3.1.53. (a) $P = rV^{-1}$ for some r

$$50 = r \, 0.106^{-1}$$

$$0.106 \times 50 = r$$

$$P = 0.106 \times 50 \, V^{-1}$$

$$V = 0.106 \times 50 \, P^{-1}$$

(b) $\frac{d}{dP} V = -0.106 \times 50 \, P^{-2}$

$$V'(50) = -0.106 \times 50 \times \frac{1}{50 \cdot 50}$$

$$= -\frac{0.106}{50}$$

At 50 kPa, an infinitesimal change in pressure results in $-\frac{0.106}{50}$ m^3 in volume. Rate dV/dP has units m^3/kPa

3.1. 57.

$$y = 2e^x + 3x + 5x^3$$

$$y' = 2e^x + 3 + 15x^2$$

If y has a tangent line with slope 2, then there exists an x, such that $2e^x + 3 + 15x^2 = 2$.

$2e^x + 15x^2 = -1$, Because $e^x > 0$, $x^2 \geq 0$ for all x's. $2e^x + 15x^2 \neq -1$.

So, y has no tangent line with slope 2.

3.2. 26. $f(x) = \dfrac{ax+b}{cx+d}$

$$f'(x) = \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

3.2. 32. $y = \dfrac{1+x}{1+e^x}$ $(0, \frac{1}{2})$

$$y' = \frac{1+e^x - e^x(1+x)}{(1+e^x)^2}$$

~~The tangent line~~ $\cancel{f(x) = y'(x)}$ $\frac{1}{2} = y'(0)\,x + C$, $x=0$

$\frac{1}{2} = C$

The tagent line is:

$$f(x) = y'(0)x + \frac{1}{2} = \frac{2-e^0(1)}{4}x + \frac{1}{2}$$

$$= \frac{1}{4}x + \frac{1}{2}$$

3.3.18.

$$\frac{d}{dx}\sec x = \lim_{h\to 0}\frac{\sec(x+h)-\sec(x)}{h} =$$

$$\lim_{h\to 0}\frac{\frac{1}{\cos(x+h)}-\sec(x)}{h} =$$

$$\lim_{h\to 0}\frac{\frac{1}{\cos(x)\cos(h)-\sin(x)\sin(h)}\ \sec(x)}{h} =$$

$$\lim_{h\to 0}\frac{\frac{\sec(x)\sec(h)}{1-\tan(x)\tan(h)}-\sec(x)}{h} =$$

$$\sec(x)\lim_{h\to 0}\frac{\frac{\sec(h)}{1-\tan(x)\tan(h)}-\cancel{\sec}\,1}{h} =$$

$$\sec(x)\lim_{h\to 0}\frac{1}{h}\left(\frac{\cos(x)}{\cos(x)\cos(h)-\sin(x)\sin(h)}-1\right) =$$

$$\sec(x)\lim_{h\to 0}\frac{1}{h}\left(\frac{\cos(x)-\cos(x)\cos(h)+\sin(x)\sin(h)}{\cos(x)\cos(h)-\sin(x)\sin(h)}\right) =$$

$$\sec(x)\lim_{h\to 0}\frac{1}{h}\left(\frac{\cos(x)-\cos(x+h)}{\cos(x+h)}\right) =$$

$$\sec(x)\lim_{h\to 0}\left(-\frac{\cos(x+h)-\cos(x)}{h}\cdot\frac{1}{\cos(x+h)}\right) =$$

$$\sec(x)\sin(x)\lim_{h\to 0}\frac{1}{\cos(x+h)} = \sec(x)\tan(x)$$

3.3. 38.

(a) $F'(\theta) = \mu W \frac{d}{d\theta} (\mu \sin\theta + \cos\theta)^{-1} =$

$\mu W (-1)(\mu \sin\theta + \cos\theta)^{-2} \frac{d}{d\theta} (\mu \sin\theta + \cos\theta)$

$= -\mu W \frac{1}{(\mu \sin\theta + \cos\theta)^2} (\mu \cos\theta - \sin\theta)$

$$= -\mu W \frac{\mu \cos\theta - \sin\theta}{(\mu \sin\theta + \cos\theta)^2}$$

(b) When $F'(\theta) = 0$

$$\mu \cos\theta = \sin\theta$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}\mu$$