UNIVERSIDAD VERACRUZANA

FACULTAD DE FÍSICA E INTELIGENCIA ARTIFICIAL

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Filtro de entropía aplicado a series de tiempo de mercados financieros

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Tesis

Filtro de entropía aplicado a series de tiempo de mercados financieros

Versión a corrección

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Resumen

En el presente trabajo de tesis se presentan y discuten los resultados obtenidos tras aplicar un filtro de entropía en series de tiempo de mercados financieros.

Las técnicas de procesamiento de datos empleadas son básicamente dos, medias móviles y entropía. Las medias móviles son empleadas para poder interpretar de una mejor manera los datos tras aplicar un filtro de entropía ya que permiten ajustar una ventana temporal de los mercados. Los indicadores de precios al mercado utilizados fueron Dow Jones, DAX Performance, BMV IPC, y Nikkei 225.

Las medias móviles son mejor conocidas por los analistas de datos como Moving Average o MAV, la cual obtiene un promedio de un determinado conjunto de datos y esto a su vez permite aminorar el ruido de la serie de tiempo, para que al aplicar el filtro de entropía el resultado obtenido no manifieste un mayor número de máximos o un comportamiento mayormente uniforme.

La selección de las ventanas de tiempo se hace manualmente por el autor, las cuales se pueden realizar para cualquier ventana de tiempo bajo dos régimenes, el filtrado de entropía en los precios del mercado a través de un Moving Average, o el filtrado de entropía en los precios del mercado sin un Moving Average.

Se utilizó software que permitiera hacer los cálculos y un análisis más preciso en menor tiempo, esto permitió identificar que es posible ajustar distribuciones estadísticas a los resultados obtenidos para la distribuciones después de haber aplicado un filtro de entropía con o sin medias móviles.

Motivación

La búsqueda de avances en la ciencia y el progreso económico de las diferentes naciones viene muchas impulsado por intereses particulares, a ellos les resulta de suma importancia estar a la vanguardia implementando los avances científicos a la tecnología, a la industria y a la economía. Aunque esta última se ve directamente beneficiada por el progreso de las dos anteriores, existe una gran cantidad de personas interesadas en el estricto funcionamiento de la economía y su progreso, que pese a estar muchas veces sujeto a intereses que van desde gobiernos, hasta sencillamente individuos estudiosos de la materia, que de igual manera buscan implementar el conocimiento de Física, Estadística o Matemáticas como herramientas para alcanzar sus diferentes objetivos, lo cual resulta de suma importancia en estos tiempos, dado que ahora se cuenta con el poder computacional necesario como para probar teoremas matemáticos, o incluso diseñar algoritmos que automaticen una predicción en tiempo real sobre los mercados.

Objetivos generales

- Aplicar un primer filtro de entropía en diferentes mercados para identificar la distribución estadística que mejor se ajuste.
- Aplicar un segundo filtro de entropía en diferentes mercados tras utilizar medias móviles para identificar la distribución estadística.
- Obtener las fechas en la serie de tiempo que manifestaron menor entropía para realizar estudios posteriores de análisis predictivo..

Contents

1 Introducción

Se puede asumir que para tener un mayor desarrollo humano hay que poseer avances en la ciencia, con base en un entendimiento de las limitaciones que existen hoy en día, para así poder revolucionar la tecnología, de manera que al tener un nuevo avance tecnológico se dé pie a nuevos avances científicos, grosso modo es así como se genera un círculo virtuoso, no obstante el trasfondo es más meritorio que codicioso.

Si bien la economía mundial se mueve a través de intereses particulares de las diferentes naciones, empresas, casas de bolsa, mercados o incluso de los intereses propios de cada individuo, la Física no es precisamente motivada por los mismos intereses, ya que al tratarse de una ciencia natural, las leyes de la naturaleza y en particular las leyes físicas que describen la naturaleza del universo son asunto interés común, y es ahí donde al unir conocimientos de propiedad universal a la economía puede parecer incluso utópico.

Es bien sabido que el estudio de la naturaleza y todo lo que le conforma ha partido de los principios básicos del razonamiento humano, hasta llegar a comprender la dinámica de los objetos que hay alrededor de cada individuo, conforme se concibe dicha descripción física de un fenómeno se puede profundizar más en el comportamiento del fenómeno, gracias a un entero entendimiento del movimiento de los cuerpos en el universo se ha podido comprender la naturaleza del movimiento en cuerpos de mayor tamaño, como lo son planetas, también se ha podido ir un paso más allá y comenzar a interpretar fenómenos tales como el modelado de un sistema de partículas con carga, e incluso adentrarse en el mundo microscópico y modelar fenómenos a escalas que resultaban difíciles de imaginar hace un siglo, y lo que es más meritorio es que hoy en día es posibile estudiar grandes cantidades de información y comprender su comportamiento.

En diferentes ámbitos, desde los microscópico-macroscópico, hasta la complejidad en el estudio de los modelos de agentes, la Física se ha encargado de explicar los fenómenos que competen a la naturaleza del universo, la Economía no es la excepción, puesto que es un fenómeno más en el que la humanidad se ve envuelta día a día, tal como lo es la termodinámica del medio en que los sere humanos habitamos el planeta, la economía es también algo de lo que nadie está exento de su sustancial carácter en el desarrollo de nuestra especie.

Desde la teoría económica de Adam Smith en 1767 (Nordhaus, Samuelson, 2004), hecho que revolucionó a la economía, y es por ello que se le conoce como el padre de la economía moderna, puesto que al día de hoy mucho de su legado se sigue aplicando de una u otra manera, evidentemente con variantes pero existe, hasta el más reciente concepto de entropía (entropía de la información o de Shannon) se encuentra particular interés en la unión de dos ciencias que describen el funcionamiento de los sistemas en que nuestra especie se desenvuelve, ya que con las herramientas de una ciencia se puede explicar cualitativa y cuantitavamente un fenómeno que pareciera ser exclusivo de una sola ciencia.

Aunque no es estrictamente necesario entender los conceptos del área de economía, es fundamental entender que los mercados son sistemas formados por un número muy grande de componentes, o participantes, cada uno viendo por su propio interés, sin saber que al hacerlo están promoviendo un concepto conocido como *la mano invisible*. Al tratarse de un fenómeno donde se experimenta alta competitividad por parte de los otros internautas, y aún conociendo el funcionamiento de los indicadores de los mercados, o estando informado sobre los acontecimientos que influyen en las decisiones de los "traders", existe una gran incertidumbre sobre si es posible predecir una tendencia que sea favorable, esto debido al gran volumen de información que muestra una serie de tiempo financiera.

Con esto en mente esta tesis esta organizada de la siguiente manera:
En el capítulo 1 se introduce al tema de la econofísica donde se tratarán los fundamentos que conforman a esta nueva rama de estudio, y se mencionan conceptos básicos de economía.

En el capítulo 2 se presentan las características de los mercados estudiados, y se describe como funcionan los mercados con base en la perspectiva de la econofísica, planteando el objetivo general de este trabajo, así como los objetivos particulares.

En el capítulo 3 se explica el método empleado para la obtención de entropía de los mercados, se describe brevemente la manera de obtener la mínima entropía de las series de tiempo financieras.

Y finalmente en el capítulo 4 se presentan los resultados y conclusiones de este trabajo.

Fundamentales de econofísica

2

Un sistema físico macroscópico no puede ser descrito solo en términos de variables mecánicas, la llamada entropía era necesaria
— **P. Richmond, J. Mimkes, S. Hutzler** (2013)

El contenido de los conceptos que se desarrollan a continuación son manifestados en el orden en que se tenga el mayor entendimiento de este trabajo de tesis, no debe confundirse la nomenclatura, ya que 1. se ocupa para todo aquello relacionado directamente con *econofísica* y 2. se ocupa para todo aquello relacionado con *Economía*.

1.1.1. Econofísica

Término acuñado por Eugene Stanley en 1997 para referirse a una rama de la Física que a partir de encontrar relaciones entre leyes físicas y variables macroscópicas propias de la Economía, busca un entendimiento más profundo de los fenómenos que en esta se presentan en los mercados.(Bertrand M. Roenher, Patterns of Speculation)

2.1.1. Economía

Es el estudio de cómo las sociedades utilizan recursos escasos para producir bienes valiosos y distribuirlos entre diferentes personas (Samuelson & Nordhaus, 2005).

1.2. Agentes

Son los individuos, compañías, países, casas de bolsa, familias, comerciantes, inversionistas, capaces de interactuar entre sí de una manera no trivial, que además se encuentran inmersos en un mecanismo auto-organizado pese a poseer intereses particulares cada uno.

1.3. Modelo de agentes

Estructuras presentes en cualquier ciencia, mismas que pese a manifestar fenómenos diametralmente diferentes poseen datos que exhiben características similares tales como; un número N de agentes, la estadística no Gaussiana en cortos periodos de tiempo, adaptabilidad, autocorrelaciones, autointeracción, y la inaccesibilidad al análisis de un solo agente en particular.

1.3.1. Modelo de agentes en econofísica

Sistemas propuestos con base en una estructura Física, y en algunas ocasiones son propuestos únicamente de manera teórica, en los que se plantea que los N agentes (partículas) se rijan bajo una distribución que les permite la interacción entre ellos mismos y a su vez la conservación de la energía, si así fuese el caso.

1.4. Sistema complejo

Compuesto por un número muy grande de agentes, los elementos que le conforman poseen dinámica en sus interacciones entre sí, esto con base en los grados de libertad que el modelo en cuestión les permita, de tal manera que sus interacciones no son triviales, sin embargo, aunque en ocasiones pueda manifestarse como un sistema caótico, también contiene regularidades como lo es la auto-organización .

2.2. Precio

Cuota requerida para el proceso de adquisición, o para que se lleve a cabo un trato de concesión de un bien. Dependiendo del contexto en el modelo de agentes y del sistema complejo que se trate, puede ser una variable dinámica en la que haya intermediarios que le modifican directa e indirectamente.

1.5. Precio en econofísica

En este contexto encontramos que es la unidad en que se «mide»una variable dinámica conformada por N agentes de un respectivo modelo, mismos que disponen de una gama de posibilidades con respecto a los grados de libertad de dicho entorno en que se lleven a cabo los tratos de adquisición, o concesión de un bien (interacciones). Los N agentes causantes de dicha variabilidad pueden ser pertencientes o no a dicho modelo, estas interacciones entre agentes internos o externos tienen una retroalimentación cuyo efecto es la principal causa de que su comportamiento sea el de un sistema complejo.

2.3. Dinero

Cada agente *i* posee una simbólica suma de recurso que utiliza para adquirir bienes o comodidades de consumo, su unidad de medida es la «moneda »aunque esta es una unidad arbitrariamente divisible. La moneda perse no puede ser producida ni consumida por los actores (Cockshitt W. P, Cottrell A. F., Michaelson G. J., Et Al, 2009). Los agente intercambian *dinero* y obtienen un incremento cuando llevan a cabo tratos de venta o cesión, y obtienen un detrimento cuando llevan a cabo un trato de adquisición o compra de un bien.

2.4. Economía financiera

Rama de la economía que analiza cómo los inversionistas racionales deben invertir sus bienes para alcanzar sus objetivos de la mejor manera posible (Samuelson & Nordhaus, 2005).

2.5. Hechos estilizados

Aunque no se profundiza en este concepto, vale la pena mencionar que este se refiere a que la experiencia recabada en econofísica relaciona una gran cantidad de datos de varias actividades que realiza la economía. Existen ciertas características universales intrínsecas (por no llamarles leyes) a los diferentes sistemas complejos que se traten, independientemente de las consecuencias del intercambio de bienes, o de la región en el planeta que uno se encuentre, o incluso dentro de una razonable brecha de observación hay independencia del tiempo también (Slanina, F. 2014).

Ejemplos de lo anteriormente descrito puede ser la cantidad de votos que tiene un candidato a la presidencia de un país, o la cantidad de gazapos que tiene una hembra conejo. Hay modelos que tienen por objetivo es explicar y predecir el comportamiento de fenómenos, en este caso, económicos, cuya tasa de acierto al ser elevada les permite ser relacionados con los llamados *hechos estilizados*, lo cual a su vez incrementa su grado de confianza.

2.6. Mercado

Un mercado es un mecanismo a través del cual compradores y vendedores interactúan para determinar precios e intercambiar bienes y servicios (Samuelson & Nordhaus, 2005).

2.7. Mercado financiero

Sistemas organizados para la compra-venta de acciones, divisas, opciones, bonos, derivados, etc. (Hernández Montoya, 2018).

Mercados cuyos productos o servicios son instrumentos financieros tales como acciones y bonos(Samuelson & Nordhaus, 2005).

1.6. Mercado financiero en econofísica

Sistemas conformados por un número muy grande de agentes que interactúan unos con otros, mismos que reaccionan a información tanto interna como externa a su entorno para determinar el mejor precio según sea el caso.

2.1.2. Economía en el contexto de econofísica

Le concierne el comportamiento de las personas así como las decisiones que tienen que tomar bajo debidas restricciones. (Hutzler, Mimkes & Richmond,2013).

Trata las cuestiones que surgen generalmente como resultado intrínseco de estos mecanismos, es decir, la toma de decisiones de un sistema conformado por agentes que llevan a cabo tratos de inversión, compra o venta, dichas cuestiones son denominadas *endógenas*, mientras que las *exógenas* son todas aquellas que si bien no están inmersas en el mecanismo en cuestión, siguen formando parte del sistema complejo, las cuales inlcuyen desde noticias de alto impacto como lo son revoluciones, el inicio de una guerra, desastres naturales, o especulaciones en los mercados financieros.

Si bien, este trabajo de tesis no se centra en economía, si resulta de suma importancia saber a qué rama de la economía se está estudiando, por esa razón se define *grosso modo* la **macroeconomía** y la **microeconomía** a continuación.

2.8. Microeconomía

Estudia la naturaleza y el comportamiento dinámico de los elementos individuales, como consumidores, dueños de compañías, consejos de administración, quienes interactúan para formar, por ejemplo, precios (Hutzler, Mimkes & Richmond,2013).

Análisis que explica el comportamiento de elementos individuales de una economía, tales como la determinación del precio de un solo producto o el comportamiento de un solo consumidor o empresa (Samuelson & Nordhaus, 2005).

2.9. Macroeconomía

Considera el comportamiento agregado de consumidores, dueños de compañías, consejos de administración, para describir cantidades como el ingreso doméstico bruto, y el nivel de empleo (Hutzler, Mimkes & Richmond,2013).

Análisis que trata el comportamiento de la economía en su totalidad con respecto al producto, el ingreso, el nivel de precios, el comercio internacional, el desempleo, y otras variable económicas agregadas. (Samuelson & Nordhaus, 2005).

1.7. Circuitos económicos

No abundaremos mucho en este tema ya que escapa del objetivo principal de este trabajo de tesis, sin embargo es importante concebir a los bienes, derivados o acciones, y más concretamente al *dinero* como la **energía** de un sistema complejo en un contexto meramente financiero, a este tipo de estructuras se les conoce como *sistemas económicos*[1].

Un **circuito económico** es un sistema que puede ser modelado de diferentes maneras, el cual está compuesto por ciclos que pueden ser tan largos como sea permitida la entrada de energía (Hutzler, Mimkes & Richmond,2013), tomando en cuenta que el balance de energía al final de cada ciclo nunca debe ser negativo. En este trabajo de tesis, se trata con *circuitos monetarios* cuya **energía** es el *precio* de cierre diario de los mercados.

Cabe destacar que conceptos como el de **esperanza** o **media** no han de sorprender, ya que la esperanza $E(\mathrm{x})$ es un concepto mejor conocido en Física por el nombre de *valor esperado*, de manera análoga y bajo este mismo contexto la media poblacional es la mejor conocida como μ. En aras de no escasear la información que solidifica este trabajo de tesis, se definen a continuación.

[1] Véase Econophysics & physical economics,Hutzler, Mimkes & Richmond,2013

1.8. Media de una distribución

La media μ de una distribución de probabilidad $P(x)$para un conjunto finito de valores numéricos $\boldsymbol{x}$, también llamada *esperanza*, *valor esperado* o coloquialmente, *media*, es el promedio de los valores $\boldsymbol{x}$ multiplicados por las probabilidades de cada uno:

$$\mu = \sum_{all\ \boldsymbol{x}} \boldsymbol{x}\, P(x)$$

(Probability, Jim Pitman 1993, pg. 162, 163)

Si bien el concepto anterior puede entenderse perfectamente para obtener la media de una distribución de valores discretos, también existe la definicion de mediana para valores continuos. A partir de la distribución *X*, sea pues *f(x)* una función de densidad para una variable, el valor esperado *E(X)* a menudo denotado solo como μ o μ_x es:

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

(Time Series Analysis, Cryer Jonathan, Chan Kung-Sik, 2008)

1.9. Mediana

Sea *m* un número que forma parte de la distribución de *X*, y sea *x* un valor númerico que forma parte de la distribución, la *mediana* es un número *m* tal que tanto la probabilidad de obtener un número *x* menor o igual que *m* y la probabilidad de obtener un número *x* mayor o igual que *m* es igual a $1/2$. (Probability, Jim Pitman 1993, pg. 165)

1.10. Moda

Sea *X* una distribución, entonces la *moda* es el valor mas probable de *X*, y puede haber más de uno valor. (Probability, Jim Pitman 1993, pg. 165)

1.11. Variable aleatoria

Una variable aleatoria *y* es una función real evaluada definida en el espacio muestra Ω tal que para cada número real c, $A_c = \{\ \omega \in \Omega \mid y(\omega) \leq c\ \} \in \mathcal{F}$, donde $\mathcal{F}$ es una suma álgebraica de eventos o subconjuntos de Ω. En otras palabras, A_c es un evento para el cual la probabilidad es definida en términos de P_r que es una medida de probabilidad definida en $\mathcal{F}$. Por completez, la función $F : \mathbb{R} \to [0,1]$, definida por $F(c) = P_r(A_c)$ es la función distribución de *y*. (New Introduction to Multiple Time Series ,Lütkepohl, H. 2005, pg. 2)

1.12. Varianza

Sea *X* una variable aleatoria. La varianza de *X* denotada como *Var(X)* es el cociente de la división entre la raíz cuadrada de la diferencia entre el valor de la variable aleatoria *X* y la media μ del valor *X* al cuadrado, dividida entre el número de observables. Esto es:

$$\mu = \mu_x = E(X)$$
$$E[(X - \mu)^2]$$

(Probability, Jim Pitman 1993, pg. 185)

1.13. Distribución normal

Hasta ahora se ha tratado el concepto de distribución de manera intuitiva y se ha etiquetado como *X*, si bien, puede trabajarse solo con uno de los elementos que le componen, es decir, un elemento *x* o x_i, así como también puede ser tratada como un todo, o sea, cuando *X* = *x*, en cualquier caso, en el presente trabajo de tesis se trabaja con una *distribución normal*.

La **distribución normal** es una curva normal que se representa por encima del eje x, el área bajo dicha curva normal se rige por los parámetros de la media μ y desviación estándar σ.

$$y = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-z^2}{2}}$$

Donde $z = \frac{x-\mu}{\sigma}$ mide el número de desviaciones estándar desde la media μ hasta el número *x*. Esta distribución posee media cero y desviación estándar uno.(Probability, Jim Pitman 1993, pg. 94)

1.14. Estandarización

Proceso aplicado a cada uno de los elementos que pertenecen a la distribución cuyo resultado es la variable aleatoria $z = \frac{x-\mu}{\sigma}$ y como consecuencia se obtiene media cero y desviación estándar uno. (Time Series Analysis, Cryer Jonathan, Chan Kung-Sik, 2008)

1.12. Proceso estacionario

Se manifiesta cuando las configuraciones de un sistema permanecen inalteradas en el tiempo, y en *Estadística* el concepto es análogo, ya que lo define como «aquel en que las leyes de la probabilidad que gobiernan el comportamiento del proceso no cambian en el tiempo »(Chan & Crier, 2008).

1.13. Proceso estocástico

Se define por familias de variables aleatorias $X(t;\omega)\} \mid t \in \mathbb{T}$, para un conjunto indicado $\mathbb{T}$ (Hassler, U. 2016) :

$$\begin{aligned} X : \mathbb{T} \times \Omega &\to \mathbb{R} \\ (t\,;\,\omega) &\mapsto X(t;\,\omega) \end{aligned}$$

Cabe mencionar que $t \in \mathbb{T}$ en el contexto de este trabajo de tesis debe ser interpretado como el «tiempo», de tal manera que cuando uno ocupa específicamente un punto o dato en el tiempo t_o el proceso estocástico devuelve simplemente una variable aleatoria,

$$\begin{aligned} X : \Omega &\to \mathbb{R} \\ \omega &\mapsto X(t_o;\,\omega) \end{aligned}$$

Es decir que se puede referir a un resultado ω_o en un conjunto, o una «trayectoria »realizada por el proceso(Hassler, U. 2016).

Por su parte, un proceso estocástico **estacionario** $x(t)$ es aquel proceso estocástico cuya función de densidad de probabilidad (FDP) $P[x(t)]$ es invariante bajo un cambio de tiempo (Mantegna R. & Stanley H., 2004).

En síntesis, es también un objeto *complejo*. Para caracterizarlo matemáticamente, se dispone de vectores aleatorios de longitud finita *n* con puntos arbitrarios en el tiempo $t_1 < ... < t_n$, así, matemáticamente se considera[2]:

$$X(t_i) \equiv (X(t_1;\,\omega), ..., X(t_n;\,\omega))' \quad , \quad t_1 < \cdots < t_n$$

[2] (Hassler, U. 2016).

1.14. Serie de tiempo

Conjunto de variables medidas secuencialmente en el tiempo (Cowpertwait P. & Metcalfe A, 2009), registradas mediante intervalos de separación entre medición y medición. Su esencia es de un proceso estocástico ya que es fácil reconocer que las observaciones realizadas son impredecibles en el tiempo.

Las series de tiempo financieras conllevan una gran cantidad de información que no es redundante. La cantidad de información es tan grande que es difícil extraer un subconjunto de información económica asociada a algún aspecto (Mantegna R. & Stanley H., 2004).

Para entender o modelar mecanismos que puedan manifestar comportamientos estocásticos para predecir futuros valores con base en el historial que manifiesta, o con otras series de tiempo cuyos factores son relacionados (Chan & D. Cryer, 2008).

En otras palabras, es una secuencia de observaciones a una cantidad o cantidades. (Ruppert D. & Matteson D, 2015).

Al utilizar series de tiempo el objetivo es obtener lo que se conoce como
«*ticks* » que en un sentido muy general representa a cualesquiera variables cotizadas de cualquier origen de cualquier instrumento financiero (Dacoragna M.M., Et Al, 2001), vale la pena mencionar que gracias al poder computacional es posible registrar cada evento (transacción o trato) minuto a minuto, de aquí que en el lenguaje coloquial se utiliza la expresión «tick a tick »o el también conocido en inglés*data-tick* (Hutzler, Mimkes & Richmond,2013). La secuencia temporal de estos *ticks* es *inhomogénea* en caso de que los intervalos varíen en tamaño, naturalmente si la serie de tiempo cuyos *ticks* registrados son de manera que sus intervalos permancen constantes, se le conoce como *homogénea*.

1.15. Retornos

Los retornos son usualmente variables de análisis más rentables que el precio perse, por varias razones. La distribución de los retornos es más simétrica y estable en el tiempo que la distribución de los precios en sí. La estructura de los retornos es cercana a estacionaria, mientras que la estructura de los precios observados en el tiempo no lo es (Dacoragna, Et Al, 2001).

El logaritmo de los precios muestra que están normalmente distribuídos, al menos aproximadamente (M.F.M. Osborne, 1959). En este trabajo de tesis se trabaja con los retornos de los registros diarios individuales de cada serie de tiempo financiera, en otras palabras, la ventana temporal entre cada «tick »es constante y se le conoce como *lag* la cual se denota como δt.

Los retornos son expresados como:

$$R_t = P_{t+1} - P_t$$

Es decir, el retorno del precio en cuestión (R_t) es equivalente al precio individual de su sucesor temporal (P_{t+1}) menos el precio del objeto temporal en turno (P_t), sin embargo, siguiendo los ideales de Osborne, hay que calcular el logaritmo de los retornos a sabiendas de que los precios se etiquetan como Z_t y son las variables de registro de las mediciones individuales diarias al cierre del mercado, donde $t \in \mathcal{T} = \{T_1, T_2, \ldots, T_M\}$. El logaritmo del precio es:

$$Y_t = \ln Z_t$$

Entonces los *retornos logarítmicos* son definidos como:

$$X_t = Y_t - Y_{t-1}$$

Cuya interpretación es la diferencia sucesiva de los logaritmos de los precios registrados en el tiempo, en otras palabras, la diferencia del logaritmo del precio en turno y su antecesor temporal. Aunque en términos computacionales es mucho más sencillo trabajar con la siguiente expresión que respeta la proposición de Osborne (Frantisek Slanina, 2014):

$$\ln R_t = \ln P_{t+1} - \ln P_t$$

2.10. Dos tipos de agentes

Los mercados que atañen a este trabajo de tesis son conformados por dos tipos de agentes o inversores, los inversionistas *fundamentales*, y los inversionistas *técnicos* (Hutlzer, Mimkes, Richmond, Econophysics & Physical economics, 2013), en obsequio a la brevedad, se define únicamente a los del segundo grupo, mismos que con la finalidad de no alejarse de la naturaleza de un *modelo de agentes*, se les presenta como *agentes técnicos*.

1.16. Tendencias o "*trends*"

Cambio en las series de tiempo que puede no ser periódico (Introductory Time Series with R, Cowpertwait, Paul S.P., Metcalfe, Andrew V, 2009, pg. 5)

1.17. Medias móviles

Sirven para indicar tendencias o "*trends*" en corto o mediano plazo dependiendo de la ventana de tiempo elegida. Mejor conocidad como MAV, por su significado en inglés, "*Moving Average*" o simplemente "*MAV*".

Asumiendo una longitud de n días para la ventana de tiempo, el periodo en cuestión se etiqueta como t y el precio de cierre que registra cada uno de los días es C_t, la media móvil simple se define como sigue:

$$MAV = \left(\frac{1}{n}\right)(C_t + C_{t-2} + \cdots C_{t-n+1})$$

Aunque suelen utilizarse con los precios de cierre, pueden ser utilizados con los precios en lo que abre un mercado, los precios más altos o los más bajos. (Mohammadali Mehralizadeh, 2017 pg.55)

2.10.1. Agentes técnicos

En Economía son mejor conocidos como "*traders*" *técnicos* o *inversionistas técnicos*. Son agentes que ignoran por completo los fundamentos en que se basan los "*traders*" *fundamentalistas*, cuyo criterio de decisión se basa en la apreciación numérica, matemática y estadística de los datos de un mercado.

El análisis que realizan estos agentes es, como su nombre lo dice, técnico, por lo que el precio, la duración de ese precio, el volumen de interacciones realizadas en un intervalo de tiempo, y las tendencias que se pueden observar en la serie de tiempo del mercado son solo unos de los muchos elementos en los que un agente técnico se basa para tomar decisiones, e incluso, predicciones.

1.16. Modelos fundamentales en econofísica

Debido a la realción que tiene la *Econofísica* con la *mecánica estadística*, es momento de sentar bases respecto a modelos como el de Boltzmann y Gibss, dado que será de suma utilidad relacionar estas bases con el tema medular de este trabajo de tesis.

1.16.1. Analogías Física-Economía

En *Física*, un sistema cerrado conserva la energía, en *Economía*, un sistema económico cerrado conserva el *dinero*.

De la anterior analogía se entiende que no hay un flujo externo de dinero, por lo que la cantidad total de dinero se conserva.

La *mecánica estadísitica* estudiada por los físicos, y la economía, tienen en común que ambas estudian grandes ensambles; colecciones de átomos, y agentes económicos, respectivamente. (Cottrell, Michelson, Wright, Classical Econophysics, pg.149)

De la anterior analogía, en una colección de átomos se asume que cada uno de ellos posee una energía cinética $\varepsilon_i \geqslant 0$, mientras que en los mercados financieros cada uno de los agentes participantes tiene un recurso llamado dinero el cual no puede ser negativo, es decir $d_i \geqslant 0$.(Cottrell, Michelson, Wright, Classical Econophysics, pg.149)

Vale la pena mencionar que un ensamble microcanónico es un conjunto en el que las características y el comportamiento de los componentes son similares, posee entropía máxima y conservación de energía. Gracias a su naturaleza física,es posible estudiarle como un todo.

Adicionalmente, en *Física*, a menudo se dice que un sistema se encuentra en equilibrio cuando la energía con que interactúan los componentes es conservada.

1.16.2. Ley de Boltzmann-Gibbs

La *ley de la distribución de la probabilidad de la energía* en la *mecánica estadística*:

$$P(\varepsilon) = \text{C}\ e^{-\varepsilon/T}$$

Donde:

ε es la energía

C es una constante de normalización

T es la temperatura (Wannier,1966) (Cottrell, Michelson, Wright, Classical Econophysics, pg.149)

Dada la similitud de un sistema descrito por la Ley de Boltzmann-Gibbs y las condiciones de un sistema económico cerrado, se puede esta ley a una descripción de la *distribución de la probabilidad del dinero*. (Cottrell, Michelson, Wright, Classical Econophysics, pg.149)

1.16.3. Distribución de la probabilidad del dinero (en equilibrio)

De acuerdo con Cottrel, Michelson y Wright (Cottrell, Michelson, Wright, Classical Econophysics, pg.149) la distribución de la probabilidad del dinero sigue la forma de la *Ley de Boltzmann-Gibbs*

$$P(d) = \mathrm{C}\ e^{-d/T}$$

Donde:

d es el dinero

C es una constante de normalización

T es el promedio de la cantidad de dinero por agente en el sistema económico.

Con base en el artículo *Price variations in a stock market with many agents* por Shubik, Pakzuski y Bak en 1997, la ley de la conservación del dinero establece que el dinero no puede ser manufacturado por agentes del sistema económico pero si puede ser transferido entre ellos. Esta descripción es análoga a la que un sistema conservativo presenta en Física, por ejemplo, cuando en un sistema cerrado sin intercambio de energía con el exterior, los átomos colisionan entre sí.

1.16.4. Ley de la conservación del dinero.

En relación con lo establecido por Shubik, Pakzuski y Bak en 1997 en el artículo ya mencionado, lo que compete en este trabajo de tesis es el resultado de la interacción entre los agentes *i* y *j*.

Sean i y j dos agentes que interactúan en el mercado, correspondiéndole a cada uno de ellos una cantidad finita de dinero d_i y d_j respectivamente, se puede denotar de la siguiente manera: $[d_i, d_j]$.

Si al llevarse a cabo la interacción entre dos agentes y el intercambio de dinero que llevan a cabo es constante, dicho intercambio se etiqueta como Δd. El resultado de la interacción de los agentes en cuestión se expresa como: $[d_i, d_j] \longrightarrow [d'_i, d'_j] = [d_i - \Delta d, d_j + \Delta d]$. Dicho lo anterior puede notarse que $d_i + d_j = d'_i + d'_j$ lo cual significa que la cantidad total de dinero en la transacción es conservada si no existe un flujo externo de dinero que modifique la cantidad total d.

En tales condiciones se asume que la distribución de probabilidad del dinero en equilibrio es invariante pese a fuertes fluctuaciones Δd entre los agentes. (Cottrell, Michelson, Wright, Classical Econophysics, pg.149)

2.11. Economía neoclásica

Rememorando los comienzos de la *Econofísica* se encuentran emblemáticos personajes como Adolphe Quétlet(1796-1874), Léon Walras (1834-1910), Vilfredo Pareto (1848-1923), y Adam Smith (1723-1790), quienes ofrecieron un conjunto de ideas que permitieron ampliar la manera en que se estudia la *Economía*, y que más allá de considerar a una *transacción* como una consecuencia de carácter determinista debida a la interacción de intercambio de bienes entre dos agentes, considera la toma de decisiones entre los agentes en función de aspectos sociales, factores externos a los mercados, e incluso de la distribucipon de la riqueza.

Esta brillante abertura permite a la *Econofísica* estudiar los sistemas complejos intrínsecos de la *Economía* a partir de que a cada sistema complejo en que se encuentra inmersa le componen un gran número de *grados de libertad* en diferentes escalas de tiempo(Richmond Econophysics pg 17).

Asi como Pareto estudió la distribución de la riqueza en el siglo XIX, Adam Smith propuso la descripción de un fenómeno constitutivo en este trabajo de tesis, llamado *la mano invisible*. Este y los siguientes conceptos son parte de la *Economía neoclásica*, su naturaleza conlleva el pensamiento de que la *Economía neoclásica* es quien da origen a la *Econofísica*.

2.11.1. La mano invisible

Idea propuesta por Adam Smith en 1776, guiada por los ideales del *laissez-faire*, que significa "dejar hacer", la cual se refiere a que cuando cada participante busca solo su propio beneficio, a menudo promueve el beneficio de la sociedad de manera más eficaz que si realmente pretendiera promoverlo, como si una mano invisible benevolente estuviera dirigiendo todo el proceso. (Economía, Samuelson Nordhauss, pg.26)

2.11.2. Teoría de juegos

Estudio que puede ser descrito con el uso de las Matemáticas para comprender la competencia y cooperación entre varias partes involucradas . El rango de aplicación va desde estrategias de guerra hasta el entendimiento de la competición económica, desde problemas económicos o sociales hasta el comportamiento de los animales en situaciones de competencia(Peters, Hans. 2015, Springer, Game Theory).

Análisis de situaciones que comprende a dos o más tomadores de decisiones con intereses al menos parcialmente en conflicto. Se puede aplicar a la interaccion de dos o más agentes en situaciones de negociación como las huelgas, o conflictos como los juegos y la guerra(Samuelson, Nordhauss, Economía).

2.11.3. Dilema del prisionero

Existen diferentes problemas que estudia la disciplina de la *teoria de juegos*, el que se presenta a continuación es probablemente el más conocido y el que mejor puede extrapolarse a este trabajo de tesis.

Sean dos prisioneros P1 y P2 que han cometido juntos un delito. El fiscal entrevista por separado a cada uno de ellos y les dice : "*Tengo suficientes pruebas sobre los dos para mandarlos un año a la cárcel. Pero haré un trato contigo: si solo confiesas tu, se te condenarpa a tres meses de cárcel, mientras que tu socio será condenado a 10 años. Si confiesan los dos, ambos serán condenados a cinco años*".

Lo importante en este caso es que cuando ambos prisioneros actúan interesadamente y confiesan, ambos serán condenados a mayores penas de cárcel. Solo son condenados a menos años de carcel cuando ambos actúan de manera colusiva y altruísta. La situación en cuestión concluye que el equilibrio no cooperativo es ineficiente.

2.11.4. Frecuencia

Si bien, en *Física* la frecuencia es medida en **Hz** debido a la ecuación:

$$f = \frac{1}{T}$$

Donde:

T es el periodo y se mide en segundos.

En el contexto de este trabajo de tesis se entiende la *frecuencia* como una proporción cuantificable de registros de observables, datos, o precios en el tiempo (Jim Pitman, Probability, Springer, 1993).

2.11.5. Fluctuación

Observable registrada en el tiempo tan grande o tan pequeña conforme con el total de observables y la frecuencia de cada una.

2.11.6. Volatilidad

En *Economia* a menudo es calculada como la desviación estándar del cambio en el precio en una apropiada ventana de tiempo. (Mantegna, Econophysics).

Aunque se conocen la *volatilidad completa o histórica*, la *volatilidad implícita*, la *volatilidad instantánea o actual*, la *volatilidad de frontera* y la *volatilidad progresiva*, siendo esta última una de las utilizadas en predicciones. Esta tesis se centra específicamente en la primera.

Es una medida de aleatoriedad de un conjunto de registros para dos puntos cualesquiera en el tiempo (Wilmott, FAQ, Quantitative Finance).

Asumiendo que existe una ventana de tiempo de interés t_i La siguiente ecuación describe la volatilidad en cuestión:

$$v(t_i) = v(\Delta t, n, p; t_i) = \left[\frac{1}{n}\sum_{j=1}^{n} |r(\Delta t; t_{i-n+j}|^p\right]^{\frac{1}{p}}$$

Donde r son los retornos que se definen en la sección **retornos**, n es el número de la cantidad de observables utilizada para calcular los retornos, y p suele tomar el valor de 2 ya que v^2 es la varianza de los retornos. (An introduction to high-frequency finance, Dacoragna, Gencay, Muller, Olsen, Pictet 2001)

3 Entropía y precios

> *Las leyes de la probabilidad, tan verdaderas en general, tan falaces en particular.*
>
> — **Edward Gibbon** (1737-1794)

La entropía en *econofísica* es la sustituta de la *función de producción* o *función de utilidad*en la economía neoclásica.

3.1 mxp

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3.2 Related Work Section 2

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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3.3 Related Work Section 3

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3.4 Conclusion

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4 System

> *Innovation distinguishes between a leader and a follower.*
>
> — **Steve Jobs**
> (CEO Apple Inc.)

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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4.1 System Section 1

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Fig. 4.1: Figure example: *(a)* example part one, *(c)* example part two; *(c)* example part three

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

4.2 System Section 2

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Fig. 4.2: Another Figure example: *(a)* example part one, *(c)* example part two; *(c)* example part three

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5 Concepts: This text is here to test a very long title, to simulate the line break behavior, to show that an extremely long tilte also works

> *Users do not care about what is inside the box, as long as the box does what they need done.*
>
> — **Jef Raskin**
>
> about Human Computer Interfaces

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6.3 Future Work

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List of Figures

List of Tables

Colophon

Esta tesis fue escrita en $\LaTeX\,2_\varepsilon$ usando el estilo *Clean Thesis*. El diseño de este estilo fue inspirado por los manuales de usuario de Apple Inc.

Descargar el estilo *Clean Thesis* en `http://cleanthesis.der-ric.de/`.

Declaración

Por la presente declaro que esta tesis contiene búsqueda de literatura e investigación original escrita por mí y que es el registro de los trabajos realizados por mí y que no se ha presentado en otra solicitud de un grado más alto. También declaro que hice todas las citas y referencias a todo el material y resultados que no son originales de este trabajo.

Declaration

I hereby declare that this thesis contains literature survey and original research written by me, that it is the record of work carried out by me and that it has not been submitted in any previous application for a higher degree. I also declare that, i have fully cited and referenced all material and results that are not original to this work.

Xalapa, 12 de abril de 2020

Versión a corrección

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