9.4. 18. (a) $\int \frac{1}{y^{1+c}}\,dy = \int k\,dt$

$$-\frac{1}{c}y^{-c} = kt + a$$

$$y^{-c} = -c(kt+a)$$

$$y = (y^{-c})^{-\frac{1}{c}} = (-c(kt+a))^{-\frac{1}{c}}$$

$$y(0) = (-c(a))^{-\frac{1}{c}} = y_0$$

$$a = -c^{-1}y_0^{-c}$$

$$y(t) = (-c(kt - c^{-1}y_0^{-c}))^{-\frac{1}{c}}$$

(b). When $y(t)$ goes to infinity, $\frac{1}{\sqrt[c]{-c(kt-c^{-1}y_0^{-c})}}$ goes to infinity,

that means $-c(kt-c^{-1}y_0^{-c})$ goes to 0. Because c is greater than 0, $kt - c^{-1}y_0^{-c}$ must go to 0, and kt goes close to $c^{-1}y_0^{-c}$.

Solving $kt = c^{-1}y_0^{-c}$, we get $t = \frac{1}{cy_0^c k}$.

So, $\lim_{t \to \left(\frac{1}{cy_0^c k}\right)^-} y(t) = \infty$

(c) $C = 0.01 \quad y_0 = 2 \quad y(3) = 16 =$

$$\left(-0.01\left(3k - 100 \cdot 2^{-0.01}\right)\right)^{-100} = 16$$

$$-0.01\left(3k - 100 \cdot 2^{-0.01}\right) = 16^{-\frac{1}{100}}$$

$$3k - 100 \cdot 2^{-0.01} = 16^{-\frac{1}{100}} \cdot (-100)$$

$$3k = 16^{-\frac{1}{100}} \cdot (-100) + 100 \cdot 2^{-0.01}$$

$$k = \frac{100}{3}\left(2^{-0.01} - 16^{-\frac{1}{100}}\right)$$

$$(c y_0^c k)^{-1} = \left(0.01 \cdot 2^{0.01} \cdot 100 \cdot \frac{1}{3}\left(2^{-0.01} - 16^{-0.01}\right)\right)^{-1}$$

$$= \left(\frac{1}{3} \cdot \left(1 - 2^{0.01}\, 2^{-0.04}\right)\right)^{-1}$$

$$= 3\left(1 - 2^{0.01-0.04}\right)^{-1}$$

$$= 3\left(1 - 2^{-0.03}\right)^{-1}$$

$$\approx 145.77 \text{ months}$$

9.5: 6. $y' - y = e^x$

$$e^{-x} y' - e^{-x} y = e^{-x} e^x$$

$$e^{-x} y = \int dx = x + C$$

$$y = \frac{x + C}{e^{-x}} = (x + C) e^x$$

9.5. 16. $t^3 \frac{dy}{dt} + 3t^2 y = \cos t$, $y(\pi) = 0$

$$\frac{dy}{dt} + \frac{3}{t} y = \frac{\cos t}{t^3}$$

$$e^{\int \frac{3}{t} dt} = e^{3\ln|t| + c} = e^{\ln|t| \cdot 3 + c} = |t|^3 e^c$$

$$t^3 y' + 3t^2 y = \cos t$$

$$t^3 y = \sin t + C$$

$$y = t^{-3} (\sin t + C)$$

$$y(\pi) = \pi^{-3} (\sin \pi + C) = 0$$

$$C = 0$$

$$y(t) = t^{-3} \sin t$$