

4.4: 48. $\lim_{x \rightarrow \infty} x^{3/2} \sin(1/x)$

$$\lim_{x \rightarrow \infty} x^{3/2} = \infty, \quad \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right) = 0$$

type: $\infty \cdot 0$

$$\lim_{x \rightarrow \infty} x^{3/2} \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{x^{-3/2}} =$$

$$\lim_{x \rightarrow \infty} \frac{\cos\left(\frac{1}{x}\right)\left(-x^{-2}\right)}{-\frac{3}{2}x^{-5/2}} =$$

$$\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right)\left(\frac{2}{3}\right)x^{-2+\frac{5}{2}} =$$

$$\lim_{x \rightarrow \infty} \frac{2}{3} \cos\left(\frac{1}{x}\right) x^{\frac{1}{2}} =$$

$$\frac{2}{3} \lim_{x \rightarrow \infty} \sqrt{x} = \infty$$

4.4: 54. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1}x}\right)$, $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$, $\lim_{x \rightarrow 0^+} \frac{1}{\tan^{-1}x} = \infty$

type: $\infty - \infty$

$$\lim_{x \rightarrow 0^+} \frac{\tan^{-1}x - x}{x + \tan^{-1}x} \quad \text{type: } \frac{0}{0}$$

$$\lim_{x \rightarrow 0^+} \frac{\tan^{-1}x - x}{x + \tan^{-1}x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x^2} - 1}{\tan^{-1}x + \frac{x}{1+x^2}} = 0$$

4.7: 22

$$D = d^2 = (x-3)^2 + (\sqrt{x}-0)^2 = x^2 - 6x + 9 + x$$

$$D' = 2x - 5 = 0$$

$$2x = 5$$

$$x = \frac{5}{2}$$

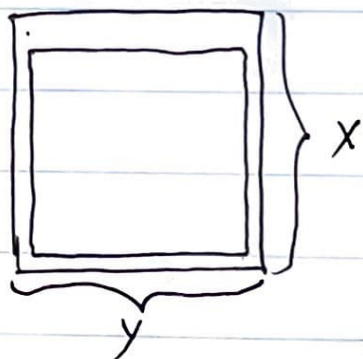
$$\text{When } x > \frac{5}{2}, \quad D' > 0$$

$$\text{When } x < \frac{5}{2}, \quad D' < 0$$

D decreases then increases.

$D\left(\frac{5}{2}\right)$ is the minimum, the point is $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

4.7: 36.



$$xy = 180, \quad y = \frac{180}{x}$$

$$(x-3)(y-2) = A$$

$$A = xy - 3y - 2x + 6 =$$

$$180 - 3\left(\frac{180}{x}\right) - 2x + 6 = 186 - 2x - \frac{540}{x}$$

$$A' = -2 - 540(-x^{-2}) = 0$$

$$A' = \frac{540}{x^2} - 2,$$

$$2x^2 = 540$$

$$A' > 0 \text{ when } x^2 < 270$$

$$x^2 = 270$$

$$A' < 0 \text{ when } x^2 > 270$$

$$x = \pm\sqrt{270}$$

A first increases, then decreases

$$= 13\sqrt{30}$$

$$y = \frac{180}{3\sqrt{30}} = 2\sqrt{30}$$

The dimension is $x = 3\sqrt{30}, y = 2\sqrt{30}$