



Tecnológico de Monterrey - Campus Monterrey

School of Engineering and Sciences

Engineering in Computational Technologies

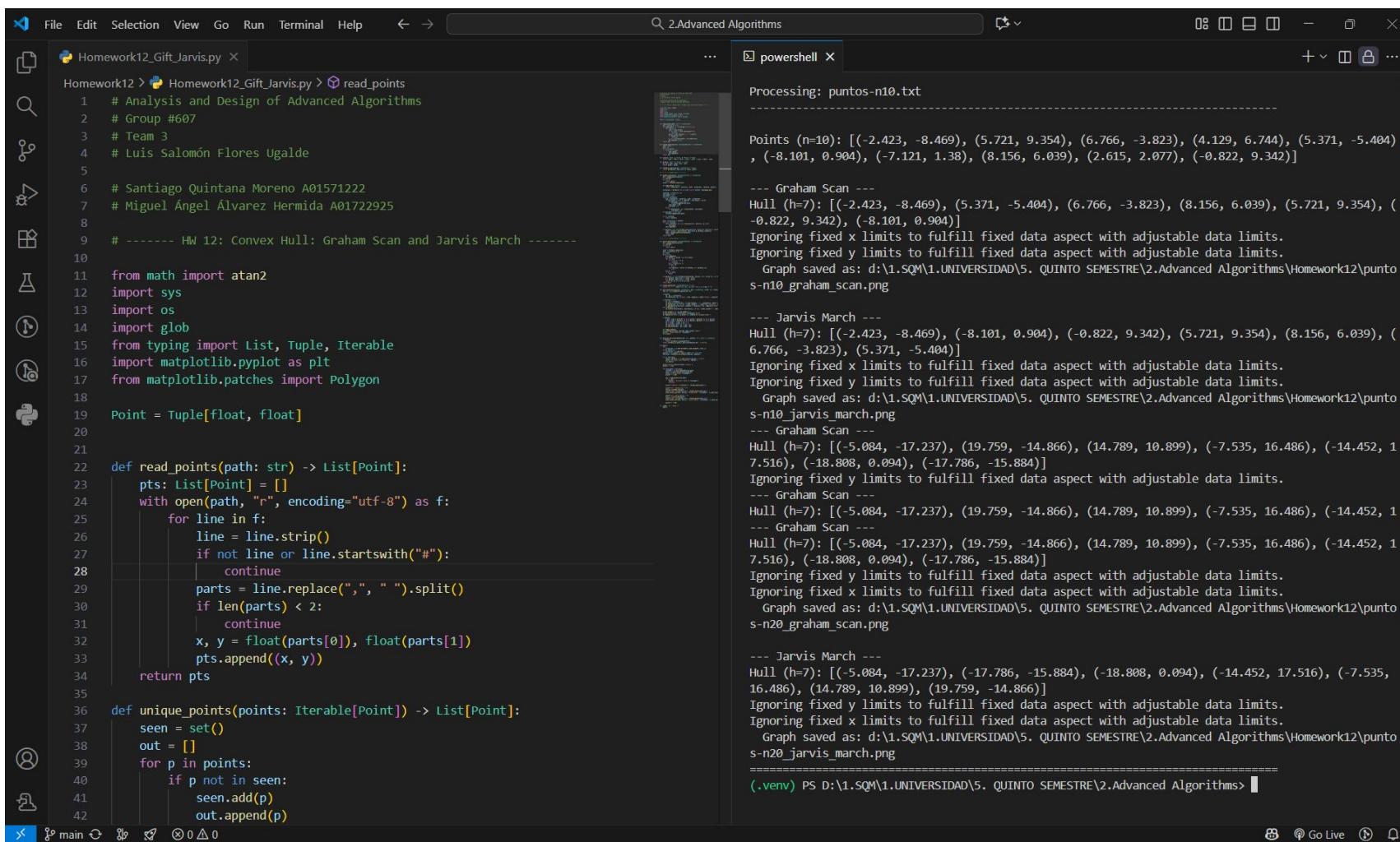
Analysis and Design of Advanced Algorithms

Homework 12: Convex Hull: Graham Scan and Jarvis March

Group: 607
Team #3

Luis Salomón Flores Ugalde

Santiago Quintana Moreno A01571222
Miguel Ángel Álvarez Hermida a01722925



Homework12_Gift_Jarvis.py x

```
43     return out
44
45     def cross(o: Point, a: Point, b: Point) -> float:
46         return (a[0] - o[0]) * (b[1] - o[1]) - (a[1] - o[1]) * (b[0] - o[0])
47
48     def dist2(a: Point, b: Point) -> float:
49         dx, dy = a[0] - b[0], a[1] - b[1]
50         return dx*dx + dy*dy
51
52     def leftmost_lowest(points: List[Point]) -> Point:
53         return min(points, key=lambda p: (p[1], p[0]))
54
55     # ----- Graham Scan -----
56
57     def graham_scan(points: Iterable[Point]) -> List[Point]:
58         pts = unique_points(points)
59         n = len(pts)
60         if n <= 1:
61             return pts[:]
62         anchor = leftmost_lowest(pts)
63
64         def angle_key(p: Point):
65             return (atan2(p[1] - anchor[1], p[0] - anchor[0]), -dist2(p, anchor))
66
67         sorted_pts = sorted((p for p in pts if p != anchor), key=angle_key)
68
69         filtered: List[Point] = []
70         last_angle = None
71         last_best = None
72         for p in sorted_pts:
73             ang = atan2(p[1] - anchor[1], p[0] - anchor[0])
74             if last_angle is None or abs(ang - last_angle) > 1e-15:
75                 if last_best is not None:
76                     filtered.append(last_best)
77                     last_angle = ang
78                     last_best = p
79             else:
80                 if dist2(anchor, p) > dist2(anchor, last_best):
81                     last_best = p
82         if last_best is not None:
83             filtered.append(last_best)
```

powershell x

Processing: puntos-n10.txt

Points (n=10): [(-2.423, -8.469), (5.721, 9.354), (6.766, -3.823), (4.129, 6.744), (5.371, -5.404), (-8.101, 0.904), (-7.121, 1.38), (8.156, 6.039), (2.615, 2.077), (-0.822, 9.342)]

--- Graham Scan ---

Hull (h=7): [(-2.423, -8.469), (5.371, -5.404), (6.766, -3.823), (8.156, 6.039), (5.721, 9.354), (-0.822, 9.342), (-8.101, 0.904)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-2.423, -8.469), (-8.101, 0.904), (-0.822, 9.342), (5.721, 9.354), (8.156, 6.039), (6.766, -3.823), (5.371, -5.404)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n20_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-5.084, -17.237), (-17.786, -15.884), (-18.808, 0.094), (-14.452, 17.516), (-7.535, 16.486), (14.789, 10.899), (19.759, -14.866)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n20_jarvis_march.png

=====

(.venv) PS D:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms> []

Homework12_Gift_Jarvis.py x

```
43     return out
44
45     def cross(o: Point, a: Point, b: Point) -> float:
46         return (a[0] - o[0]) * (b[1] - o[1]) - (a[1] - o[1]) * (b[0] - o[0])
47
48     def dist2(a: Point, b: Point) -> float:
49         dx, dy = a[0] - b[0], a[1] - b[1]
50         return dx*dx + dy*dy
51
52     def leftmost_lowest(points: List[Point]) -> Point:
53         return min(points, key=lambda p: (p[1], p[0]))
54
55     # ----- Graham Scan -----
56
57     def graham_scan(points: Iterable[Point]) -> List[Point]:
58         pts = unique_points(points)
59         n = len(pts)
60         if n <= 1:
61             return pts[:]
62         anchor = leftmost_lowest(pts)
63
64         def angle_key(p: Point):
65             return (atan2(p[1] - anchor[1], p[0] - anchor[0]), -dist2(p, anchor))
66
67         sorted_pts = sorted((p for p in pts if p != anchor), key=angle_key)
68
69         filtered: List[Point] = []
70         last_angle = None
71         last_best = None
72         for p in sorted_pts:
73             ang = atan2(p[1] - anchor[1], p[0] - anchor[0])
74             if last_angle is None or abs(ang - last_angle) > 1e-15:
75                 if last_best is not None:
76                     filtered.append(last_best)
77                     last_angle = ang
78                     last_best = p
79             else:
80                 if dist2(anchor, p) > dist2(anchor, last_best):
81                     last_best = p
82         if last_best is not None:
83             filtered.append(last_best)
```

powershell x

Processing: puntos-n10.txt

Points (n=10): [(-2.423, -8.469), (5.721, 9.354), (6.766, -3.823), (4.129, 6.744), (5.371, -5.404), (-8.101, 0.904), (-7.121, 1.38), (8.156, 6.039), (2.615, 2.077), (-0.822, 9.342)]

--- Graham Scan ---

Hull (h=7): [(-2.423, -8.469), (5.371, -5.404), (6.766, -3.823), (8.156, 6.039), (5.721, 9.354), (-0.822, 9.342), (-8.101, 0.904)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-2.423, -8.469), (-8.101, 0.904), (-0.822, 9.342), (5.721, 9.354), (8.156, 6.039), (6.766, -3.823), (5.371, -5.404)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-5.084, -17.237), (-17.786, -15.884), (-18.808, 0.094), (-14.452, 17.516), (-7.535, 16.486), (14.789, 10.899), (19.759, -14.866)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

=====

(.venv) PS D:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms> []

Homework12 > Homework12_Gift_Jarvis.py > read_points

```
57 def graham_scan(points: Iterable[Point]) -> List[Point]:  
84  
85     if not filtered:  
86         return [anchor]  
87  
88     hull: List[Point] = [anchor]  
89     for p in filtered:  
90         while len(hull) >= 2 and cross(hull[-2], hull[-1], p) <= 0:  
91             hull.pop()  
92         hull.append(p)  
93  
94     if len(hull) >= 3 and all(abs(cross(hull[i-2], hull[i-1], hull[i])) < 1e-15 for i in range(2, len(hull))):  
95         ends = [anchor, max(filtered, key=lambda q: dist2(q, anchor))]  
96     else:  
97         ends = unique_points(sorted(filtered))  
98     return ends  
99  
100    # ----- Jarvis March -----  
101  
102    def jarvis_march(points: Iterable[Point]) -> List[Point]:  
103        pts = unique_points(points)  
104        n = len(pts)  
105        if n <= 1:  
106            return pts[:]  
107  
108        start = leftmost_lowest(pts)  
109        hull: List[Point] = [start]  
110        p = start  
111        while True:  
112            hull.append(p)  
113            q = pts[0] if pts[0] != p else pts[1]  
114            for r in pts:  
115                if r == p or r == q:  
116                    continue  
117                ori = cross(p, q, r)  
118                if ori > 0:  
119                    q = r  
120                elif abs(ori) < 1e-15 and dist2(p, r) > dist2(p, q):  
121                    q = r  
122                p = q  
123                if p == start:  
124                    break
```

powershell x

Processing: puntos-n10.txt

Points (n=10): [(-2.423, -8.469), (5.721, 9.354), (6.766, -3.823), (4.129, 6.744), (5.371, -5.404), (-8.101, 0.904), (-7.121, 1.38), (8.156, 6.039), (2.615, 2.077), (-0.822, 9.342)]

--- Graham Scan ---

Hull (h=7): [(-2.423, -8.469), (5.371, -5.404), (6.766, -3.823), (8.156, 6.039), (5.721, 9.354), (-0.822, 9.342), (-8.101, 0.904)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-2.423, -8.469), (-8.101, 0.904), (-0.822, 9.342), (5.721, 9.354), (8.156, 6.039), (6.766, -3.823), (5.371, -5.404)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n20_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-5.084, -17.237), (-17.786, -15.884), (-18.808, 0.094), (-14.452, 17.516), (-7.535, 16.486), (14.789, 10.899), (19.759, -14.866)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n20_jarvis_march.png

(.venv) PS D:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms> []

Homework12_Gift_Jarvis.py

```

102  def jarvis_march(points: Iterable[Point]) -> List[Point]:
103      if p == start:
104          break
105
106      if len(hull) > 2 and all(abs(cross(hull[0], hull[1], r)) < 1e-15 for r in
107          a = min(pts, key=lambda x: (x[0], x[1]))
108          b = max(pts, key=lambda x: (x[0], x[1]))
109          return [a, b] if a != b else [a]
110
111      return hull
112
113  def format_points(pts: Iterable[Point]) -> str:
114      return "[" + ", ".join(f"{{x:.6g}, {y:.6g}}" for x, y in pts) + "]"
115
116  def plot_convex_hull(points: List[Point], hull: List[Point], title: str, file:
117      fig, ax = plt.subplots(figsize=(10, 8))
118
119      if points:
120          px, py = zip(*points)
121          ax.scatter(px, py, c='blue', s=50, alpha=0.6, label='Points', zorder=1)
122
123          if len(hull) >= 2:
124              hx, hy = zip(*hull)
125              ax.plot(list(hx)+[hx[0]], list(hy)+[hy[0]], 'r-', linewidth=2, label='Hull')
126              ax.scatter(hx, hy, c='red', s=100, marker='^', label='Hull Vertices',
127              ax.add_patch(Polygon(hull, alpha=0.2, facecolor='red', edgecolor='none'))
128
129          elif len(hull) == 1:
130              ax.scatter([hull[0][0]], [hull[0][1]], c='red', s=100, marker='^', label='Hull Vertices')
131
132          ax.set_xlabel('X'); ax.set_ylabel('Y')
133          ax.set_title(title, fontsize=14, fontweight='bold')
134          ax.legend(loc='best'); ax.grid(True, alpha=0.3); ax.axis('equal')
135
136          if points:
137              x_min, x_max = min(p[0] for p in points), max(p[0] for p in points)
138              y_min, y_max = min(p[1] for p in points), max(p[1] for p in points)
139              mx = (x_max - x_min) * 0.1 or 1
140              my = (y_max - y_min) * 0.1 or 1
141              ax.set_xlim(x_min - mx, x_max + mx)
142              ax.set_ylim(y_min - my, y_max + my)
143
144              plt.tight_layout()
145              plt.savefig(filename, dpi=150, bbox_inches='tight')
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163

```

powershell

Processing: puntos-n10.txt

```

Points (n=10): [(-2.423, -8.469), (5.721, 9.354), (6.766, -3.823), (4.129, 6.744), (5.371, -5.404),
, (-8.101, 0.904), (-7.121, 1.38), (8.156, 6.039), (2.615, 2.077), (-0.822, 9.342)]

```

--- Graham Scan ---

```

Hull (h=7): [(-2.423, -8.469), (5.371, -5.404), (6.766, -3.823), (8.156, 6.039), (5.721, 9.354),
, (-0.822, 9.342), (-8.101, 0.904)]

```

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

```

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto
s-n10_graham_scan.png

```

--- Jarvis March ---

```

Hull (h=7): [(-2.423, -8.469), (-8.101, 0.904), (-0.822, 9.342), (5.721, 9.354), (8.156, 6.039),
, (6.766, -3.823), (5.371, -5.404)]

```

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

```

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto
s-n10_jarvis_march.png

```

--- Graham Scan ---

```

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 1
7.516), (-18.808, 0.094), (-17.786, -15.884)]

```

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

--- Graham Scan ---

```

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 1
7.516), (-18.808, 0.094), (-17.786, -15.884)]

```

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

```

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto
s-n20_graham_scan.png

```

--- Jarvis March ---

```

Hull (h=7): [(-5.084, -17.237), (-17.786, -15.884), (-18.808, 0.094), (-14.452, 17.516), (-7.535,
16.486), (14.789, 10.899), (19.759, -14.866)]

```

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

```

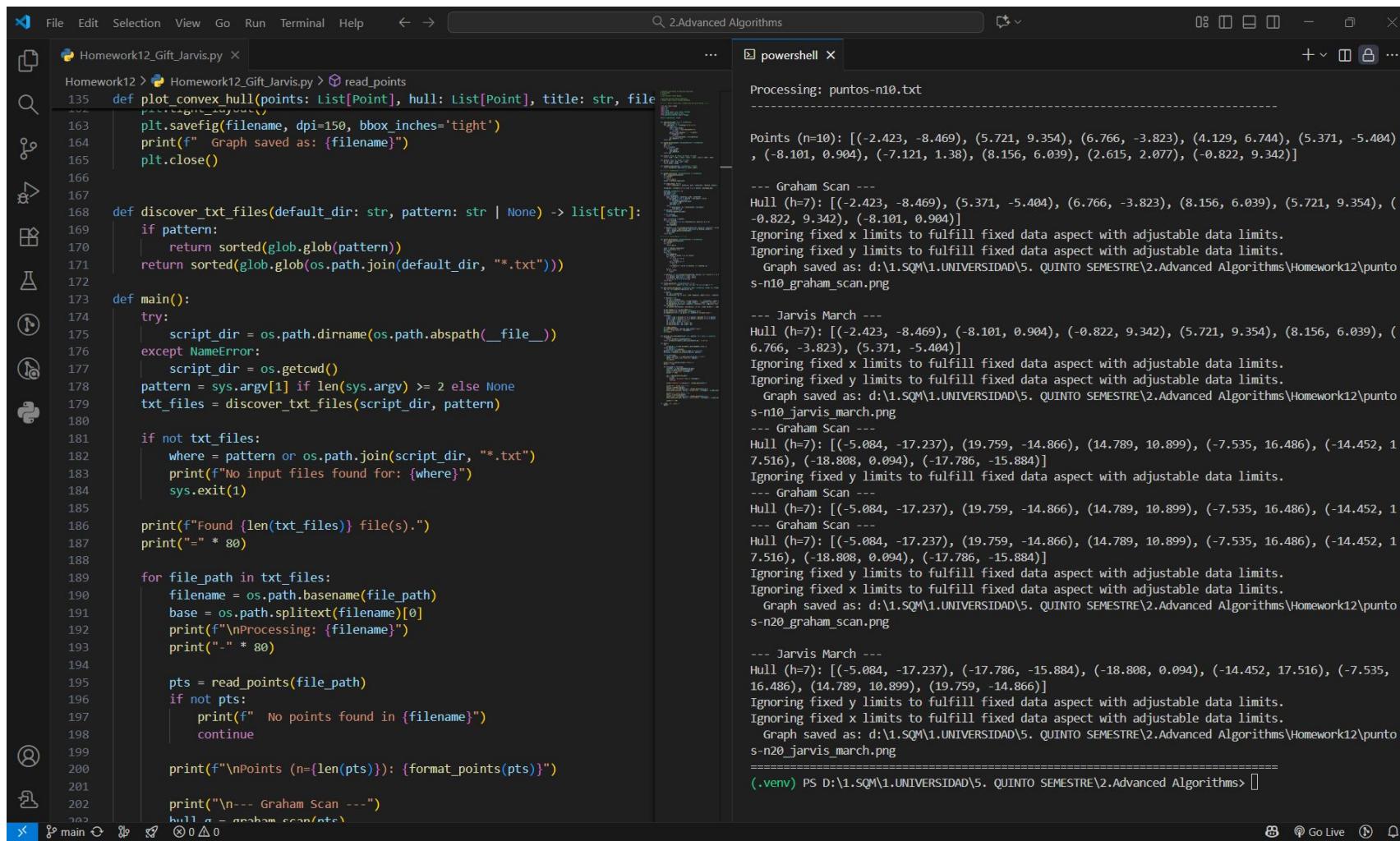
Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto
s-n20_jarvis_march.png

```

```

(.venv) PS D:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms> []

```



Homework12 > Homework12_Gift_Jarvis.py > read_points

```
173 def main():
194
195     pts = read_points(file_path)
196     if not pts:
197         print(f" No points found in {filename}")
198         continue
199
200     print(f"\nPoints (n={len(pts)}): {format_points(pts)}")
201
202     print("\n--- Graham Scan ---")
203     hull_g = graham_scan(pts)
204     print(f"Hull (h={len(hull_g)}): {format_points(hull_g)}")
205     plot_convex_hull(pts, hull_g, f"Graham Scan - {filename}", os.path.j
206
207     print("\n--- Jarvis March ---")
208     hull_j = jarvis_march(pts)
209     print(f"Hull (h={len(hull_j)}): {format_points(hull_j)}")
210     plot_convex_hull(pts, hull_j, f"Jarvis March - {filename}", os.path.j
211
212     print("-" * 80)
213
214 if __name__ == "__main__":
215     main()
216
```

powershell x

Processing: puntos-n10.txt

Points (n=10): [(-2.423, -8.469), (5.721, 9.354), (6.766, -3.823), (4.129, 6.744), (5.371, -5.404), (-8.101, 0.904), (-7.121, 1.38), (8.156, 6.039), (2.615, 2.077), (-0.822, 9.342)]

--- Graham Scan ---

Hull (h=7): [(-2.423, -8.469), (5.371, -5.404), (6.766, -3.823), (8.156, 6.039), (5.721, 9.354), (-0.822, 9.342), (-8.101, 0.904)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-2.423, -8.469), (-8.101, 0.904), (-0.822, 9.342), (5.721, 9.354), (8.156, 6.039), (6.766, -3.823), (5.371, -5.404)]

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

--- Graham Scan ---

Hull (h=7): [(-5.084, -17.237), (19.759, -14.866), (14.789, 10.899), (-7.535, 16.486), (-14.452, 17.516), (-18.808, 0.094), (-17.786, -15.884)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_graham_scan.png

--- Jarvis March ---

Hull (h=7): [(-5.084, -17.237), (-17.786, -15.884), (-18.808, 0.094), (-14.452, 17.516), (-7.535, 16.486), (14.789, 10.899), (19.759, -14.866)]

Ignoring fixed y limits to fulfill fixed data aspect with adjustable data limits.

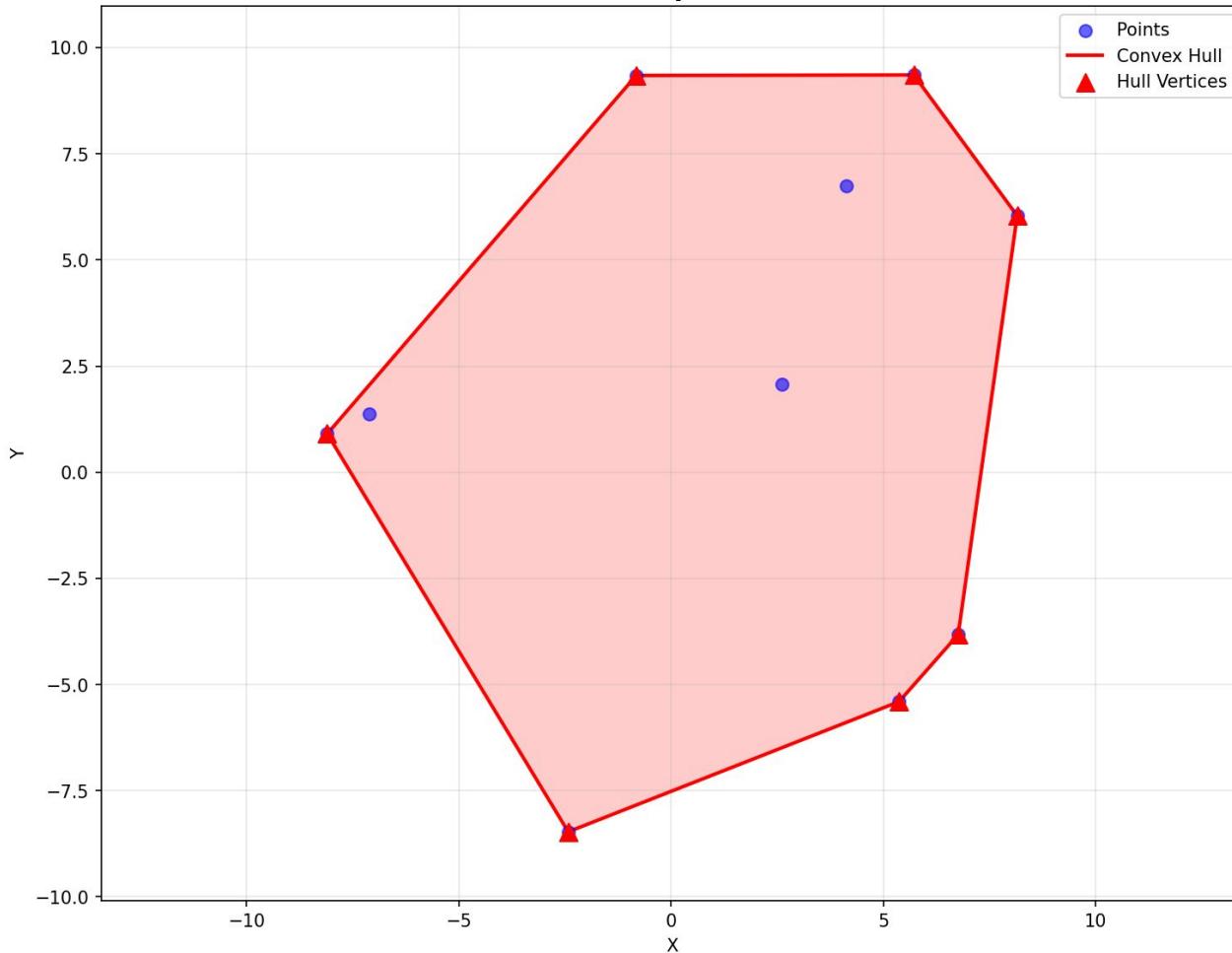
Ignoring fixed x limits to fulfill fixed data aspect with adjustable data limits.

Graph saved as: d:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms\Homework12\punto s-n10_jarvis_march.png

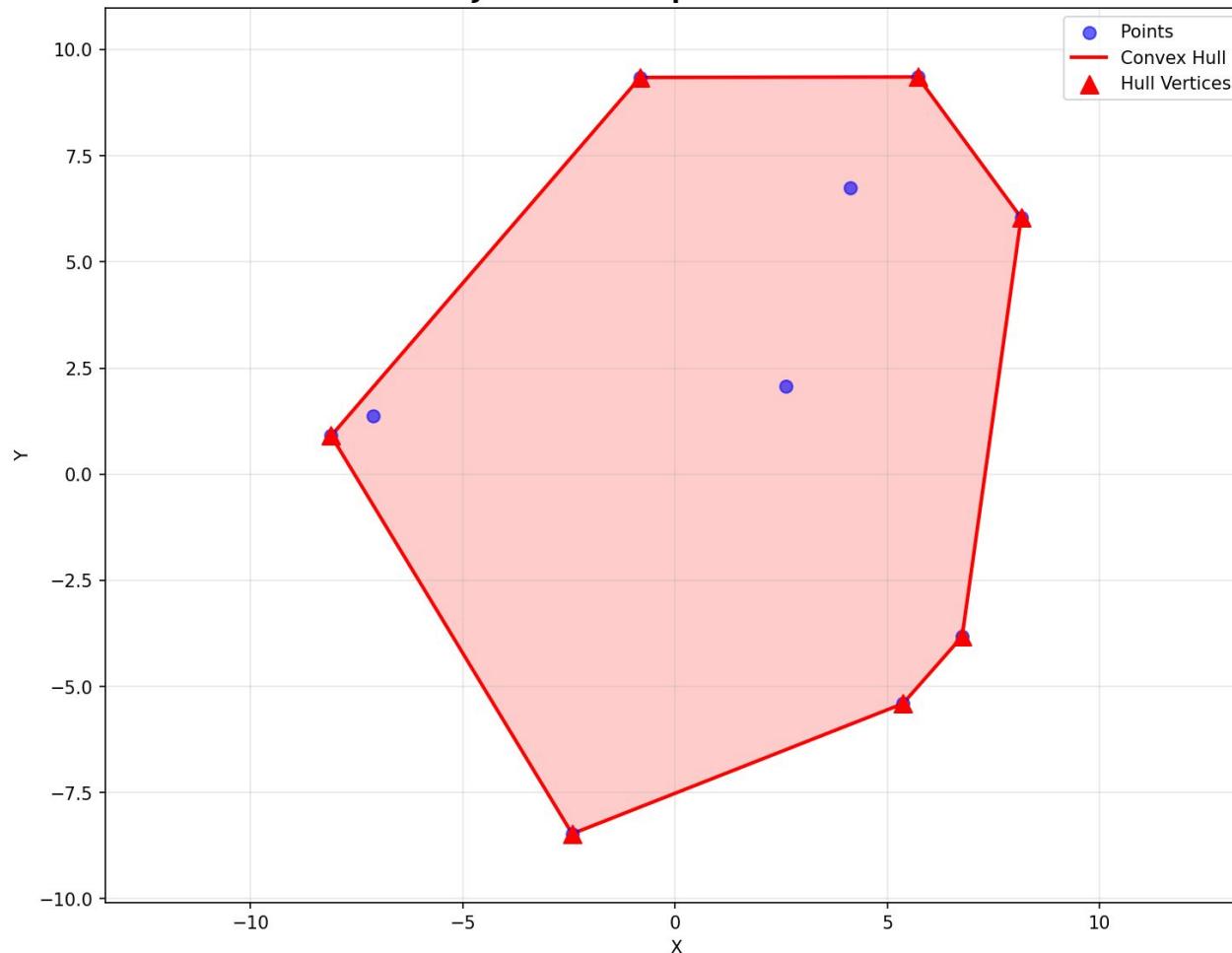
=====

(.venv) PS D:\1.SQM\1.UNIVERSIDAD\5. QUINTO SEMESTRE\2.Advanced Algorithms> █

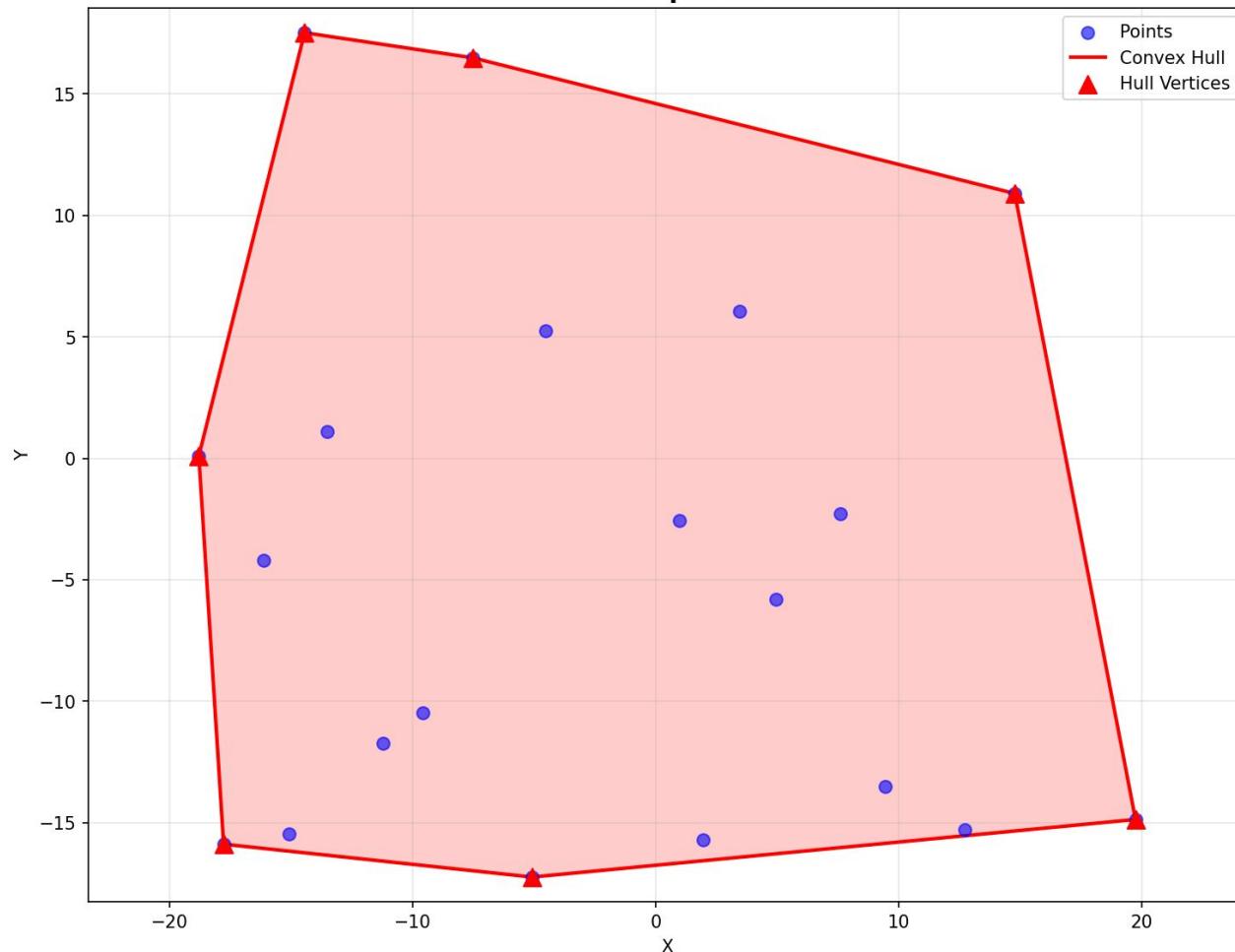
Graham Scan - puntos-n10.txt



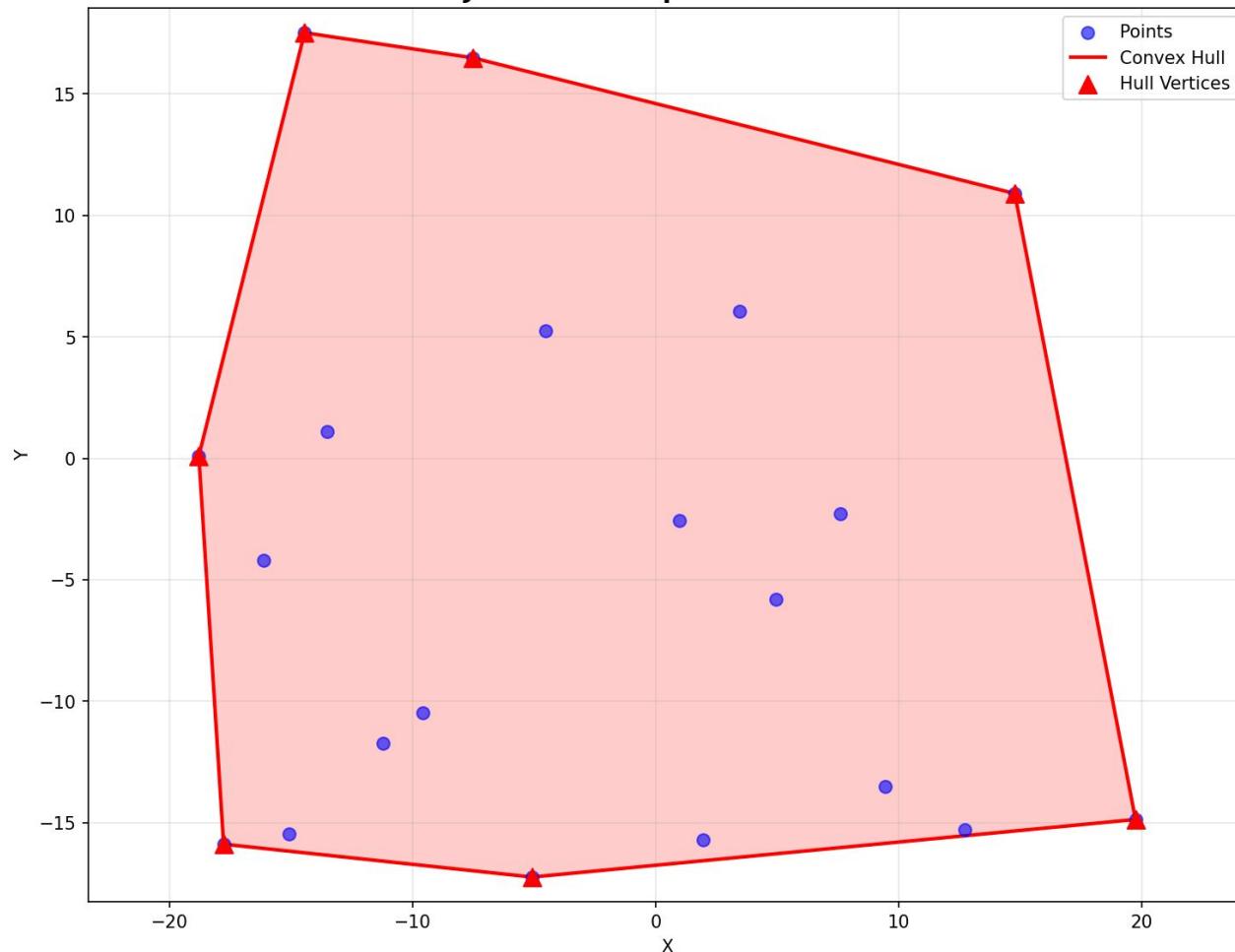
Jarvis March - puntos-n10.txt



Graham Scan - puntos-n20.txt



Jarvis March - puntos-n20.txt



<https://colab.research.google.com/drive/1XT17WAUIMplMIcWttz-zvIWGABLCDE3v?usp=sharing>