

9.1, 10.

(a) When  $\frac{dv}{dt} = 0$ ,  $v = 0$ , or

$$v^2 - (1+a)v + a = 0$$

$$(v-1)(v-a) = 0$$

$$v = 1 \quad \text{or} \quad v = a$$

(b)  $\frac{dv}{dt} = -v(v-a)(v-1)$

When  $\frac{dv}{dt} > 0$ ,  $v \in (a, 1) \cup (-\infty, 0)$

(c) ~~When  $\frac{dv}{dt} = 0$~~

When  $\frac{dv}{dt} < 0$ ,  $v \in (0, a) \cup (1, \infty)$

9.2. 4.  $y' = x(2-y)$

	$(0, 0)$	$(-1, 1)$	$(-2, -2)$	$(1, 1)$	$(2, 2)$	$(-1, 1)$
$y'$	0	-3	-8	1	0	-1

Not IV, because  $y' = 0$  at  $(0, 0)$ .

Not II, because  $y' < 0$  at  $(-2, -2)$ .

Not III, because  $y' < 0$  at  $(-1, 1)$ .

It's I.

9.2. 6.  $y' = \sin x \sin y$

	$(0, 0)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$(-\frac{\pi}{2}, -\frac{\pi}{2})$	$(\frac{\pi}{2}, -\frac{\pi}{2})$	$(\frac{\pi}{2}, \frac{\pi}{2})$
$y'$	0	-1	1	-1	1

Not IV, because  $y' = 0$  at  $(0, 0)$ .

Not I, because I is solution of  $y' = x(2-y)$ .

Not III, because  $y' = 0$  at  $(0, 0)$ .

It's II

$$9.2. \quad 24. \quad (a) \quad \hat{y}(0.2) = y'(0)(0.2) + y(0)$$

$$= 0.2$$

$$\hat{y}(0.4) = y'(0.2)(0.2) + \hat{y}(0.2)$$

$$= 0.2 \cos(0.4) + 0.2 \approx 0.38$$

$$\hat{y}(0.6) = y'(0.4)(0.2) + \hat{y}(0.4)$$

$$= \cos(0.6 + 0.2 \cos(0.4))(0.2) + 0.2 \cos(0.4) + 0.2$$

$$\approx 0.53$$

$$(b) \quad \hat{y}(0.1) = y'(0)(0.1) + y(0)$$

$$= 0.1$$

$$\hat{y}(0.2) = \frac{2}{3} y'(0.1)(0.1) + \hat{y}(0.1)$$

$$= \cos(0.2)(0.1) + 0.1 \approx 0.20$$

$$\hat{y}(0.3) = y'(0.2)(0.1) + \hat{y}(0.2) =$$

$$0.1 \cos(0.2) + 0.1 + 0.1 \cos(0.3 + 0.1 \cos(0.2)) \approx 0.29$$

$$\hat{y}(0.4) = \hat{y}(0.3) + 0.1 y'(0.3) =$$

$$\approx 0.2999 + 0.1 \cos(0.3 + 0.29019)$$

$$= 0.37327$$

$$\hat{y}(0.4) =$$

$$\hat{y}(0.5) = \hat{y}(0.4) + 0.1 \frac{2}{3} y'(0.4)$$

$$= 0.37327 + 0.1 \cos(0.4 + 0.37327)$$

$$= 0.44483$$

$$\hat{y}(0.6) = \hat{y}(0.5) + 0.1 y'(0.5)$$

$$= 0.44483 + 0.1 \cos(0.5 + 0.44483)$$

$$= 0.503$$

9.3. 4.  $y' + xe^y = 0$

$$\frac{dy}{dx} + xe^y = 0$$

$$dy = -xe^y dx$$

$$\int e^{-y} dy = -\int x dx$$

$$+e^{-y} = +\frac{1}{2}x^2 + C$$

$$+y = -\ln\left(\frac{1}{2}x^2 + C\right)$$

9.3. 12.  $\frac{dy}{dx} = \frac{x \sin x}{y}$ ,  $y(0) = -1$

$$\int y dy = \int x \sin x dx$$

$$\frac{1}{2} y^2 = \sin x - x \cos x + C$$

$$y = \pm \sqrt{2(\sin x - x \cos x + C)}$$

$$y(0) = -1 = -\sqrt{2(0 - 0 + C)} = -\sqrt{2C}$$

$$C = \frac{1}{2}$$

$$y = -\sqrt{2(\sin x - x \cos x + \frac{1}{2})}$$

$$\int x \sin x dx$$

$$u = x \quad dv = \sin x dx$$

$$du = dx \quad v = -\cos x$$