15.3 : 24   Z goes from 1 to 7, let $r^2 = x^2 + y^2$

when $Z = 1$, $r = 0$,

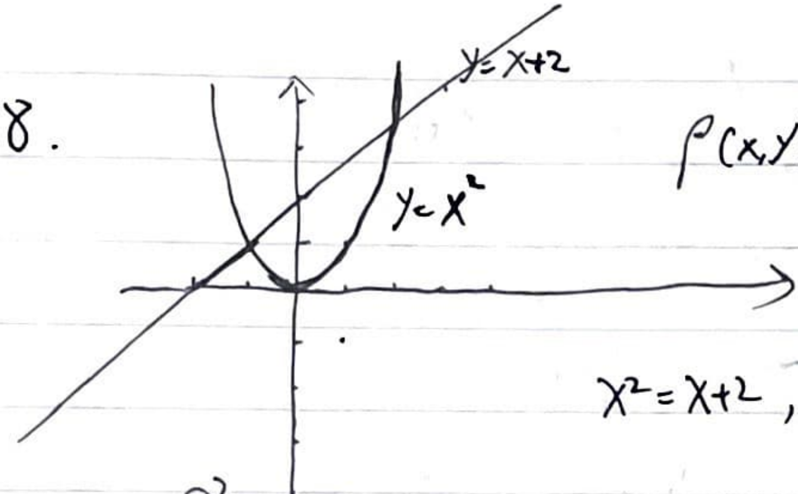
when $Z = 7$, $r = \sqrt{3}$

$$\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{3}} (1 + 2r^2)\, r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{3}} r + 2r^3 \, dr\,d\theta =$$

$$\int_0^{\frac{\pi}{2}} \left(\frac{r^2}{2} + \frac{1}{2}r^4\right)\Big|_0^{\sqrt{3}} d\theta = \int_0^{\frac{\pi}{2}} \frac{3}{2} + \frac{9}{2} \, d\theta = \int_0^{\frac{\pi}{2}} 6 \, d\theta =$$

3π

15.4 : 8.



$\rho(x,y) = kx^2$

$x^2 = x+2, \quad x=-1 \text{ or } x=2$

$$\int_{-1}^{2}\int_{x^2}^{x+2} kx^2 \, dy\, dx =$$

$$\int_{-1}^{2} \left(kx^2 y \Big|_{x^2}^{x+2} \right) dx =$$

$$\int_{-1}^{2} kx^2(x+2) - kx^4 \, dx =$$

$$\int_{-1}^{2} kx^3 + 2kx^2 - kx^4 \, dx =$$

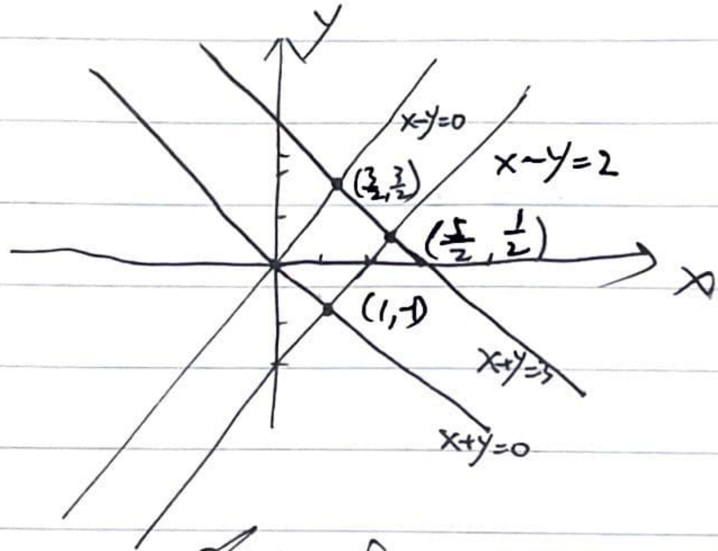
$$\left(\frac{k}{4}x^4 + \frac{2k}{3}x^3 - \frac{k}{5}x^5\right)\Big|_{-1}^{2} =$$

$$2^2 k + \frac{16}{3}k - \frac{32}{5}k - \left(\frac{k}{4} - \frac{2}{3}k + \frac{1}{5}k\right) =$$

$$\left(4 + \frac{16}{3} - \frac{32}{5} - \frac{1}{4} + \frac{2}{3} - \frac{1}{5}\right)k = \left(10 - \frac{33}{5} - \frac{1}{4}\right)k =$$

$$= \left(\frac{200}{20} - \frac{132}{20} - \frac{5}{20}\right)k = \frac{63}{20}k$$

15.9: 24



$$\begin{cases} x-y=2 \\ x+y=0 \end{cases}$$

$2x=2$

$x=1,\ y=-1$

$$\begin{cases} x+y=3 \\ x-y=0 \end{cases}$$

$2x=3$

$x=\frac{3}{2},\ y=\frac{3}{2}$

$$\begin{cases} x-y=2 \\ x+y=3 \end{cases}$$

$2x=5$

$x=\frac{5}{2},\ y=\frac{1}{2}$

$u=x+y$

$v=x-y$

$2x=u+v \qquad x=\frac{1}{2}u+\frac{1}{2}v$

$2y=u-v \qquad y=\frac{1}{2}u-\frac{1}{2}v$

$$\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Determinant is $-\frac{1}{2}$

$$\iint_R u e^{uv}\left(-\tfrac{1}{2}\right) du\, dv = \int_0^2\int_0^3 u e^{uv}\left(-\tfrac{1}{2}\right) du\, dv =$$

$$\int_0^3\int_0^2 u e^{uv}\left(-\tfrac{1}{2}\right) dv\, du = -\frac{1}{2}\int_0^3\int_0^2 u e^{uv}\, dv\, du =$$

$$-\frac{1}{2}\int_0^3 \left(e^{uv}\Big|_0^2\right) du = -\frac{1}{2}\int_0^3 e^{2u}-1\, du = -\frac{1}{2}\left(\frac{1}{2}e^{2u}-u\right)\Big|_0^3 =$$

$$-\frac{1}{2}\left(\left(\frac{1}{2}e^6-3\right)-\left(\frac{1}{2}e^0\right)\right) = -\frac{1}{2}\left(\frac{1}{2}e^6-\frac{7}{2}\right) = \frac{1}{4}\left(7-e^6\right)$$