4.1: 56. $f(t) = \frac{\sqrt{t}}{1+t^2}$, $[0, 2]$

$$f'(t) = \frac{\frac{1}{2}t^{-\frac{1}{2}}(1+t^2) - t^{\frac{1}{2}} \cdot 2t}{(1+t^2)^2} = 0$$

$$\frac{1}{2}\frac{1}{\sqrt{t}}(1+t^2) = 2\sqrt{t} \cdot t$$

$$\frac{1}{2}(1+t^2) = 2t^2$$

$$1+t^2 = 4t^2$$

$$1 = 3t^2$$

$$\frac{1}{3} = t^2$$

$$t = \pm\frac{1}{\sqrt{3}}$$

$f(0) = 0$,

$f(\frac{1}{\sqrt{3}}) = (3^{-\frac{1}{2}})^{\frac{1}{2}} \cdot (1+\frac{1}{3})^{-1} = 3^{-\frac{1}{4}} \cdot \frac{3}{4} = 3^{3/4} \cdot \frac{1}{4}$

$f(2) = \frac{\sqrt{2}}{5}$

$$f(\tfrac{1}{\sqrt{3}}) > f(2) > f(0)$$

The maximum value is $f(\frac{1}{\sqrt{3}}) = 3^{3/4} \cdot \frac{1}{4}$,

the minimum value is $f(0) = 0$

4.1: 72

$$F = \frac{\mu w}{\mu \sin\theta + \cos\theta}$$

$$\frac{dF}{d\theta} = \mu w \, (-1) \left(\mu \sin\theta + \cos\theta\right)^{-2} \left(\mu \cos\theta - \sin\theta\right)$$

$$= -\frac{\mu w (\mu \cos\theta - \sin\theta)}{(\mu \sin\theta + \cos\theta)^2} = 0$$

$$\mu \cos\theta = \sin\theta$$

$$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$F(\theta = 0) = \mu w$,

$F(\theta = \pi/2) = w$,

$F(\theta = \tan^{-1}\mu) = \mu w \left(\sqrt{1+\mu^2}\right)^{-1} = \frac{\mu}{\sqrt{1+\mu^2}} w$,

$1 + \mu^2 > \mu^2$,

$\sqrt{1+\mu^2} > \sqrt{\mu^2} = \mu$,

$\frac{\mu}{\sqrt{1+\mu^2}} < 1$,

$F(\theta = \tan^{-1}\mu) < F(\theta = \frac{\pi}{2})$

$\mu \geq 0$

$1 + \mu^2 \geq 1$

$\sqrt{1+\mu^2} \geq \sqrt{1} = 1$

$\mu \geq \frac{\mu}{\sqrt{1+\mu^2}}$

$F(\theta = 0) > F(\theta = \tan^{-1}\mu)$

$F'(\theta) = 0$ in $[0, \pi/2]$ only at $\theta = \tan^{-1}\mu$, and $F(\tan^{-1}\mu)$ is less than $F(0)$ and $F(\frac{\pi}{2})$, so $F(\tan^{-1}\mu)$ is the minimum for θ in $[0, \pi/2]$

4.2: 14 $f(x) = 1/x$, $[1, 3]$,

$f(x)$ is continuous and differentiable in $[1, 3]$.

$$\frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-1}{2} = -\frac{1}{3}$$

$$f'(x) = -x^{-2} = -\frac{1}{3}$$

$$\frac{1}{x^2} = \frac{1}{3},$$

$$x = \pm\sqrt{3}$$

$x = \sqrt{3}$ in $[1, 3]$

4.3: 50 $f(x) = \dfrac{x^2-4}{x^2+4}$

(a) $\lim_{x\to\infty} f(x) = 1$,

$\lim_{x\to-\infty} f(x) = 1$

The horizontal asymptotes is $y=1$.

There are no vertical asymptotes.

(b) $f'(x) = \dfrac{2x(x^2+4)-2x(x^2-4)}{(x^2+4)^2} = 0$

$2x(x^2+4) = 2x(x^2-4)$,

$x = 0$

$f'(x) < 0$ in $(-\infty, 0)$, $f(x)$ decreases.

$f'(x) > 0$ in $(0, \infty)$, $f(x)$ increases.

(c) $f(0) = -1$ is the minimum value.

(d) $$f'(x) = \frac{16x}{(x^2+4)^2}$$

$$f''(x) = \frac{16(x^2+4)^2 - 2(x^2+4)(2x)(16x)}{(x^2+4)^4} =$$

$$= \frac{16(x^2+4)^2 - 64x^2(x^2+4)}{(x^2+4)^4} = 0$$

$$x^2+4 = 4x^2$$
$$4 = 3x^2$$
$$\pm\frac{2}{\sqrt{3}} = x$$

$f''(x) > 0$ in $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$, $f(x)$ concave up

$f''(x) < 0$ in $\left(-\infty, -\frac{2}{\sqrt{3}}\right) \cup \left(\frac{2}{\sqrt{3}}, \infty\right)$. $f(x)$ concave down.

Inflection points are $f\left(-\frac{2}{\sqrt{3}}\right) = f\left(\frac{2}{\sqrt{3}}\right) = \frac{\frac{4}{3}-4}{\frac{4}{3}+4} = -\frac{1}{2}$

(e).