5.5. 34. $\int \frac{\cos(\pi/x)}{x^2}\,dx$

$u = \pi/x = \pi x^{-1}$

$du = -\pi x^{-2}\,dx = -\frac{\pi}{x^2}\,dx$

$-\int \frac{\cos(u)}{\pi}\,du = -\frac{\sin(u)}{\pi} + C$

$= -\frac{\sin(\pi/x)}{\pi} + C$

5.5. 60 $\int_0^1 xe^{-x^2}\,dx$

$u = e^{-x^2} \quad du = -2xe^{-x^2}\,dx$

$\int_0^1 xe^{-x^2}dx = \int_{x=0}^{x=1} -\frac{1}{2}\,du = -\frac{1}{2}u\Big|_{x=0}^{x=1} = -\frac{1}{2}e^{-x^2}\Big|_0^1 =$

$-\frac{1}{2}e^{-1} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2e} + \frac{1}{2}$

7.1. 26 $\int_1^2 w^2 \ln w \, dw$

$u = \ln w \qquad dv = w^2 dw$

$du = \frac{1}{w} dw \qquad v = \frac{1}{3} w^3$

$$\int_1^2 w^2 \ln w \, dw = \frac{1}{3} w^3 \ln w \Big|_1^2 - \int_1^2 \frac{1}{3} \frac{w^3}{w} dw$$

$$= \frac{1}{3} 2^3 \ln(2) - \frac{1}{3} \ln(1) - \frac{1}{3} \int_1^2 w^2 dw$$

$$= \frac{8}{3} \ln(2) - \frac{1}{3} \left(\frac{1}{3} w^3\right)\Big|_1^2$$

$$= \frac{8}{3} \ln(2) - \frac{1}{3} \cdot \frac{1}{3} \cdot (8-1)$$

$$= \frac{8}{3} \ln(2) - \frac{1}{9} \cdot 7$$

$$= \frac{8}{3} \ln(2) - \frac{7}{9}$$

7.1. 38 $\int \cos(\ln x)\, dx$

$$u = \ln(x)$$
$$du = \frac{1}{x}\, dx$$
$$x\, du = dx$$

$$\int \cos(\ln x) dx = \int \cos(u)\, x\, du$$

$$u = \ln(x)$$
$$e^u = x$$

$$\int \cos(u)\, x\, du = \int \cos(u) e^u\, du$$

$a = e^u$ $\qquad$ $db = \cos(u)\, du$

$da = e^u du$ $\qquad$ $b = \sin(u)$

$$\int \cos(u) e^u du = e^u \sin(u) - \int \sin(u) e^u du$$

$c = e^u$ $\qquad$ $de = \sin(u)\, du$

$dc = e^u du$ $\qquad$ $e = -\cos(u)$

$$\int \sin(u) e^u du = e^u(-\cos(u)) - \int e^u(-\cos(u))\, du$$
$$= -e^u \cos(u) + \int e^u \cos(u)\, du$$

$$\int \cos(u) e^u du = e^u \sin(u) + e^u \cos(u) - \int e^u \cos(u)\, du$$

$$\int \cos(u) e^u du = \frac{1}{2} e^u \sin(u) + \frac{1}{2} e^u \cos(u) + C$$
$$= \frac{x}{2} \sin(\ln(x)) + \frac{x}{2} \cos(\ln(x)) + C$$

7.2 : 36 $\quad \int_{\pi/4}^{\pi/2} \cot^3 x \, dx =$

$$\int_{\pi/4}^{\pi/2} \cos^3 x \sin^{-3} x \, dx =$$

$$\int_{\pi/4}^{\pi/2} \cos x \,(1-\sin^2 x)\, \sin^{-3} x \, dx =$$

$$\int_{\pi/4}^{\pi/2} (\cos x - \cos x \sin^2 x) \sin^{-3} x \, dx =$$

$$\int_{\pi/4}^{\pi/2} \cos x \sin^{-3} x - \cos x \sin^2 x \sin^{-1} x \, dx =$$

$$\int_{\pi/4}^{\pi/2} \cos x \sin^{-3} x \, dx - \int_{\pi/4}^{\pi/2} \cos x (\sin x)^{-1} \, dx$$

let $u = \sin x$, $du = \cos x \, dx$

$$\int_{x=\pi/4}^{x=\pi/2} u^{-3} \, du - \int_{x=\pi/4}^{x=\pi/2} u^{-1} \, du =$$

$$\cancel{-\tfrac{1}{4} u} \quad -\frac{1}{2} u^{-2} \Big|_{x=\pi/4}^{x=\frac{\pi}{2}} - \ln|u| \Big|_{x=\pi/4}^{x=\frac{\pi}{2}}$$

When $x = \frac{\pi}{2}$, $u = \sin\frac{\pi}{2} = 1$,
when $x = \frac{\pi}{4}$, $u = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\left(-\frac{1}{2} u^{-2} - \ln|u|\right)\Big|_{\frac{1}{\sqrt{2}}}^{1} = -\left(-\frac{1}{2}\left(2^{-\frac{1}{2}}\right)^{-2} - \ln\left(2^{-\frac{1}{2}}\right)\right) + \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2}\left(1 - \ln(2)\right)$$

7-3: **8.** $\int \frac{dt}{t^2\sqrt{t^2-16}} =$

$$\int \frac{dt}{t^2\sqrt{16\left(\left(\frac{t}{4}\right)^2-1\right)}} = \int \frac{1}{4t^2\sqrt{\left(\frac{t}{4}\right)^2-1}}\,dt$$

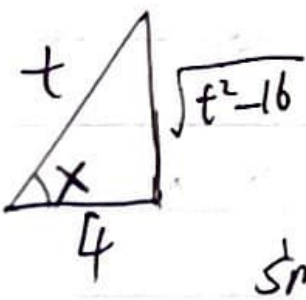
$$\frac{t}{4} = \sec X$$

$$t = 4\sec X$$

$$dt = 4\sec X\tan X\,dx$$

$$\int \frac{1}{4t^2\sqrt{\left(\frac{t}{4}\right)^2-1}}\,dt = \int \frac{1}{4(4^2\sec^2 X)\tan X}\,4\sec X\tan X\,dx$$

$$= \int \frac{1}{4^2\sec X}\,dx = \frac{1}{4^2}\int \cos X\,dx$$



$$= \frac{1}{4^2}\sin X + C$$

$$\sin X = \frac{\sqrt{t^2-16}}{t}$$

$$\frac{1}{4^2}\sin X + C = \frac{\sqrt{t^2-16}}{4^2 t} + C = \frac{\sqrt{t^2-16}}{16t} + C$$