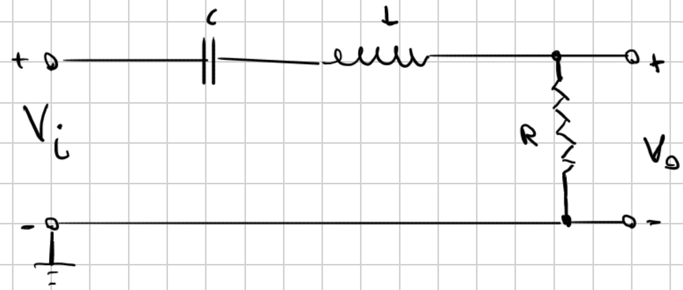
$[s = \sigma + j\omega]$

a)



$$H(s) = \frac{V_o}{V_I}$$

- $Z_c = \frac{1}{sC}$ • $Z_L = sL$ • $Z_R = R$ $\qquad V_o = Z_R \qquad V_i = Z_c + Z_L + Z_R$

$$H(s) = \frac{Z_R}{Z_c + Z_L + Z_R} = \frac{R}{1/sC + sL + R} = \frac{R.sC}{s^2 CL + RsC + 1} = \frac{R\,s\cancel{C}}{\cancel{C}L} \cdot \frac{1}{s^2 + \frac{Rs}{L} + \frac{1}{cL}}$$

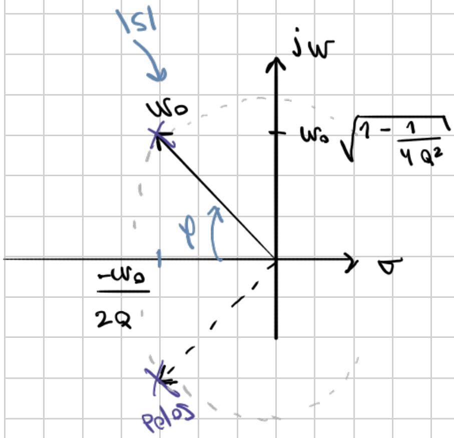
$\frac{R\,s}{L} \to \frac{\omega_o}{Q}.s \qquad \frac{Rs}{L} \to \frac{\omega_o}{Q} \qquad \frac{1}{cL} \to \omega_o^2$

$$\to \boxed{T(s) = \frac{\omega_o/Q \,.\, s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}} \to \text{Polos} \quad s^2 + \frac{\omega_o}{Q} s + \omega_o^2 = 0$$

$$s = \omega_o \left(\underbrace{\frac{-1}{2Q}}_{> -1 \,\to\, Q > 1/2} \pm j \sqrt{1 - \frac{1}{4Q^2}} \right)$$

$$\omega_o = \frac{1}{\sqrt{cL}} \qquad Q = \frac{\sqrt{cL}\,L}{R}$$

Normalizado $\omega_o = 1$



Veamos que:

$\Leftarrow$ • $\frac{-\omega_o}{2Q}$ Indica el amortiguamiento (parte real)

• $\omega_o \sqrt{1 - \frac{1}{4Q^2}}$ freq. oscilatoria (Pte. Im.)

$\to \cos\varphi = \frac{\sigma}{|s|} \qquad |s| = \omega_o \to \cos\varphi = \frac{\frac{\omega_o}{2Q}}{\omega_o} = \frac{1}{2Q}$

Con $Q = 1/2$ se juntan los polos

P/valores altos de Q $\varphi \to 90°$, p/valores bajos $\varphi \to 0°$

$$Q = \frac{1}{2\cos\varphi}$$

Dejamos fijo $\omega_o \to \omega_o^2 = 1 = \frac{1}{cL} \to c = \frac{1}{L}$

$$\varphi = \cos^{-1}(2Q)$$

$\to$ Estado estacionario: $s = j\omega$? Esto sirve p/ análisis en frecuencia

$$T(j\omega) = \frac{\omega_o/Q \,.\, \omega j}{(j\omega)^2 + j\omega \frac{\omega_o}{Q} + \omega_o^2} = \frac{\omega_o/Q \,.\, \omega j}{\omega_o^2 - \omega^2 + j\omega \frac{\omega_o}{Q}}$$

$$|T| = \frac{\omega \,.\, \omega_o/Q}{\sqrt{(\omega_o^2 - \omega^2)^2 + \left(\frac{\omega\,\omega_o}{Q}\right)^2}}$$

• A bajas frecuencias $\omega \ll \omega_o$

$$\to |T(j\omega)| = \frac{\omega}{Q\,\omega_o}$$

$$|T| = \frac{\omega \,.\, \cancel{\omega_o}/Q}{\sqrt{(\underbrace{\omega_o^2 - \omega^2}_{\to \omega_o^4})^2 + \left(\frac{\cancel{\omega\,\omega_o}}{Q}\right)^2}} \quad \to 0$$

• A altas frecuencias $\omega \gg \omega_o$

$$\to |T(j\omega)| = \frac{\omega_o}{Q\,\omega}$$

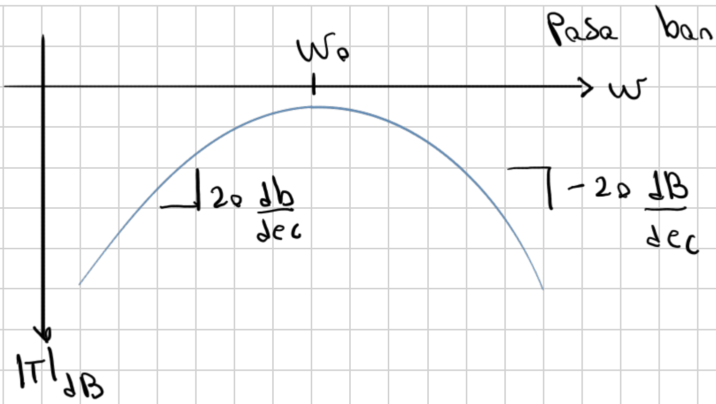
$$|T| = \frac{\cancel{\omega} \,.\, \omega_o/Q}{\sqrt{(\underbrace{\omega_o^2 - \omega^2}_{\to \omega^4})^2 + \left(\frac{\omega\,\omega_o}{Q}\right)^2}} \quad \to 0$$

- En resonancia $w = w_0$

$$|T(jw)| = 1$$

Graficamos el patron de dB.

Pasa banda:



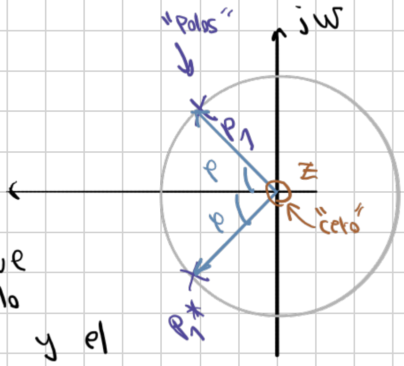
→ Analisis de fase

$$T(s) = \frac{s \cdot w_0/Q}{s^2 + s\frac{w_0}{Q} + w_0^2}$$

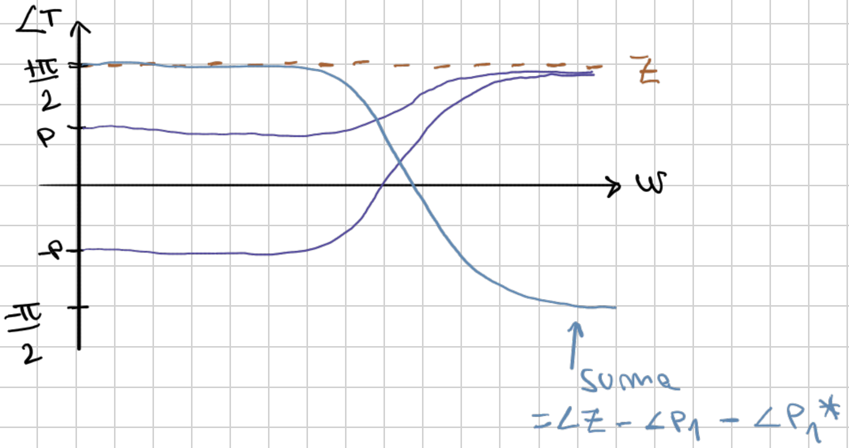
$$T(s) = \frac{s\, w_0/Q}{(s-P_1)(s-P_1^*)}$$ ← cero

Poles

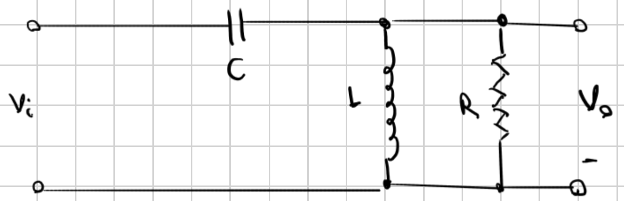
⇒ graficamente:



Podemos ver que cuando $w \to \infty$ cada polo aporta $-\pi/2$ a la fase y el cero aporta $\pi/2$



b)



$V_o = Z_{LR} = \dfrac{Z_L \cdot Z_R}{Z_L + Z_R}$ ¿Pq?

$V_i = Z_c + Z_{LR} = Z_c + \dfrac{Z_L \cdot Z_R}{Z_L + Z_R}$

$= \dfrac{1}{sC} + \dfrac{sLR}{sL+R}$

$$H(s) = \frac{Z_{LR}}{Z_c + Z_{LR}} = \frac{\dfrac{sLR}{sL+R}}{\dfrac{1}{sC} + \dfrac{sLR}{sL+R}} \cdot sC(sL+R) = \frac{s^2 cLR}{sL+R+s^2 cLR}$$

$$= \frac{s^2}{s^2 + \dfrac{s}{RC} + \dfrac{1}{LC}}$$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ $\quad \dfrac{\omega_0}{Q} = \dfrac{1}{RC}$ $\quad Q = \dfrac{RC}{\sqrt{LC}}$

Estado estacionario $\rightarrow s = j\omega$

$$\Rightarrow T(j\omega) = -\frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega\dfrac{\omega_0}{Q}}$$

$$|T| = \frac{\omega^2}{\sqrt{(\omega_0^2-\omega^2)^2 + \left(j\omega\dfrac{\omega_0}{Q}\right)^2}}$$

- A bajas f $\omega \ll \omega_0$

$|T| = \dfrac{\omega^2}{\omega_0^2}$

- Resonancia ($\omega = \omega_0$)

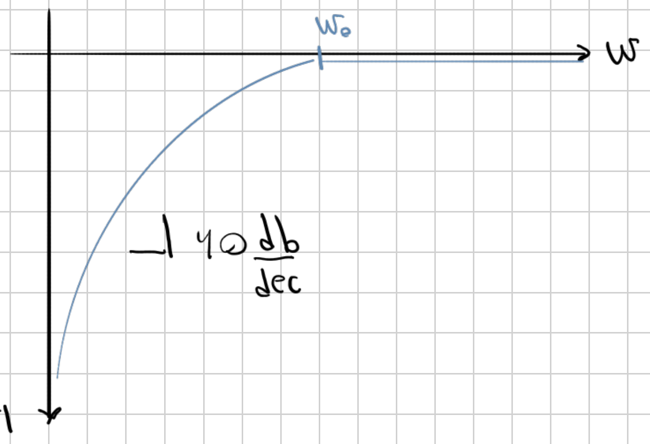
$|T| = 1$

- A altas f $\omega \gg \omega_0$

$|T| = 1$

Pasa altos ☺

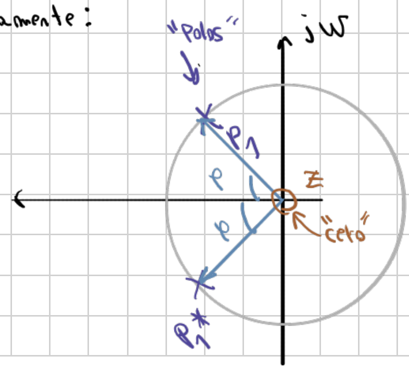
Graficamos el patrón de dB $\rightarrow$



→ Analisis de fase

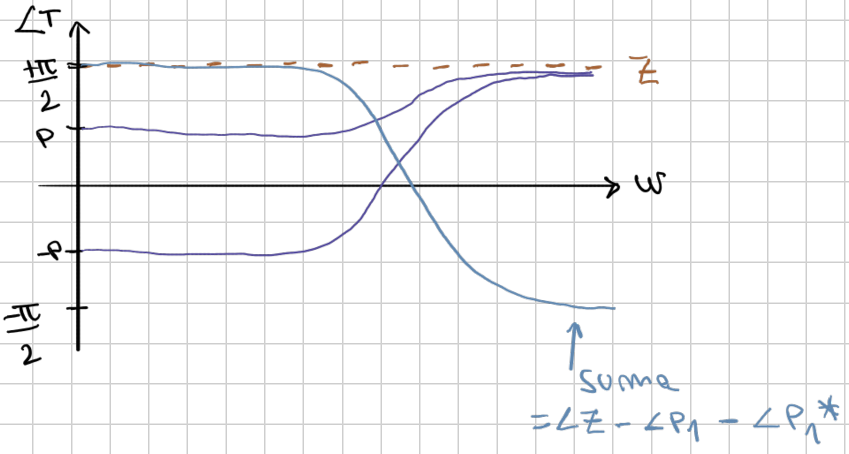
$$T(s) = \frac{s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2} \rightarrow T(s) = \frac{s^2}{(s-p_1)(s-p_1^*)}$$

$\Rightarrow$ graficamente:



Mismo analisis que para a)

Podemos ver que cuando $\omega \rightarrow \infty$ cada polo aporta $-\pi/2$ a la fase y el cero aporta $\pi/2$



∠T
π/2
φ
ω
-φ
-π/2
Z
Suma
= ∠Z − ∠P₁ − ∠P₁*