11.9: 8. $f(x) = \frac{x}{2x^2+1} = x \frac{1}{1+2x^2} = x \sum (-2x^2)^n$

$|-2x^2| < 1$

$|x^2| < \frac{1}{2}$

$-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$

11.9: 16. $f(x) = x^2 \tan^{-1}(x^3)$

$g(x) = \tan^{-1}(x^3)$

$g'(x) = \frac{1}{1+x^6} = \sum (-x^6)^n$

$\int g'(x)\,dx = \sum \int (-x^6)^n\,dx = \sum \int (-1)^n x^{6n}\,dx =$

$\sum (-1)^n \frac{x^{6n+1}}{6n+1} + C$

$g(0) = 0$, $C = 0$

$g(x) = \tan^{-1}(x^3) = \sum (-1)^n \frac{x^{6n+1}}{6n+1}$

$f(x) = x^2 g(x) = \sum (-1)^n \frac{x^{6n+3}}{6n+1}$

$|x^{6n}| < 1$

$|x| < 1$

$x \in (-1, 1)$

11.10: 24. $f(x) = \cos x$, $a = \pi/2$

$f(x) = \cos x \quad f(a) = 0$

$f'(x) = -\sin x \quad f'(a) = -1$

$f''(x) = -\cos x \quad f''(a) = 0$

$f'''(x) = \sin x \quad f'''(a) = 1$

$f^4(x) = \cos x \quad f^4(a) = 0$

$$f(x) = 0 - \frac{1}{1!}\left(x-\frac{\pi}{2}\right) - 0 + \frac{1}{3!}\left(x-\frac{\pi}{2}\right)^3 + 0 - \cdots =$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)!}\left(x-\frac{\pi}{2}\right)^{2n+1} \cdot (-1)^{n+1}$$

$$\cancel{\lim_{n\to\infty} \frac{\left(x-\frac{\pi}{2}\right)^{2n+1}}{(2n+1)!} = 0}$$

$$\left| \frac{\left(x-\frac{\pi}{2}\right)^{2(n+1)+1}}{(2(n+1)+1)!} \quad \frac{(2n+1)!}{\left(x-\frac{\pi}{2}\right)^{2n+1}} \right| = \left| \frac{\left(x-\frac{\pi}{2}\right)^{2n+3}}{\left(x-\frac{\pi}{2}\right)^{2n+1}} \cdot \frac{(2n+1)!}{(2n+3)!} \right| =$$

$$\left| \frac{\left(x-\frac{\pi}{2}\right)^2}{(2n+3)(2n+2)} \right|$$

, as $n$ goes to infinity,

it goes to zero, no matter what $x$ is.

$x \in (-\infty, \infty)$

11.10: 36. $f(x) = \sin(\pi x/4)$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin\left(\frac{\pi x}{4}\right) = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{\pi x}{4}\right)^{2n+1}}{(2n+1)!} =$$

$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{\pi}{4}\right)^{2n+1} \frac{x^{2n+1}}{(2n+1)!}$$