7-4. 18. $\displaystyle\int_1^2 \frac{3x^2+6x+2}{x^2+3x+2}\,dx$

$$\begin{array}{r l} & \quad 3 \\ x^2+3x+2 \,\big) & 3x^2+6x+2 \\ & \underline{3x^2+9x+6} \\ & \quad -3x-4 \end{array}$$

$$\int_1^2 3 + \frac{-3x-4}{x^2+3x+2}\,dx = \int_1^2 3 - \frac{3x+4}{x^2+3x+2}\,dx$$

$$\frac{3x+4}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$3x+4 = Ax+2A+Bx+B$$

$$\begin{cases} 2A+B=4 \\ 3x=(A+B)x \end{cases} \qquad A=1, \quad B=2$$

$$\int_1^2 3 - \frac{3x+4}{x^2+3x+2}\,dx = \int_1^2 3 - \left(\frac{1}{(x+1)} + \frac{2}{x+2}\right)dx$$

$\cancel{\ln|x+1| + 2\ln|x+2|}$

$$= 3 - \ln|x+1|\Big|_1^2 - 2\ln|x+2|\Big|_1^2$$

$$= 3 - \left(\ln\left|\frac{3}{2}\right| + 2\ln\left|\frac{4}{3}\right|\right) = 3 - \ln\left(\frac{8}{3}\right)$$

7.4. 50. $\int \frac{e^x}{(e^x-2)(e^{2x}+1)}\,dx$ let $e^x = u$, $du = u\,dx$

$\int \frac{1}{(u-2)(u^2+1)}\,du$ $\frac{1}{(u-2)(u^2+1)} = \frac{A}{u-2} + \frac{Bu+C}{u^2+1}$

$1 = A(u^2+1) + (Bu+c)(u-2)$
$= Au^2 + A + Bu^2 - 2C + uC - 2Bu$
$= (A+B)u^2 + A - 2C + (C-2B)u$

$\begin{cases} A+B=0 \\ A-2C=1 \\ C-2B=0 \end{cases}$ ~~$\Rightarrow$ $C=2B$~~ $A=\frac{1}{5}$, $B=-\frac{1}{5}$, $C=-\frac{2}{5}$

$\int \frac{+\frac{1}{5}}{u-2} + \frac{-\frac{1}{5}u-\frac{2}{5}}{u^2+1}\,du = \frac{1}{5}\ln|u-2| - \int \frac{\frac{1}{5}u+\frac{2}{5}}{u^2+1}\,du =$

$\frac{1}{5}\ln|u-2| - \left(\frac{1}{5}\int \frac{u}{u^2+1}\,du + \frac{2}{5}\int \frac{1}{u^2+1}\,du\right) =$

$\frac{1}{5}\ln|u-2| - \frac{1}{10}\ln|u^2+1| - \frac{2}{5}\tan^{-1}(u) + C =$

$\frac{1}{5}\left(\ln\left|\frac{u-2}{(u^2+1)^{1/2}}\right| - 2\tan^{-1}(u)\right) + C =$

$\frac{1}{5}\ln\left|\frac{e^x-2}{\sqrt{e^{2x}+1}}\right| - 2\tan^{-1}(e^x) + C$

7-8. 18. $\int_2^{\infty} \frac{dv}{v^2+2v-3} = \int_2^{\infty} \frac{1}{(v+3)(v-1)}\, dv$

$$\frac{1}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1} \qquad 1 = Av - A + Bv + 3B = (A+B)v + 3B - A$$

$$\begin{cases} A+B=0 \\ 3B-A=1 \end{cases}, \quad B=\frac{1}{4}, \quad A=-\frac{1}{4}$$

$$\int_2^{\infty} -\frac{1}{4}\left(\frac{1}{v+3}\right) + \frac{1}{4}\left(\frac{1}{v-1}\right) dv = \frac{1}{4}\int_2^{\infty} \frac{1}{v-1}\, dv - \frac{1}{4}\int_2^{\infty} \frac{1}{v+3}\, dv =$$

$$\lim_{t\to\infty} \frac{1}{4}\left(\int_2^t \frac{1}{v-1}\, dv - \int_2^t \frac{1}{v+3}\, dv\right) = \lim_{t\to\infty} \frac{1}{4}\left(\ln|v-1|\Big|_2^t - \ln|v+3|\Big|_2^t\right) =$$

$$\lim_{t\to\infty} \frac{1}{4} \ln\left|\frac{v-1}{v+3}\right|\Bigg|_2^t = \lim_{t\to\infty} \frac{1}{4}\left(\ln\left|\frac{t-1}{t+3}\right| - \ln\left|\frac{1}{5}\right|\right) =$$

$$\lim_{t\to\infty} \frac{1}{4} \ln\left|\frac{5t-5}{t+3}\right| = \lim_{t\to\infty} \frac{1}{4} \ln\left|\frac{5t}{t+3} - \frac{5}{t+3}\right| = \frac{1}{4} \ln|5|$$

7-8. 22. $\int_1^{\infty} \frac{\ln x}{x^2}\, dx \qquad u = \ln x \quad dv = \frac{1}{x^2}dx$

$du = \frac{1}{x}dx \quad v = -x^{-1}$

$$\int_1^{\infty} \frac{\ln x}{x^2}\, dx = -\frac{\ln x}{x}\Big|_1^{\infty} + \int_1^{\infty} x^{-2}\, dx = -x^{-1}\Big|_1^{\infty} - \frac{\ln x}{x}\Big|_1^{\infty} =$$

$$\lim_{t\to\infty} \left(-\frac{1}{x} - \frac{\ln x}{x}\right)_1^t = 0 + 1 = 1$$