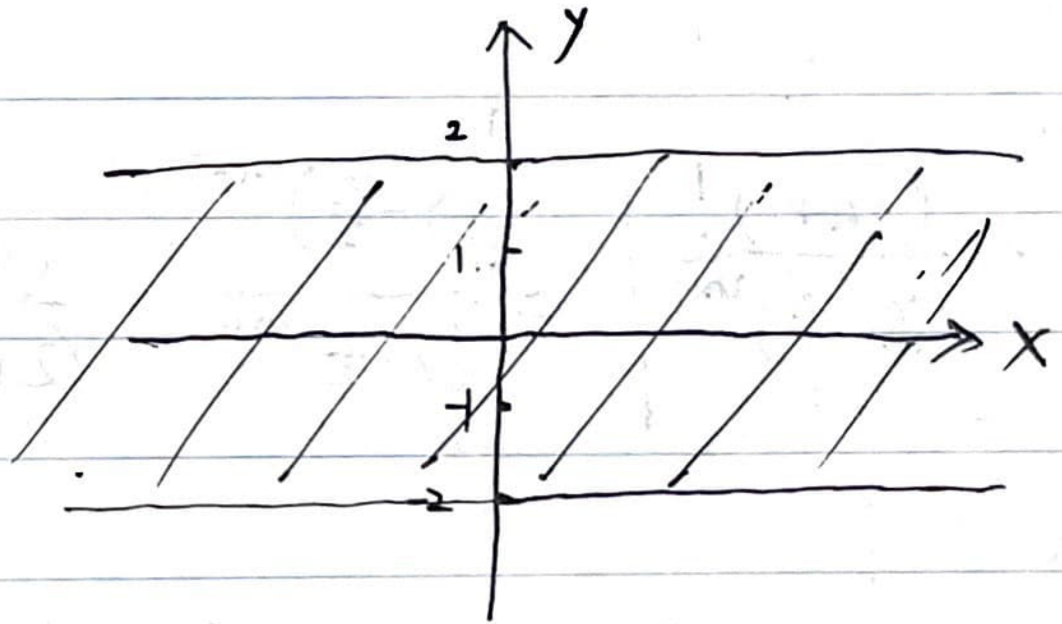
14.1 : 10. $F(x,y) = 1 + \sqrt{4-y^2}$

(a) $F(3,1) = 1 + \sqrt{4-1} = 1 + \sqrt{3}$

(b) $4 - y^2 \geq 0$, $4 \geq y^2$, $y \in [-2, 2]$,

$x \in (-\infty, \infty)$



(c) $\sqrt{4-y^2} \geq 0$, $1 + \sqrt{4-y^2} \geq 1$, $f(x,y) \geq 1$

14.2: 36 $\quad f(x,y,z) = \sqrt{y-x^2}\ \ln z$

$$\left\{ (x,y,z) \;\middle|\; y-x^2 \geqslant 0,\ \ z>0 \right\}$$

14.3: 10.

$f(2,1) \approx 10$

$f(2\tfrac{2}{3},1) \approx 12$

$(12-10)/\tfrac{2}{3} = 2\times\tfrac{3}{2} = 3$

$f_x(2,1) \approx 3$

$f(2,1) \approx 10$

$f(2,1\tfrac{7}{8}) \approx 8$

$(8-10)/\tfrac{7}{8} = 2\times\tfrac{8}{7} \approx 2.3$

$f_y(2,1) \approx 2.3$

14.3: 48. $\quad x^2-y^2+z^2-2z=4$

$f(x,y,z) = x^2-y^2+z^2-2z$

$f'(x,y,z) = 0 = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial f}{\partial z}\dfrac{dz}{dt}.$

Viewing y as a constant. $\dfrac{\partial f}{\partial x}\dfrac{dx}{dt} = -\dfrac{\partial f}{\partial z}\dfrac{dz}{dt}$, $\dfrac{\partial z}{\partial x} = -\dfrac{f_x}{f_z}$

Viewing x as a constant, $\dfrac{\partial f}{\partial y}\dfrac{dy}{dt} = -\dfrac{\partial f}{\partial z}\dfrac{dz}{dt}$, $\dfrac{\partial z}{\partial y} = -\dfrac{f_y}{f_z}$

$f_x = 2x,\ f_y = -2y,\ f_z = 2z-2$

$\dfrac{\partial z}{\partial x} = -\dfrac{f_x}{f_z} = -\dfrac{2x}{2z-2}$, $\quad \dfrac{\partial z}{\partial y} = -\dfrac{f_y}{f_z} = \dfrac{2y}{2z-2}$

14.5: 22 $\quad T = \frac{v}{2u+v}, \quad u = pq\sqrt{r}, \quad v = p\sqrt{q}\, r$

$$\frac{\partial T}{\partial p} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial p} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial p}$$

$$\frac{\partial T}{\partial q} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial q} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial q}$$

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial r}$$

$$\frac{\partial T}{\partial u} = \frac{\partial}{\partial u} v(2u+v)^{-1} = -2v(2u+v)^{-2}$$

$$\frac{\partial T}{\partial v} = \frac{\partial}{\partial v} v(2u+v)^{-1} = (2u+v)^{-1} + v(-1)(2u+v)^{-2}$$

$$\frac{\partial u}{\partial p} = q\sqrt{r}, \quad \frac{\partial u}{\partial q} = p\sqrt{r}, \quad \frac{\partial u}{\partial r} = \frac{1}{2}pq\, r^{-\frac{1}{2}}$$

$$\frac{\partial v}{\partial p} = \sqrt{q}\, r, \quad \frac{\partial v}{\partial q} = \frac{1}{2}pr\, q^{-\frac{1}{2}}, \quad \frac{\partial v}{\partial r} = p\sqrt{q}$$

$$\frac{\partial T}{\partial p} = -\frac{2v}{(2u+v)^2} q\sqrt{r} + \frac{2u}{(2u+v)^2}\sqrt{q}\, r = \frac{2u\sqrt{q}\, r - 2v q\sqrt{r}}{(2u+v)^2} =$$

$$\frac{2pq\sqrt{r}\sqrt{q}\, r - 2p\sqrt{q}\, r q\sqrt{r}}{(2pq\sqrt{r} + p\sqrt{q}\, r)^2} = 0$$

$$\frac{\partial T}{\partial q} = \left(p\sqrt{r}(-2v) + \frac{1}{2}pr q^{-\frac{1}{2}}(2u)\right) / (2u+v)^2 =$$

$$\left(pq\sqrt{r}\, pr q^{-\frac{1}{2}} - 2p\sqrt{q}\, r\, p\sqrt{r}\right) / (2u+v)^2 = \frac{p^2\sqrt{q}\sqrt[2]{r^3}}{(2pq\sqrt{r} + p\sqrt{q}\, r)^2}$$

$$\frac{\partial T}{\partial r} = \left(-\frac{1}{2}pq r^{-\frac{1}{2}} \cdot 2p\sqrt{q}\, r + p\sqrt{q} \cdot 2pq\sqrt{r}\right) / (2u+v)^2 = \frac{p^2\sqrt[2]{q^3}\sqrt{r}}{(2pq\sqrt{r} + p\sqrt{q}\, r)^2}$$