

11.6: 16.

$$\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}} \quad \lim_{n \rightarrow \infty} \left| \frac{(n+1)^{10}}{(-10)^{n+2}} \cdot \frac{(-10)^{n+1}}{n^{10}} \right| =$$
$$\lim_{n \rightarrow \infty} \left| \left(\frac{n+1}{n} \right)^{10} \cdot \frac{1}{(-10)} \right| = \frac{1}{10} < 1$$

It is convergent.

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$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1} \right)^{5n} \quad \lim_{n \rightarrow \infty} \left| \left(\frac{-2n}{n+1} \right)^{5n \left(\frac{1}{n} \right)} \right| =$$
$$\lim_{n \rightarrow \infty} \left| 2^5 \left(\frac{n}{n+1} \right)^5 \right| > 1$$

It is divergent

11.8: 16

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)2^n} (x-1)^n$$

$$\text{let } u = x-1$$

$$\lim_{n \rightarrow \infty} \left| \sqrt[n]{\frac{(-1)^n}{(2n+1)2^n} u^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{-1}{\sqrt[n]{(2n+1)2^n}} u \right| =$$

$$\lim_{n \rightarrow \infty} \left| \frac{u}{2 \cdot \sqrt[n]{2n+1}} \right| =$$
$$\left| \frac{1}{2} u \right|$$

$$-1 < \frac{1}{2} u < 1$$

$$-2 < u < 2$$

$$-2 < x-1 < 2$$

$$-1 < x < 3$$

The radius is $3 - (-1) = 4$

When $x = -1$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)2^n} (-2)^n = \sum_{n=1}^{\infty} \frac{1}{2n+1}$, which is divergent.

When $x = 3$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)2^n} (2)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n+1}$, which is convergent.

The interval is $x \in (-1, 3]$

11.8: 20. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \cdot \sqrt{n}}$ let, $u = 2x-1$

$$\lim_{n \rightarrow \infty} \left| \frac{(2x-1)^n}{5^n \cdot \sqrt{n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{u}{5 \cdot \sqrt{n}} \right| = \left| \frac{u}{5} \right| \lim_{n \rightarrow \infty} \left| \frac{1}{\sqrt{n}} \right| = \left| \frac{u}{5} \right| \cdot 0 = 0$$

$$\left| \frac{u}{5} \right| < 1 \Leftrightarrow \left| \frac{2x-1}{5} \right| < 1 \Leftrightarrow -1 < \frac{2x-1}{5} < 1 \Leftrightarrow$$

$$-5 < 2x-1 < 5 \Leftrightarrow -4 < 2x < 6 \Leftrightarrow$$

$$-2 < x < 3$$

The radius is $3 - (-2) = 5$

When $x = -2$, $\sum_{n=1}^{\infty} \frac{(2(-2)-1)^n}{5^n \cdot \sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-5)^n}{5^n \cdot \sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$,

which is convergent.

When $x = 3$, $\sum_{n=1}^{\infty} \frac{(6-1)^n}{5^n \cdot \sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, which is divergent

The interval is ~~\mathbb{R}~~ $[-2, 3]$

$$[-2, 3)$$