NLP Study Notes

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Contents

Chapter 1

Shortest-Path Algorithms and Dynamic Programming

1.1 Graphs

1.2 Dynamic programming

When designing a DP algorithm, there are two things to consider:

1. Deconstruct a big problem into smaller (recursive) sub-problems.
2. Store intermediate results.

1.2.1 DP coding problems

- Nth Fibonacci Number

- Longest Increasing Sub-sequence
- Coin Change

1.3 The Viterbi algorithm

Chapter 2

Logistic Regression

2.1 The importance of establishing a baseline

We draw a function that shows decreased marginal accuracy with increasing model complexity. From this graph, we observe an upper limit. This limit helps us making informed decisions like:

1. Is this project feasible? (the requirement is 75% accuracy but the upper limit is 72%.)
2. Is it cost-effective to add model complexity?

Furthermore, if we use a complex model upfront without setting a baseline but the accuracy is bad, then it's hard for us to tell whether there was a mistake when building the model or it's because the problem is too complex.

2.2 Understanding LR

graph of 1d data draft*

Why sigmoid?

2.3 From likelihood to cost function

The likelihood function is defined as $l(\theta|D) = f(D|\theta)$. f can be either a PMF or a PDF. $|$ is used instead of ; because we employ the Bayesian view (not frequenist) and see θ as a random variable. l is a function of θ and doesn't integrate to 1 (with respect to θ).

The likelihood function of logistic regression is

$$\prod_{i=1}^{n} \sigma(wx_i + b)^{y_i} (1 - \sigma(wx_i + b))^{1-y_i}.$$

(see derivation) Maximizing the likelihood is equal to minimizing the negative log-likelihood:

$$cost(w, b) = -\sum_{i=1}^{n} y_i \ln \sigma(wx_i + b) + (1 - y_i) \ln(1 - \sigma(wx_i + b)).$$

And we get KL divergence, or binary cross-entropy, which is convex. (Why is it convex?)

2.3.1 On KL-divergence and cross-entropy

2.3.2 Logistic loss

If the outcome space is $y = \{-1, 1\}$ instead of $y = \{0, 1\}$, then

$$p(y_i = 1|f(x_i)) = \sigma(f(x_i)) = \frac{1}{1 + e^{-f(x_i)}}$$
$$p(y_i = -1|f(x_i)) = 1 - \sigma(f(x_i)) = \frac{1}{1 + e^{f(x_i)}}.$$

In both cases

$$p(y_i|f(x_i)) = \frac{1}{1 + e^{-y_i f(x_i)}}.$$

The negative log-likelihood is

$$\sum_{i=1}^{n} \log(1 + e^{-y_i f(x_i)}).$$

Which is called the log loss/logistic loss and it's the same thing as the cross-entropy loss.

2.4 Implement LR with mini-batch GD

The cost function can't be solved analytically, hence we use gradient descent. The derivative of the sigmoid function is:

$$\sigma(x)(1-\sigma(x)).$$

Knowing this facilitates the calculation of the gradient:

$$\frac{\partial l(w,b)}{\partial w} = \sum_{i=1}^{n}(\sigma(wx_i+b)-y_i)x_i$$
$$\frac{\partial l(w,b)}{\partial b} = \sum_{i=1}^{n}\sigma(wx_i+b)-y_i.$$

Now we update the parameters:

$$w^{t+1} = w^t - \eta_t \sum_{i=1}^{n}(\sigma(wx_i+b)-y_i)x_i$$
$$b^{t+1} = b^t - \eta_t \sum_{i=1}^{n}\sigma(wx_i+b)-y_i.$$

Now we've got the updates using GD. The updates using mini-batch GD and stochastic GD become apparent. The former is:

$$w^{t+1} = w^t - \eta_t \sum_{x_i,y_i \in batch}(\sigma(wx_i+b)-y_i)x_i$$
$$b^{t+1} = b^t - \eta_t \sum_{x_i,y_i \in batch}\sigma(wx_i+b)-y_i.$$

Between GD and stochastic GD, mini-batch GD finds the balance between robustness and efficiency. Moreover, it works well with GPU, and it helps escaping the saddle point.

code draft*

Chapter 3

Generalization

3.1 When w goes to infinity

When the problem is linearly separable, as w goes to infinity:

$$\lim_{w\to\infty} p\left(y_i = 1|x_i; w, b\right) = \lim_{w\to\infty} \frac{1}{1+e^{-(wx_i+b)}} = 1 \; for \; wx_i + b > 0,$$
$$\lim_{w\to\infty} p(y_i = 0|x_i; w, b) = \lim_{w\to\infty} \frac{e^{-(wx_i+b)}}{1+e^{-(wx_i+b)}} = 0 \; for \; wx_i + b < 0.$$

At this time, MLE is the largest:

$$MLE = \arg\max_{w,b} \prod_{i=1}^{n} p\left(y_i = 1|x_i; w, b\right)^{y_i} p\left(y_i = 0|x_i; w, b\right)^{1-y_i}.$$

It is consistent with our goal of maximizing the likelihood function to aim for a large w. For a linearly separable problem, w doesn't converge, and regularization gives bounded solution.

For a non-linearly separable problem, w can converge (mathematically, why?). But when there are too many features, the non-separable becomes the separable, again, w goes to infinity, and uncertainty regions shrink to 0. At this point, limiting the

magnitude of w leads to better generalization and gives back uncertainty regions. How are all these happening? 1 2 Graphically, higher degree terms variables with smaller w doesn't disappear, but go 'out of range', e.g. $y = 6x_1 + 3x_2^2$ vs $y = 6x_1 + 0.1x_2^2$. draft*

We don't discuss feature selection here, why don't we just use feature selection? Is there an algorithm for separability testing?

3.2 L1 and L2 regularization

3d geometric moving representation of l1 and l2 and why l1 makes some parameters 0. draft*

There are some disadvantages of l1 regularization:

1. It's not differentiable everywhere, so gradient descent doesn't work, in this case we can use subgradient descent (I don't need to know the details).

2. When a group of collinear features exist, it randomly selects one feature, but we want the best feature. The lecturer says using elastic net can counter this problem but I don't know how. It's another topic. draft*

3.3 K-fold CV

When dataset is small, we can increase k. One extreme case is leave-one-out CV.

3.4 MLE, MAP and L1, L2

MLE:

$$p(D|\theta).$$

MAP:

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} \propto p(D|\theta)p(\theta).$$

MAP estimator:

$$\theta_{MAP} = \arg\max_{\theta} \ prior \ \cdot \ likelihood.$$

Assume prior is $p(\theta) \sim N(0, \sigma^2)$,

$$\begin{aligned} p(\theta) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\theta^2}{2\sigma^2}\right) \\ &\propto \exp\left(-\frac{\theta^2}{2\sigma^2}\right), \\ \arg\max_{\theta} \ \log(p(\theta)) &= \arg\max_{\theta} \ \log\left(\exp\left(-\frac{\theta^2}{2\sigma^2}\right)\right) \\ &= \arg\max_{\theta} \ -\frac{\theta^2}{2\sigma^2}, \\ \theta_{MAP} &= \arg\min_{\theta} -\log \ likelihood + \frac{1}{2\sigma^2}\theta^2. \end{aligned}$$

This looks very familiar. MAP estimator with Gaussian prior equals adding a l2 regularization term to the cost function (and how does the λ coefficient relates to the variance? draft*).

Similarly when $p(\theta) \sim Laplace(0, b)$, the resulting cost function is added by l1 term.

Chapter 4

Theory of Convex Optimization

4.1 Mathematical optimization

A mathematical optimization problem has the form:

$$\begin{aligned} &\text{minimize } f_0(x) \\ &\text{subject to } f_i(x) \leq b_i,\ i = 1, 2, \ldots, m, \end{aligned}$$

in which $f_0, f_1, \ldots, f_i$ are functions that map R^d to R.

In a linear programming problem, $f_0, f_1, \ldots, f_i$ are linear, which means

$$f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y).$$

For a convex optimization problem, we have

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y).$$

, with $\alpha \geq 0, \beta \geq 0$ and $\alpha+\beta = 1$. Comparing linear programming problem and convex optimization problem, because convexity is less restricted, all linear problems are convex optimization problems. (I have problem understanding this, this link helps. draft*)

4.2 Least-squares, linear programming and convex optimization

Generally, mathematical optimization problems are hard to solve, but we present 3 exceptions in this section.

4.2.1 Least-squares

4.2.2 Linear programming

4.2.3 Convex optimization

4.3 Non-linear optimization

Non-linear problems are optimization problems that are not linear, but not known to be convex.

Finding a global minima is time-consuming, and finding a local minima is more of an art than a science.

Formulating a non-linear problem is relatively straightforward, but the difficulty lies in solving it. While for a convex optimization problem, solving it is straightforward, but the challenge is in problem formulation.

Chapter 5

SVM

5.1 Decision theory and the statistical learning framework

input space: X
action space: A
output space: y
loss:

$$l : A \times y \to R$$

decision function:

$$f : X \to A$$

risk:

$$R(f) = E[l(f(x), y)]$$

Bayesian decision function:

$$f^* = \arg\min_f R(f)$$

empirical risk:

$$\hat{R}_n(f) = \frac{1}{n}\sum_{i=1}^{n} l(f(x_i), y_i)$$

empirical risk minimizer:

$$\hat{f} = \arg\min_f \hat{R}_n(f)$$

hypothesis space: F
constrained empirical risk minimizer:

$$\hat{f}_n = \arg\min_{f\in F} \hat{R}$$

constrained risk minimizer:

$$f_F^* = \arg\min_{f\in F} R.$$

5.2 Define SVM

input space: R^d
action space: R
output space: $\{-1, 1\}$
hypothesis space:

$$F = \{f(x) = w^T x | w \in R^d\}$$

loss/hinge loss:

$$l\left(f(x), y\right) = \max(1 - y_i f(x_i), 0) = \left(1 - y_i f(x_i)\right)_+$$

l2 regularization:

$$min_{w\in R^d} \sum_{i=1}^{n} \left(1 - y_i f_w(x_i)\right)_+ + \lambda \|w\|_2^2,$$

How does minimizing w relates to minimizing the risk function? draft*
Can we derive the loss function from a probability distribution like we did for logistic regression? draft*
$f_w(x_i)$ is called the score, and $y_i f_w(x_i)$ is called the margin.

graph of logistic loss, hinge loss, perceptron loss, zero-one loss, square loss with respect to margin draft*

5.3 Duality

5.4 Kernel

Chapter 6

Naive Bayes, Decision Tree and Random Forest

6.1 Naive Bayes

Chapter 7

Text Processing

The text analytic workflow:

raw data → word segmentation → cleaning (stopwords, lowercase, special character) → normalization (stemming, lemmazation) → feature extraction (tf-idf, word2vec) → modeling (classification, similarity measures)

7.1 Segmentation

7.1.1 Tools for Chinese words segmentation

- Jieba: https://github.com/fxsjy/jieba
- SnowNLP: https://github.com/isnowfy/snownlp
- LTP: http://www.ltp-cloud.com/
- HanNLP: https://github.com/hankcs/HanLP/

7.1.2 Max matching algorithm