\;.\;\operatorname{sen}\left(\dfrac{1}{x^2+y^2}\right), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

- f es diferenciable en $(0,0)$ ya que el Δf puede verifica que ...

$$\Delta f = f(0+\Delta x,\, 0+\Delta y) - f(0,0)$$

$$= f(\Delta x, \Delta y) = (\Delta x^2 + \Delta y^2) \cdot \operatorname{sen}\left(\frac{1}{\Delta x^2 + \Delta y^2}\right)$$

$$= \Delta x^2 \cdot \operatorname{sen}\left(\frac{1}{\Delta x^2 + \Delta y^2}\right) + \Delta y^2 \cdot \operatorname{sen}\left(\frac{1}{\Delta x^2 + \Delta y^2}\right)$$

$$= \Delta x \cdot \underbrace{\operatorname{sen}\left(\frac{1}{\Delta x^2 + \Delta y^2}\right) \Delta x} + \Delta y \cdot \underbrace{\operatorname{sen}\left(\frac{1}{\Delta x^2 + \Delta y^2}\right) \Delta y}$$

$$= \underbrace{\Delta x \cdot \varepsilon_1 + \Delta y \cdot \varepsilon_2}_{\varepsilon_{1,2} \to 0 \text{ cuando } \Delta x, \Delta y \to 0}$$

- DD PP no son continuas en $(0,0)$

$$f_x = \begin{cases} 2x \cdot \operatorname{sen}\left(\frac{1}{x^2+y^2}\right) + \cancel{(x^2+y^2)} \cdot \cos\left(\frac{1}{x^2+y^2}\right) \cdot \frac{-2x}{(x^2+y^2)^{\cancel{2}}}, & (x,y) \neq 0 \\ 0, & (x,y) = 0 \end{cases}$$

$\downarrow$

$$f_x = \begin{cases} 2x \cdot \operatorname{sen}\left(\frac{1}{x^2+y^2}\right) + \frac{-2x}{(x^2+y^2)} \cdot \cos\left(\frac{1}{x^2+y^2}\right), & (x,y) \neq 0 \\ 0, & (x,y) = 0 \end{cases}$$

$\rightarrow$ $f_x(0,0) = 0$, pero:

$$\lim_{(x,y)\to(0,0)} \left[2x \cdot \operatorname{sen}\left(\frac{1}{x^2+y^2}\right) + \frac{-2x}{(x^2+y^2)} \cdot \cos\left(\frac{1}{x^2+y^2}\right)\right]$$

$$\lim_{(x,y)\to(0,0)} \left[2x \cdot \operatorname{sen}\left(\frac{1}{x^2+y^2}\right)\right] + \left[\underbrace{\lim_{(x,y)\to(0,0)} \left(\frac{-2x}{(x^2+y^2)}\right)} \cdot \lim_{(x,y)\to(0,0)} \left(\cos\left(\frac{1}{x^2+y^2}\right)\right)\right]$$

$\downarrow$

$$\lim_{\substack{(0,y)\to(0,0) \\ x=0}} \frac{-2x}{(x^2+y^2)} = \frac{0}{y^2} = 0$$

$$\lim_{\substack{x\to 0 \\ y=x}} \frac{-2x}{(x^2+y^2)} = \frac{-2\cancel{x}}{2x^{\cancel{2}}} = \frac{-2}{2x} = \infty$$

entonces todo el lím no existe

(análogamente p/ f_y)