1.4. 4.(a) $\frac{x^{2n}\cdot x^{3n-1}}{x^{n+2}} = \frac{x^{5n-1}}{x^{n+2}} = x^{5n-1-n-2} = x^{4n-3}$

4.(b) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \frac{(ab^{\frac{1}{2}})^{\frac{1}{2}}}{(ab)^{\frac{1}{3}}} = \frac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = a^{\frac{1}{6}}b^{-\frac{1}{12}}$

$= \frac{\sqrt[6]{a}}{\sqrt[12]{b}}$

1.4. 30.(a) Let $f(t)$ be the number of bacteria with respect to time t. And $f(t) = 500e^{rt}$ with some number r.

$500e^{r(t+\frac{1}{2})} = 2\cdot 500e^{re}$

$500e^{rt}e^{\frac{1}{2}r} = 2\cdot 500e^{rt}$

$e^{\frac{1}{2}r} = 2$

$\frac{1}{2}r = \log(2)$

$r = 2\log(2)$

So, $f(t) = 500e^{2\log(2)t} = 500\cdot 4^{t}$

$f(3) = 500\cdot 4^{3} = 500\times 64 = 64000/2$

$= 32000$

30. (b) $f(t) = 500 \cdot 4^{t} = 500 \cdot 2^{2t}$

30. (c) $f\left(\frac{40}{60}\right) = 500 \cdot 4^{\frac{2}{3}} = 500 \cdot 2^{\frac{4}{3}}$

30. (d) $f(t) = 500 \cdot 4^{t} = 100\,000$

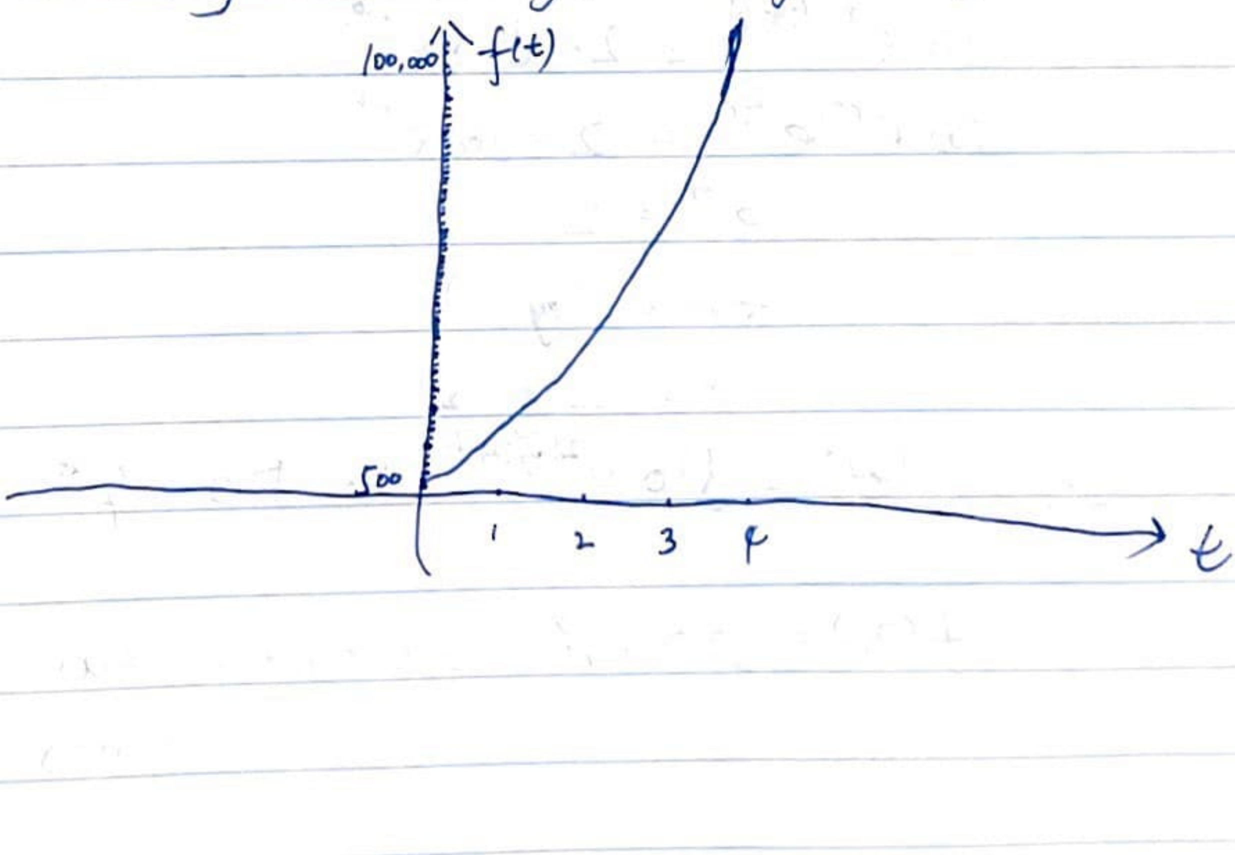
$4^{t} = 200$

We know that $2^{8} = 256$ and $2^{7} = 128$

So, $2^{7} < 2^{2t} < 2^{8}$, and $7 < 2t < 8$.

So, $3.5 < t < 4$.

Taking the average we get $t \approx 3.75$.



1.5. 24. $f(x) = 1 + \sqrt{2+3x} = y$.

For $f(x)$ to be defined in $\mathbb{R}$, $2+3x$ must be greater than or equal to 0.

$$2+3x \geq 0$$
$$3x \geq -2$$

$x \geq -\frac{2}{3}$ is the domain, $y \geq 1$ is range

Solving for x in terms of y, we get:

$$\sqrt{2+3x} = y-1$$
$$2+3x = (y-1)^2$$
$$3x = (y-1)^2 - 2$$
$$\cancel{x = \frac{1}{3}(y^2 - 2y)}$$
$$x = \frac{1}{3}(y^2 - 2y - 1),$$

$f^{-1}(x) = \frac{1}{3}(x^2 - 2x - 1)$, $x \geq 1$ is domain,

$f^{-1}(x) \geq -\frac{2}{3}$ is range

1.5. 36. (a) $\log_5 \frac{1}{125} = \log_5 (125^{-1}) = -\log_5 (125)$
$= -\log_5 (5^3) = -3$

36. (b) $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = -2$

1.5. 40. $\ln b + 2\ln c - 3\ln d$

$$= \ln(b \cdot c^2 / d^3)$$

1.5. 52. (a) $\ln(x^2-1) = 3$

$$e^{\ln(x^2-1)} = e^3$$

$$x^2-1 = e^3$$

$$x^2 = e^3+1$$

$$x = \pm\sqrt{e^3+1}$$

52. (b) $e^{2x} - 3e^x + 2 = 0$

Let $e^x = u$,

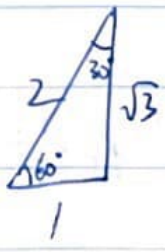
$u^2 - 3u + 2 = 0$,

$u = 1$ or $u = 2$.

When $e^x = 1$, $x = 0$,

when $e^x = 2$, $x = \log(2)$

1.5. 64. (a) $\tan^{-1}\sqrt{3}$

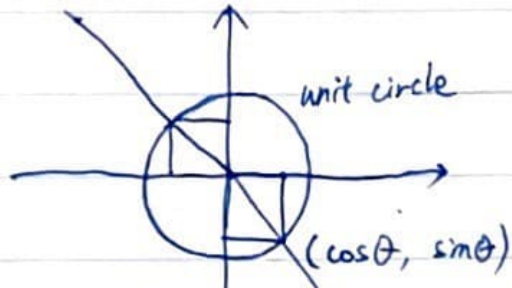


$$\sqrt{3} = \frac{\sqrt{3}}{2} \div \frac{1}{2}$$

$$= \frac{\sin 60^\circ}{\cos 60^\circ} = \tan 60^\circ = \tan\frac{\pi}{3}$$

$$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

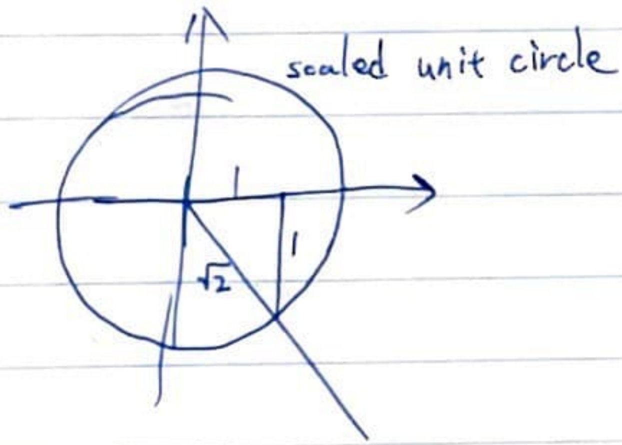
64. (b). $\arctan(-1)$



When $\theta = -\frac{\pi}{4}$,
$\tan(\theta) = -1$, arctan and arcsim have the same range, so $\text{artan}(-1) = -\frac{\pi}{4}$

~~65~~.

1.5. 66. (a). $\sin^{-1}(-1/\sqrt{2})$
$= -\frac{\pi}{4}$,
because the range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



66. (b) $\cos^{-1}(\sqrt{3}/2)$
The range is $[0, \pi]$,

$$\cos 30^\circ = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$